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Self-Similar Solutions with Electro-Thermal Processes for Plasmas of Arbitrary Beta*

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Magnetic Fields in Laser Plasmas
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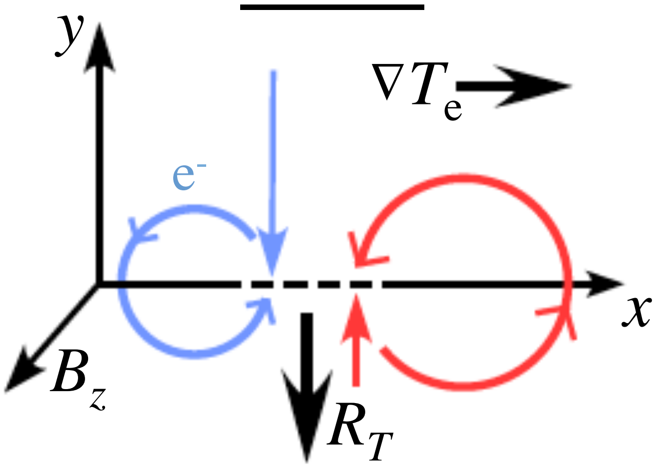
Electro-thermal effects in 1-D

NRL PPD

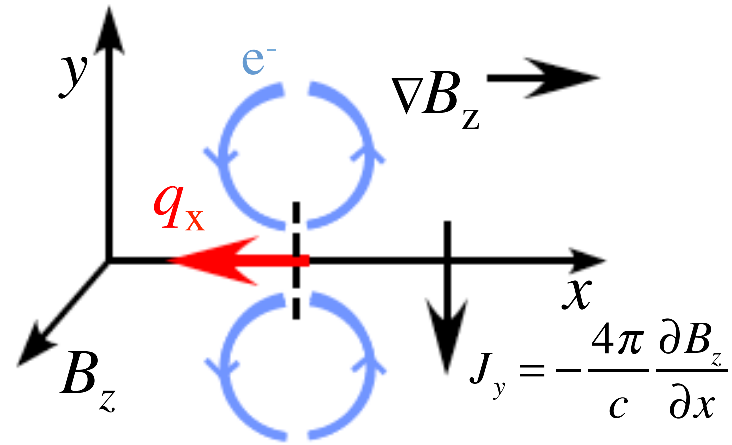
Nernst



left side slower e^- experience stronger opposing collisional drag than right side faster e^-

$$R_{Ty} = -\beta_\wedge^{uT}\,\hat{b}\times\nabla T_e$$

Ettinsghausen



conduction E accelerates e^- on right and decelerates e^- on left

$$q_x = -T_e\frac{\beta_\wedge^{uT}}{en_e}\hat{b}\times\vec{J}$$

Thomson

heating (or cooling) effects in a current carrying plasma with a ∇T_e

$$\vec{E}_T\cdot\vec{J} = +\frac{\beta_\wedge^{uT}}{en_e}\frac{\partial T_e}{\partial x}\frac{c}{4\pi}\frac{\partial B_z}{\partial x}$$

$$\beta_\wedge^{uT} = n_e\frac{\chi_e(\beta_1''\chi_e^2+\beta_o'')}{\chi_e^4+\delta_1\chi_e^2+\delta_o},\quad \chi_e = \omega_e\tau_e$$

Braginskii, Rev. Plasma Phys. 1, ed. Leontovich, 205, 1965.

