Magnetic Field Dynamics in Imploding Plasmas *

Magnetic Field Workshop

LLE Rochester 2018

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Key aspects in MagLIF

- Magnetization of the fuel is intended to
  - ✓ Provide magnetic insulation: Reduce thermal losses.
  - ✓ Enhance α-particle energy deposition.

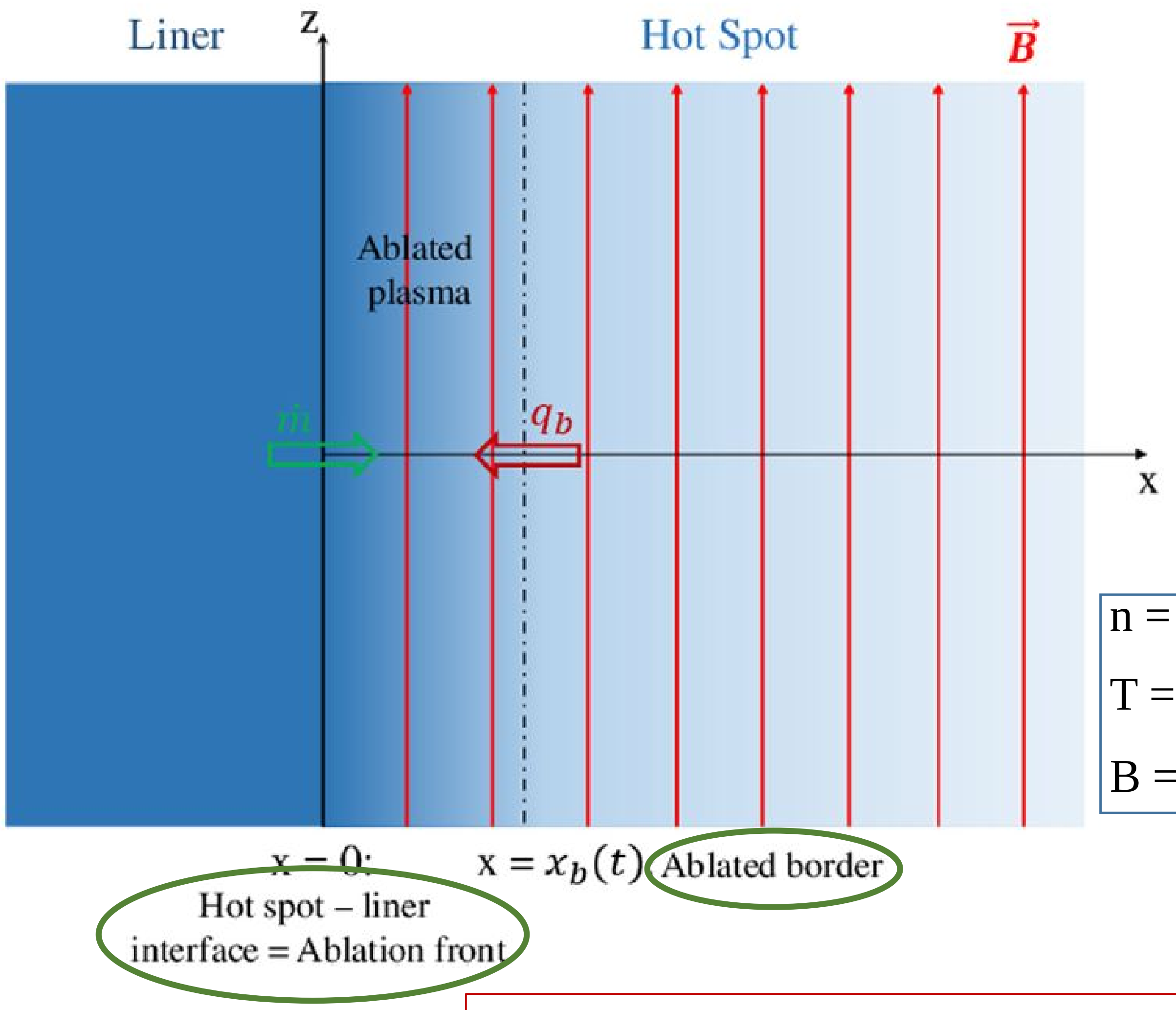
- The magnetic field is compressed and amplified.
  - ✓ The Nernst term degrades the magnetic flux conservation.
  - ✓ Understanding the plasma dynamics close to the liner becomes essential.

Scheme of the problem

n = ∞

T = 0

B = 0

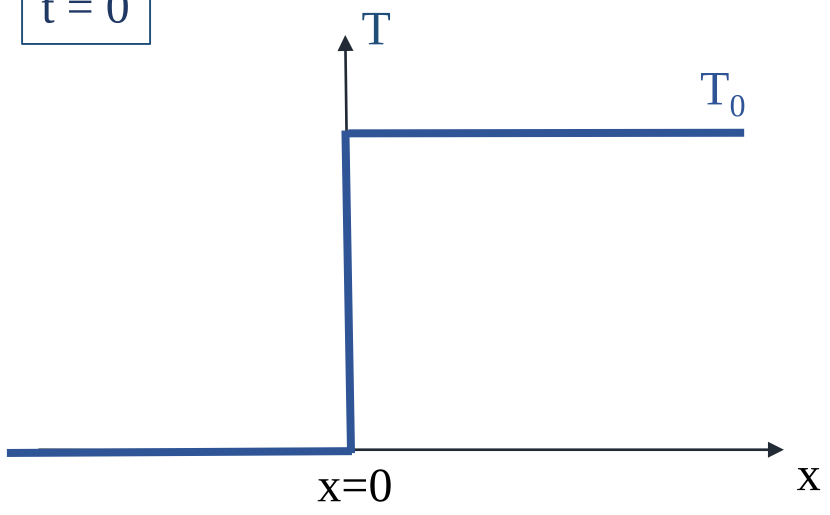


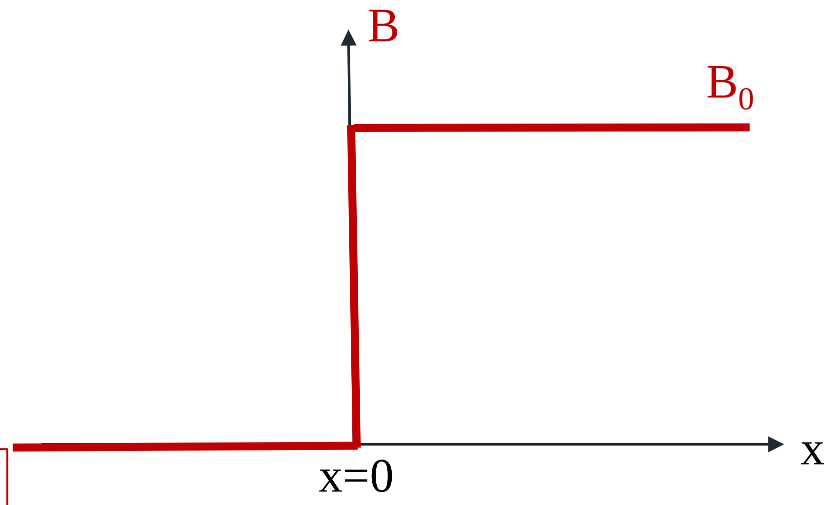
$n = n_0$

$T = T_0$

$B = B_0$

t = 0





A. L. Velikovich, J. L. Giuliani and S. T. Zalesak, Phys. Plasmas 22, 2015.

Outline

1. Magnetization effects
2. Magnetic pressure effects
3. Liner material effects
4. Ion diffusion

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2. Magnetic pressure effects
3. Liner material effects
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Governing equations

Planar geometry

Simplifying assumptions

1. Low Mach number (subsonic)

$$\mathrm{Ma} \ll 1$$

2. Large thermal to magnetic pressure ratio

$$\beta = 8\pi p_0 / B_0^2 \gg 1$$

3. Planar geometry

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0, \quad p = 2nT,$$

$$p = p_0,$$

$$v = \frac{\gamma - 1}{\gamma p_0} \chi_\perp \frac{\partial T}{\partial x}$$

$$\frac{\partial B}{\partial t} + \frac{\partial}{\partial x}(vB) = \frac{\partial}{\partial x}\left(\underbrace{D_{m\perp} \frac{\partial B}{\partial x}}_{\text{Joule}} + \underbrace{\frac{c\beta_\wedge^{uT}}{enB} \frac{\partial T}{\partial x} B}_{\text{Nernst}} \right).$$

