Numerical Simulation of Sound-Wave Generation in Two-Ion Plasma

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The filamentation and SBS of laser beams have been observed in numerous ICF experiments. We are interested in how sound-wave nonlinearities modify the evolution of these instabilities. A standard model for nonlinear sound waves consists of the ion-fluid and Poisson equations. In this model the electron fluid is assumed to respond adiabatically to the electric field produced by the motion of ion fluid. We have written the code to solve the system of ion-fluid equations for two ion species and Poisson equation. To simulate the generation of sound waves by backward and forward SBS a linear pondermotive potential term was added to the Poisson equation. We will describe the results of preliminary simulations of sound-wave generation in a two-ion plasma.
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Numerical Simulation and Analysis of Sound-Waves in a Two-Ion Plasma

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Abstract

The filamentation and SBS of laser beams have been observed in numerous ICF experiments. We are interested in how sound-wave nonlinearities modify the evolution of these instabilities. A standard model for nonlinear sound waves consists of the ion-fluid and Poisson equations. In this model the electron fluid is assumed to respond adiabatically to the electric field produced by the motion of ion fluid. We have written the code to solve the system of ion-fluid equations for two ion species and Poisson equations. To simulate the generation of sound waves by backward and forward SBS a linear ponderomotive potential term was added to the Poisson equation. We will describe the results of preliminary simulations of sound-wave generation in a two-ion plasma.

Introduction

- The filamentation and SBS of laser beams have been observed in numerous ICF experiments.
- We are interested in how these instabilities grow and saturate in two-ion plasmas.
- This study concerns linear and nonlinear sound waves in two-ion plasma.

Sound waves are governed by the ion-fluid and Poisson equations

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- **Ion-fluid equations include mass and momentum conservation equations for both species**

$$\partial_t n_s + \partial_x (n_s v_s) = 0,$$

$$\partial_t v_s + v_s \partial_x v_s + \alpha_s \partial_x \phi / \beta_s = 0,$$

where $s = l$ **denotes light ions and** $s = h$ **denotes heavy ions**

$$\alpha_l = 1; \beta_l = 1; \alpha_h = Z_h / Z_l; \beta_h = m_h / m_l$$

- **Assuming a Boltzman electron response we obtain Poisson equation**

$$\partial^2_{xx} \phi - \exp(\phi) + \delta \sum_{s=l,h} \alpha_s \gamma_s n_s = 0,$$

where $\delta = n_{l0} Z_l / n_{e0}$; $\gamma_l = 1$ **and** $\gamma_h = \dfrac{n_{h0}}{n_{l0}}$

"e" denotes electrons
"0" denotes values in equilibrium state.

We solved the IFP equations numerically using standard schemes

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- We solved the fluid equations using the leap-frog scheme

$$n_{s_j}^{i+2} = \left(n_{s_{j+1}}^{i} + n_{s_{j-1}}^{i}\right)\Big/2 - \Delta t\left(n_{s_{j+1}}^{i+1} v_{s_{j+1}}^{i+1} - n_{s_{j-1}}^{i+1} v_{s_{j-1}}^{i+1}\right)\Big/\Delta x,$$

$$v_{s_j}^{i+2} = \left(v_{s_{j+1}}^{i} + v_{s_{j-1}}^{i}\right)\Big/2 - \Delta t\left[\left(v_{s_{j+1}}^{i+1}\right)^2\Big/2 + \alpha_s \phi_{j+1}^{i+1}\Big/\beta_s - \left(v_{s_{j-1}}^{i+1}\right)^2\Big/2 - \alpha_s \phi_{j-1}^{i+1}\Big/\beta_s\right]\Big/\Delta x,$$

  where i (j) denotes the number of time (spatial) steps.

- We solved the Poisson equation using Newton iteration; with the new n_l and n_h known, we wrote the new ϕ as the current ϕ plus $\Delta\phi$ and iterated

$$\left(-\Delta\phi_{j-1} + 2\,\Delta\phi_j - \Delta\phi_{j+1}\right)\Big/\Delta x^2 + \exp\phi_j \cdot \Delta\phi_j$$

$$= \left(\phi_{j-1} - 2\,\phi_j + \phi_{j+1}\right)\Big/\Delta x^2 - \exp\phi_j + \delta \sum_{s=l,h} \alpha_s \gamma_s n_{sj}$$

  until the accuracy goal $|\Delta\phi| < \varepsilon$ was achieved.