Early studies of the Nernst effect in heated plasmas bounded by liners

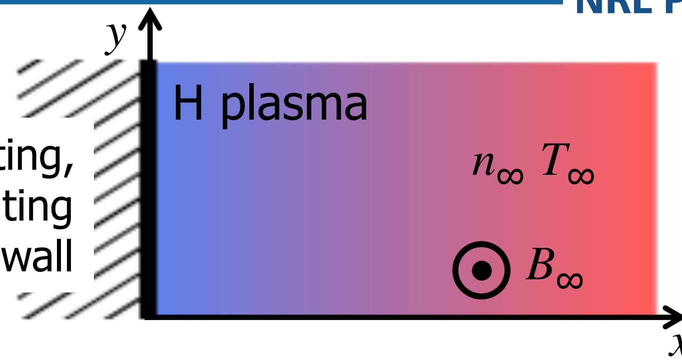
NRL PPD

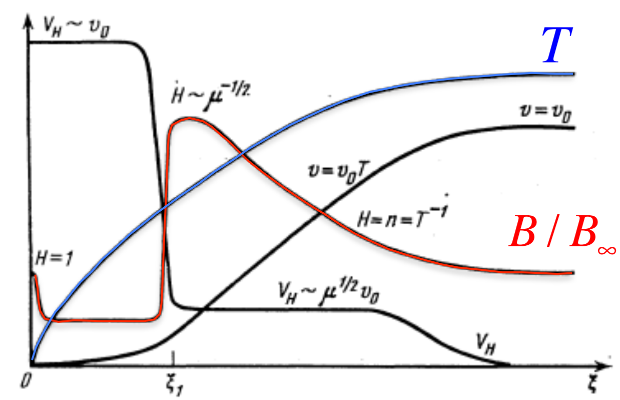
Vekshtein, Doklady, 271, 1983 & JETP, 84, 1983.

For $\beta_{\infty} = \dfrac{2nT}{B^2/8\pi} >> 1,$

effective magnetic diffusivity with Nernst

$$D_{eff}^{(B)} \sim \frac{cT_{e\,\infty}}{10eB_{\infty}} = 1.6 D_{Bohm}$$





Garanin, JAppMechTechPhy, 28, 816, 1987.

steady state self-similar solution with
Braginskii coefficients and constant total pressure;

Garanin & Mamyshev, JAMTP, 31, 28, 1990.

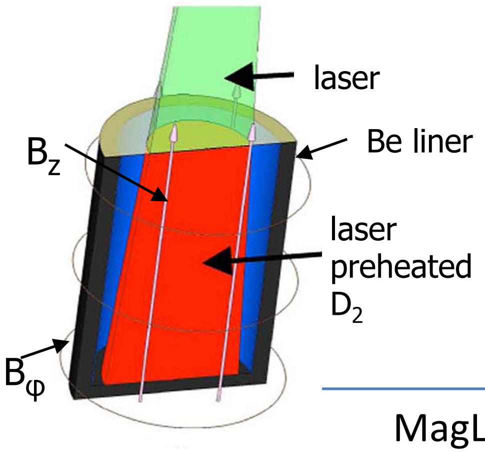
As above, but included wall vaporization.

On-going experiments with laser heated plasmas inside liners with embedded axial magnetic fields

NRL PPD

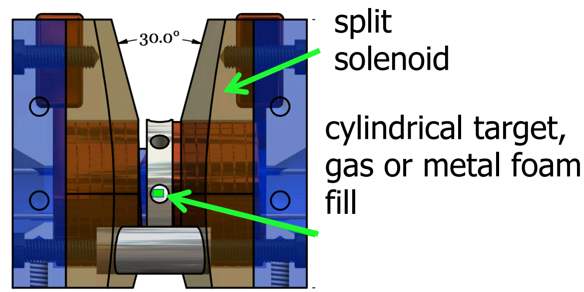
ICF on Z

Slutz, et al., PoP, **17**, 056393, 2010.
Sefkow, et al., PoP, **21**, 072711, 2014.
Ryutov, et al., PoP, **19**, 062706, 2012.



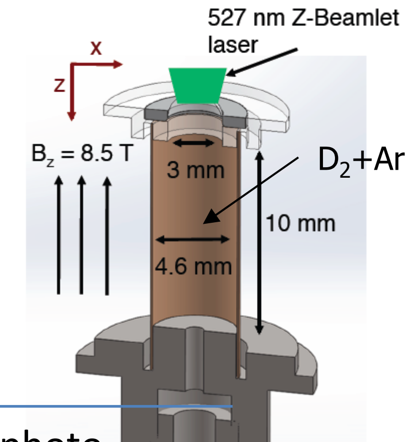
K-shell rad sources

Kemp, LLNL LDRD
Kemp, et al., PoP, **23**, 101204, 2016.
Kemp, et al., APS-DPP, 2017.



photoionization on Z Beamlet

Carpenter, et al., APS-DPP, 2017.
Mancini, private comm.



	MagLIF (preheat)	MagLIF (imploded)	rad source	photo-ionization
n_e (cm^{-3})	3e20	1e23	1e21 (Z~10)	2e20
T_e (eV)	300	8000	1000	600
B (T)	30	13,000	10	8.5
β	80	4.6	4.4e3	1.1e3
$\chi_e = \omega_e \tau_e$	4.2	608	0.24	4.5
Lm	760	8.9e5	2.4e4	1.5e4

Outline

NRL PPD

Self-similar analyses provide “**exact**” solutions to a **simplified** problem – multi-physics codes provide **approximate** solutions to an “**exact**” problem.

