The Nernst Effect, the Thermal Dynamo and Flux Limitation

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v}_{\text{eff}} \times \vec{B}) + \vec{v}_\eta \times (\nabla \times \vec{B}) + \frac{\eta_\perp}{\mu_0}\nabla^2 \vec{B} + \frac{\nabla P_e \times \nabla n_e}{n_e^2 e}$$

$$\vec{v}_{\text{eff}} = \vec{v} + \vec{v}_j + \vec{v}_q$$

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$$\vec{v}_q = -\frac{\beta_\wedge \nabla k T_e}{eB} - \frac{\beta_\parallel - \beta_\perp}{eB}\vec{b} \times \nabla k T_e$$

$$\vec{v}_\eta = -\frac{\nabla \eta_\perp}{\mu_0}$$

J. R. Davies
University of Rochester
Laboratory for Laser Energetics

Meeting on Magnetic Fields in Laser Plasmas
Rochester, NY
23–24 April 2018

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Summary

To simulate magnetized spherical compressions resistive MHD routines with the Nernst term have been added to Draco*

- **Resistive and electrothermal terms lead to velocity-like terms proportional to the electron temperature gradient that can compress magnetic field and produce a thermal dynamo**
- **Thermal flux limiting leads to a divergent solution for the magnetic field as it gives a divergent solution for the electron temperature gradient**
- **The Nernst velocity had to be flux limited to reproduce the measured increase in neutron yield due to magnetization**
- **Thermal and Nernst flux limitation should include all three components of the heat flux ($q_\perp$, $q_\parallel$ and $q_\wedge$)**

*J. R. Davies *et al* Phys. Plasmas **22** 112703 (2015).

Collaborators

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[1]Now at NCKU, Taiwan
[2]Now at University of Michigan

Outline

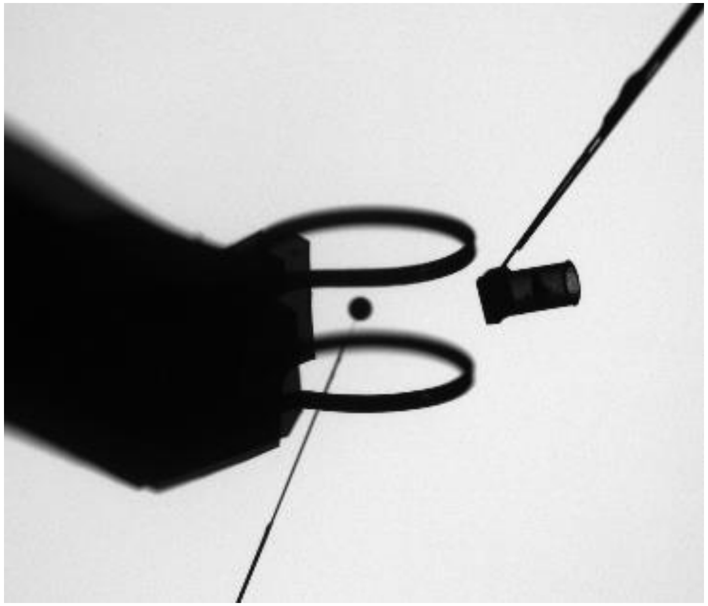
1. Motivation
2. Formulation of the magnetic field evolution using an effective velocity
3. Issues with implementation in 2D Draco simulations of magnetized, spherical compressions on OMEGA; thermal and Nernst flux limitation

Outline

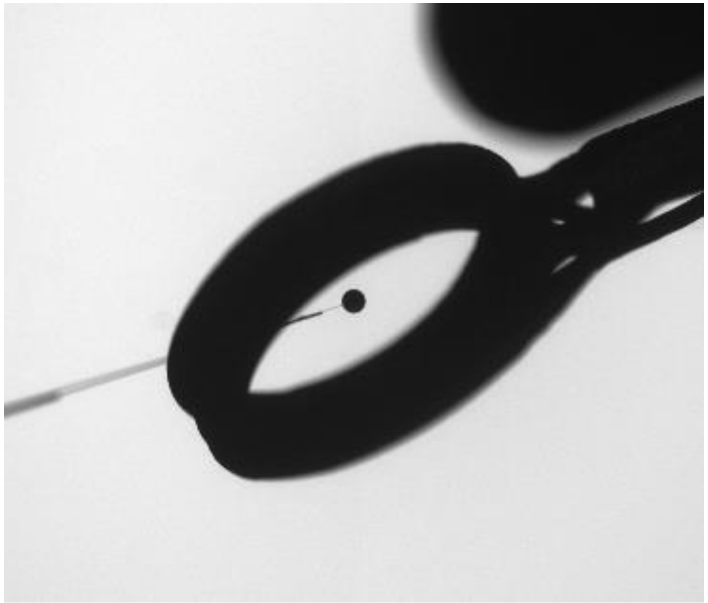
1. **Motivation**
2. **Formulation of the magnetic field using an effective velocity**
3. **Issues with implementation in 2D Draco simulations of magnetized, spherical compressions on OMEGA; thermal and Nernst flux limitation**

Axial magnetic fields up to 15 T have been applied to warm spherical implosions on OMEGA on 7 shot days

MIFEDS gen 1 Cu foil giving 6 T
Rings 1, 2, and 4
EP proton probing



MIFEDS gen 2 multi-turn wire on a 3D printed mount giving 15 T
Rings 1, 2, and 3 (PDD*)



$B_{z0} \geq 6$ T increased neutron yield by ~ 30% on average**

*Polar direct drive
P.-Y. Chang *et al* Phys. Rev. Lett. **107 035006 (2011);
M. Hohenberger *et al* Phys. Plasmas **19** 056306 (2012)

Outline

1. Motivation
2. **Formulation of the magnetic field evolution using an effective velocity**
3. Issues with implementation in 2D Draco simulations of magnetized, spherical compressions on OMEGA; thermal and Nernst flux limitation

Braginskii's formulation of Ohm's law with all 6 terms from the resistivity and thermoelectric tensors but not the 5 terms from the electron stress tensor

- **Includes electron-ion and electron-electron collisions with a single, constant lnΛ, a straight, uniform magnetic field, and first order perturbation from a Maxwellian distribution**

