The importance of including XMHD physics in HED codes

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National Nuclear Security Administration

Outline

- XMHD/Hall physics: why it’s important and why it’s difficult to include in codes
- Numerical algorithms: the relaxation method
- Simulation results exemplifying Hall dynamics
  - Electrode and Polarity asymmetry
  - Instability and reconnection affects
  - Flux penetration
  - Transition to vacuum and ablation effects
  - Flux compression and generation and force-free currents

Importance of XMHD Physics

Low-density plasma can have great impact on global dynamics

- For magnetized systems the Hall effect is most important at low density
- Low-density plasma can carry large currents
- Hall effect is necessary for a physical transition to vacuum

Generalized Ohm's Law

$$\partial_t \mathbf{J} + \nabla \cdot \mathbf{F} - \frac{e}{m_e} \nabla p_e = \frac{n_e e^2}{m_e} \left(\mathbf{E} + \mathbf{u} \times \mathbf{B} - \frac{1}{n_e e} \mathbf{J} \times \mathbf{B} - \eta \mathbf{J} \right)$$

Various terms

MHD $\mathbf{E} + \mathbf{u} \times \mathbf{B} - \eta \mathbf{J}$

Electron inertia $\partial_t \mathbf{J} + \nabla \cdot \mathbf{F}$

Electron pressure $\frac{e}{m_e} \nabla p_e$

Hall $\frac{1}{n_e e} \mathbf{J} \times \mathbf{B}$ Focus of talk

Importance of XMHD Physics

Parameters characterizing the Hall effect

- The important length scale is the ion inertial length $\lambda_i = \frac{c}{\omega_{pi}} = \sqrt{\frac{m_i}{ne^2\mu_0}}$
- Length scale below which B is decouples from mass flow to electron flow
- Hall parameter h is a measure of electron-ion collisionality $h = \Omega_e \tau_{ei} = \frac{B}{ne\eta}$
- Hall important when $\lambda_i \gtrsim 1$ and $h \gtrsim 1$ are satisfied
- MHD is valid when $\lambda_i \ll L$ and $h \ll 1$
- The conductivity is anisotropic for $h \gtrsim 1$

Hall term breaks Anode-Cathode symmetry

Consider Hall MHD Ohm's law

$$\mathbf{E} + \mathbf{u} \times \mathbf{B} - \frac{1}{n_e e}\mathbf{J} \times \mathbf{B} - \eta\mathbf{J} = 0$$

Transform from cathode to anode or visa versa

$$\mathbf{J} \rightarrow -\mathbf{J} \quad \mathbf{B} \rightarrow -\mathbf{B} \quad \mathbf{E} \rightarrow -\mathbf{E} \quad \mathbf{u} \rightarrow \mathbf{u}$$

Only the sign of the Hall term is changed, thus breaking A-K symmetry

$$\mathbf{E} + \mathbf{u} \times \mathbf{B} + \frac{1}{n_e e}\mathbf{J} \times \mathbf{B} - \eta\mathbf{J} = 0$$

Hall term is necessary for physical transition to vacuum

Cross-field conductivity along $\mathbf{E}$ if B is not zero (Pederson conductivity)

$$\sigma_p = \frac{ne^2}{m_e}\left(\frac{\nu_{ei}}{\underbrace{\Omega_e^2}_{\text{Hall}} + \nu_{ei}^2}\right)$$

Electron-ion collision frequency obeys

$$\lim n_e \to 0 \quad \Rightarrow \quad \nu_{ei} \to 0$$

without Hall

$$\lim n_e \to 0 \quad \Rightarrow \quad \sigma_p \to \frac{1}{\eta_s} \neq 0$$

with Hall

$$\lim n_e \to 0 \quad \Rightarrow \quad \sigma_p \to 0$$

Reason why MHD codes often use a model for the conductivity that goes to zero in the low-density limit. Hall greatly affects ablation.

Hall term can produce rapid flux penetration

Kingsep et al (1993) showed Hall term can give quasi-1D propagation of the magnetic field along contours of constant density. The result is a Burger's equation for the out of plane magnetic field B_z .

$$\partial_t B_z - \frac{1}{e\mu_0}\partial_y \left(\frac{1}{n}\right) \partial_x \left(\frac{B_z^2}{2}\right) = \frac{\eta}{\mu_0}\partial_x^2 B_z$$

Solution is a shock with propagation speed equal to the Hall velocity

$$v_H = \frac{B}{ne\mu_0 L_n} > v_A \qquad \text{for} \;\; L_n < \lambda_i$$

Flux Amplification by Force-Free Currents

An under-investigated effect of the Hall term

- Assume no flow ($\mathbf{u} = 0$)
- Assume large Hall parameter (electrons strongly magnetized)
- Assume current driven by an inductive axial electric field E_z
- Then the Hall-MHD Ohm's law gives approximately

$$\mathbf{J} = \mathbf{B}\frac{B_z E_z}{B^2 \eta}$$

- Shows that current is force free
- Shows that an axial driving electric field can drive azimuthal current to amplify the axial magnetic field. Current takes path of least resistance.

Importance of XMHD Physics

If XMHD effects are so important why not always include them?

- Answer: Because of time step restrictions
- Consider Ampere-Maxwell equation and simplified GOL to see why

$$\partial_t \mathbf{E} = c^2 \left(\nabla \times \mathbf{B} - \mu_0 \mathbf{J} \right)$$

$$\partial_t \mathbf{J} = \frac{n_e e^2}{m_e} \left(\mathbf{E} + \mathbf{u} \times \mathbf{B} \right) - \frac{e}{m_e} \mathbf{J} \times \mathbf{B} - \nu_{ei} \mathbf{J}$$

- These equations have two characteristic frequencies
- Electron inertial response is at the electron plasma frequency ω_{pe}
- Hall effect produces frequencies up to the electron cyclotron frequency Ω_e

Numerical algorithms

How can we get around the problem of high frequencies?

