3-D Extended-MHD Effects in Pre-magnetised ICF Implosions

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Chimera

Magnetic transport:
- Bulk plasma advection
- Resistive diffusion
- Nernst
- Cross-gradient-Nernst
- Hall (+ collisional correction)
- Cross-gradient Hall

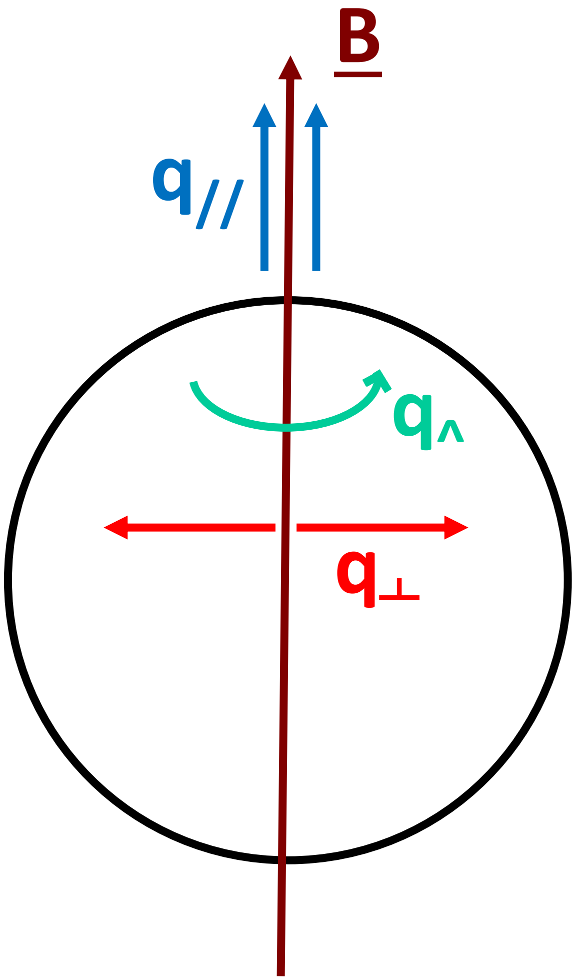
Thermal transport:
- Anisotropic thermal conduction
- Righi-Leduc
- Ettingshausen
- Anisotropic thermal transport with current (+ collisional correction)

Chimera

Magnetic transport:
- Bulk plasma advection
- Resistive diffusion
- Nernst
- **Cross-gradient-Nernst** ➔ **Magnetic field twisting**
- ~~Hall (+ collisional correction)~~
- ~~Cross-gradient Hall~~

Thermal transport:
- Anisotropic thermal conduction
- **Righi-Leduc** ➔ **Cooling**
- ~~Ettingshausen~~
- ~~Anisotropic thermal transport with current (+ collisional correction)~~