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$$

$$\vec{E} = -\vec{v} \times \vec{B} + \frac{\vec{j}}{n_e e} \times \vec{B} - \frac{\nabla \cdot \mathrm{P}_e}{n_e e} + \eta_{\|}\vec{b}(\vec{b} \cdot \vec{j}) + \eta_{\perp}\vec{b} \times (\vec{b} \times \vec{j}) - \eta_{\wedge}(\vec{b} \times \vec{j})$$
$$- \frac{k\beta_{\|}}{e}\vec{b}\left(\vec{b} \cdot \nabla T_e\right) - \frac{k\beta_{\perp}}{e}\vec{b} \times \left(\vec{b} \times \nabla T_e\right) - \frac{k\beta_{\wedge}}{e}\left(\vec{b} \times \nabla T_e\right)$$

Removing terms with zero curl and grouping like terms gives just velocity, diffusion and source terms

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v}_{\text{eff}} \times \vec{B}) + \vec{v}_\eta \times (\nabla \times \vec{B}) + \frac{\eta_\perp}{\mu_0}\nabla^2 \vec{B} + \frac{\nabla P_e \times \nabla n_e}{n_e^2 e}$$

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$$\vec{v}_\eta = -\frac{\nabla \eta_\perp}{\mu_0}$$

Effective fluid velocity seen by the magnetic field

$$\frac{\partial \vec{B}}{\partial t} = \boxed{\nabla \times (\vec{v}_{\text{eff}} \times \vec{B})} + \vec{v}_\eta \times (\nabla \times \vec{B}) + \frac{\eta_\perp}{\mu_0}\nabla^2 \vec{B} + \frac{\nabla P_e \times \nabla n_e}{n_e^2 e}$$

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A resistive velocity that moves magnetic field to regions of lower resistivity (no $\nabla \cdot v_\eta$ term)

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v}_{\text{eff}} \times \vec{B}) + \boxed{\vec{v}_\eta \times (\nabla \times \vec{B})} + \frac{\eta_\perp}{\mu_0}\nabla^2 \vec{B} + \frac{\nabla P_e \times \nabla n_e}{n_e^2 e}$$

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Magnetic diffusion

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v}_{\text{eff}} \times \vec{B}) + \vec{v}_\eta \times (\nabla \times \vec{B}) + \boxed{\frac{\eta_\perp}{\mu_0}\nabla^2 \vec{B}} + \frac{\nabla P_e \times \nabla n_e}{n_e^2 e}$$

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Source term

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v}_{\text{eff}} \times \vec{B}) + \vec{v}_\eta \times (\nabla \times \vec{B}) + \frac{\eta_\perp}{\mu_0} \nabla^2 \vec{B} + \boxed{\frac{\nabla P_e \times \nabla n_e}{n_e^2 e}}$$

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The effective fluid velocity seen by the magnetic field has three distinct sources

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v}_{\text{eff}} \times \vec{B}) + \vec{v}_\eta \times (\nabla \times \vec{B}) + \frac{\eta_\perp}{\mu_0}\nabla^2 \vec{B} + \frac{\nabla P_e \times \nabla n_e}{n_e^2 e}$$

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The ion fluid velocity

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v}_{\text{eff}} \times \vec{B}) + \vec{v}_\eta \times (\nabla \times \vec{B}) + \frac{\eta_\perp}{\mu_0}\nabla^2 \vec{B} + \frac{\nabla P_e \times \nabla n_e}{n_e^2 e}$$

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The drift velocity or Hall term…

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v}_{\text{eff}} \times \vec{B}) + \vec{v}_\eta \times (\nabla \times \vec{B}) + \frac{\eta_\perp}{\mu_0}\nabla^2 \vec{B} + \frac{\nabla P_e \times \nabla n_e}{n_e^2 e}$$

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…increased by the cross field resistivity term

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v}_{\text{eff}} \times \vec{B}) + \vec{v}_\eta \times (\nabla \times \vec{B}) + \frac{\eta_\perp}{\mu_0}\nabla^2 \vec{B} + \frac{\nabla P_e \times \nabla n_e}{n_e^2 e}$$

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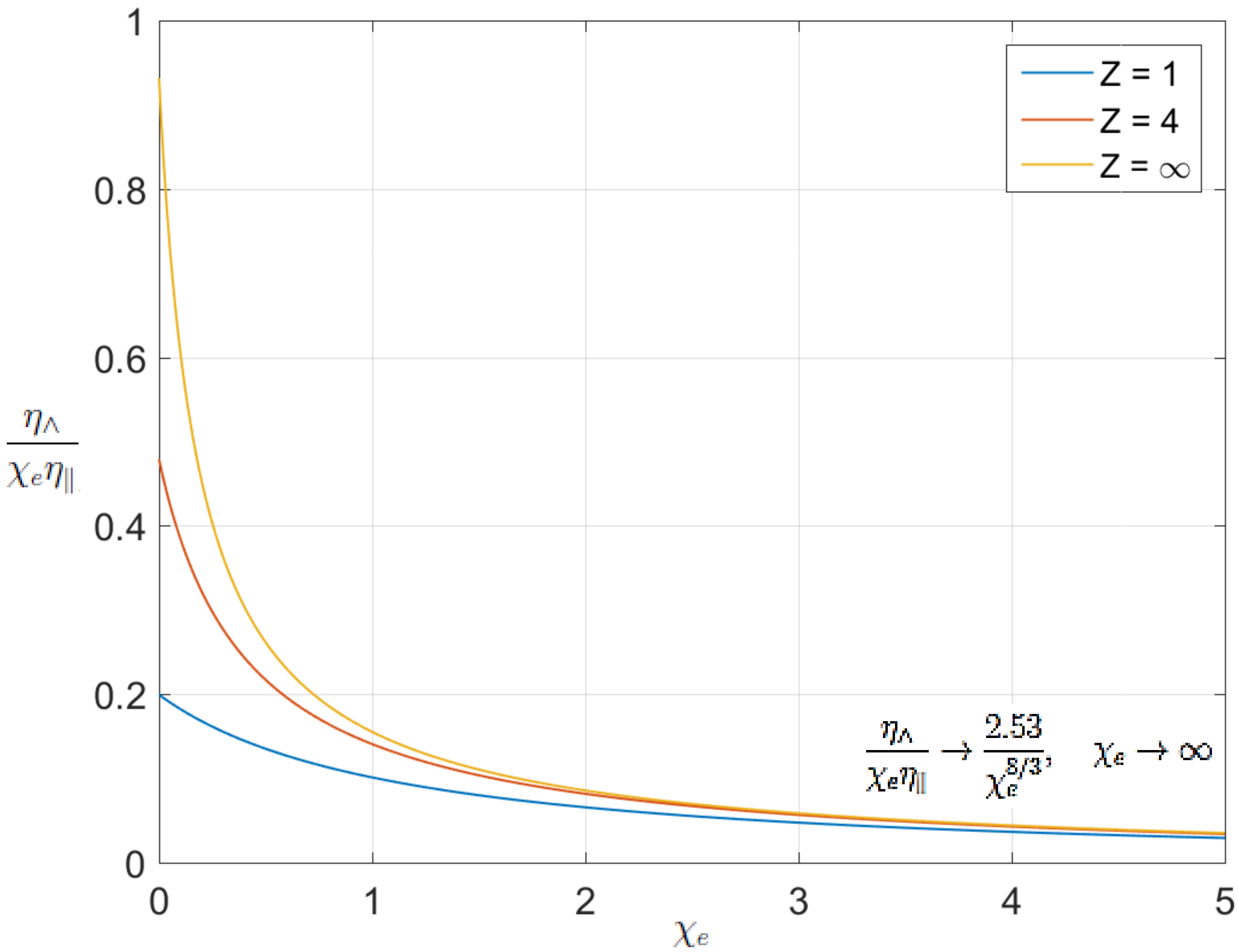
The cross field resistivity term is only significant for Z > 1 and weakly magnetized plasma