- Subcycle the fast equations, i.e. use a smaller timestep for these (J. Huba)
- Use GOL w/o electron inertia and neglect displacement current to eliminate E in Faradays's law to get

$$\partial_t \mathbf{B} = \nabla \times \left(\left[\mathbf{u} - \frac{1}{n_e e \mu_0} (\nabla \times \mathbf{B}) \right] \times \mathbf{B} \right)$$

- Evaluate B in the complicated curl operator at the advanced time level
- Use a sophisticated singular linear system solver to solve for B at the advanced time level. Requires careful preconditioning to work (L. Chacon).
- This results in a global linear system to solve as well as a nonlinear iteration scheme due to the nonlinear B dependence. Expensive!

Numerical algorithms

Apply relaxation method to full GOL

- Keep the Ampere-Maxwell and GOL equations in their original form and view as evolution equations for E and J respectively
- Evaluate E and J at the advanced (n+1) time level for the *source* terms only

$$\mathbf{J}^{n+1} = \mathbf{J}^* + \frac{ne^2 Z\Delta t}{m_e}\left(\mathbf{E}^{n+1} + \mathbf{u}^* \times \mathbf{B}^* - \frac{1}{n_e^* eZ}\mathbf{J}^{n+1} \times \mathbf{B}^* + \eta^* \mathbf{J}^{n+1}\right)$$

$$\mathbf{E}^{n+1} = \mathbf{E}^* + c^2\Delta t(\nabla \times \mathbf{B}^* - \mu_0 \mathbf{J}^{n+1})$$

*means time advanced using explicit source and flux terms

- Solving for the advanced time-level J and E and taking the limit of large timestep we obtain

$$\mathbf{J}^{n+1} = \frac{1}{\mu_0}\nabla \times \mathbf{B}^*$$

$$\mathbf{E}^{n+1} = -\mathbf{u}^* \times \mathbf{B}^* + \frac{1}{n_e^* \mu_0}(\nabla \times \mathbf{B}^*) \times \mathbf{B}^* + \frac{\eta^*}{\mu_0}\nabla \times \mathbf{B}^*$$

Numerical algorithms

Spatial discretization: Discontinuous Galerkin (DG)

Assume equations are in conservation form: Hyperbolic plus source

$$\partial_t Q + \nabla \cdot \mathbf{F}(Q) = S(Q)$$

Use a local Cartesian basis. Then in each cell

$$Q(\mathbf{x}, t) = \sum_{i=1}^{N} q_i(t)\phi_i(\mathbf{x})$$

Lowest order 3D basis has 8 internal points corresponding to the local cell basis functions

$$\phi_i(\mathbf{x}) \in \{1, x, y, z, xy, xz, yz, xyz\}$$

where $-1 \leq xyz \leq 1$

Numerical algorithms

Spatial discretization: Discontinuous Galerkin (DG)

Use orthogonality to arrive at evolution equations for basis coefficients q_i (t).
For lowest order 3D DG need to do 8-point internal and 4-point face Gaussian Legendr

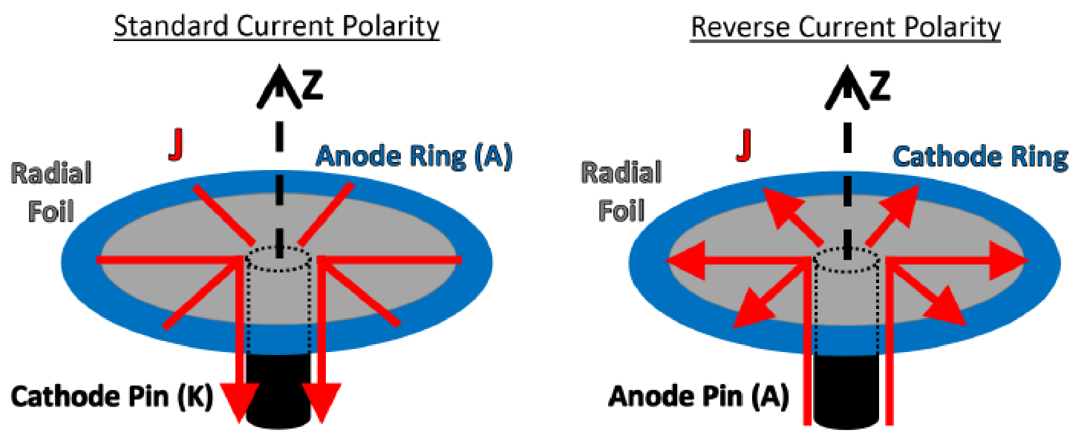
Since the method uses discontinuous basis functions a physical prescription for the numerical flux across a cell interface must be used. This can be local Lax-Friedrichs or one can use an approximate Riemann solver. We use a Harten-Lax-van Leer with contact wave solver called an HLLC solver.