Similarity solutions provide **verification tests** for multi-physics codes:
e.g., LASNEX, HYDRA, PERSEUS

a) Magnetized, hot, β>> 1 plasma bounded by a cold wall:

b) Magnetized, hot, arbitrary β plasma bounded by a cold wall:

1D MHD, planar MHD eqns at large plasma beta β_∞ includes the Nernst electro-thermal term

NRL PPD

$$n_i = n_e = n \qquad T_e = T_i = T$$

continuity: $$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0$$

momentum: $$2nT = \text{constant}$$

thermal: $$\frac{\partial}{\partial t}(3nT) + \frac{\partial}{\partial x}(3nTu_x) + 2nT\frac{\partial u_x}{\partial x} = \frac{\partial}{\partial x}\left[\ \underbrace{(\kappa_\perp^{(e)} + \kappa_\perp^{(i)})\frac{\partial T}{\partial x}}_{\textbf{thermal conduction}}\ \right]$$

Maxwell: $$\frac{\partial B_z}{\partial t} = -c\frac{\partial E_y}{\partial x} \quad \text{and} \quad J_y = -\frac{c}{4\pi}\frac{\partial B_z}{\partial x}$$

E field: $$E_y = \frac{u_x}{c}B_z + \eta_\perp J_y - \frac{\beta_\wedge^{uT}}{en}\frac{\partial T}{\partial x}$$

induction: $$\frac{\partial B_z}{\partial t} + \frac{\partial}{\partial x}(u_x B_z) = \frac{\partial}{\partial x}\left[\ \underbrace{\frac{c^2\eta_\perp}{4\pi}\frac{\partial B_z}{\partial x}}_{\textbf{magnetic diffusion}} + \underbrace{\frac{c\beta_\wedge^{uT}}{en}\frac{\partial k_B T}{\partial x}}_{\textbf{Nernst}}\ \right]$$

Self-similar solution for heat & magnetic flux loss to a cold liner from a planar, isobaric, plasma of large β_∞