E. M. Epperlein and M. G. Haines Phys. Fluids **29** 1029 (1985)

A velocity from the electrothermal terms, which has two distinct components…

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v}_{\text{eff}} \times \vec{B}) + \vec{v}_\eta \times (\nabla \times \vec{B}) + \frac{\eta_\perp}{\mu_0}\nabla^2 \vec{B} + \frac{\nabla P_e \times \nabla n_e}{n_e^2 e}$$

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The Nernst velocity moves field lines in the direction of electron heat flow

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v}_{\text{eff}} \times \vec{B}) + \vec{v}_\eta \times (\nabla \times \vec{B}) + \frac{\eta_\perp}{\mu_0}\nabla^2 \vec{B} + \frac{\nabla P_e \times \nabla n_e}{n_e^2 e}$$

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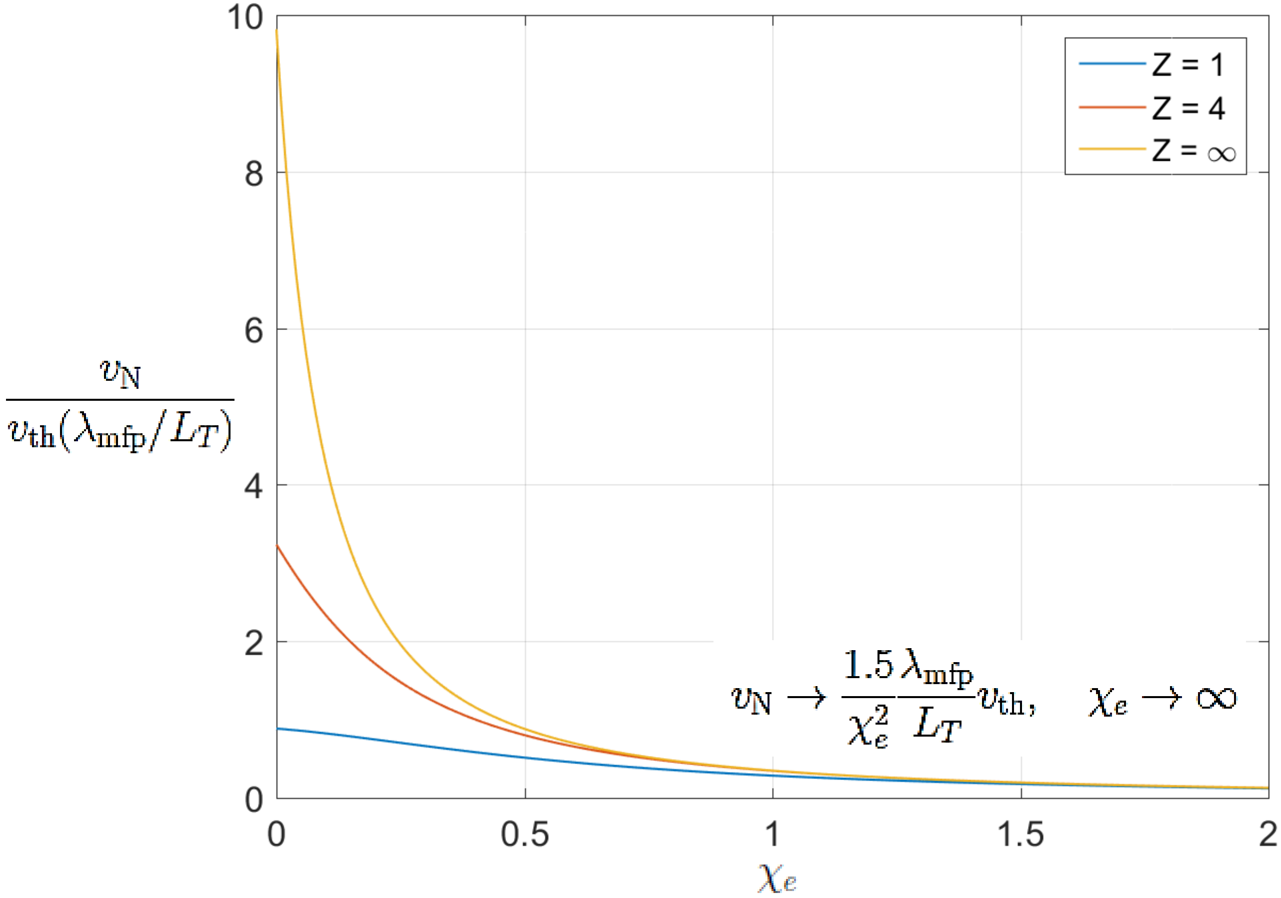
$$\vec{v}_\eta = -\frac{\nabla \eta_\perp}{\mu_0}$$