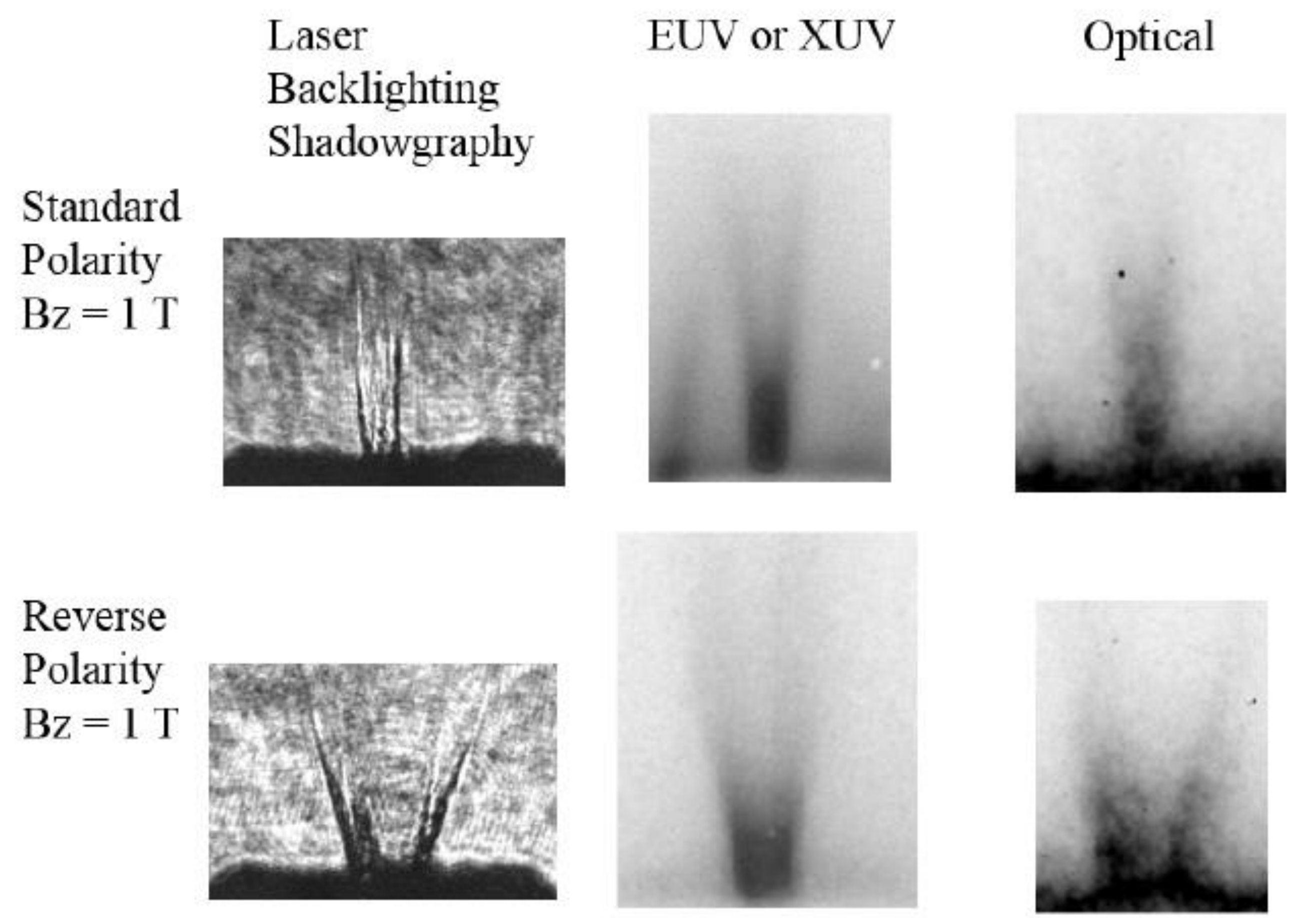
There are numerous other considerations, e.g. :

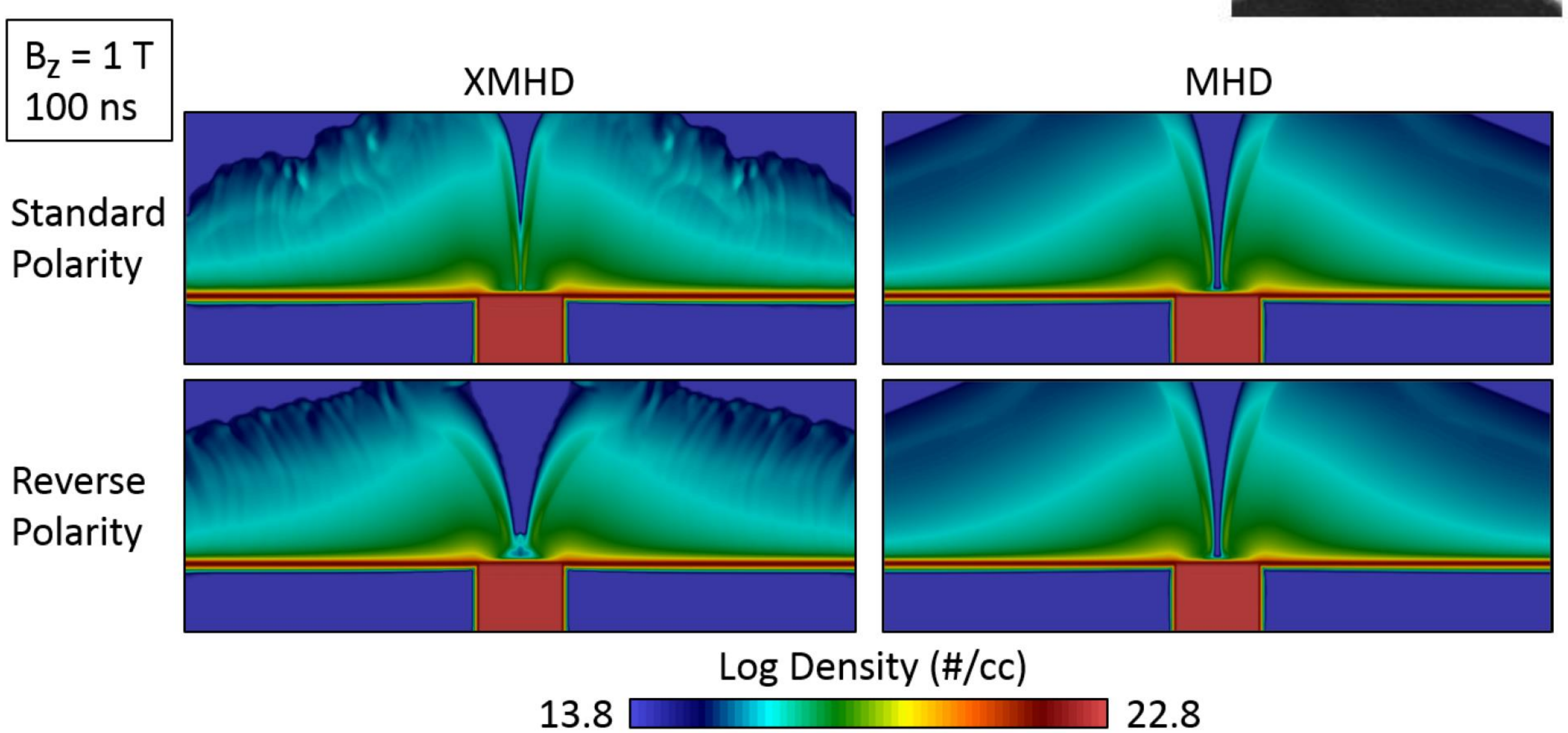
- the equation of state
- a limiter to preserve the positivity of the density and pressure
- a resistivity model
- boundary conditions