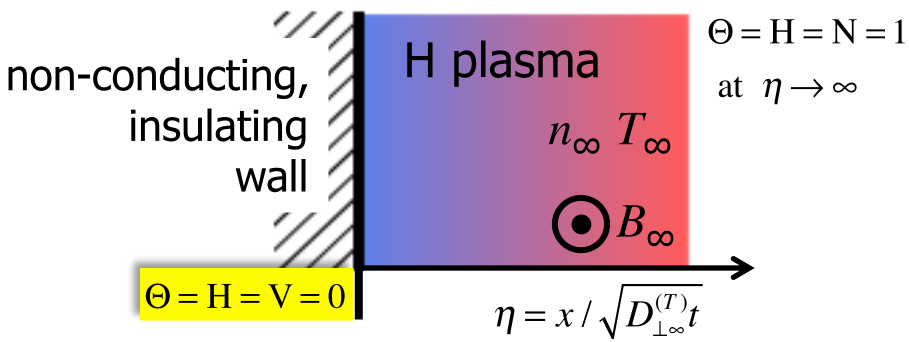
NRL PPD

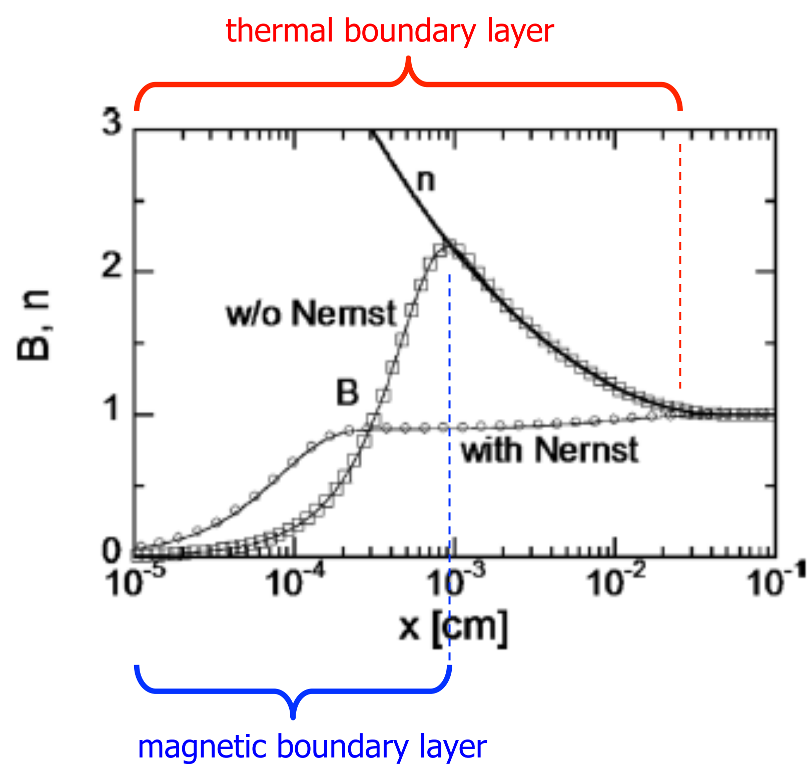
Self-similar ansatz

$$\eta \equiv \frac{x}{\sqrt{D_{\perp\infty}^{(T)} t}} \qquad \mathrm{V}(\eta) = \frac{2u_x(x,t)}{\sqrt{D_{\perp\infty}^{(T)}/t}} \qquad \mathrm{N}(\eta) = \frac{n(x,t)}{n_\infty} \qquad \Theta(\eta) = \frac{T(x,t)}{T_\infty} \qquad \mathrm{H}(\eta) = \frac{B_z(x,t)}{B_\infty}$$

$$D_{\perp\infty}^{(T)} = \frac{\kappa_{\perp\infty}^{(e)} + \kappa_{\perp\infty}^{(i)}}{3n_\infty}$$

boundary conditions





Velikovich, Giuliani, & Zalesak, PoP, 22, 042702, 2015.

Magnetic & thermal losses to an unmagnetized, cold liner at large β_∞

NRL PPD

$$D_\perp^{(T)} = \frac{\kappa_\perp^{(e)} + \kappa_\perp^{(i)}}{3n} = \frac{T\tau_e}{3m_e} f_1(\chi_e, \ln\Lambda)$$

$$D_\perp^{(B)} = \frac{c^2\eta_\perp}{4\pi} = \frac{c^2 m_e}{4\pi e^2 n\tau_e} f_2(\chi_e, \ln\Lambda)$$

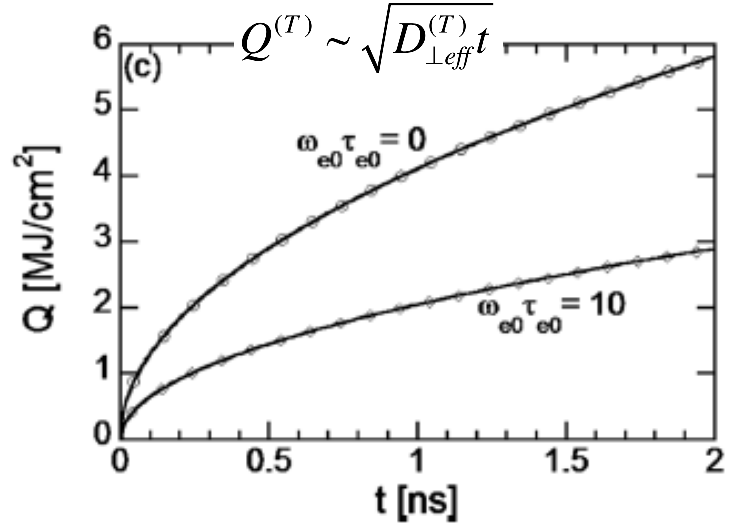
Heat loss is dominated
by thermal conduction & advection,