Heat-flow

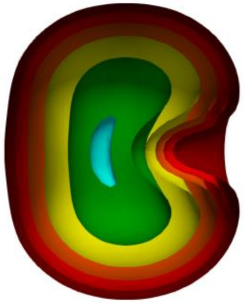


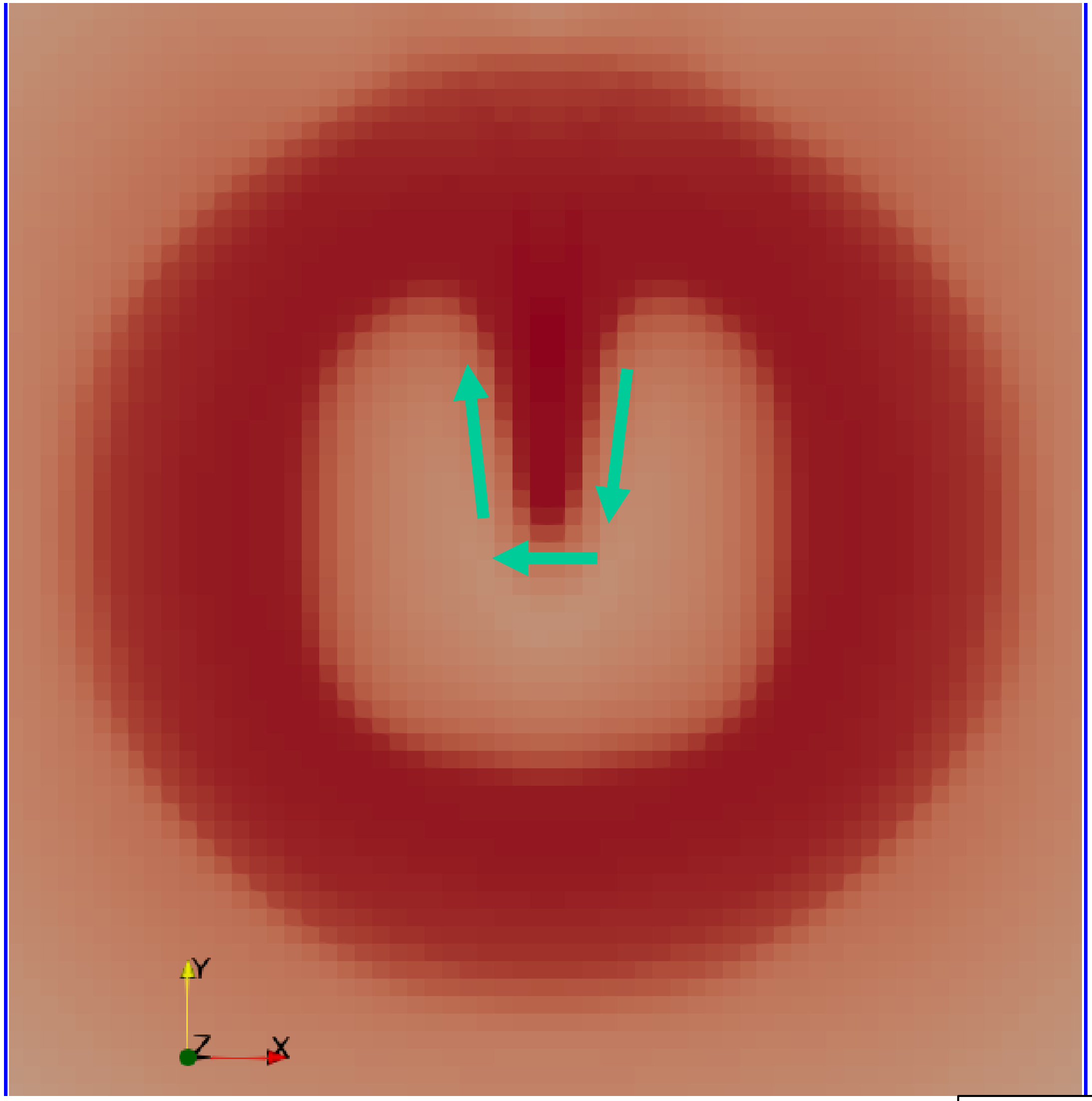
Righi-Leduc

$$\underline{q}_{\wedge} = -\kappa_{\wedge}\, \underline{\hat{b}} \times \nabla T_e$$

- $|\underline{q}_{\wedge}| = 0$ at poles
- $|\underline{q}_{\wedge}|$ maximum at waist
- Azimuthal asymmetry required for $\nabla \cdot \underline{q}_{\wedge} \neq 0$

Righi-Leduc – Fill-tube

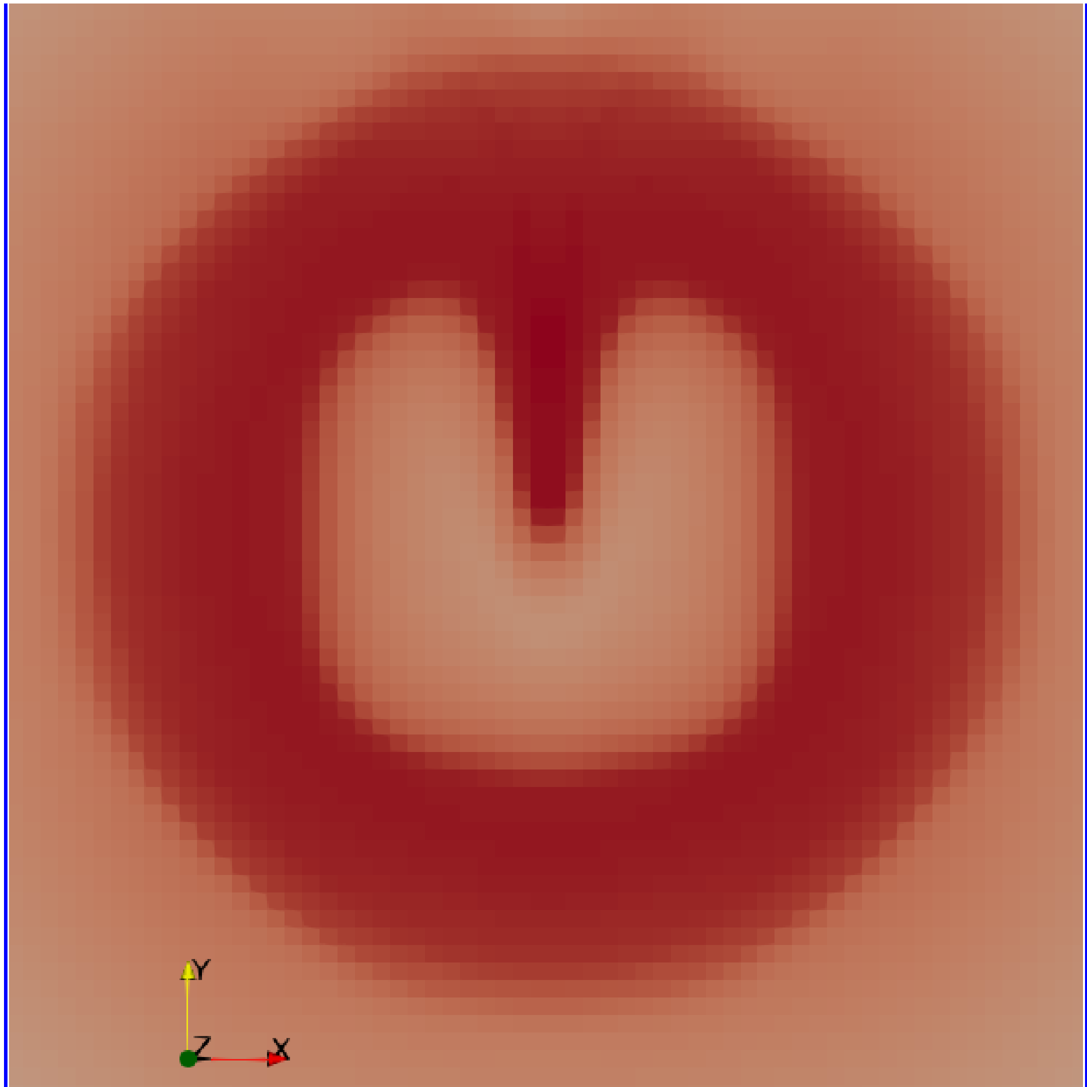




B

$$\underline{q}_{\wedge} = -\kappa_{\wedge}\, \underline{\hat{b}} \times \nabla T_e$$