Simulation Results

Radial Foil Simulations: Anode-Cathode asymmetry







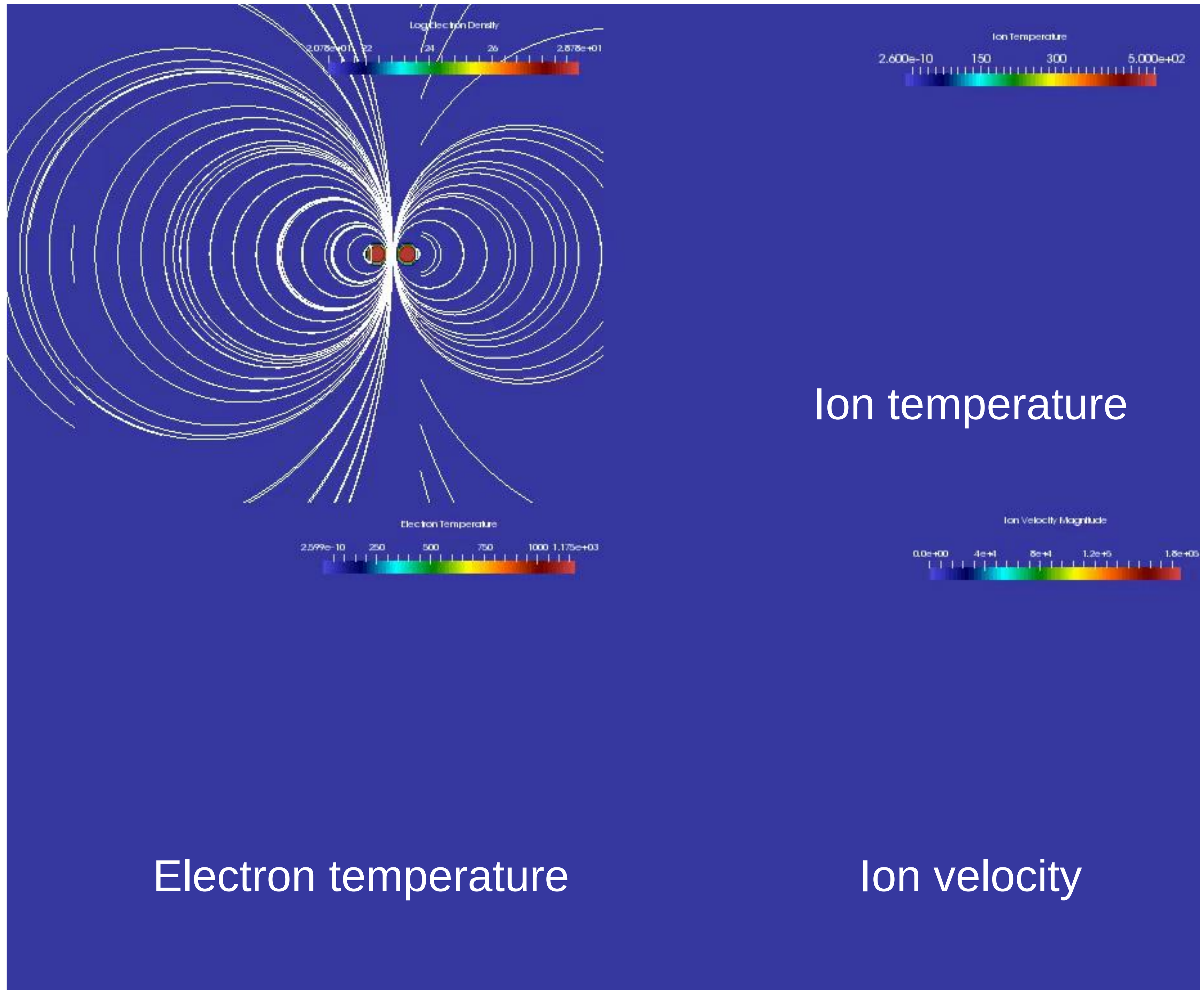
Courtesy Tom Byvank
Cornell PhD 2018

Simulation Results

Simulated Magnetosphere: Fast reconnection

Comparison of MHD and Hall MHD shows significantly faster reconnection for Hall

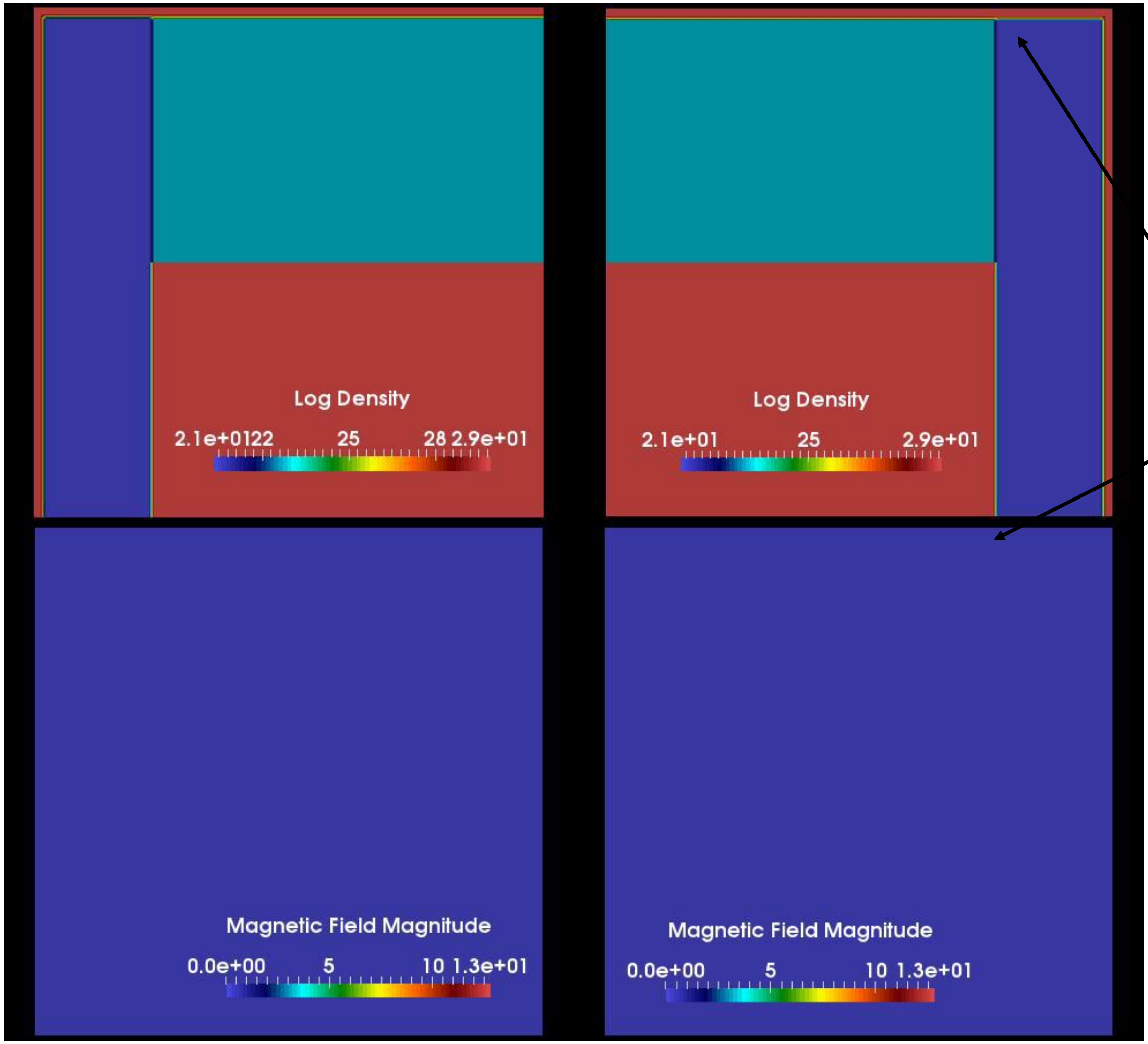
Experiment possible using Omega MIFEDS?