The Nernst velocity is much less than the electron thermal velocity and falls with magnetization

$$\boxed{v_{\rm N} = \frac{\beta_\wedge}{\chi_e}\frac{\lambda_{\rm mfp}}{L_T} v_{\rm th}}$$



An unnamed and usually unmentioned term

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v}_{\text{eff}} \times \vec{B}) + \vec{v}_\eta \times (\nabla \times \vec{B}) + \frac{\eta_\perp}{\mu_0}\nabla^2 \vec{B} + \frac{\nabla P_e \times \nabla n_e}{n_e^2 e}$$

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The cross field velocity $v_\wedge$ or thermal dynamo: directed around field lines so can wind them up like a dynamo

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v}_{\mathrm{eff}} \times \vec{B}) + \vec{v}_\eta \times (\nabla \times \vec{B}) + \frac{\eta_\perp}{\mu_0}\nabla^2 \vec{B} + \frac{\nabla P_e \times \nabla n_e}{n_e^2 e}$$

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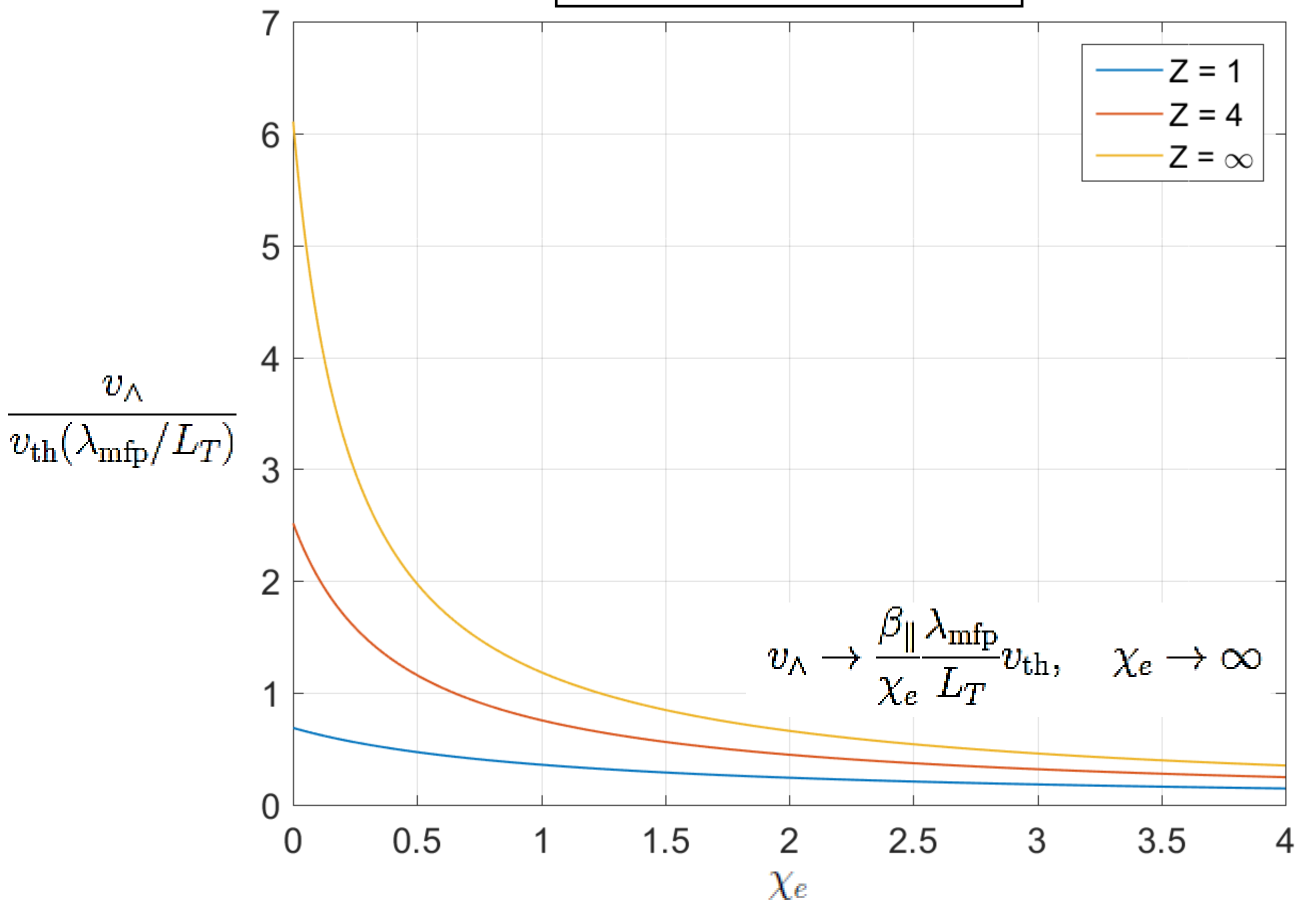
$$\vec{v}_\eta = -\frac{\nabla \eta_\perp}{\mu_0}$$

The cross field velocity shows similar behavior to the Nernst velocity but falls more slowly with magnetization

$$v_\wedge = \frac{\beta_\parallel - \beta_\perp}{\chi_e}\frac{\lambda_{\text{mfp}}}{L_T} v_{\text{th}}$$



Haines' formulation of Ohm's law* provides a simple physical picture for the effective velocity

- **Neglects electron-electron scattering and changes electron-ion scattering from $1/v^3$ to $1/v^2$ because it makes the tensors vanish**
  - **$\ln\Lambda$ depends on v so real scaling is somewhere between $1/v^3$ and $1/v^2$**

$$\vec{E} = -\vec{v}\times\vec{B} + \frac{\vec{j}}{n_e e}\times\vec{B} - \frac{\vec{q_e}\times\vec{B}}{2.5P_e} - \frac{\nabla P_e}{n_e e} + \eta\vec{j} - \nabla\frac{kT_e}{e}$$