Depentent variables	Indepentent variables
T, B	x, t

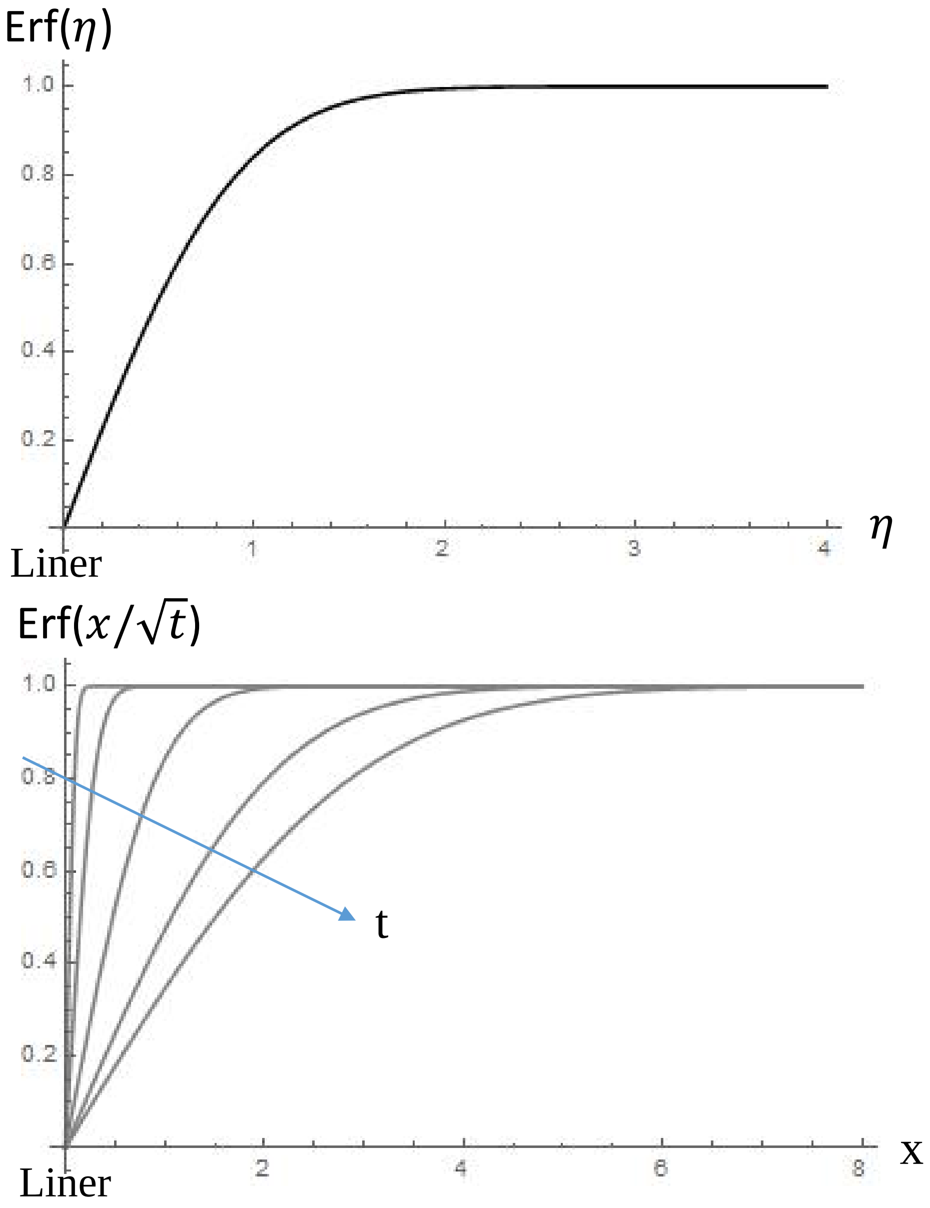
Self-similarity

- ✓ The motion is triggered by the transport processes: heat conduction.
- ✓ The structure of the equations admits a self-similar solution in form of a diffusive wave.
- ✓ Self-similar variable:

$$\eta = \frac{x}{\sqrt{\kappa_0 t}} \quad (\eta \geq 0).$$

- ✓ The independent variables only depend on one variable:

$$\mathrm{T}(x,t) \rightarrow T(\eta)$$



Normalization

✓ We normalize the variables:

$$\theta = \frac{T}{T_0}, \quad \phi = \frac{B}{B_0}.$$

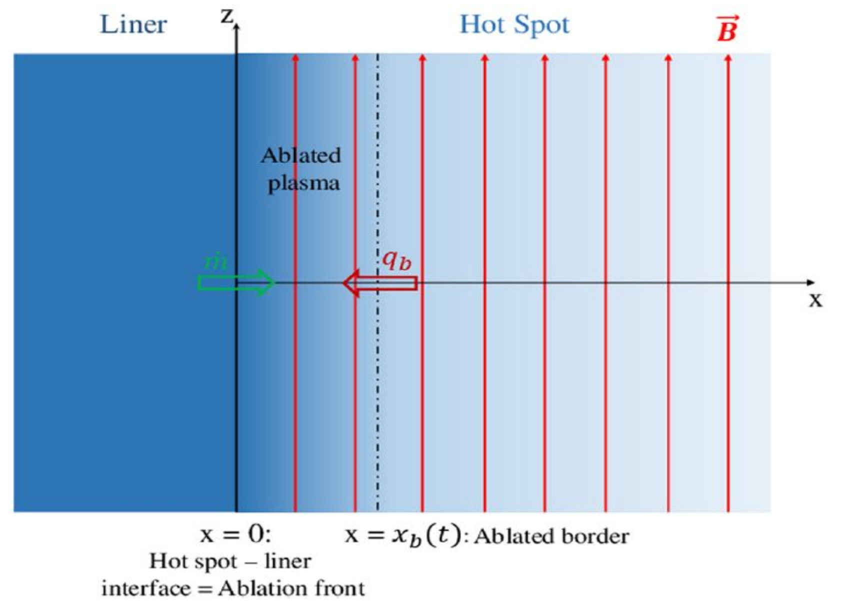
✓ The equations can be written as:

Cont.
$$\eta \frac{\mathrm{d}\theta}{\mathrm{d}\eta} + 2\theta^2 \frac{\mathrm{d}}{\mathrm{d}\eta}\left(\mathcal{P}_c \theta^{3/2} \frac{\mathrm{d}\theta}{\mathrm{d}\eta}\right) = 0.$$

Ind.
$$-\eta \frac{\mathrm{d}\phi}{\mathrm{d}\eta} + \frac{\mathrm{d}}{\mathrm{d}\eta}\left[2\mathcal{P}_c(1-\mathcal{P}_n)\theta^{5/2}\frac{\mathrm{d}\theta}{\mathrm{d}\eta}\phi\right] = \frac{2}{\mathrm{Le}}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{\mathcal{P}_d}{\theta^{3/2}}\frac{\mathrm{d}\phi}{\mathrm{d}\eta}\right).$$

✓ Coupled through the functions $P_c(x_e), P_c(x_e)$ and $P_d(x_e)$.

$$x_e = x_{e0}\theta^{5/2}\phi.$$



$$\eta = 0: \quad \theta = 0, \quad \phi = 0.$$

$$\eta = \infty: \quad \theta = 1, \quad \phi = 1.$$

✓ There are two free parameters:

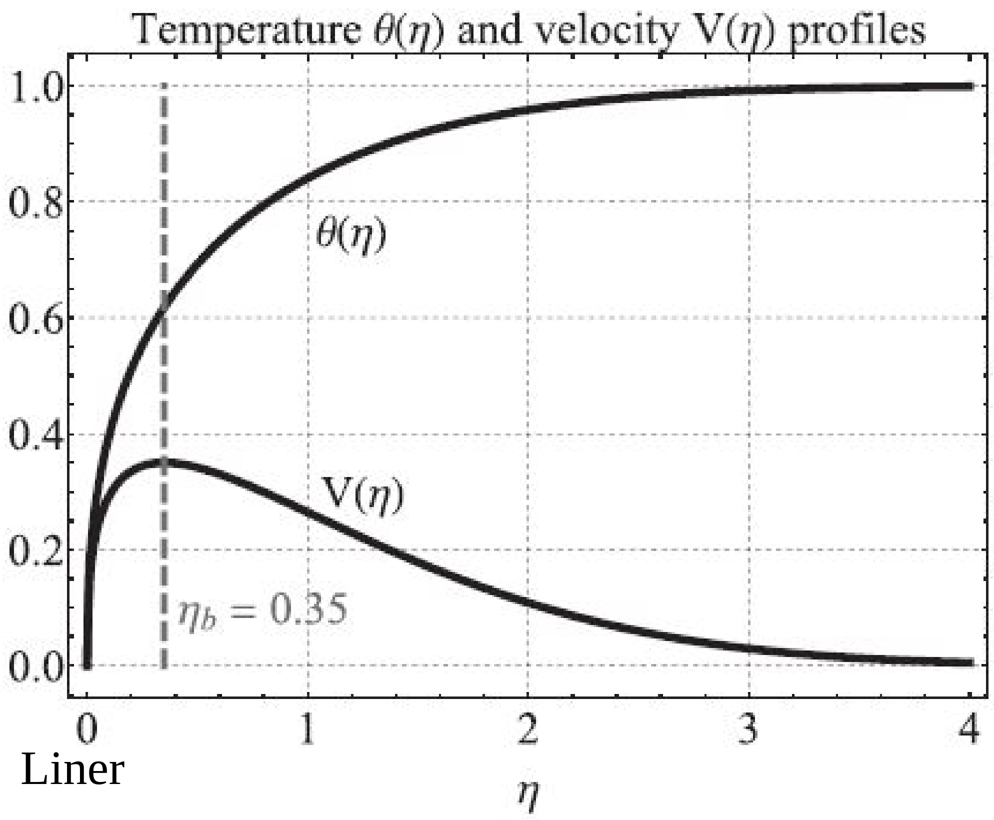
1) Magnetic Lewis number:

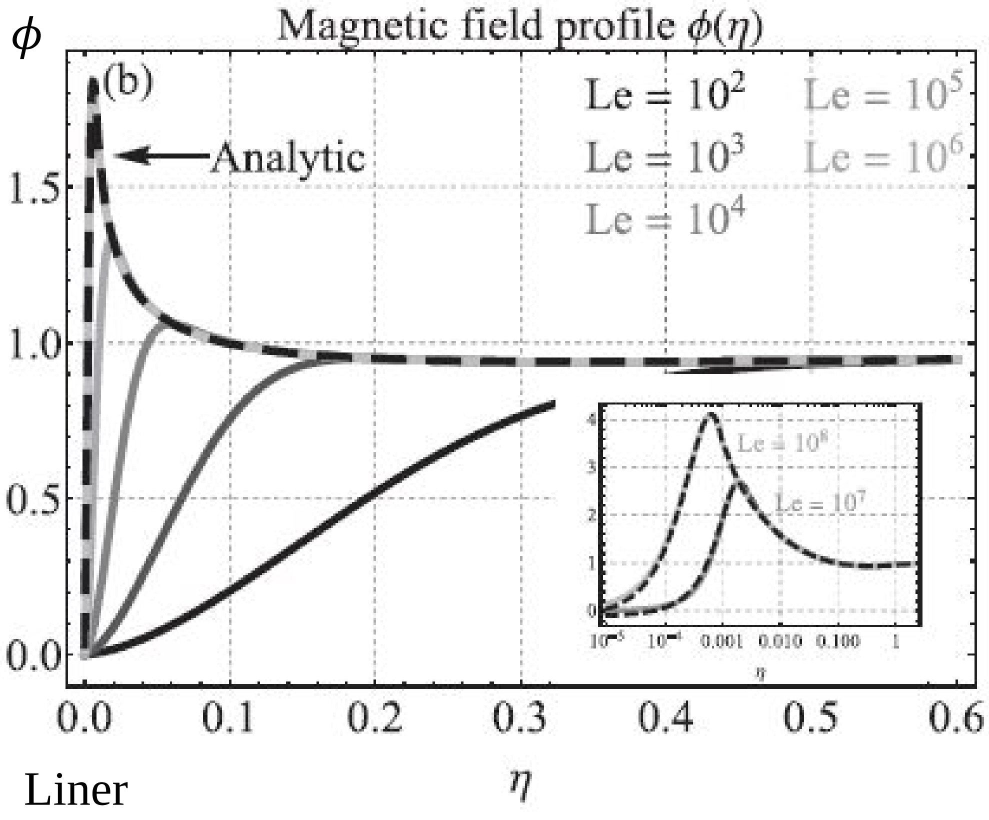
$$\mathrm{Le} = \frac{\text{Thermal conduction}}{\text{Magnetic diffusiviy}}$$

2) Magnetization downstream: x_{e0}

Unmagnetized Plasma.

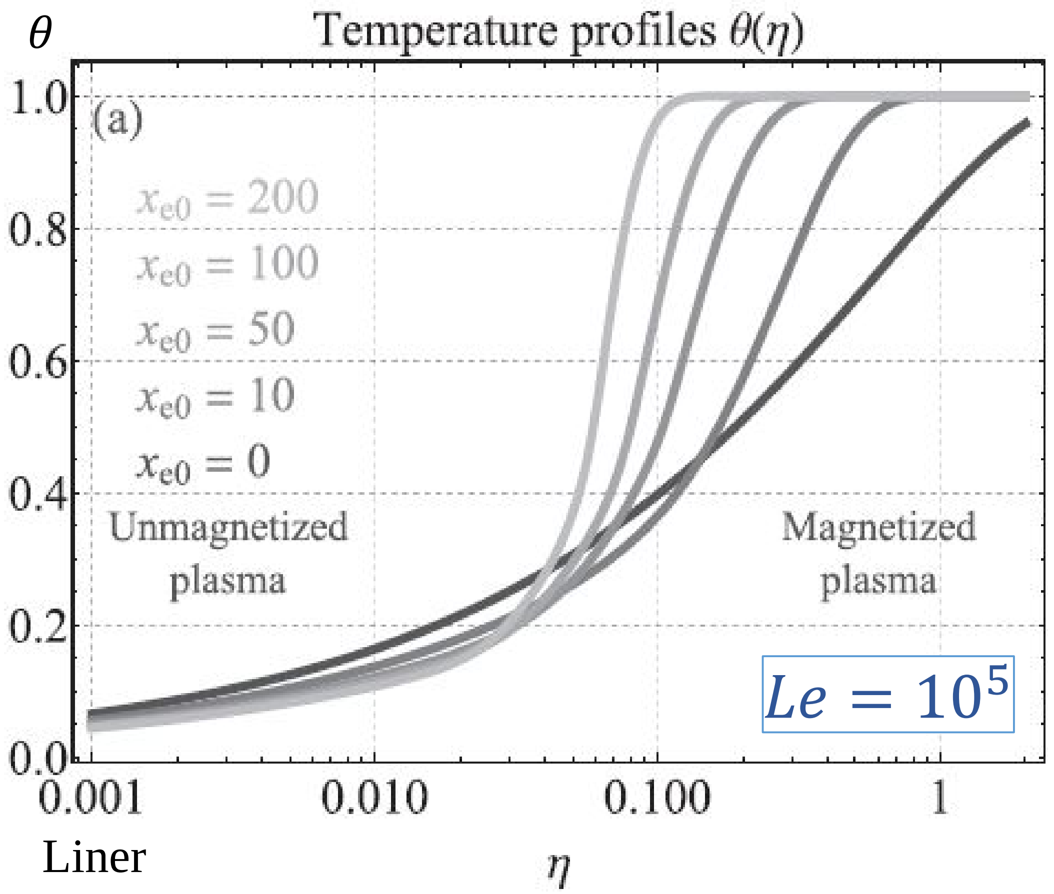
$x_{e0} \ll 1$

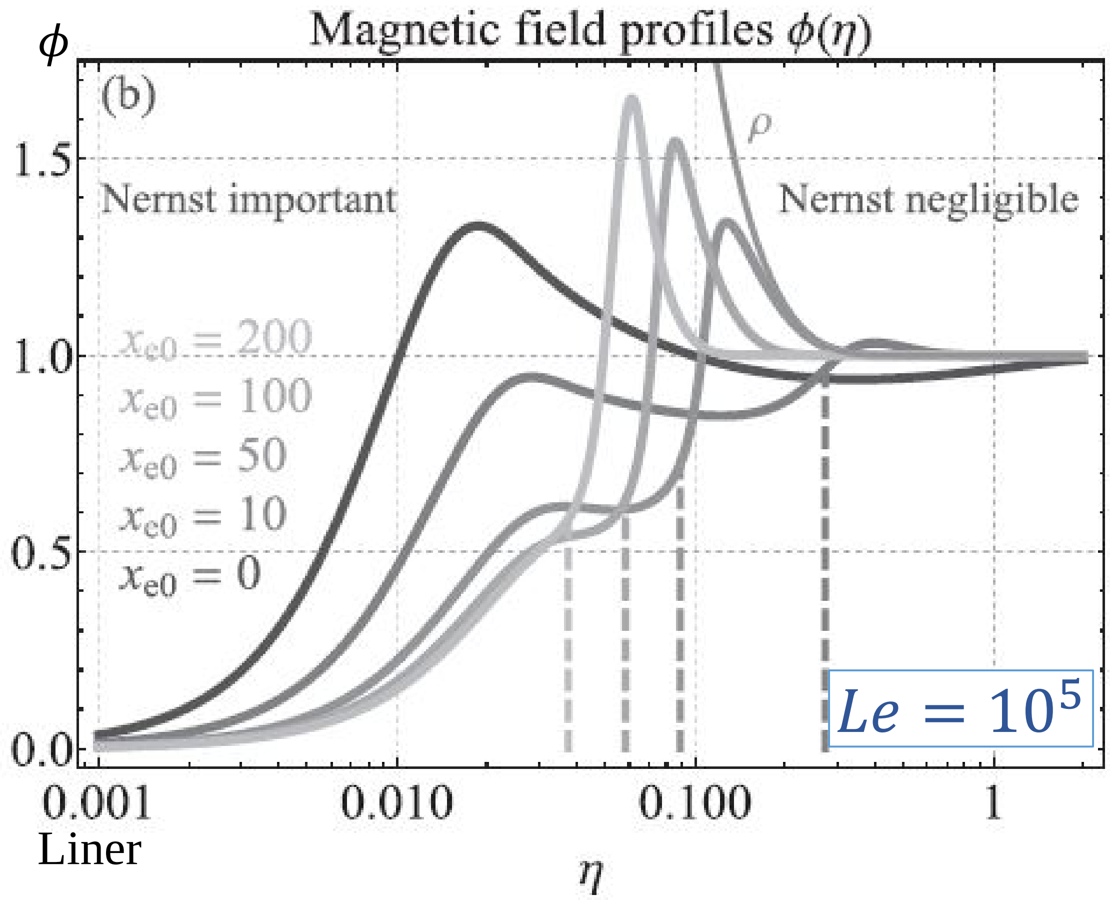




✓ Magnetic field convected towards the liner.

Magnetized Plasma.