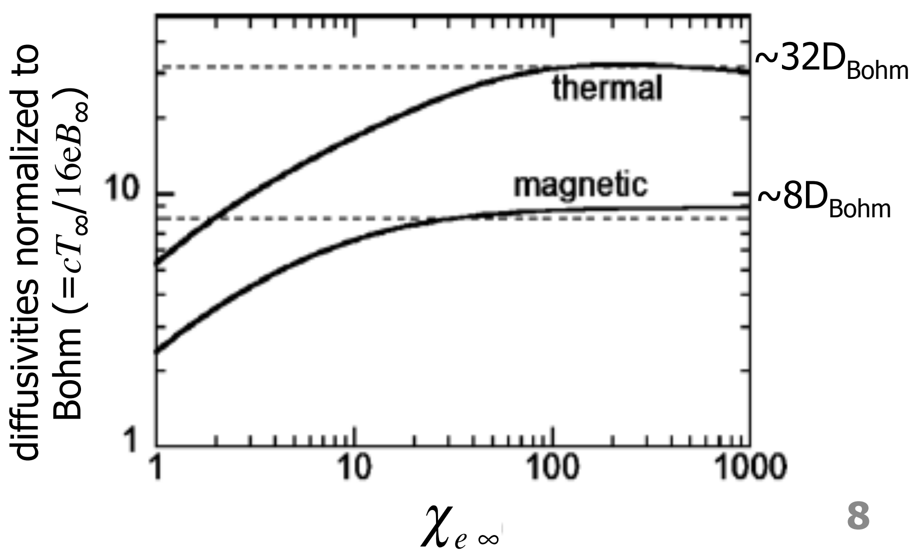
Magnetic flux loss dominated
by advection & Nernst effect.

$$D_{\perp eff}^{(T)} \sim \frac{2cT_\infty}{eB_\infty} = 32 D_{Bohm}$$

$$D_{\perp eff}^{(B)} \sim \frac{cT_\infty}{2eB_\infty} = 8 D_{Bohm}$$

El-Nadi & Rosenbluth, PoF, 16, 2036, 1973





1D MHD, planar eqns have three thermo-electric transport terms for arbitrary β_∞

NRL PPD

$$n_i = n_e = n \quad T_e = T_i = T$$

continuity: $$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nu) = 0$$

momentum: $$\frac{\partial}{\partial t}(\rho u_x) + \frac{\partial}{\partial x}(\rho u_x^2) + \frac{\partial}{\partial x}\left(2nT + \frac{B_z^2}{8\pi}\right) = 0$$

for self-similar solutions

$$2nT + B_z^2/8\pi = \text{constant}$$

thermal:

$$\frac{\partial}{\partial t}(3nT) + \frac{\partial}{\partial x}(3nTu_x) + 2nT\frac{\partial u_x}{\partial x} = \frac{\partial}{\partial x}\Big[\underbrace{(\kappa_\perp^{(e)} + \kappa_\perp^{(i)})\frac{\partial T}{\partial x}}_{\text{thermal conduction}} - \underbrace{\frac{\beta_\wedge^{uT}}{en}TJ_y}_{\text{Ettingshausen}}\Big] + \underbrace{\eta_\perp J_y^2}_{\text{Joule heating}} - \underbrace{\frac{\beta_\wedge^{uT}}{en}\frac{\partial T}{\partial x}J_y}_{\text{Thomson}}$$

induction: $$\frac{\partial B_z}{\partial t} + \frac{\partial}{\partial x}(u_x B_z) = \frac{\partial}{\partial x}\Big[\underbrace{\frac{c^2\eta_\perp}{4\pi}\frac{\partial B_z}{\partial x}}_{\text{magnetic diffusion}} + \underbrace{\frac{c\beta_\wedge^{uT}}{en}\frac{\partial k_B T}{\partial x}}_{\text{Nernst}}\Big]$$

For arbitrary β_∞, one has Ettingshausen, Thomson, & Joule in thermal eqn, in addition to Nernst in induction

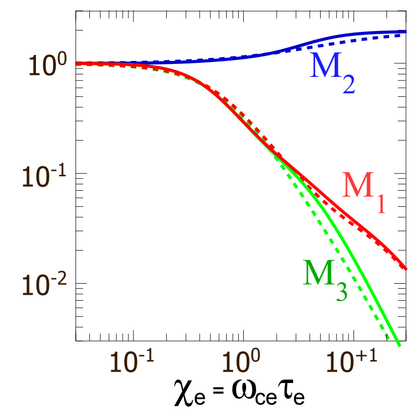
momentum: $$N\Theta = 1 + (1-\mathrm{H}^2)/\beta_\infty$$

continuity: $$\frac{dV}{d\eta} = (V - \frac{\eta}{2})\frac{d\ln\Theta}{d\eta} + (V - \frac{\eta}{2})\frac{2\mathrm{H}}{\beta_\infty + (1-\mathrm{H}^2)}\frac{d\mathrm{H}}{d\eta}$$

thermal:

$$(1+\frac{1-\mathrm{H}^2}{\beta_\infty})\left[(V-\frac{\eta}{2})\frac{d\ln\Theta}{d\eta} + \frac{2}{3}\frac{dV}{d\eta}\right] = \frac{d}{d\eta}\Big[\underbrace{\Theta^{5/2}\frac{M_1}{M_4}\frac{d\Theta}{d\eta}}_{\text{thermal conduction}} + \underbrace{\frac{2}{3}\frac{\tilde{\beta}_\wedge^{(ET)}}{\beta_\infty}\Theta\frac{d\mathrm{H}}{d\eta}}_{\text{Ettingshausen}}\Big] + \underbrace{\frac{4}{3\mathrm{Lm}_{\|\infty}\beta_\infty}\frac{M_2 M_4}{\Theta^{3/2}}\left(\frac{d\mathrm{H}}{d\eta}\right)^2}_{\text{Joule heating}} + \underbrace{\frac{2}{3}\frac{\tilde{\beta}_\wedge^{(ET)}}{\beta_\infty}\frac{d\Theta}{d\eta}\frac{d\mathrm{H}}{d\eta}}_{\text{Thomson}}$$

induction: $$(V-\frac{\eta}{2})\frac{d\mathrm{H}}{d\eta} + \mathrm{H}\frac{dV}{d\eta} = \frac{d}{d\eta}\Big[\underbrace{\frac{1}{\mathrm{Lm}_{\|\infty}}\frac{M_2 M_4}{\Theta^{3/2}}\frac{d\mathrm{H}}{d\eta}}_{\text{magnetic diffusion}} + \underbrace{\frac{1}{2}\tilde{\beta}_\wedge^{(ET)}\frac{d\Theta}{d\eta}}_{\text{Nernst}}\Big]$$



electro-thermal: $$\tilde{\beta}_\wedge^{(ET)} = \zeta\frac{\mathrm{H}\Theta^{5/2}}{1+(1-\mathrm{H}^2)/\beta_\infty}\frac{M_3}{M_4} \qquad V_{Nernst} = -\frac{1}{2}\frac{\tilde{\beta}_\wedge^{(ET)}}{\mathrm{H}}\frac{d\Theta}{d\eta}$$

e^- Hall parameter:

$$\chi_e = \omega_{ce}\tau_e = \sqrt{\frac{\mathrm{Lm}_{\|\infty}}{\lambda\beta_\infty}}\frac{\mathrm{H}\Theta^{5/2}}{1+(1-\mathrm{H}^2)/\beta_\infty}\frac{1}{M_4}$$

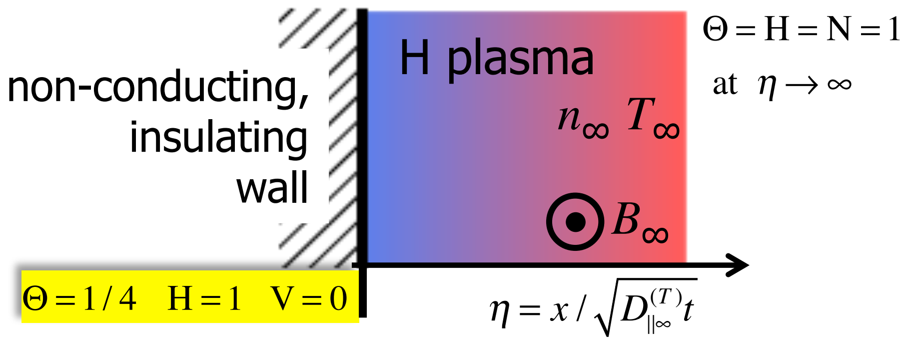
Coulomb log:

$$\ln\Lambda = M_4(\Theta,\mathrm{H},\beta_\infty)\ln\Lambda_\infty \qquad \mathrm{Lm}_{\|\infty} = \left(\frac{D_{\|\infty}^{(T)}}{D_{\|\infty}^{(B)}}\right) \sim \frac{1}{2}\beta_\infty\chi_{e\infty}^2 \propto \frac{T^4}{n}$$

Giuliani & Velikovich, submitted to IEEE TPS, Feb., 2018.