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- **Magnetic field is frozen to the conduction electrons due to their lower collision frequency**

*M. G. Haines, Plasma Phys. Control. Fusion **28** 1705 (1986)

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$$\vec{v}_j = -\frac{\vec{j}}{n_e e}$$

Nernst velocity ($\beta_\wedge$) = perpendicular heat flux

$$\vec{v}_q = \frac{\vec{q_e}}{2.5P_e} = \boxed{-\frac{\kappa_\perp \nabla T_e}{2.5P_e}} - \frac{\kappa_\wedge \vec{b} \times \nabla T_e}{2.5P_e} - \frac{\vec{j} kT_e}{2.5P_e e}$$

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$$\vec{v}_j = -\frac{\vec{j}}{n_e e}$$

Cross field velocity ($\beta_{//}$ - $\beta_{\perp}$) = cross field heat flux

$$\vec{v}_q = \frac{\vec{q}_e}{2.5P_e} = -\frac{\kappa_\perp\nabla T_e}{2.5P_e}\boxed{-\frac{\kappa_\wedge\vec{b}\times\nabla T_e}{2.5P_e}}-\frac{\vec{j}kT_e}{2.5P_e e}$$

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UR LLE

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$$\vec{v}_j = -\frac{\vec{j}}{n_e e}$$

Increase in drift velocity ($\eta_\wedge$) = thermoelectric term

$$\vec{v}_q = \frac{\vec{q_e}}{2.5P_e} = -\frac{\kappa_\perp \nabla T_e}{2.5P_e} - \frac{\kappa_\wedge \vec{b} \times \nabla T_e}{2.5P_e} \boxed{- \frac{\vec{j} k T_e}{2.5 P_e e}}$$

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- **Magnetic field is frozen to the conduction electrons due to their lower collision frequency**
- **Use this form to calculate the Nernst and cross field velocities, removing the need to calculate β, and to provide a basis for flux limitation of v_q to $f_N v_{th}/2.5$**

*M. G. Haines, Plasma Phys. Control. Fusion **28** 1705 (1986)

Even with azimuthal symmetry velocity and magnetic field remain three-dimensional

- **Spherical coordinates with $\partial/\partial\phi \equiv 0$ as used in Draco showing only the source terms for v_ϕ and B_ϕ:**

$$v_{\phi\text{eff}} = v_\phi - \frac{1+\eta_\wedge/\chi_e\eta_\parallel}{\mu_0 n_e e r}\left(\frac{\partial}{\partial r}(rB_\theta) - \frac{\partial B_r}{\partial \theta}\right) + \frac{\kappa_\wedge}{2.5P_e}\frac{\partial T_e}{\partial r}b_\theta - \frac{\kappa_\wedge}{2.5P_e r}\frac{\partial T_e}{\partial \theta}b_r$$

$$r\frac{\partial B_\phi}{\partial t} = \frac{\partial}{\partial r}(rv_\phi B_r) + \frac{\partial}{\partial \theta}(v_\phi B_\theta) + \frac{1}{n_e^2 e}\left(\frac{\partial P_e}{\partial r}\frac{\partial n_e}{\partial \theta} - \frac{\partial n_e}{\partial r}\frac{\partial P_e}{\partial \theta}\right)$$

$$\mu_0 r\rho\frac{\partial v_\phi}{\partial t} = B_r\frac{\partial}{\partial r}(rB_\phi) + \frac{B_\theta}{\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta B_\phi)$$

  - **Added both generation terms for B_ϕ to Draco but stopped short of adding plasma rotation**
  - **The effective velocity is used in the advection routines, requiring no artificial resistivity**
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The cross field velocity will always be present in spherical and cylindrical compressions…

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…and will wind up an azimuthal magnetic field at the ends

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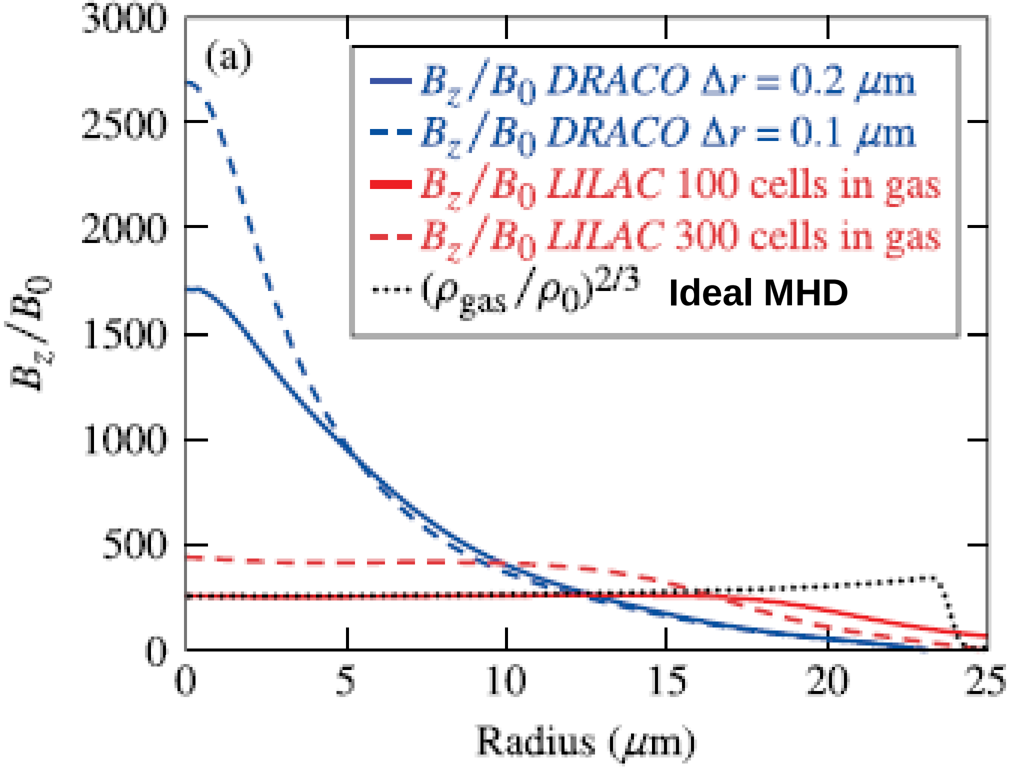
Outline