Unmagnetized region | Magnetized region

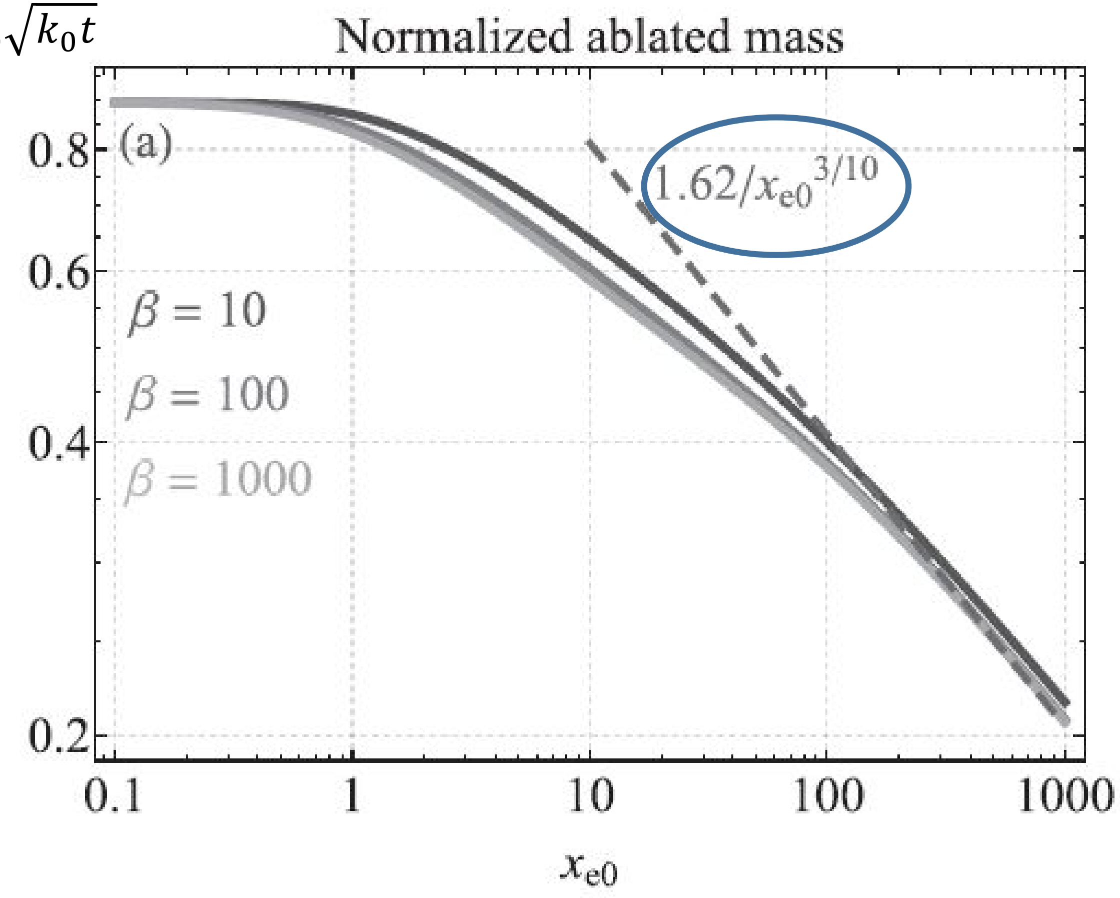
Unmagnetized region | Magnetized region

Mass ablation & Magnetic flux losses

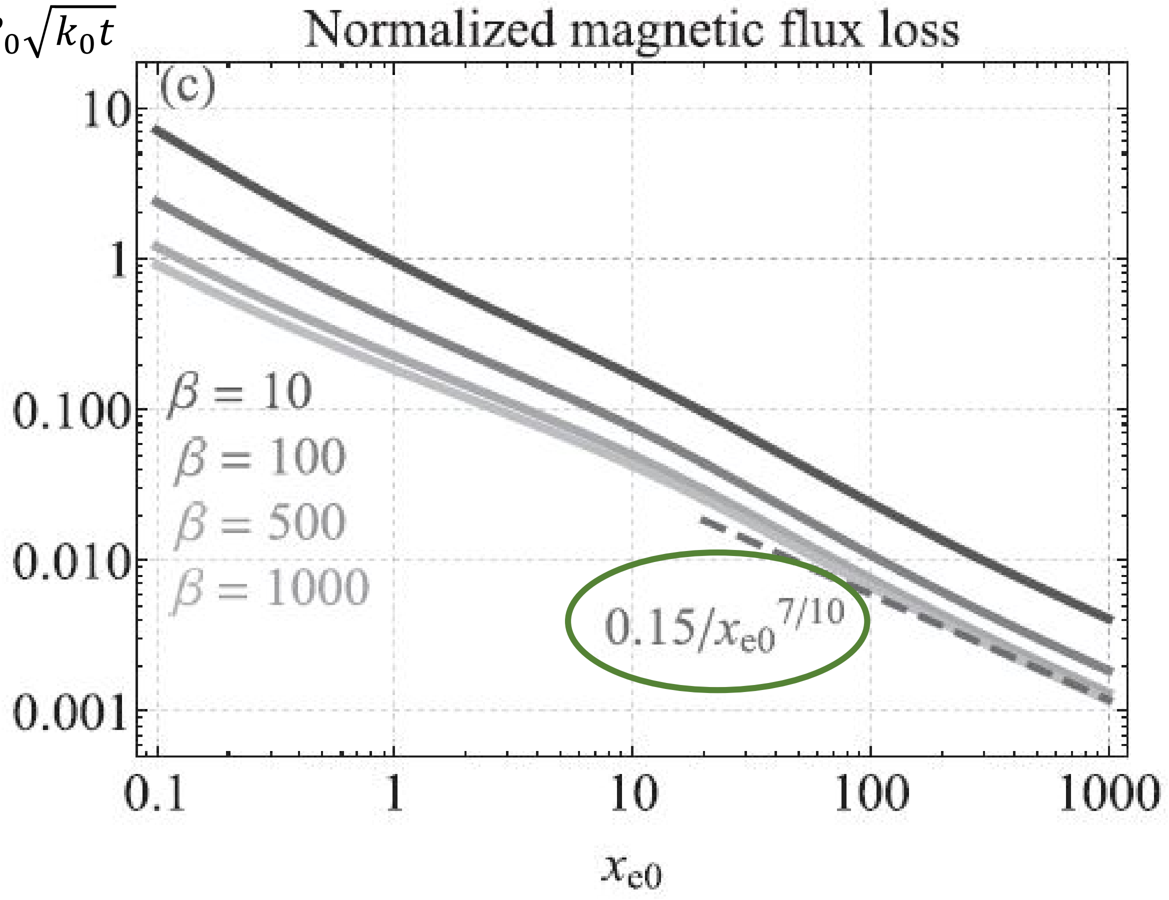
$$m = \int_0^\infty (n - n_0)\mathrm{d}x,$$

$$\Phi = \int_0^\infty (B_0 - B)\mathrm{d}x.$$

$\frac{m}{n_0\sqrt{k_0 t}}$



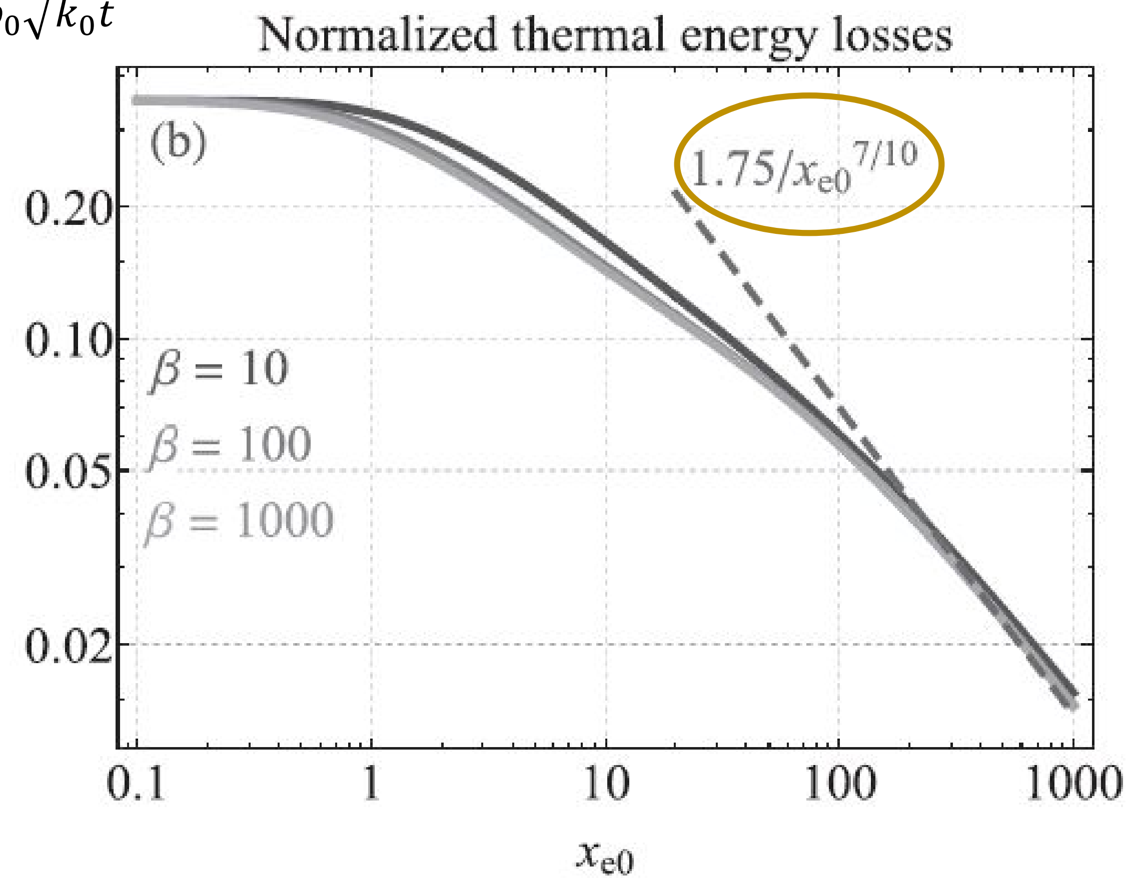
$\frac{\phi}{B_0\sqrt{k_0 t}}$

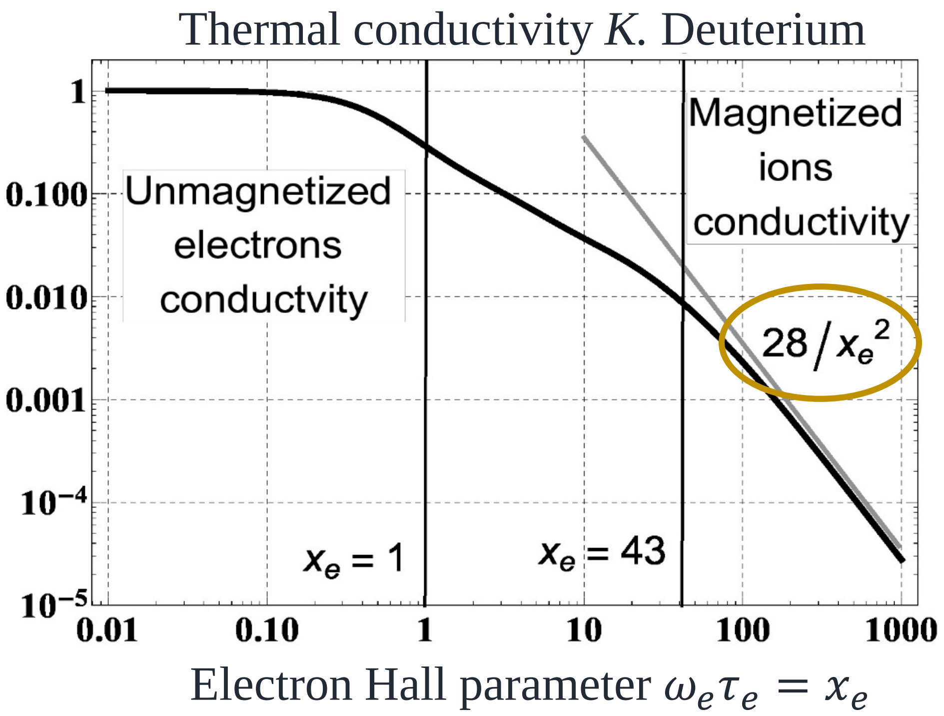


Thermal energy losses

$$\mathcal{E} = \int_0^\infty \frac{p_0}{\gamma - 1}\,\mathrm{d}x - \int_{x_b}^\infty \frac{p_0}{\gamma - 1}\,\mathrm{d}x,$$

$$\frac{(\gamma - 1)\varepsilon}{p_0\sqrt{k_0 t}}$$





✓ The presence of a cold unmagnetized liner makes the thermal energy losses scale as $x_{e0}^{-7/10}$ instead of x_{e0}^{-2}.