No Righi-Leduc

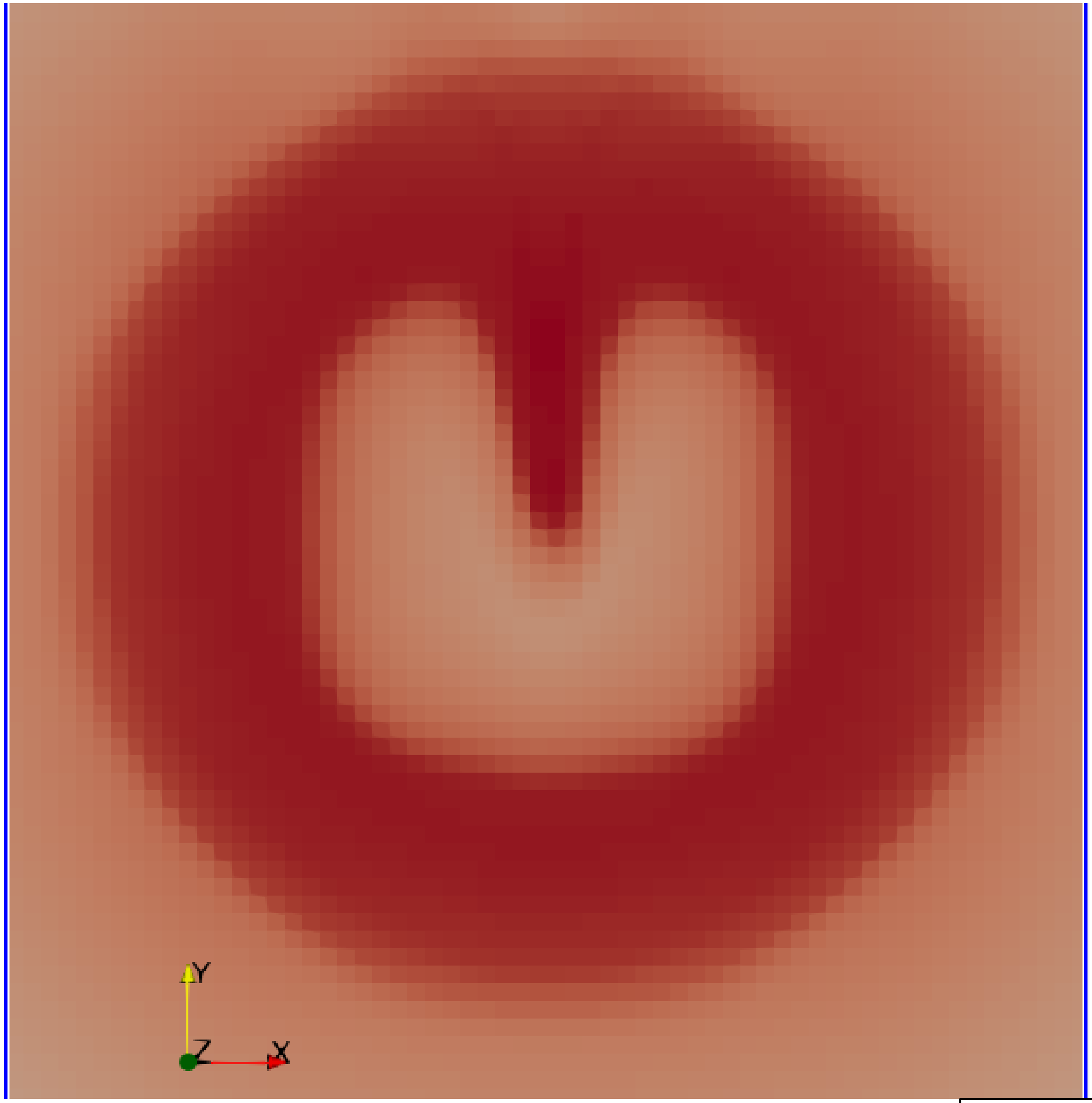


Small deflection of tip

B ⊙

$$\underline{q}_\wedge = -\kappa_\wedge \underline{\hat{b}} \times \nabla T_e$$

Righi-Leduc

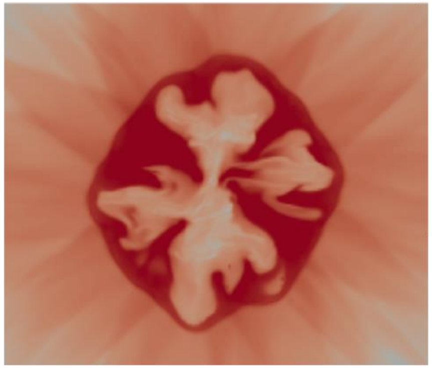


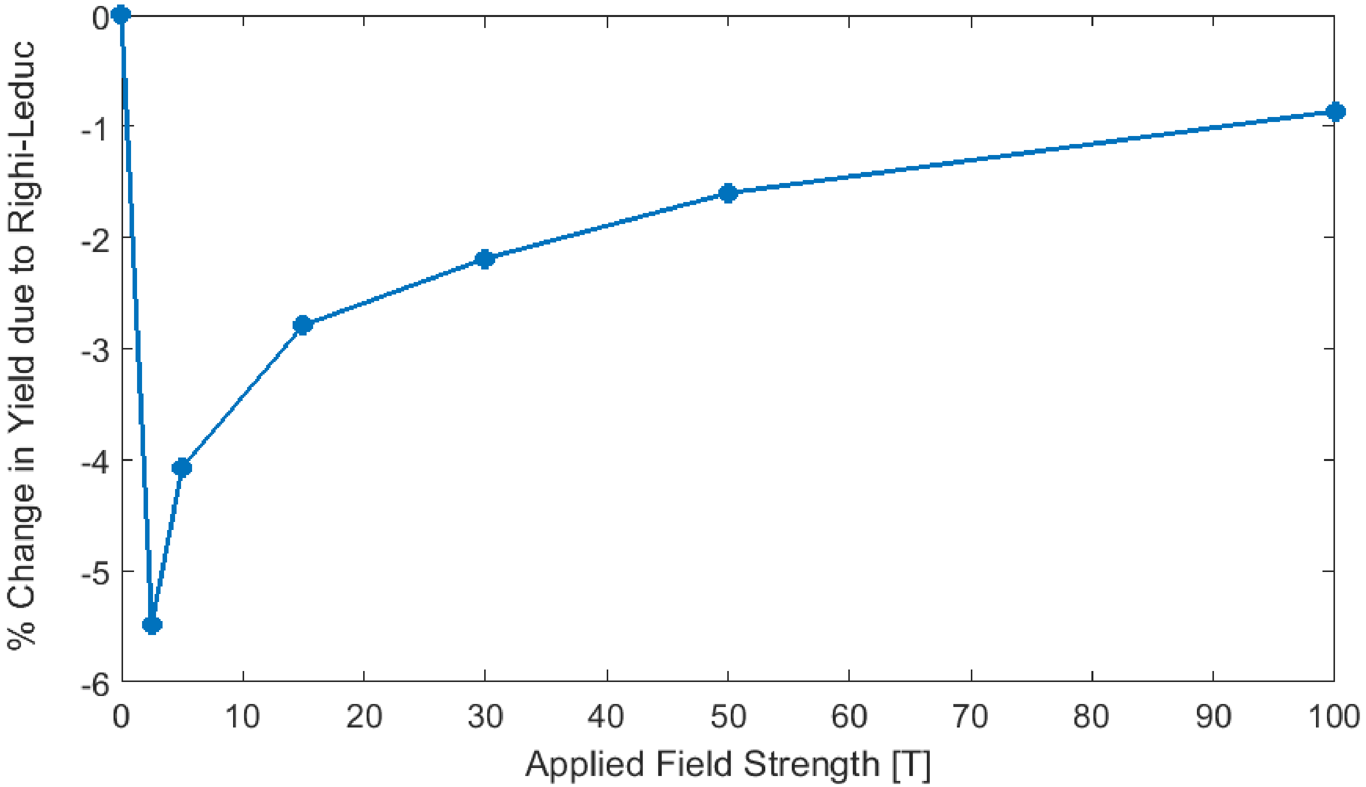
Small deflection of tip

B ⊙

$$\underline{q}_{\wedge} = -\kappa_{\wedge}\, \underline{\hat{b}} \times \nabla T_e$$

Righi-Leduc – Multi-mode

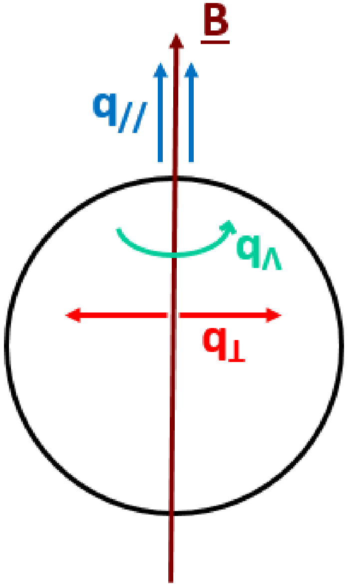




Summary: Righi-Leduc

- Effect:
  - Deflection of perturbations ➔ small
  - Increased cooling ➔ <6% yield reduction

- Why small?
  - 3-D implosion needed for $\nabla \cdot \underline{q}_{\wedge} \neq 0$
  - Capsules mainly 3-D in stagnation
    - $\omega_e \tau_e \gg 1$ ➔ $\underline{q}_{\wedge}$ small



Cross-Gradient-Nernst

Cross-Gradient-Nernst

Braginskii:

$$\left[\frac{\partial \underline{B}}{\partial t}\right]_{\nabla T_e} = \nabla \times \frac{\underline{\underline{\beta}} \cdot \nabla T_e}{e}$$

Cross-Gradient-Nernst

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$$\underline{v}_{N\perp} = -\frac{\beta_\wedge}{e|\underline{B}|} \nabla T_e$$

Nernst

Cross-Gradient-Nernst

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Nernst

$$\underline{v}_{N\wedge} = -\frac{\beta_{//} - \beta_\perp}{e|\underline{B}|}(\underline{\hat{b}} \times \nabla T_e)$$

Cross-Gradient-Nernst

Davies, et. al, PoP 2015

Cross-Gradient-Nernst

Braginskii:

$$\left[\frac{\partial \underline{B}}{\partial t}\right]_{\nabla T_e} = \nabla \times \frac{\underline{\underline{\beta}} \cdot \nabla T_e}{e}$$

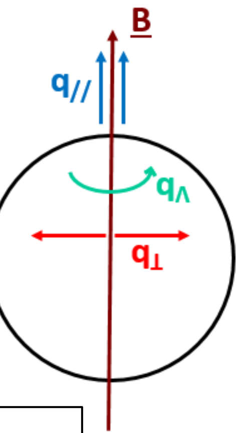
$$\underline{v}_{N\perp} = -\gamma_{\perp} \nabla T_e$$