1. Motivation
2. Formulation of the magnetic field evolution using an effective velocity
3. **Issues with implementation in 2D Draco simulations of magnetized, spherical compressions on OMEGA; thermal and Nernst flux limitation**

Before adding the Nernst term, just resistivity was found to lead to a divergent solution for the field

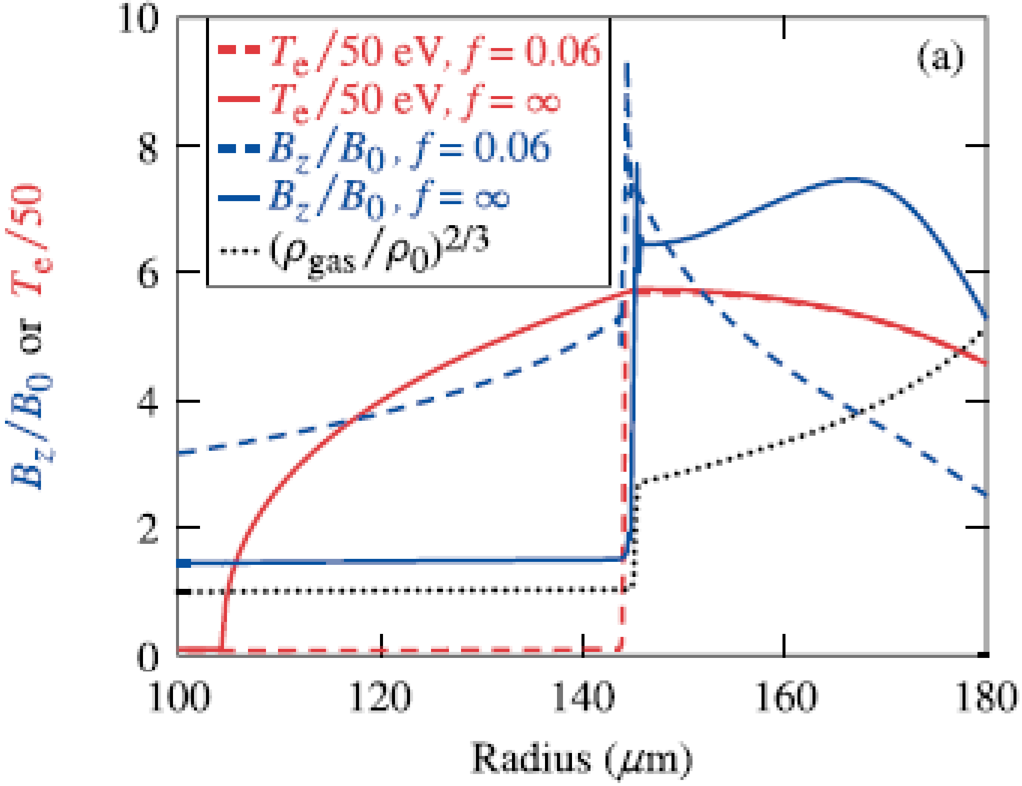
1D Runs



Draco is Eulerian
Lilac is Lagrangian

The cause was flux limiting forming a step in electron temperature at the shock front and hence a diverging v_η

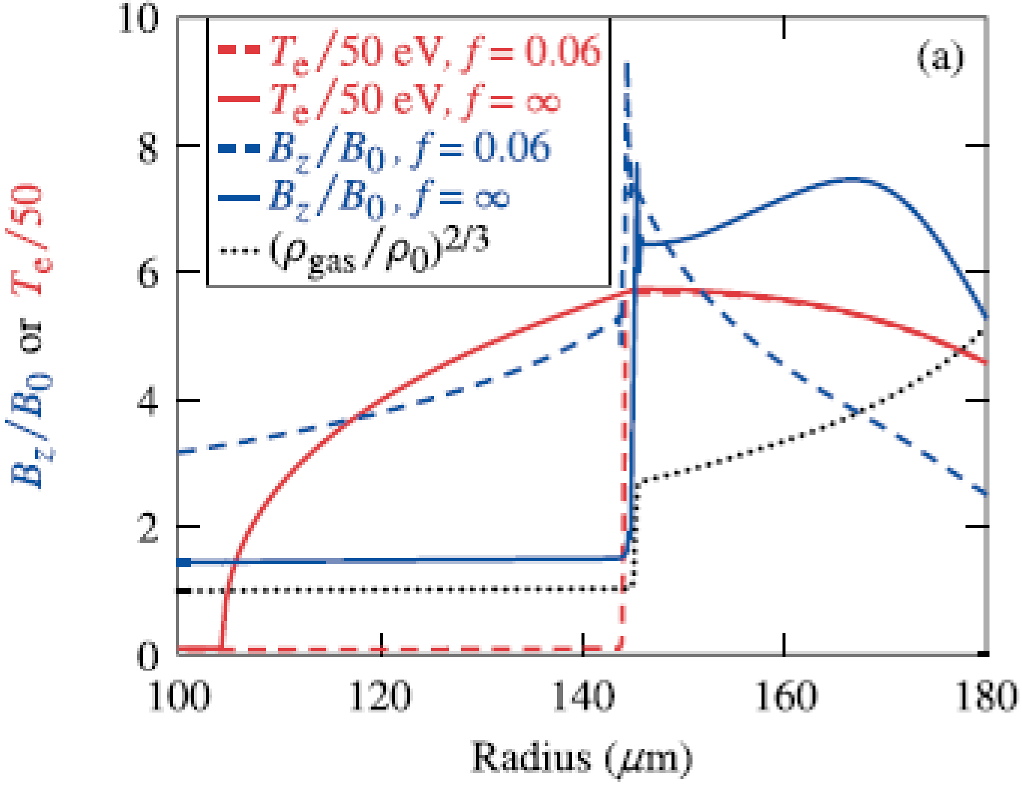
1D Draco Runs



The magnetic vector potential is used so v_η is not calculated explicitly but appears when calculating $\nabla \times A_\phi$