Dispersion is modeled correctly

- Linear KDV equation associated with IFP equations:

$$\partial_t \phi + M_0\left(\partial_x \phi + \partial^3_{xxx}\ \phi/2\right) = 0,$$

where the linear Mach number

$$M_0 = \left(\delta \sum_{s=h,l} \alpha_s^2 \gamma_s \Big/ \beta_s\right)^{1/2} \le 1$$

- Initial conditions:

$$\phi(0,x) = a \cdot \exp\left(-x^2/2L^2\right)$$

$$v_s(0,x) = \alpha_s \cdot \phi(0,x)/M_0\,\beta_s$$

$$n_s(0,x) = 1 + v_s(0,x)/M_0$$

- Analytical solution of linear KDV equation:

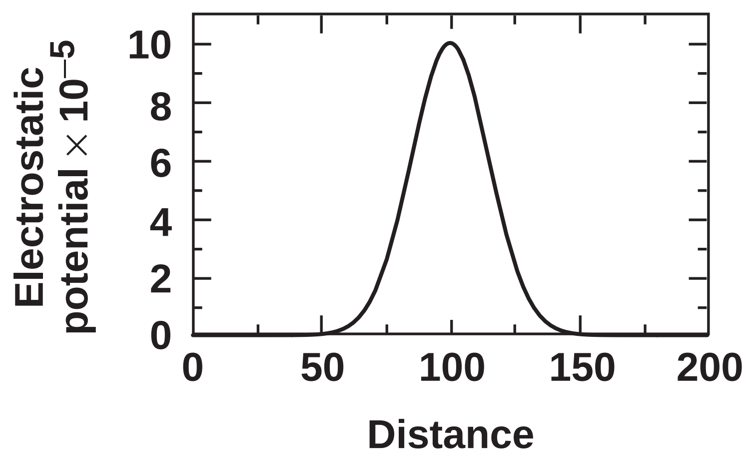
$$\phi(t,x) = a\sqrt{2\pi}\ \exp\left[\left(1+6\zeta^3\xi\right)/12\zeta^6\right]$$

$$\times \mathrm{Ai}\left[\left(1+4\zeta^3\xi\right)/4\zeta^4\right]/\zeta,$$

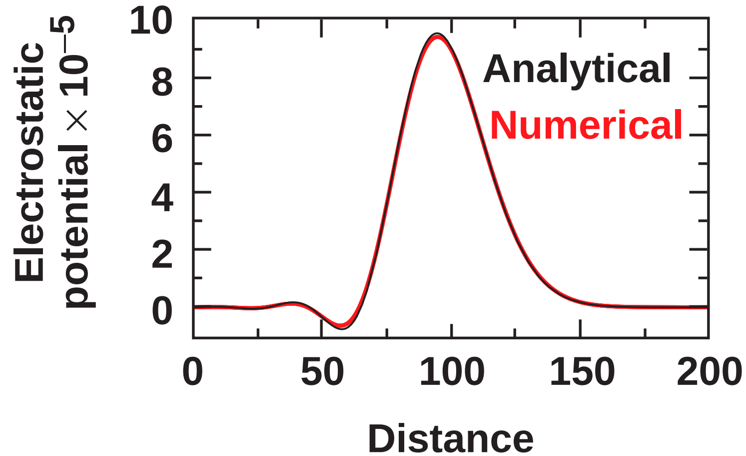
where

$$\zeta = [3M_0 t/2]^{1/3}/L, \quad \xi = (x - M_0 t)/L$$

Initial conditions



Solutions of linear KDV and IFP equations



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Nonlinearity and dispersion cancel each other for specific wave profiles

- **Solitary-wave solutions satisfy the equation**

$$\left(d_{\xi}\,\phi\right)^{2} = V(\phi),$$

where $\xi = x - Mt,\; M = M_{0}\,(1+\varepsilon).$

– **The potential function**

$$V(\phi) = 2\left[\exp\phi + M\delta \sum_{s=1,h} \gamma_{s}\,\beta_{s}\left(M^{2} - 2\alpha_{s}\,\phi/\beta_{s}\right)^{1/2} - 1 - M^{2}\delta \sum_{s=1,h} \gamma_{s}\,\beta_{s}\right].$$