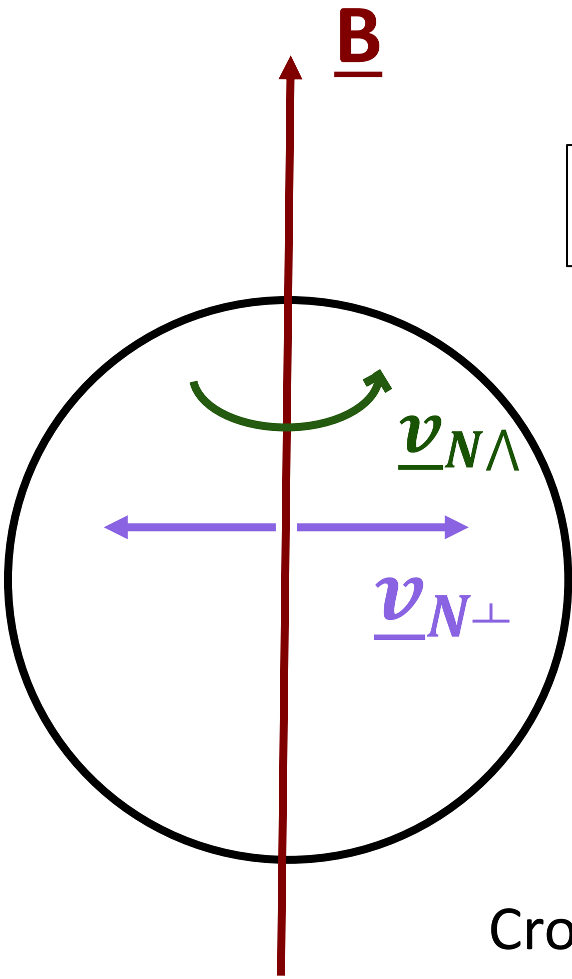
Nernst

$$\underline{v}_{N\wedge} = -\gamma_{\wedge} (\hat{\underline{b}} \times \nabla T_e)$$

Cross-Gradient-Nernst

Cross-Gradient-Nernst





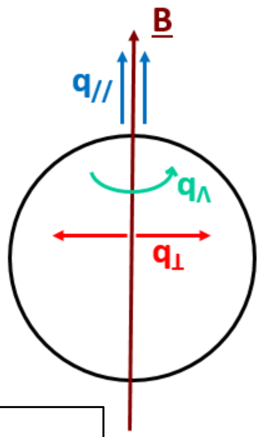
$$\underline{\boldsymbol{v}}_{N\wedge} = -\gamma_\wedge(\hat{\underline{\boldsymbol{b}}} \times \nabla T_e)$$

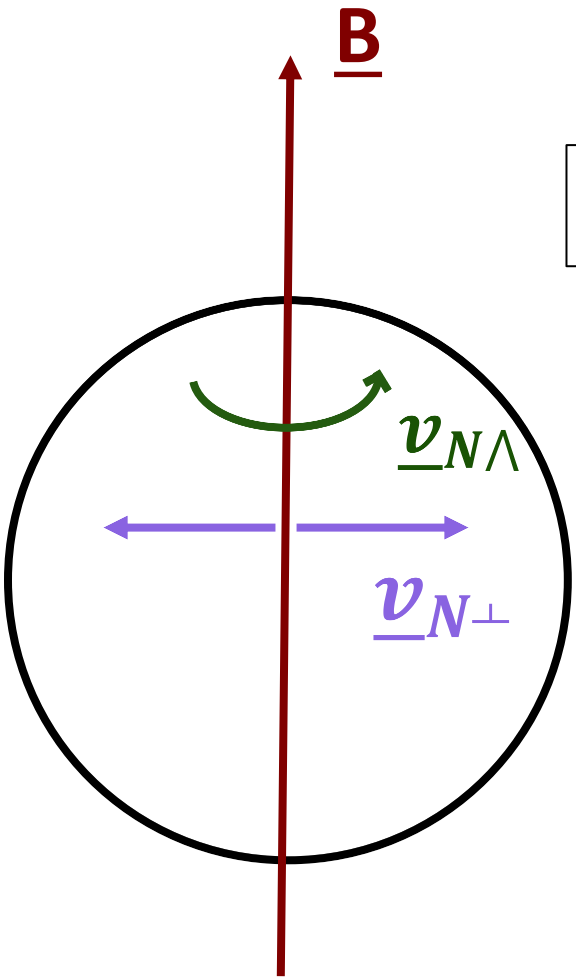
$\underline{v}_{N\perp}$ = **Nernst**

$\underline{v}_{N\wedge}$ = **Cross-gradient-Nernst**

Cross-gradient-Nernst $\gtrsim$ Nernst at all times!

Cross-Gradient-Nernst

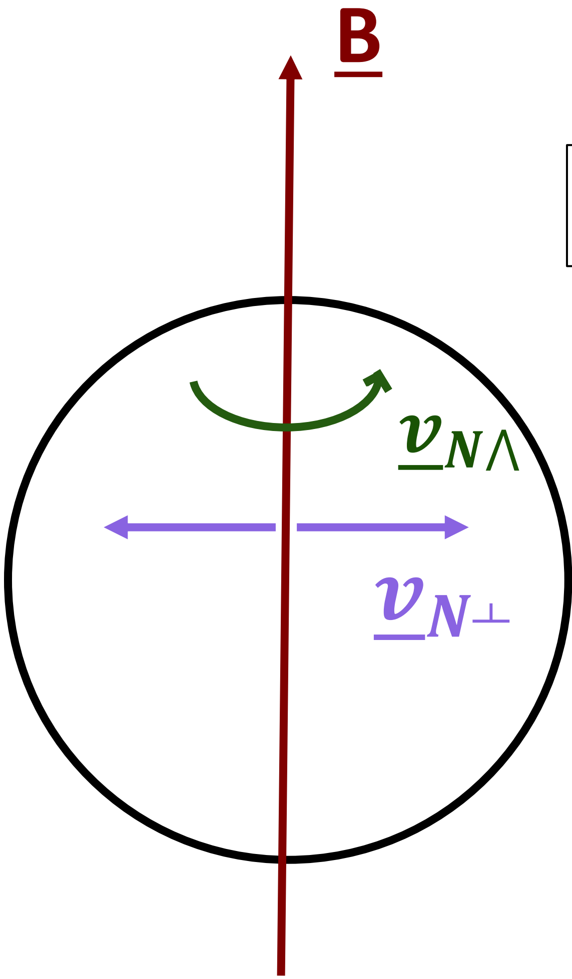




$$\underline{v}_{N\wedge} = -\gamma_{\wedge}(\hat{\underline{b}} \times \nabla T_e)$$