The cause was flux limiting forming a step in electron temperature at the shock front and hence a diverging v_η

1D Draco Runs

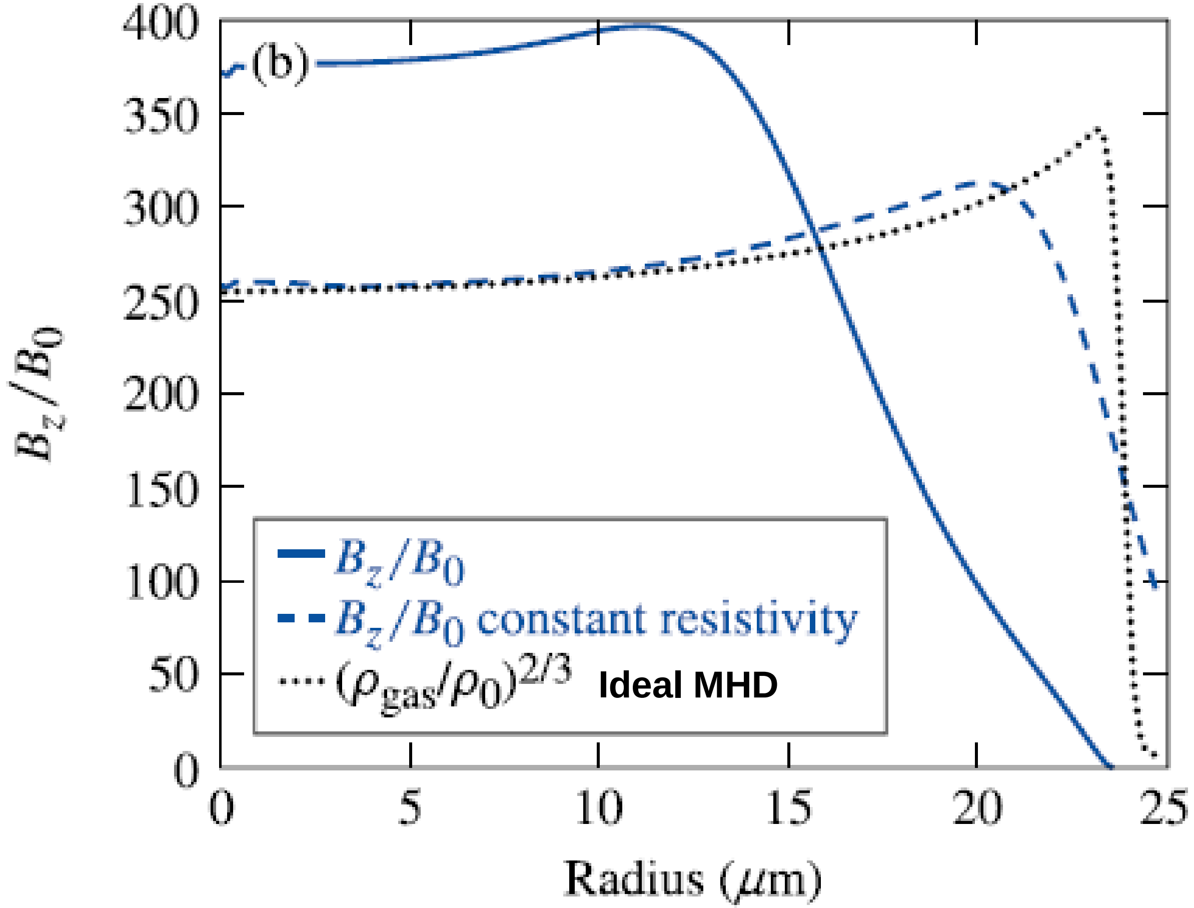


Solution: turn off thermal flux limiting in the gas

(Kept thermal flux limiting in the shell to maintain the same absorption)

The compressed magnetic field without thermal flux limiting in the gas and with adequate grid resolution is *increased* by resistivity