Simulation Results

Simulation of Pinching Plasma: Fast flux penetration

MHD

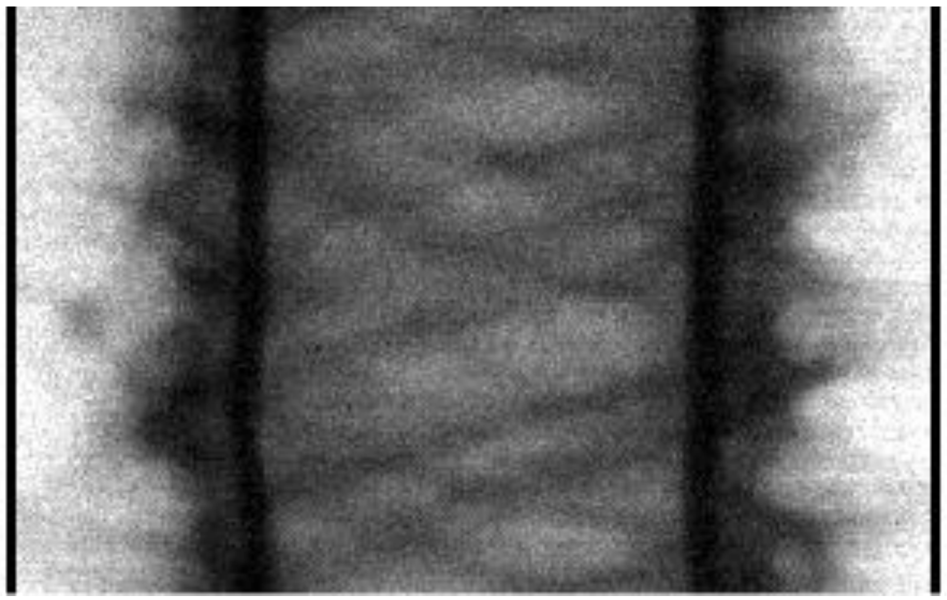


Look here

XMHD

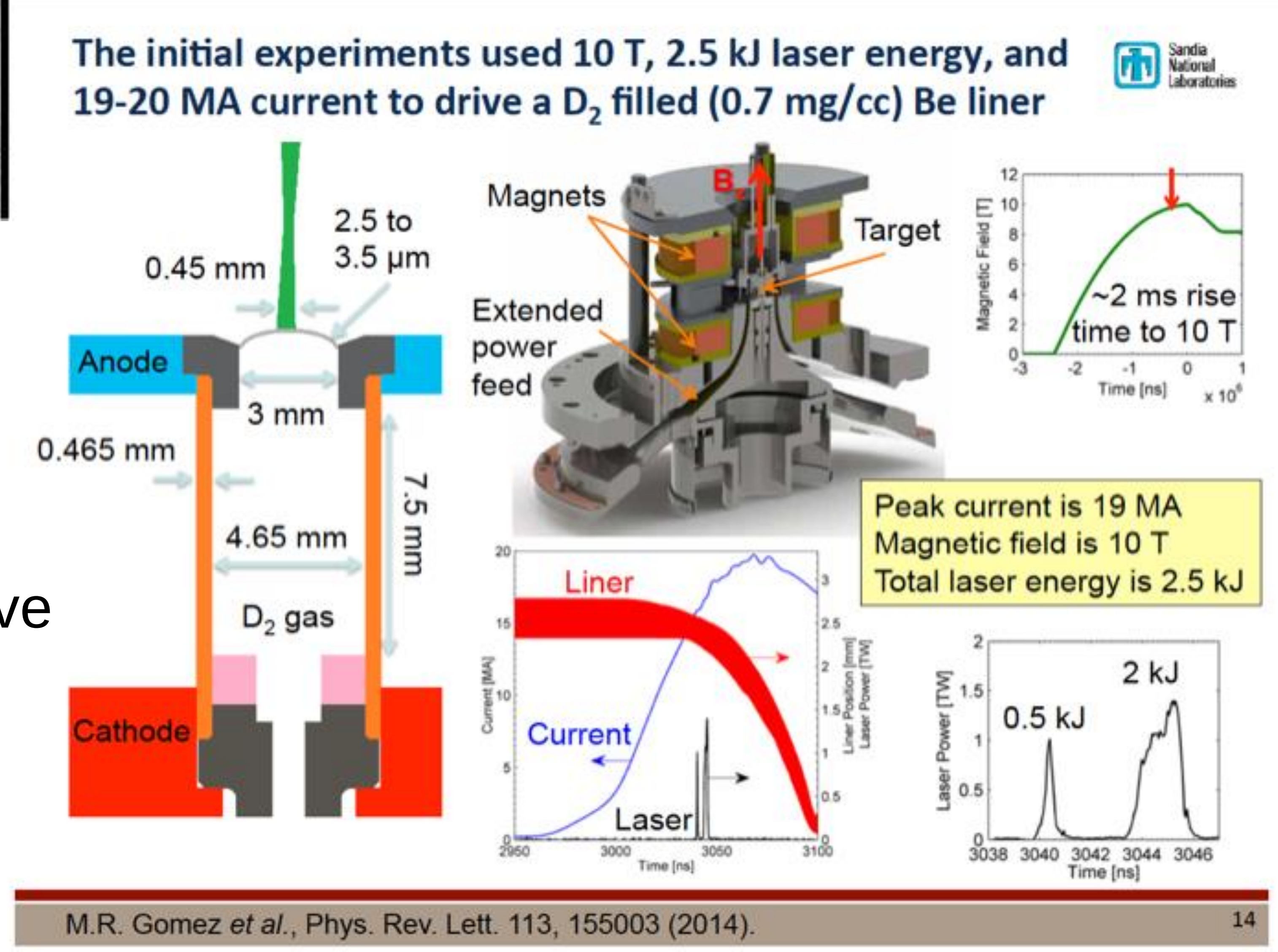
Simulation Results

MagLIF Simulations



Helical instability observed by radiography

Perform XMHD modeling of the liner and feed section and compare to MHD



A number of MHD Lab codes have failed to reproduce the instability without introducing an initial helical seed perturbation