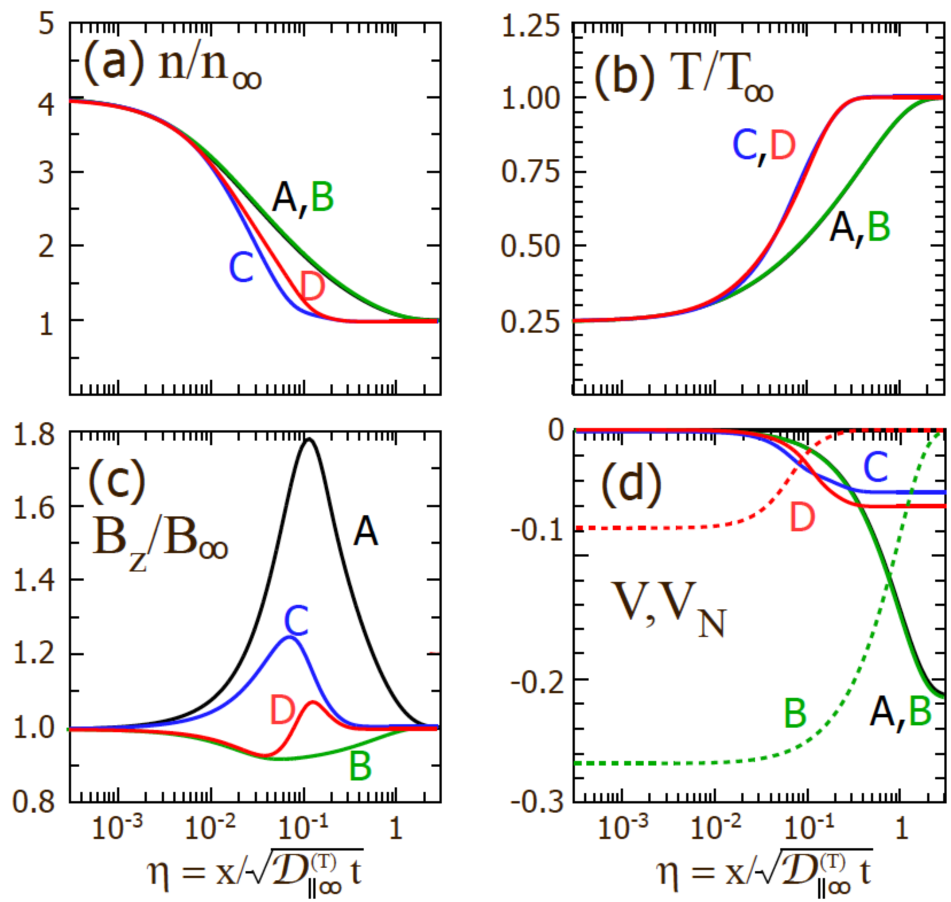
n, T, & V profiles depend on β_∞, weakly on $\tilde{\beta}_\wedge^{(ET)}$, but B profile varies strongly with β_∞ and $\tilde{\beta}_\wedge^{(ET)}$

boundary conditions ~ MagLIF preheat stage



$$n_\infty = 3\times10^{20}\ \mathrm{cm}^{-3},\ T_\infty = 300\ \mathrm{eV},\ \Rightarrow \mathrm{Lm}_{\|\infty} = 763$$

	B_∞ (T)	β_∞	$\chi_{e\infty}$	electro-thermal
A	1.4	3.7e4	0.2	no
B	1.4	3.7e4	0.2	yes
C	140	3.7	20.	no
D	140	3.7	20.	yes