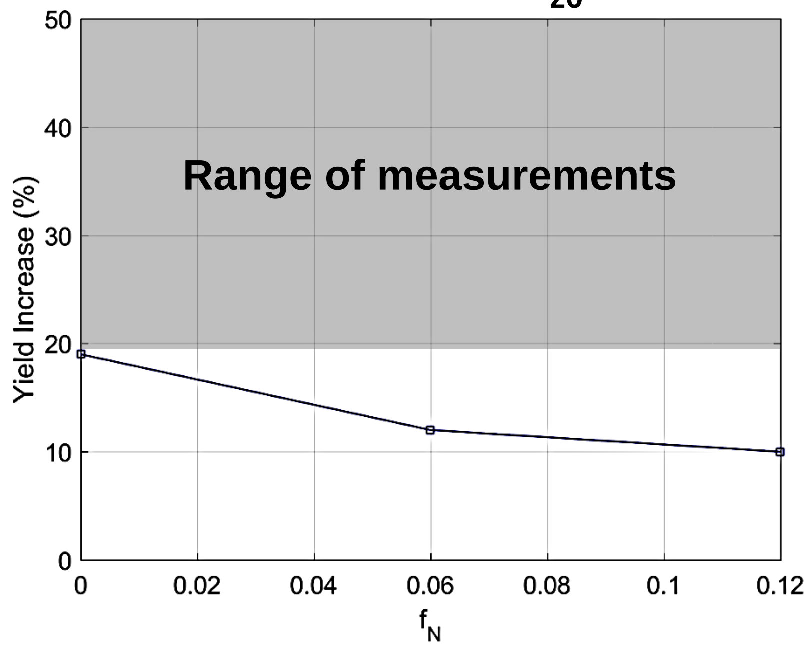
1D Draco Runs



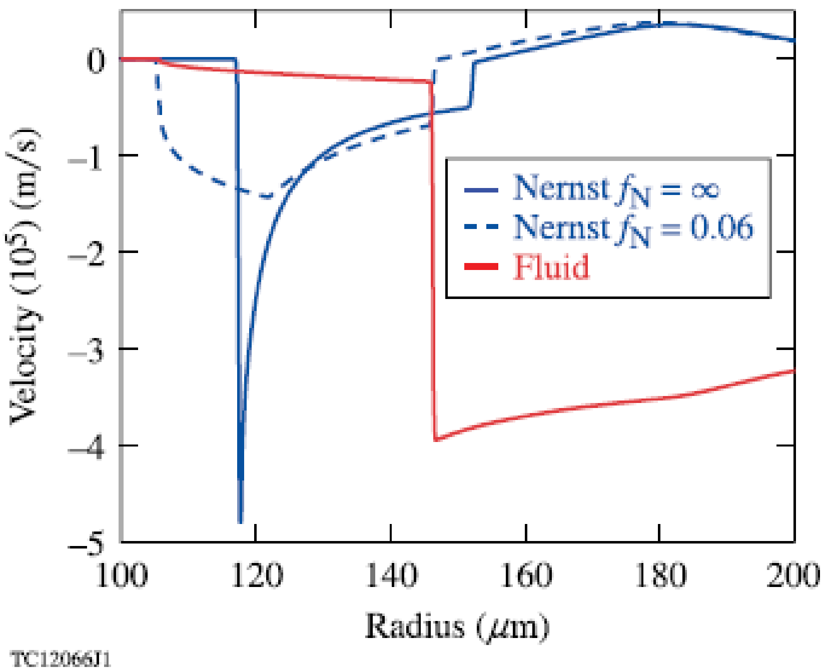
TC12062J1

Adding the Nernst term caused the increase in yield to fall well below measured values because field compression became detached from fuel compression

2D Draco Runs B_{z0} = 6 T



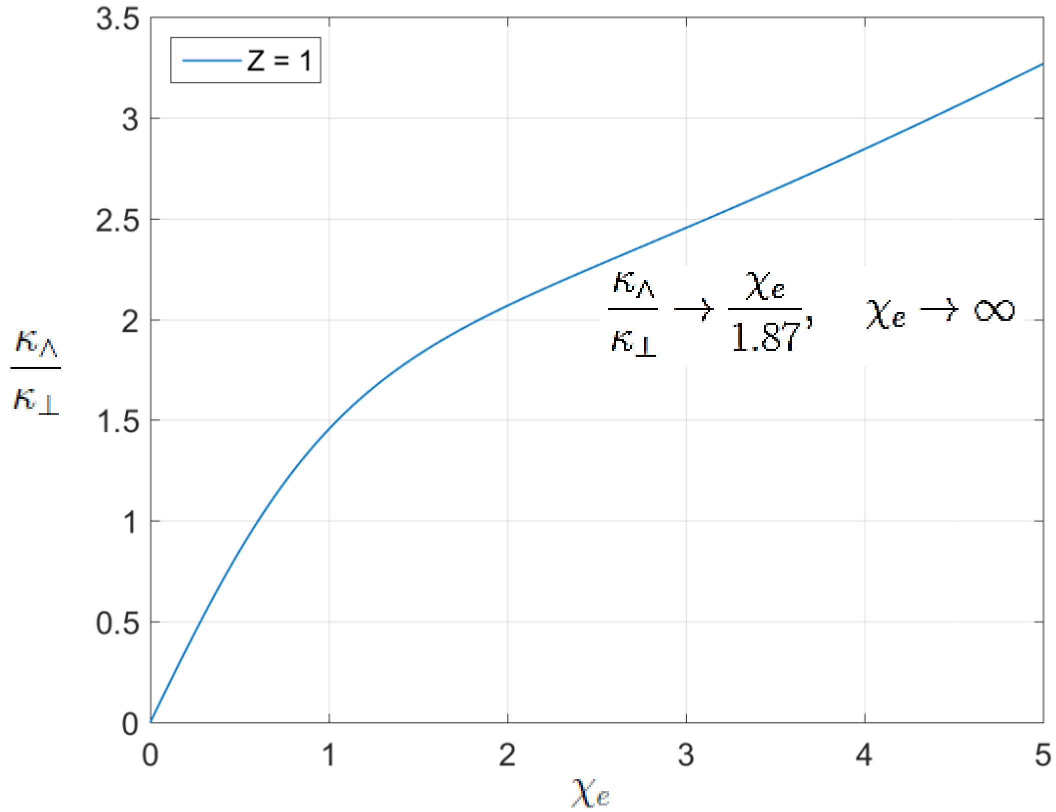
1D Draco Runs



Solution: flux limit the Nernst term with a separate flux limiter f_N < 0.12 maintaining the Nernst velocity below the shock velocity

Flux limiting should use the absolute value of heat flux requiring the calculation of the cross field heat flux

- **Without the cross field heat flux the flux limited Nernst velocity does not fall with increasing magnetic field, as the non-flux limited value does**



We now have 4 free parameters: the thermal flux limiter and the Nernst flux limiter in the gas and in the shell

- **The thermal flux limiter in the gas had to be removed to prevent the electron temperature gradient from diverging, which causes the compressed magnetic field to diverge**
- **The thermal flux limiter in the shell was left at 0.06 to lower the laser absorption closer to measured values**
- **The Nernst flux limiter in the gas had to be set to < 0.12 to agree with measured increases in neutron yields**
- **The Nernst flux limiter in the shell does not really matter because the magnetic field in the corona is negligible**
  - **Used 0.06 so that thermal flux limiting does not enhance the Nernst term**
- **A kinetic treatment of electron thermal transport and magnetic field evolution is really required**

To simulate magnetized spherical compressions resistive MHD routines with the Nernst term have been added to Draco*

- **Resistive and electrothermal terms lead to velocity-like terms proportional to the electron temperature gradient that can compress magnetic field and produce a thermal dynamo**
- **Thermal flux limiting leads to a divergent solution for the magnetic field as it gives a divergent solution for the electron temperature gradient**
- **The Nernst velocity had to be flux limited to reproduce the measured increase in neutron yield due to magnetization**
- **Thermal and Nernst flux limitation should include all three components of the heat flux ($q_\perp$, $q_{\parallel}$ and $q_\wedge$)**

*J. R. Davies *et al* Phys. Plasmas **22** 112703 (2015).