Comparison with previous work

- A. L. Velikovich, J. L. Giuliani and S. T. Zalesak, "*Magnetic flux and heat losses by diffusive, advective, and Nernst effects in magnetized liner inertial fusión-like plasma*", Phys. Plasmas 22, 2015.

	Mass ablation	Heat flux at the liner wall	Thermal energy losses	Magnetic flux losses
Velikovich	$m = 0$	$q\|_{x=0} \sim \frac{1}{x_{e0}^{1/2}}$	$Q \sim \frac{1}{x_{e0}^{1/2}}$	$\Phi \sim \frac{1}{x_{e0}^{1/2}}$
Present work	$m \sim \frac{1}{x_{e0}^{3/10}}$	$q\|_{x=0} = 0$	$\mathcal{E} \sim \frac{1}{x_{e0}^{7/10}}$	$\Phi \sim \frac{1}{x_{e0}^{7/10}}$

Velikovich

- ✓ Liner: solid state
- ✓ No ablation

Present work

- ✓ Liner: cold dense plasma
- ✓ Ablation

Outline

1. Magnetization effects
2. Magnetic pressure effects
3. Liner material effects
4. Ion diffusion

Magnetic pressure effects: Finite β

- Continuity:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0,$$

- Momentum:

$$p + \frac{B^2}{8\pi} = p_T = p_{T0},$$

- Energy:

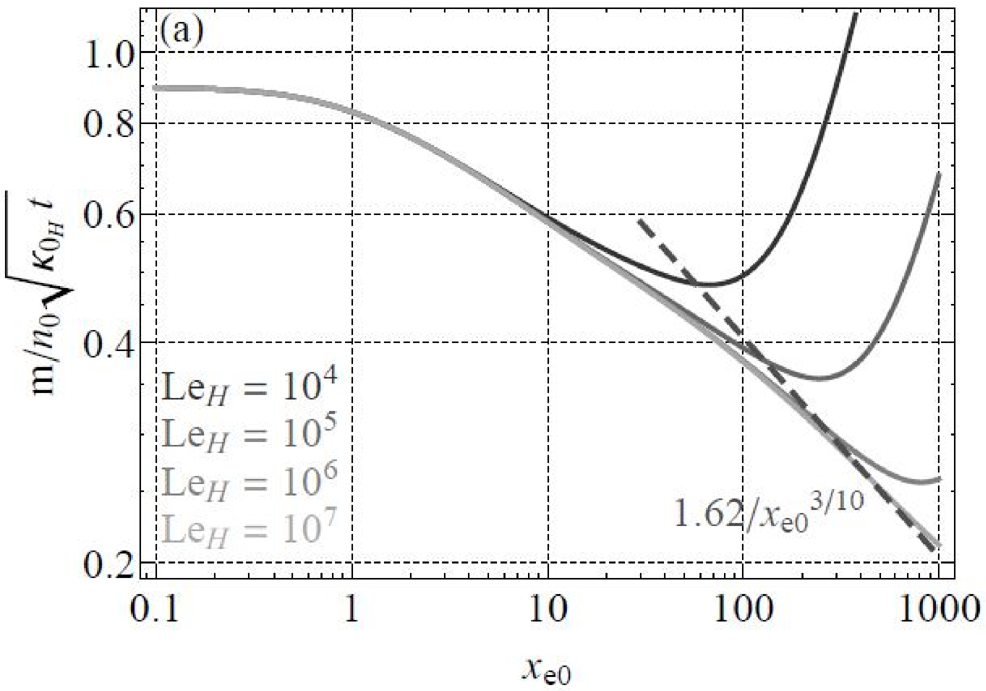
$$\frac{\partial}{\partial t}\left(\frac{1}{\gamma-1}p + \frac{B^2}{8\pi}\right) + \frac{\partial}{\partial x}\left[\left(\frac{\gamma}{\gamma-1}p + \frac{B^2}{4\pi}\right)v\right] = \frac{\partial}{\partial x}\left[\underbrace{\chi_\perp \frac{\partial T}{\partial x}}_{\text{Cond.}} + \frac{c\beta_\wedge^{uT}}{4\pi en}\left(\underbrace{B\frac{\partial T}{\partial x}}_{\text{Nernst}} + \underbrace{T\frac{\partial B}{\partial x}}_{\text{Ettingsh.}}\right) + \underbrace{\frac{D_{m\perp}}{4\pi}B\frac{\partial B}{\partial x}}_{\text{Joule}}\right],$$

- Induction:

$$\frac{\partial B}{\partial t} + \frac{\partial}{\partial x}(vB) = \frac{\partial}{\partial x}\left(\underbrace{D_{m\perp}\frac{\partial B}{\partial x}}_{\text{Joule}} + \underbrace{\frac{c\beta_\wedge^{uT}}{en}\frac{\partial T}{\partial x}}_{\text{Nernst}}\right),$$

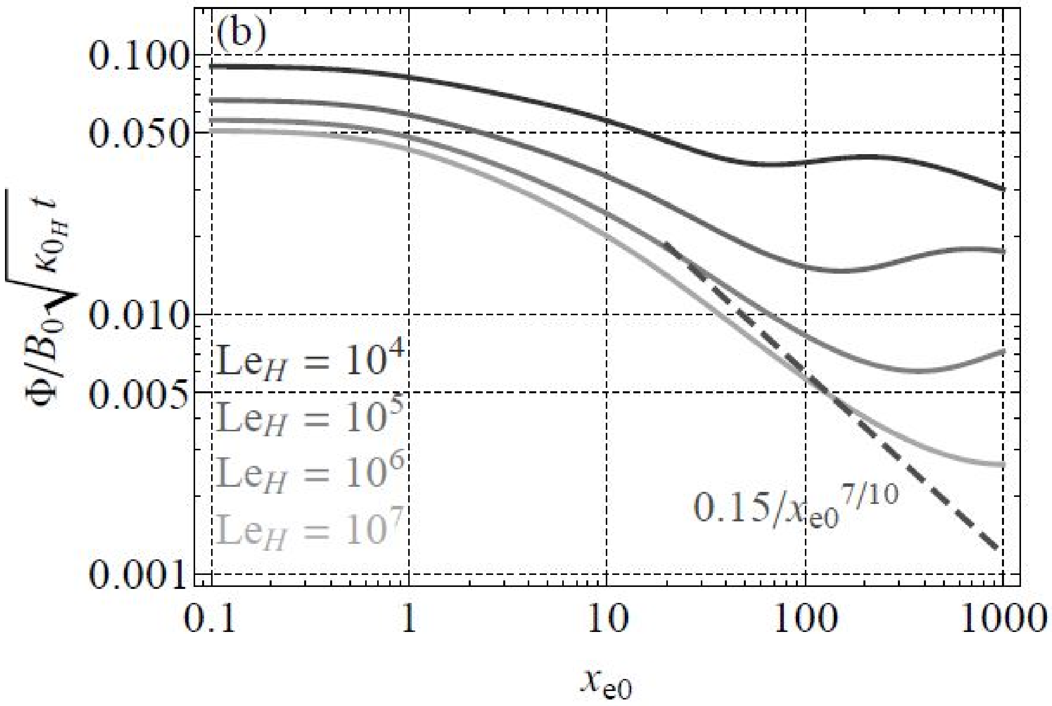
Mass ablation and Magnetic flux losses

Mass ablation



- ✓ Magnetic pressure enhances mass ablation.

Magnetic flux losses



- ✓ Magnetic flux conservation is degraded.