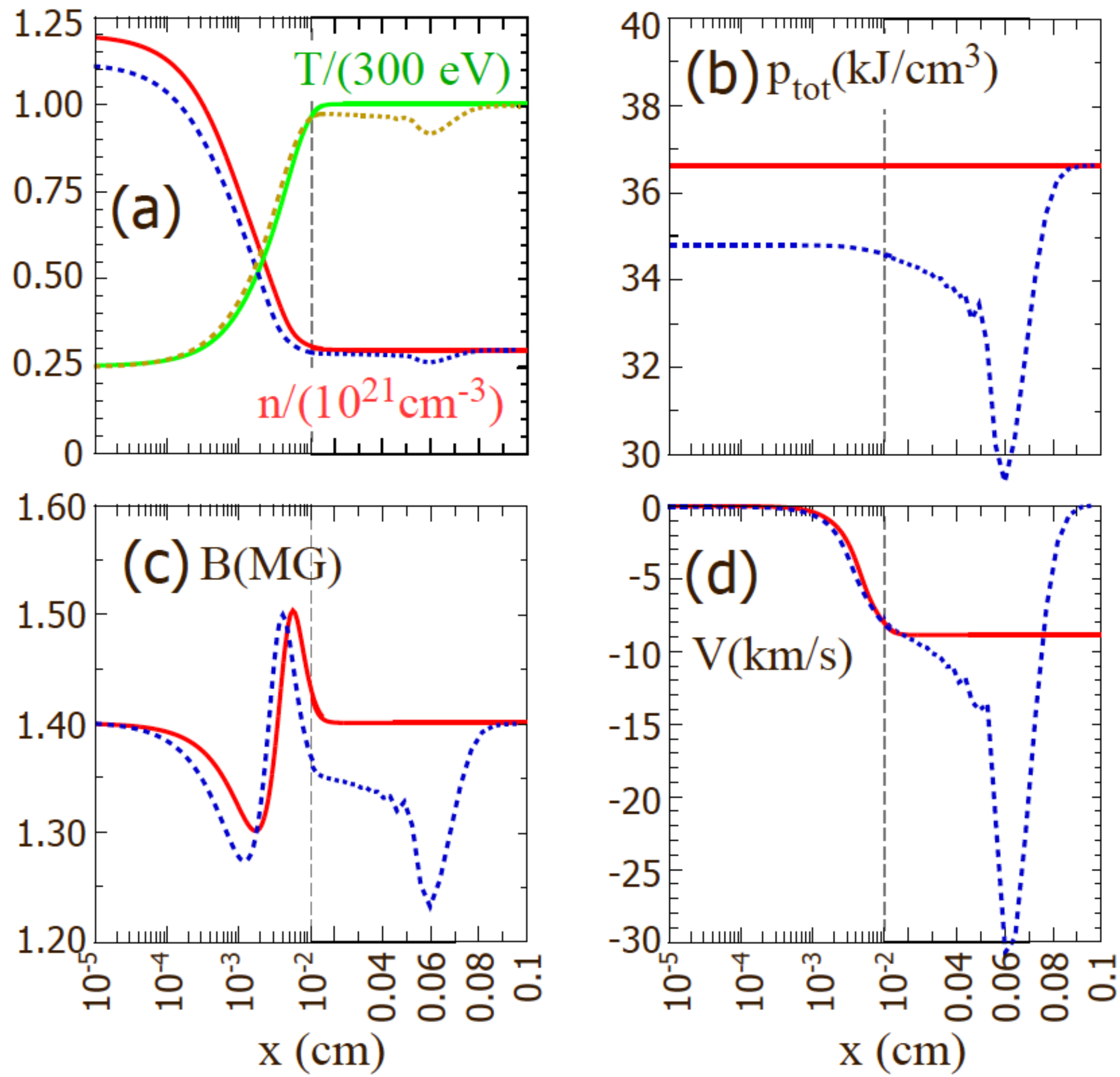


Case D as a MHD code verification test: $\beta_\infty = 3.7$, $\chi_e = 20$ at 3 ns.

NRL PPD

self-similar

non-isobaric, MHD code

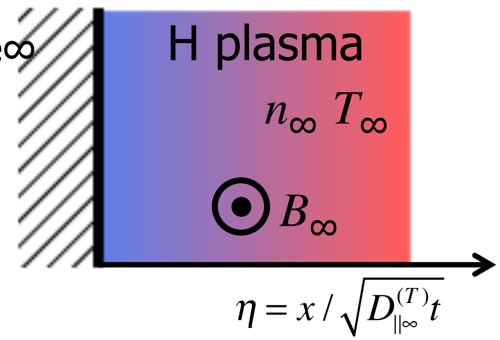


Summary

NRL PPD

Early studies showed anomalously high thermal transport from a hot, magnetized plasma to a cold wall due to Nernst.

Planar, isobaric, self-similar solutions with large β_∞ and $\chi_{e\infty}$ show that heat loss by thermal conduction & advection, magnetic flux loss by Nernst & advection:



$$\mathrm{Lm}_{\|\infty} \approx \frac{1}{2}\beta_\infty \chi^2_{e\,\infty} \qquad D^{(T)}_{\perp eff} \sim 32 D_{Bohm} \qquad D^{(B)}_{\perp eff} \sim 8 D_{Bohm}$$

For low β_∞ plasmas, the Ettingshausen, Thomson, and Joule terms enter the thermal energy eqn, in addition to Nernst in induction.

Four self-similar solutions appropriate to the conditions following laser pre-heat:

n, T, & V profiles depend on β_∞, weakly on electro-thermal effects; but B varies strongly with both β_∞ and electro-thermal effects.

A verification test for electro-thermal physics in MHD codes without isobaric constraint in self-similar solutions.