- $|\underline{v}_{\wedge}| = \mathbf{0}$ at poles
- $|\underline{v}_{\wedge}|$ maximum at waist
- Azimuthal asymmetry required for $\nabla \cdot \underline{v}_{\wedge} \neq \mathbf{0}$

Cross-Gradient-Nernst



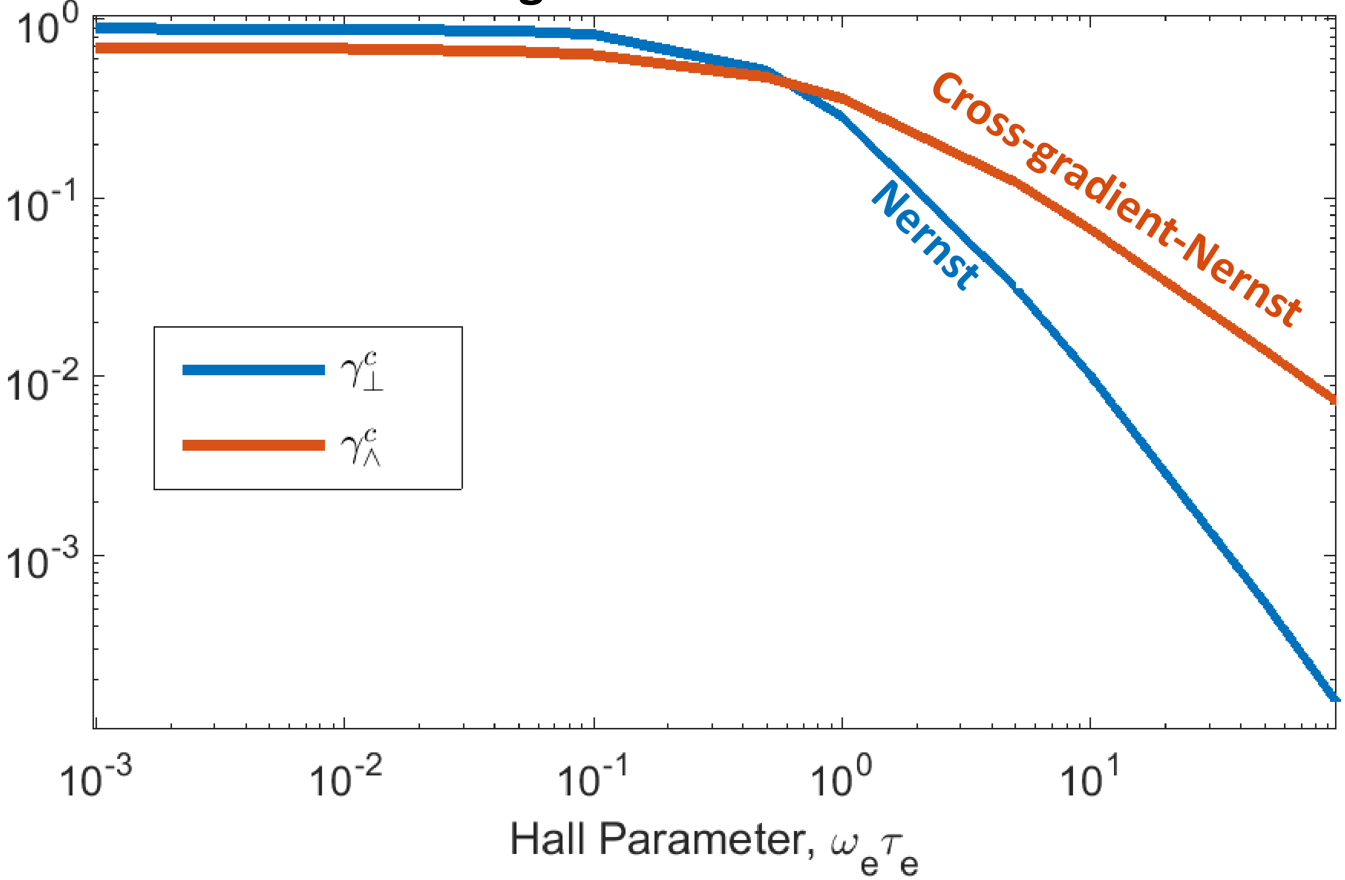
$$\underline{v}_{N\wedge} = -\gamma_{\wedge}(\hat{\underline{b}} \times \nabla T_e)$$

$\underline{v}_{N\wedge}$ has 2 effects:

- $\nabla \cdot \underline{v}_{N\wedge}$ ➔ 3-D compression/rarefaction
- $\partial \underline{v}_{N\wedge}/\partial z$ ➔ 2-D twisting

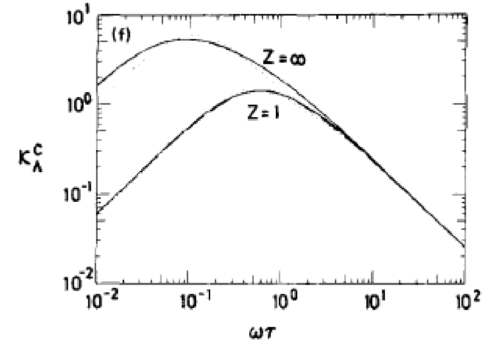
Cross-Gradient-Nernst

Coefficient magnitudes:

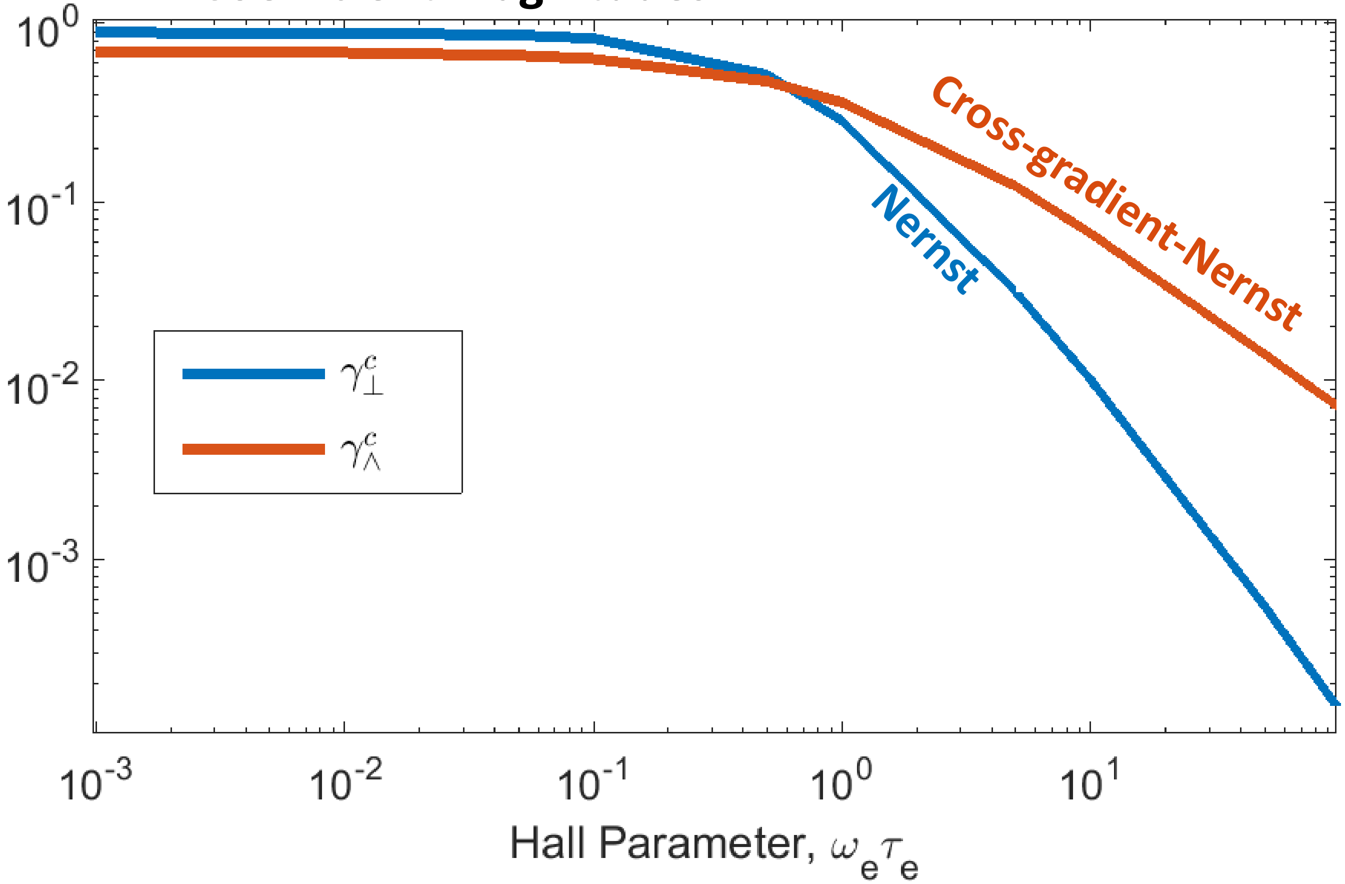


Cross-Gradient-Nernst

Epperlein and Haines 1986



Coefficient magnitudes:



Cross-Gradient-Nernst

Cross-gradient-Nernst summary:

$$\underline{v}_{N\perp} = -\gamma_{\perp} \;\; \nabla T_e$$

- As large as Nernst
- Modifies field even in 2-D

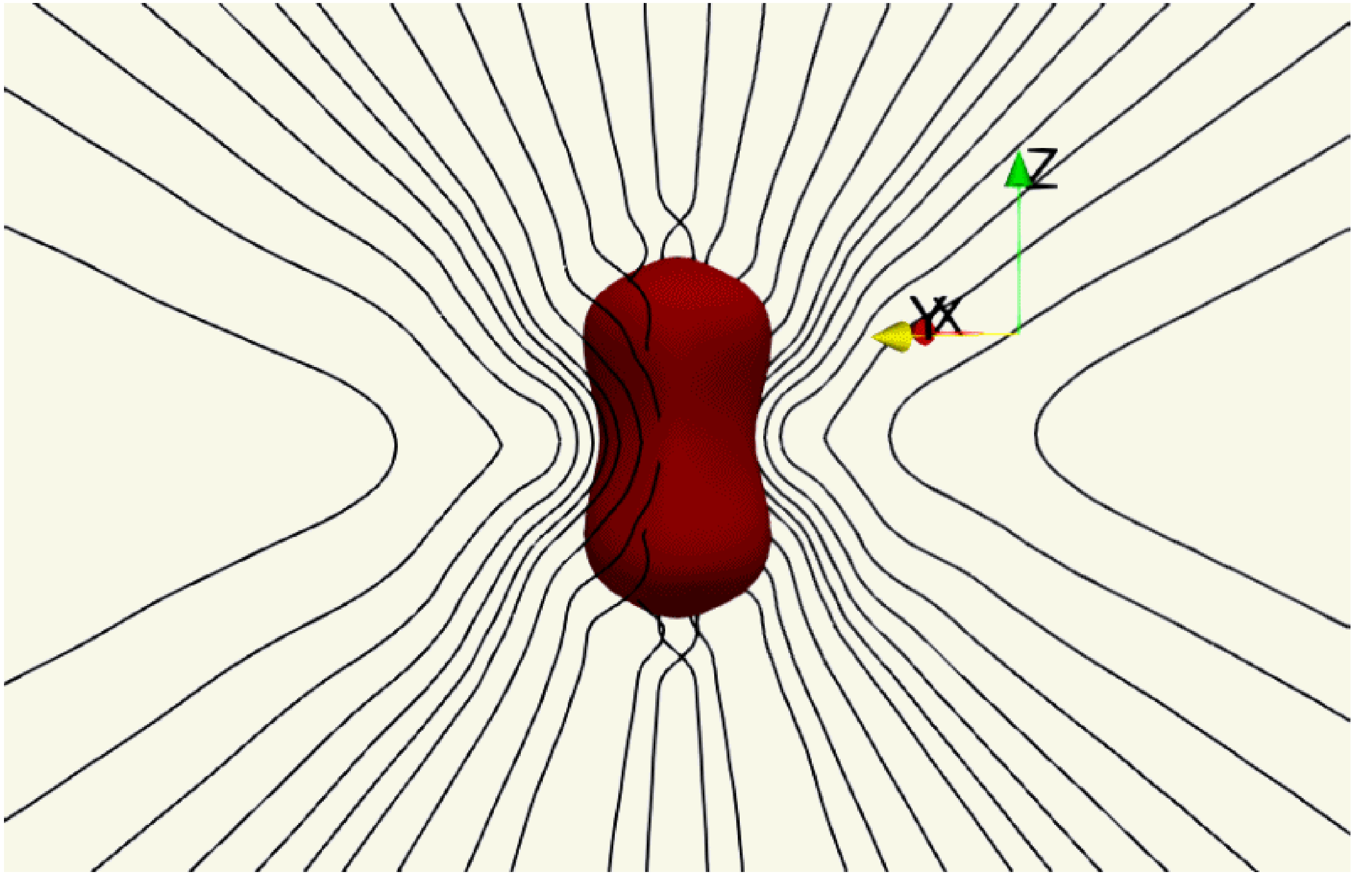
$$\underline{v}_{N\wedge} = -\gamma_{\wedge} \;\; (\underline{\hat{b}} \times \nabla T_e)$$

Where is cross-gradient-Nernst large?

- Ablation front
- In-flight compressed fuel
- Hot-spot

Field-line Twisting

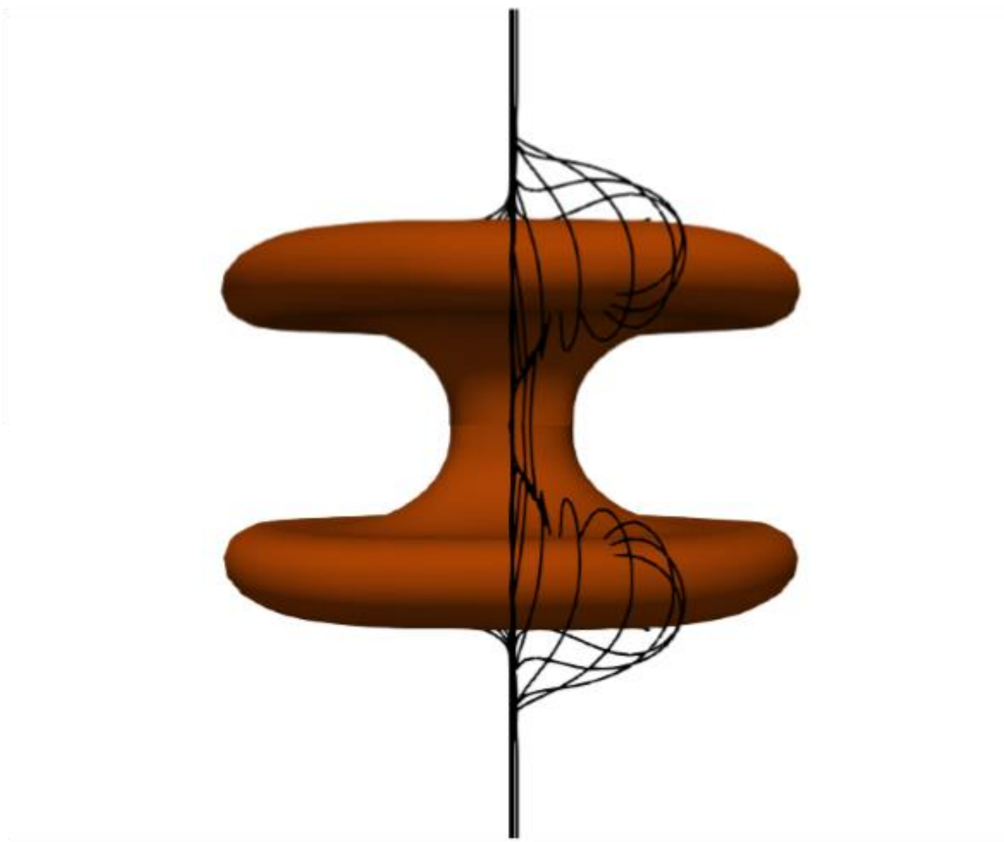
In-flight cross-gradient-Nernst in compressed fuel:



Increases energy containment!

Field-line Twisting

In-flight cross-gradient-Nernst in compressed fuel:



Increases energy containment!

Effects?

- Increased hot-spot thermal containment
- Ablation front twisting, modifying thermal smoothing
- Magnetic tension ➔ azimuthal velocities?
- Other '3-D' extended-MHD phenomena become important in 2-D, e.g. Righi-Leduc
- Fast ignition ➔ Increased charged particle guiding?
- MagLIF ➔ important if variations along cylindrical axis