Thermal energy losses

- ✓ The thermal energy losses can be expressed as

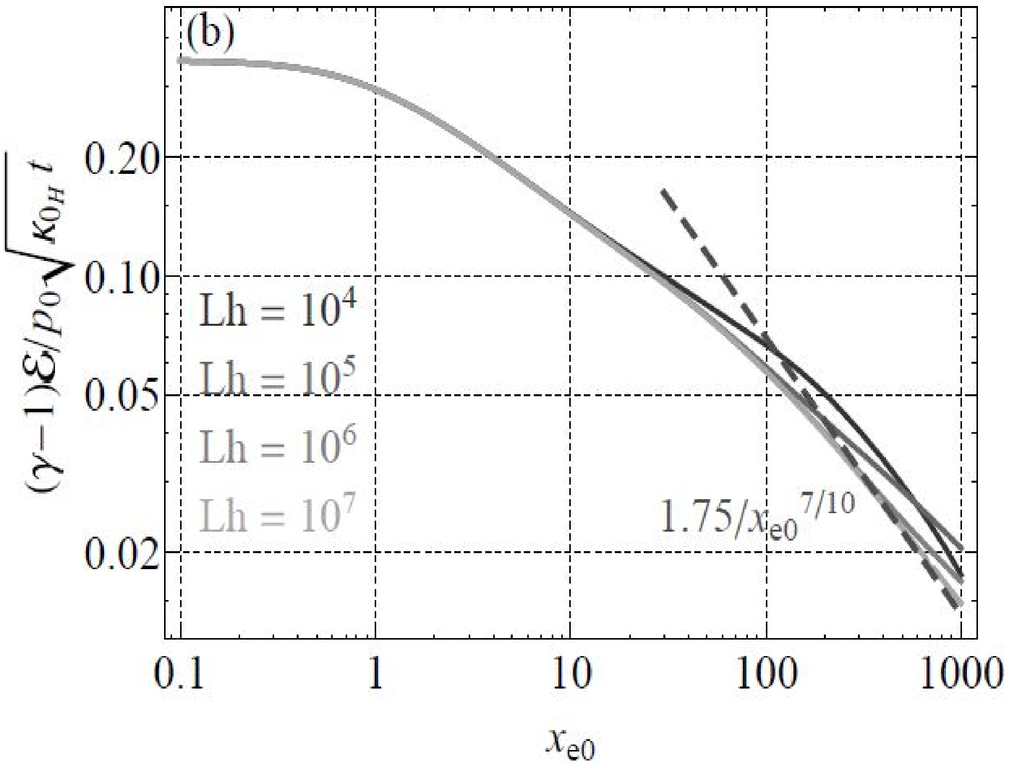
$$\mathcal{E} = \int_0^\infty \frac{p_0}{\gamma - 1} \mathrm{d}x - \int_{x_b}^\infty \frac{p}{\gamma - 1} \mathrm{d}x.$$

- ✓ And computed as

$$\mathcal{E} = \frac{p_0}{\gamma - 1} \sqrt{\kappa_0 t} \left(\eta_b - \int_{\eta_b}^\infty \frac{1 - \phi^2}{\beta} \mathrm{d}\eta \right)$$

Transport terms through the ablated border

Magnetic energy converted dissipated into thermal energy



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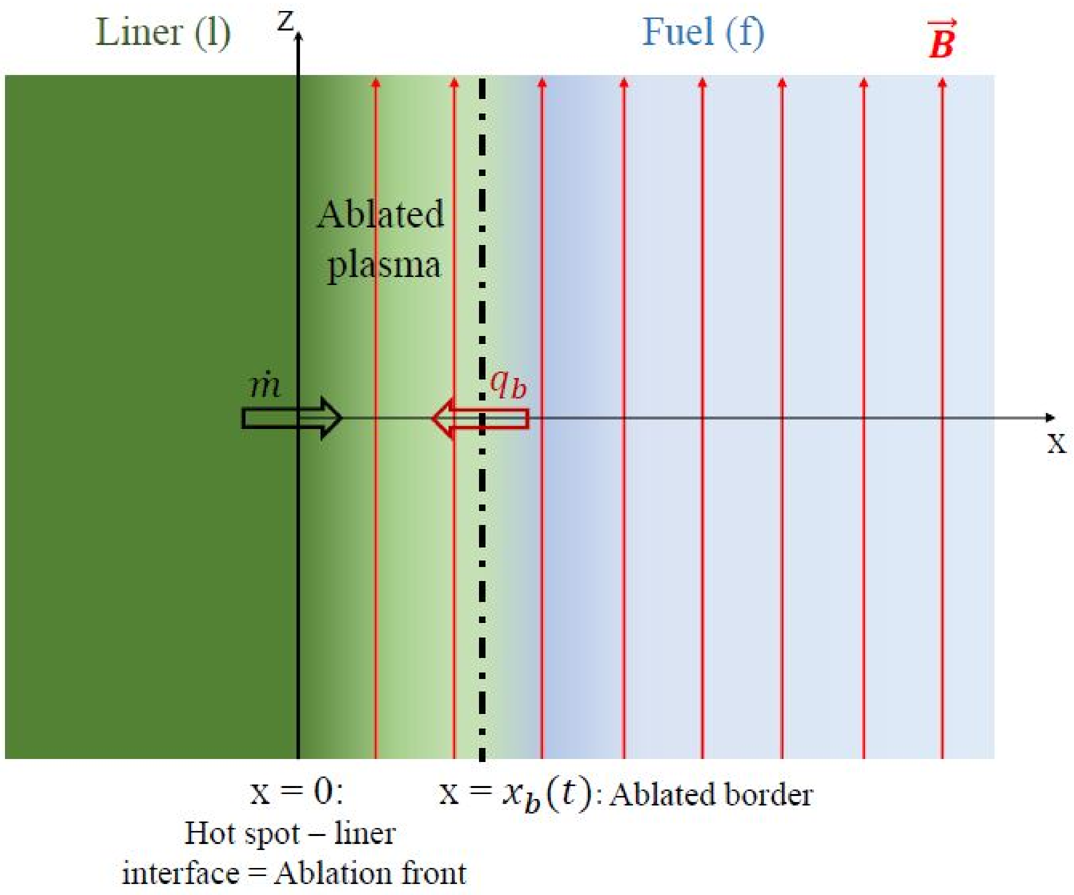
Effect of the liner material



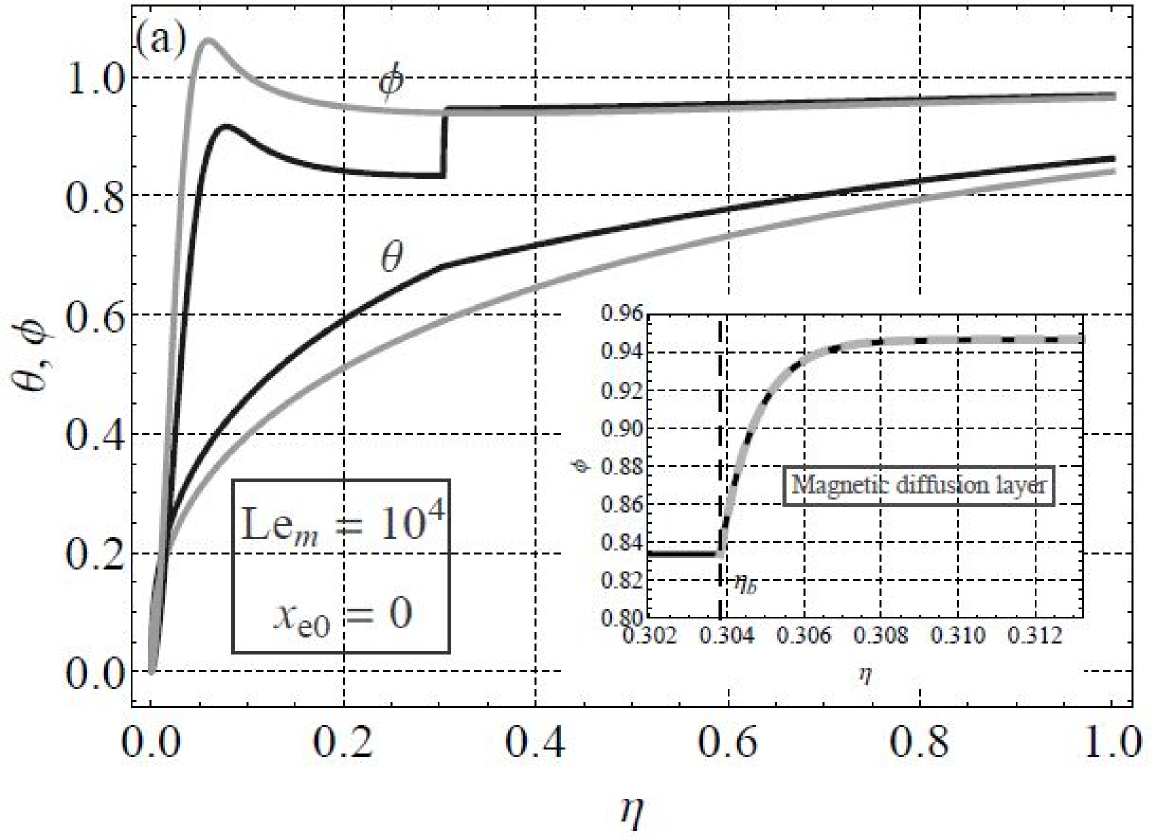
- ✓ **Effect** of the atomic number Z:
  - ✓ Reduction of thermal conductivity: $\chi_e \sim \frac{1}{Z}$
  - ✓ Enhacement of magnetic diffusivity: $\nu_m \sim Z$