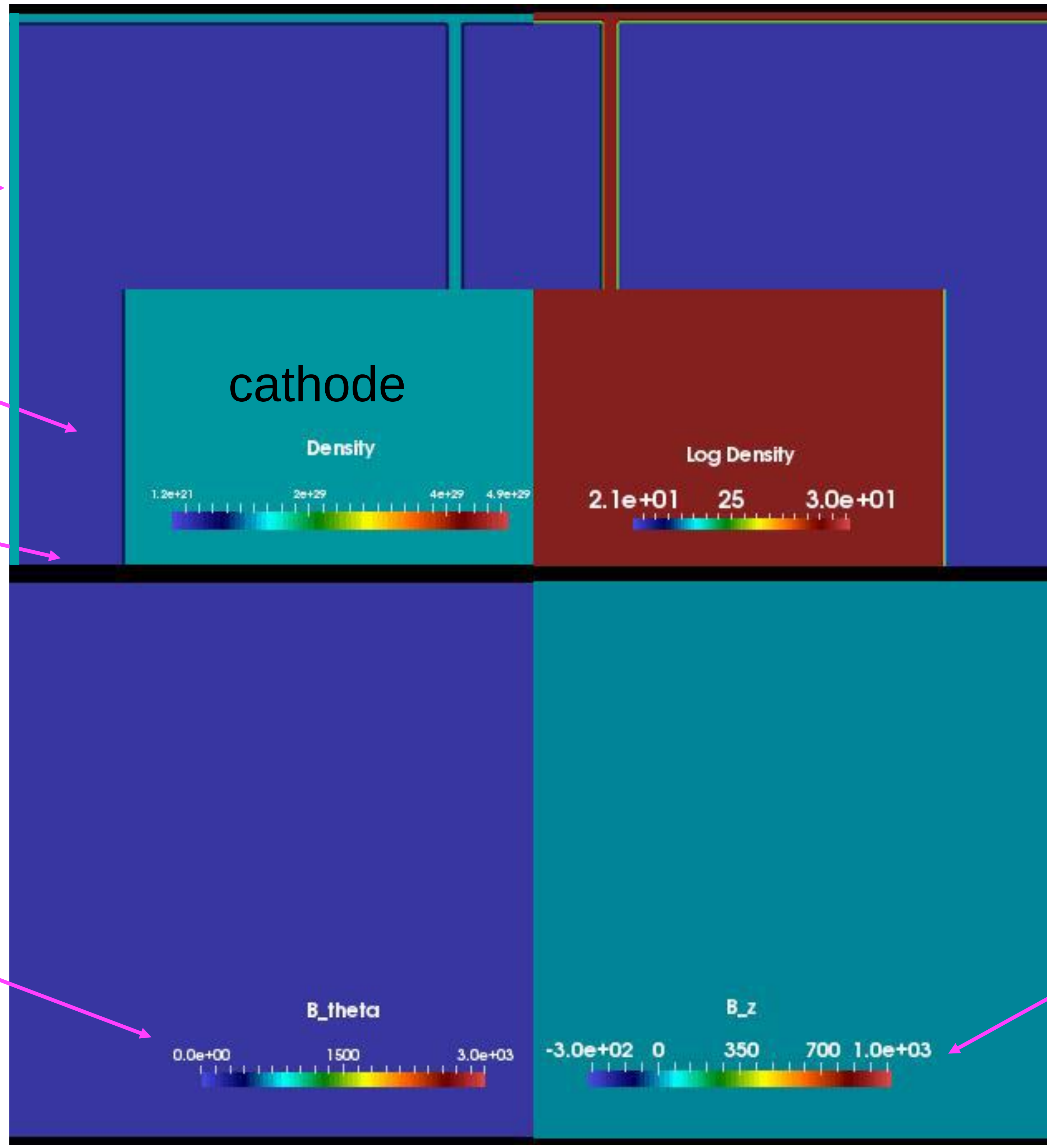
MagLIF: Axial flux compression

density max scale is 5×10^{29} m^{-3}

feed gap

B_θ applied here

azimuthal magnetic field max scale is 3000 T



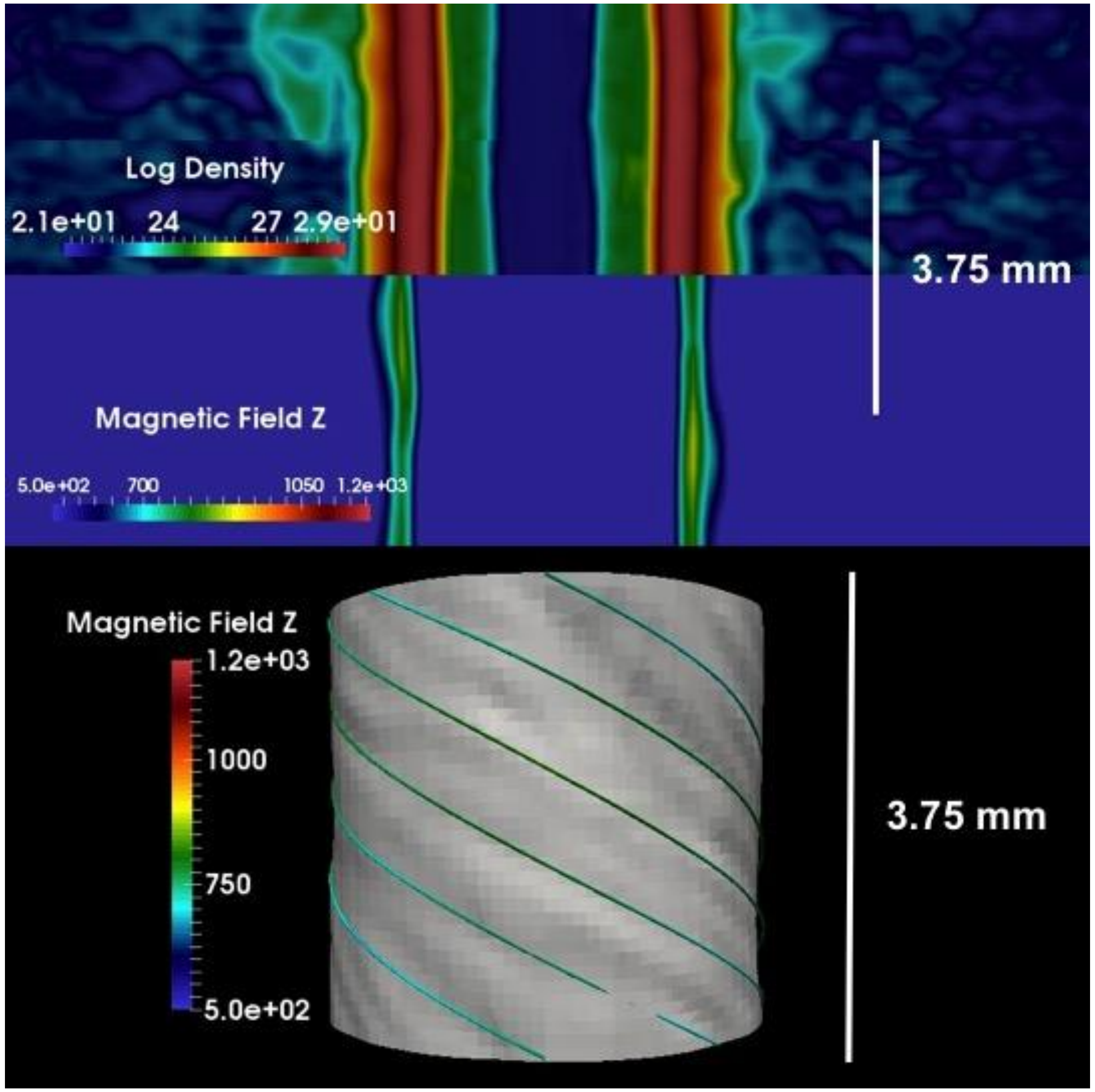
log density max scale is 30

conducting boundary

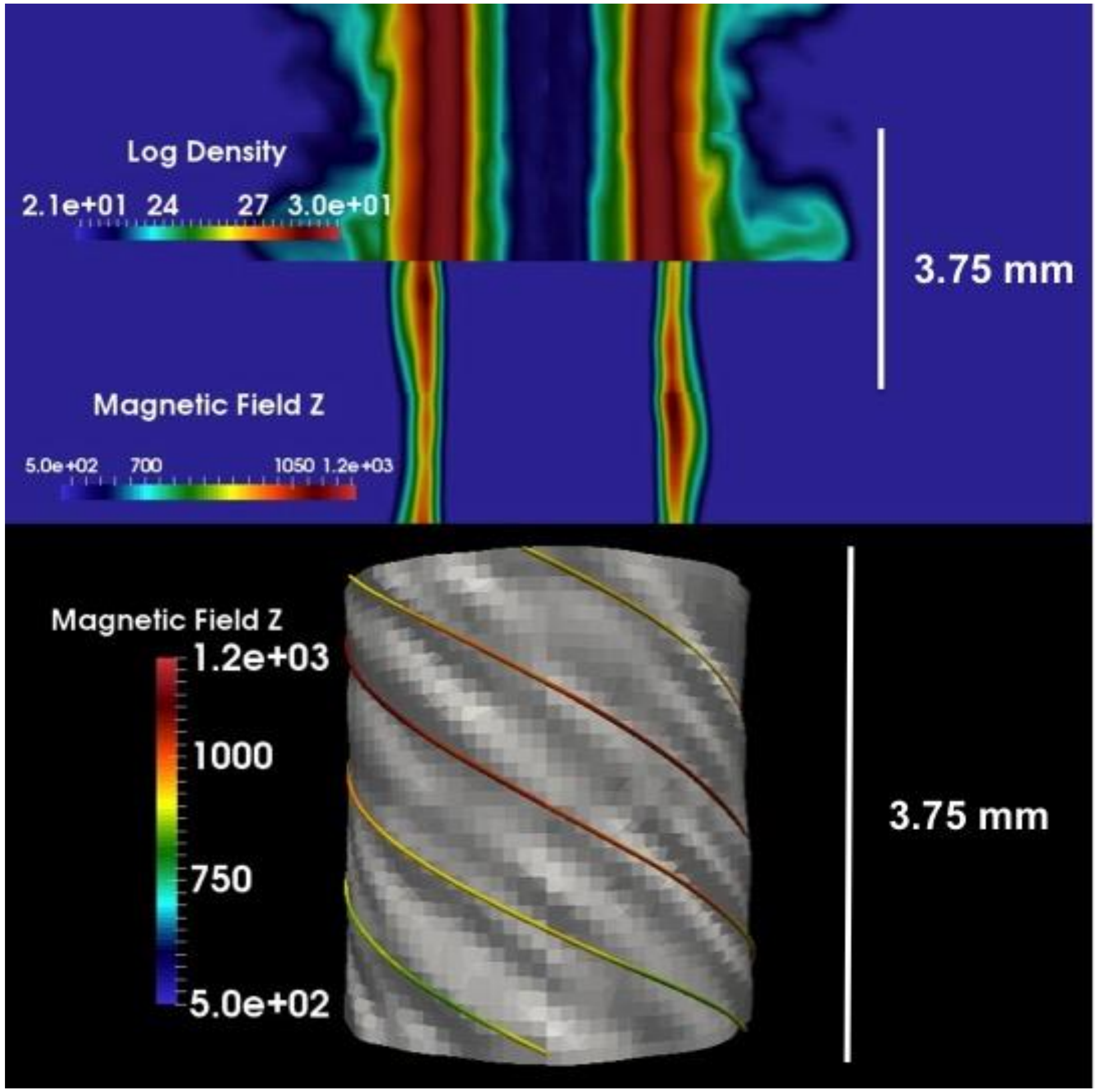
axial magnetic field max scale is 1000 T