If we neglect ion diffusion

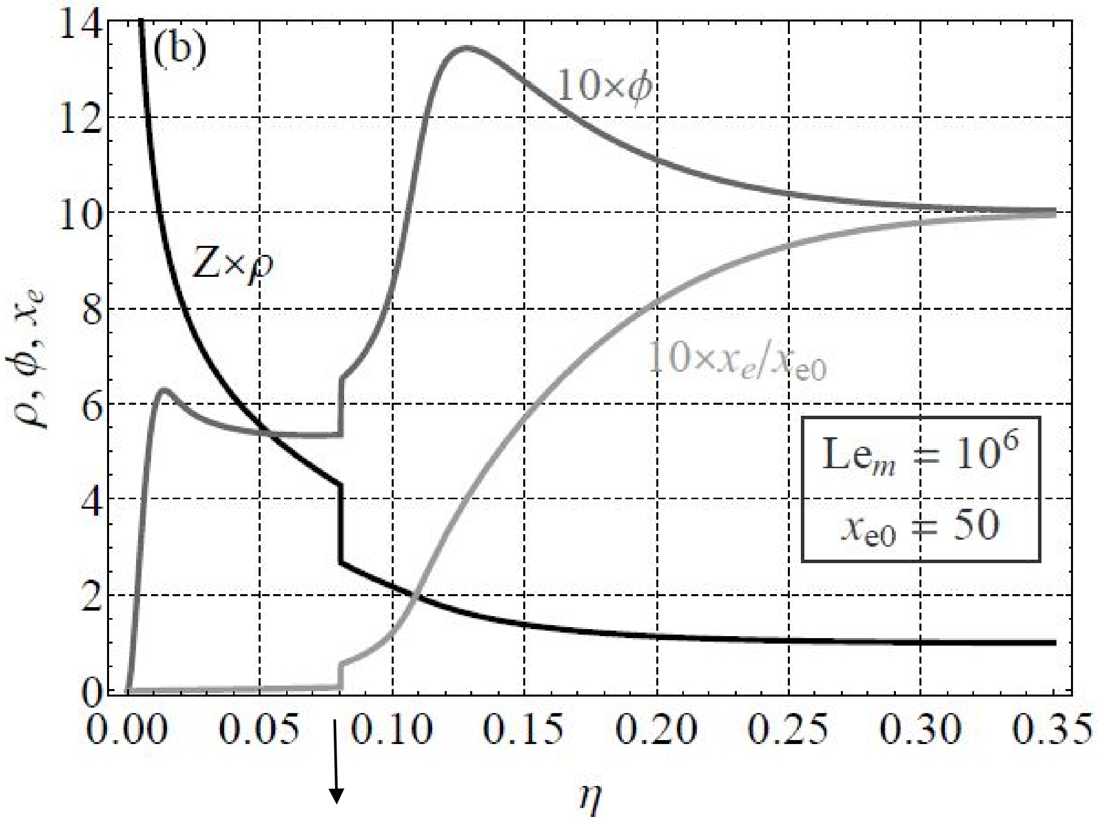
- ✓ The ablated border is a contact discontinuity:
  - Density is discontinuous.
  - Thermal conductivity is discontinuous.
  - Nernst velocity is discontinuous.

Profiles for Beryllium liner

Unmagnetized



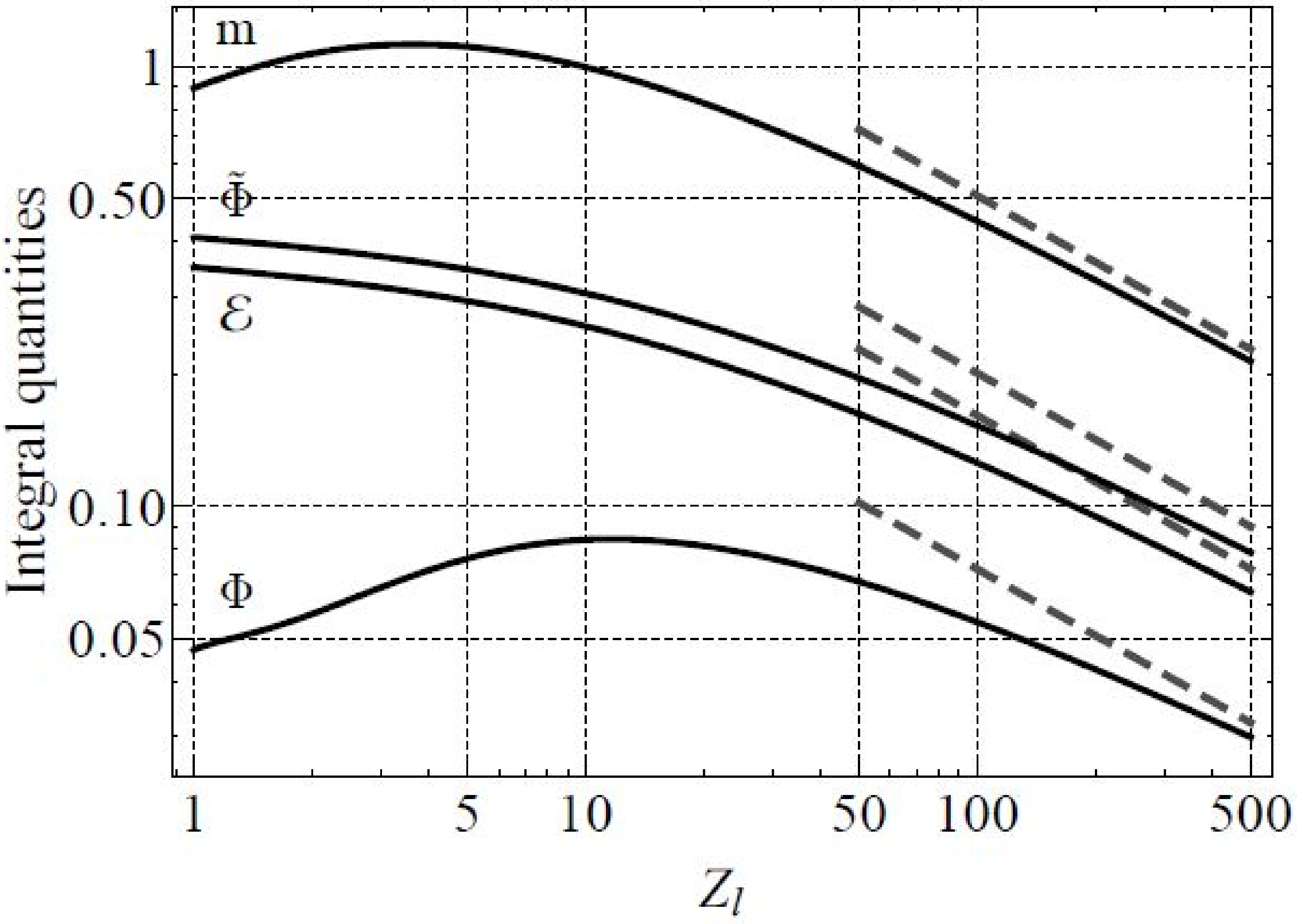
Magnetized



$\eta = \eta_b$

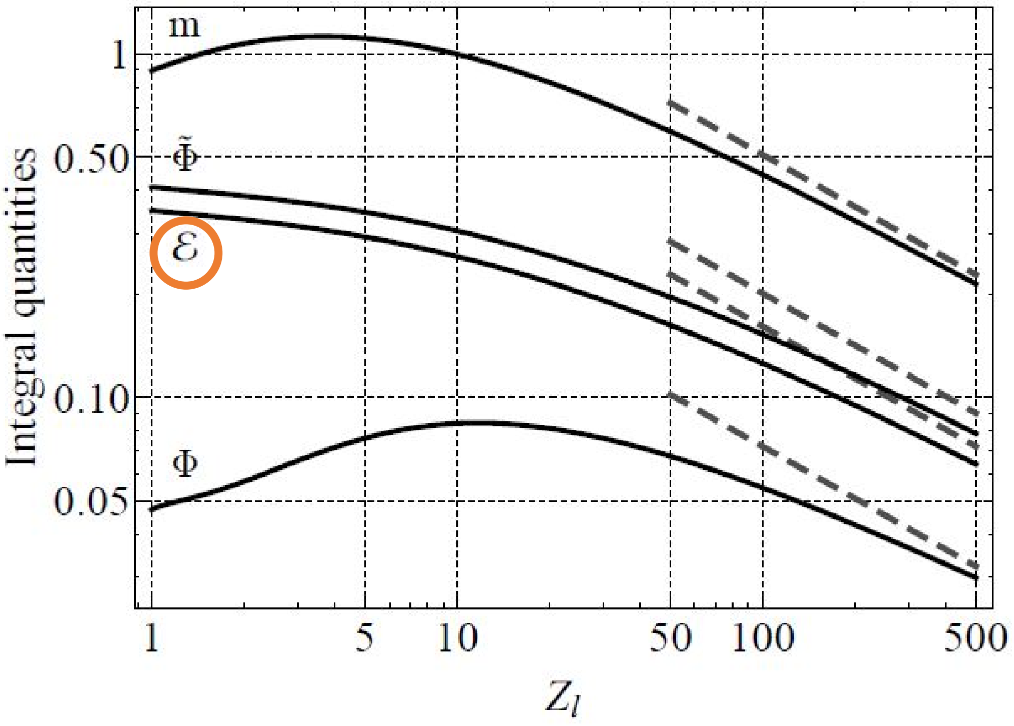
✓ Potential hydrodynamic instability.