MagLIF: Axial flux amplification

MHD at 100ns



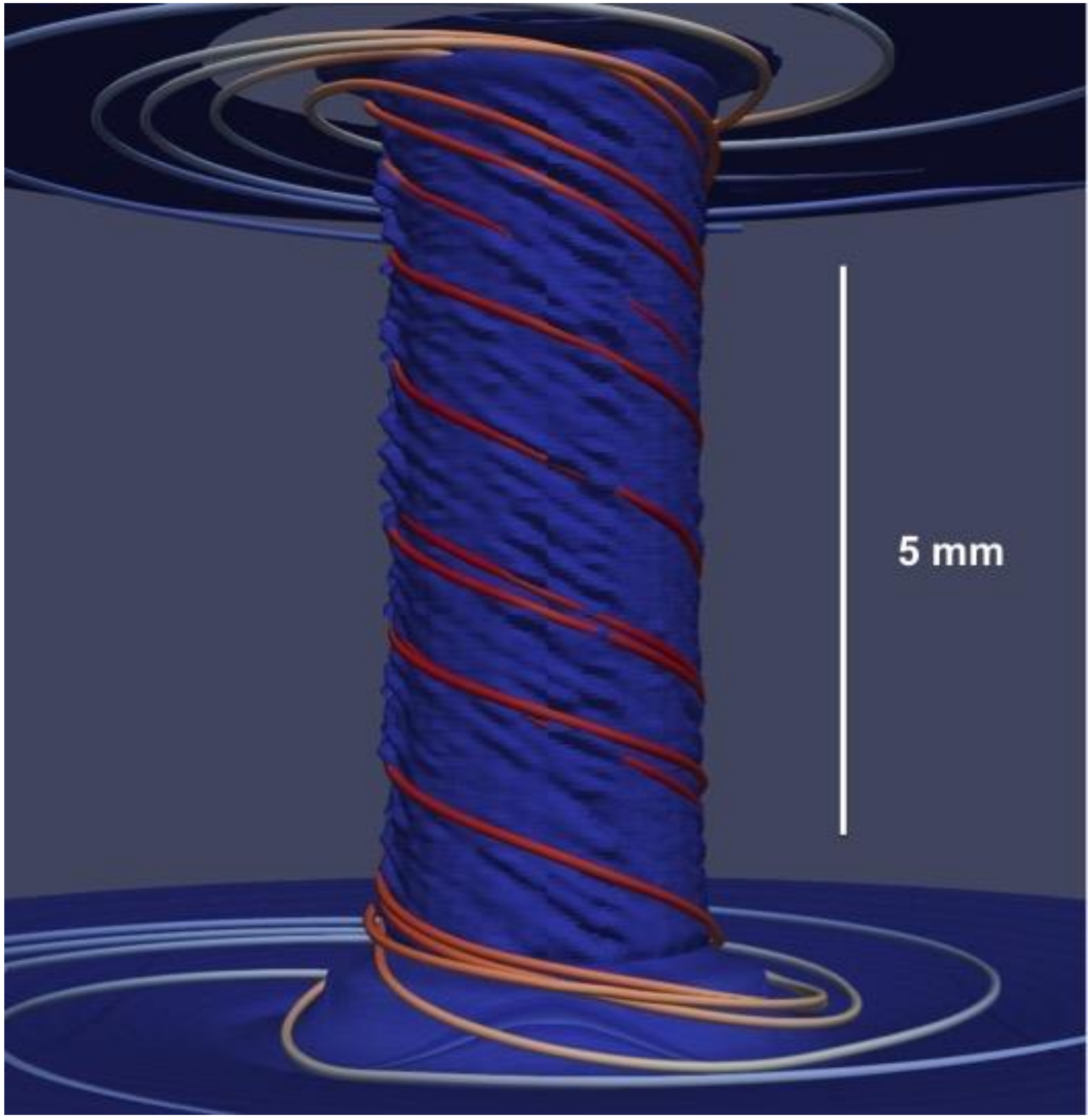
XMHD at 100ns



Simulation including ablation from feed

Simulation by
Matt Martin using
PERSEUS Code
performed at
Sandia Nat. Labs.

4096 cores
36 hours



Clearly shows field-aligned MRT instability

wavelength agrees with radiography

Importance of XMHD Physics

How the Hall effect can impact HED experiments

- The Hall effect is important in low density plasma
- Most (all?) HED experiments have surrounding low-density plasma
- This low-density plasma can carry large currents
- The large currents in low-density regions can produce large forces
- Even force-free currents can produce dramatic differences in the global dynamics of the dense plasma