Integral quantities

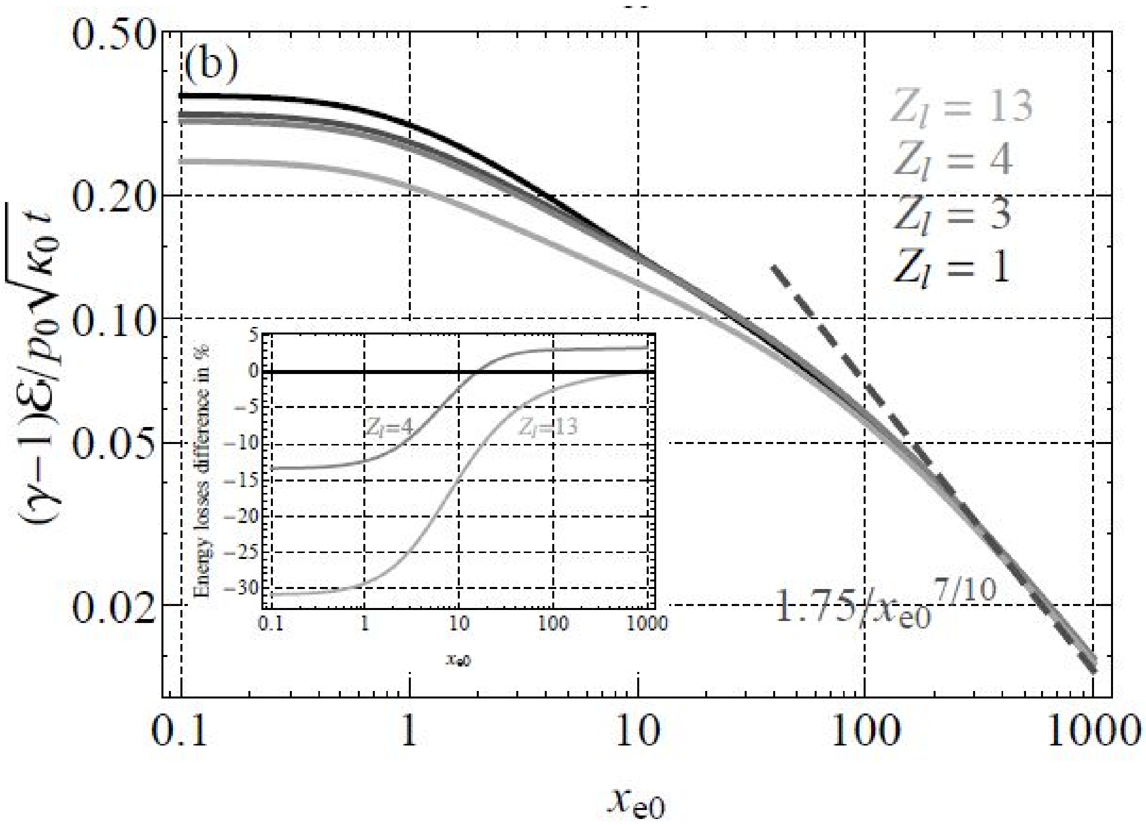


Thermal energy losses

Unmagnetized



$$\mathcal{E} = \int_0^\infty \frac{p_0}{\gamma - 1}\mathrm{d}x - \int_{x_b}^\infty \frac{p_0}{\gamma - 1}\mathrm{d}x$$



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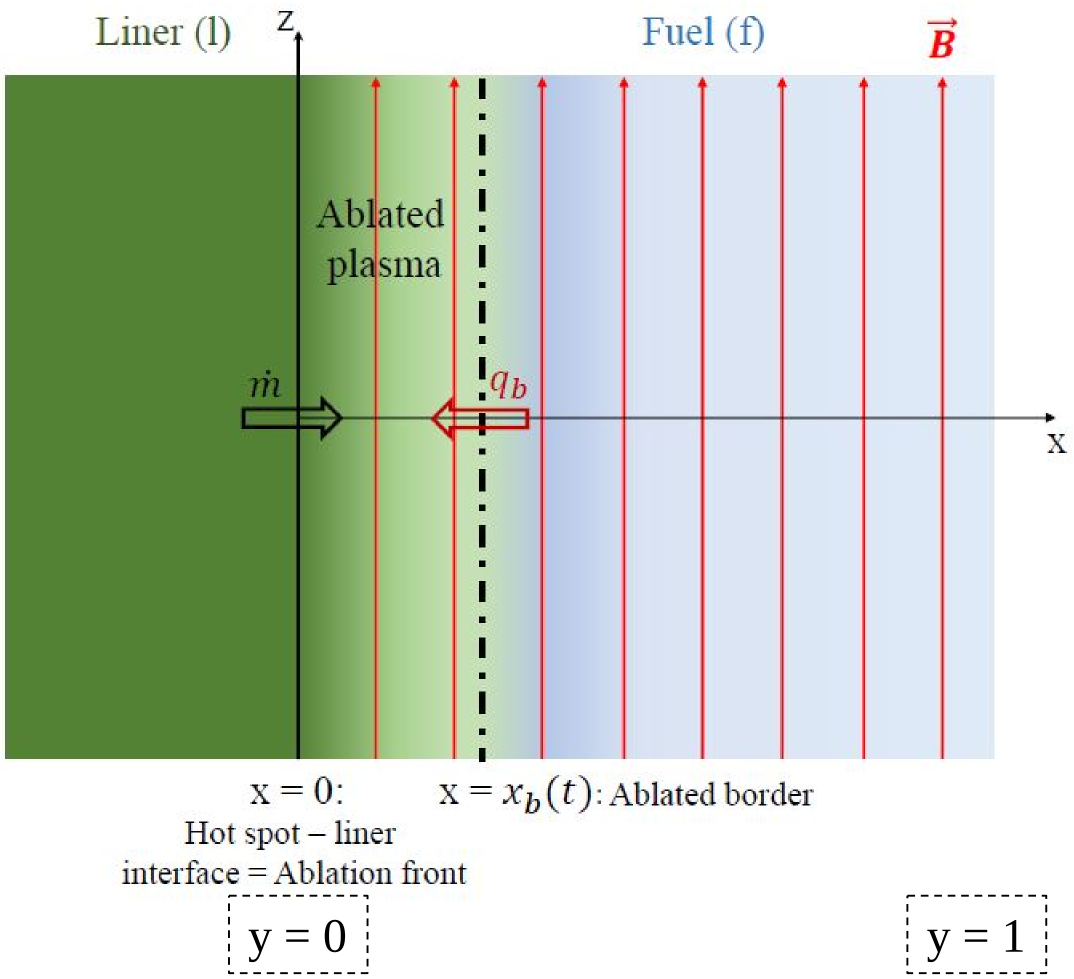
Ion mass flux



Fuel concentration Eq:

$$\rho \frac{\partial y}{\partial t} + \rho v \frac{\partial y}{\partial x} = -\frac{\partial}{\partial x}(\rho y v_1)$$

Diffusion velocity

$$v_1 = -\rho D \left(\frac{\mathrm{d}y}{\mathrm{d}x} + k_p \frac{\mathrm{d}\log p_i}{\mathrm{d}x} + \frac{e k_E}{T}\frac{\mathrm{d}U}{\mathrm{d}x} + k_T \frac{\mathrm{d}\log T}{\mathrm{d}x} \right)$$

Classical

Barodiffusion

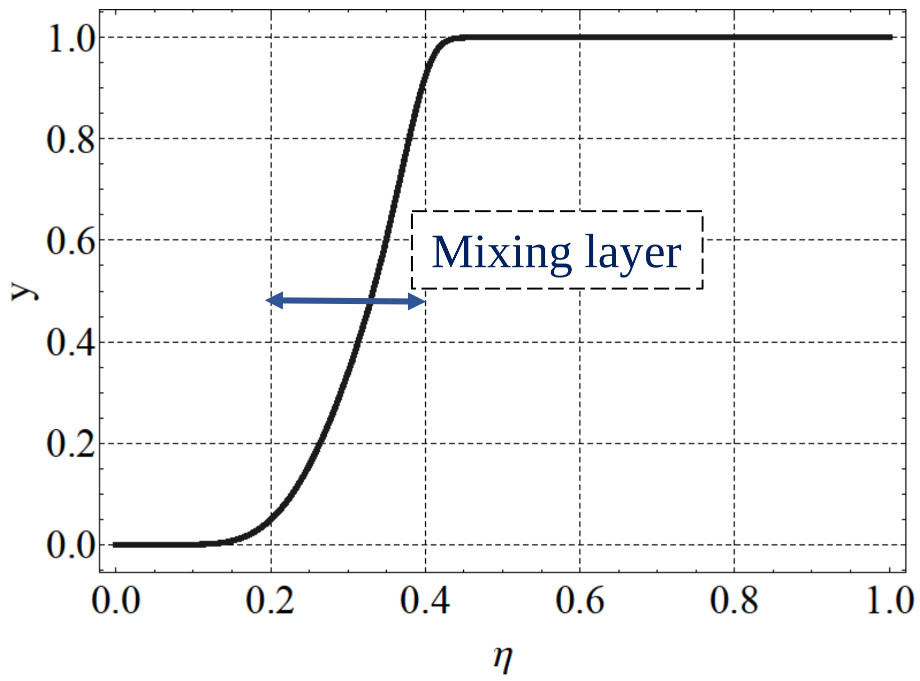
Electrodiffusion

Thermodiffusion

- ✓ Molvig, Simakov & Vold. PoP 21, 092709 (2014)
- ✓ Simakov & Molvig. PoP 23, 032115 (2016)
- ✓ Simakov & Molvig. PoP 23, 032116 (2016)

Mixing layer

- ✓ Fuel concentration profile for Beryllium liner



- ✓ The diffusion eq.is governed by the Lewis number.
- ✓ The Lewis number only depends on the liner material.

$$Le \sim \frac{Z_l}{Z_f}$$

- ✓ The width of the mixing layer scales as

$$\epsilon \sim \frac{1}{\sqrt{Le}}$$

	Lythium	Beryllium	Aluminum
Le	350	630	6600

Conclusions

Magnetization effects

1. Magnetization reduces the effect of the Nernst velocity by confining it to the outer region of the hot spot.
2. The thermal energy losses are reduced by magnetization as $x_e^{-7/10}$.

Magnetic pressure effects

1. Magnetic pressure enhances mass ablation.
2. When the magnetic pressure becomes important, the magnetic flux losses are no longer reduced with magnetization.

Liner material effects

1. A sharp discontinuity appears at the interface between fuel and ablated liner.
2. Increasing the atomic number of the liner reduces energy and magnetic flux losses.

THANK YOU FOR YOUR ATTENTION

ANY QUESTION?

Publications

- ✓ F. García-Rubio and J. Sanz, "*Mass ablation and magnetic flux losses through a magnetized plasma-liner wall interface*", Phys. Plasmas 24 (2017).

- ✓ F. García-Rubio and J. Sanz, "*Magnetic pressure effects in a plasma-liner interface*", Phys. Plasmas 25, 042114 (2018).

- ✓ F. García-Rubio and J. Sanz, "*Ion diffusion and liner material effect in a MagLIF fusion-like plasma*", to be submitted on Phys. Plasmas.