- **Ordinary differential equation for stationary wave profile was solved**
  – **numerically**
  – **analytically by expending ϕ in powers of ε**

$$\phi(\xi) = a\,\varepsilon\,\operatorname{sech}^{2}(k\xi), \qquad a = \left[\frac{\delta}{2M_{0}^{4}}\left(1 + \frac{\alpha^{3}\gamma}{\beta^{2}}\right) - \frac{1}{6}\right]^{-1}, \qquad k = \left(\varepsilon/2\,M_{0}\right)^{1/2}.$$

Solitary-wave propagation is modeled correctly

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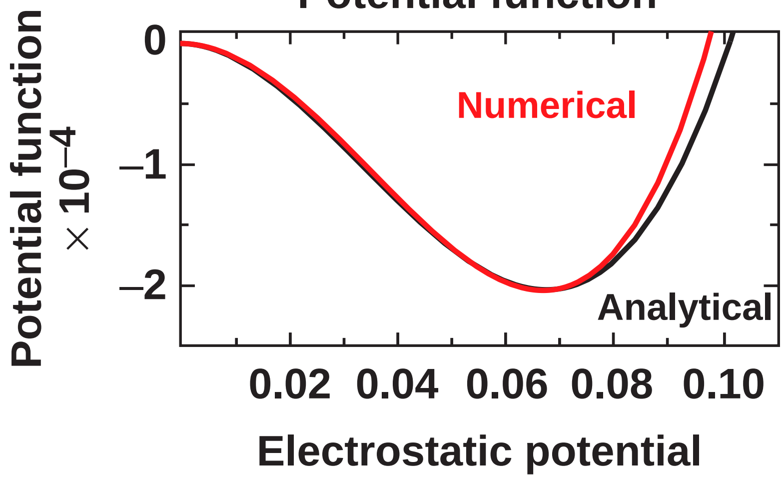
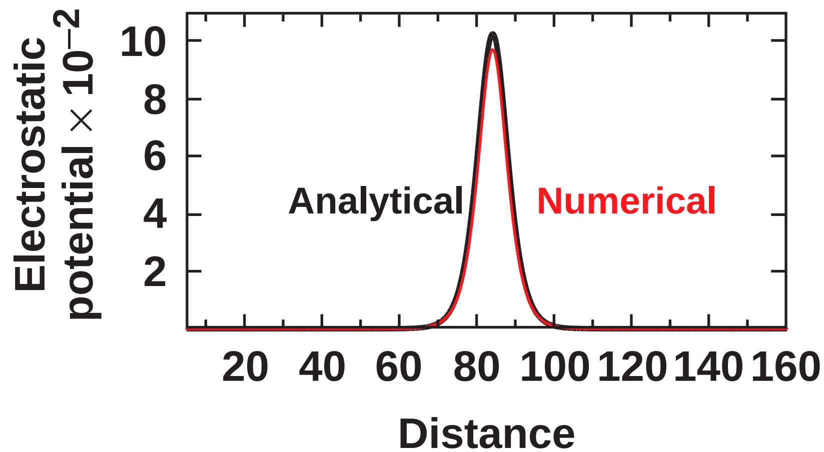
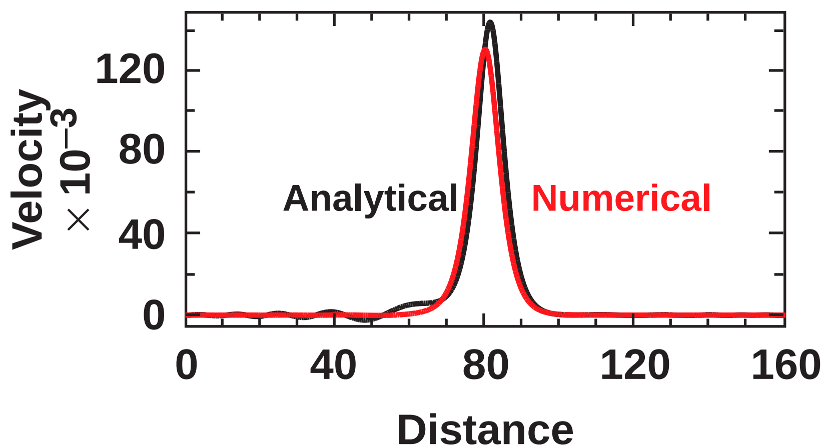
Potential function



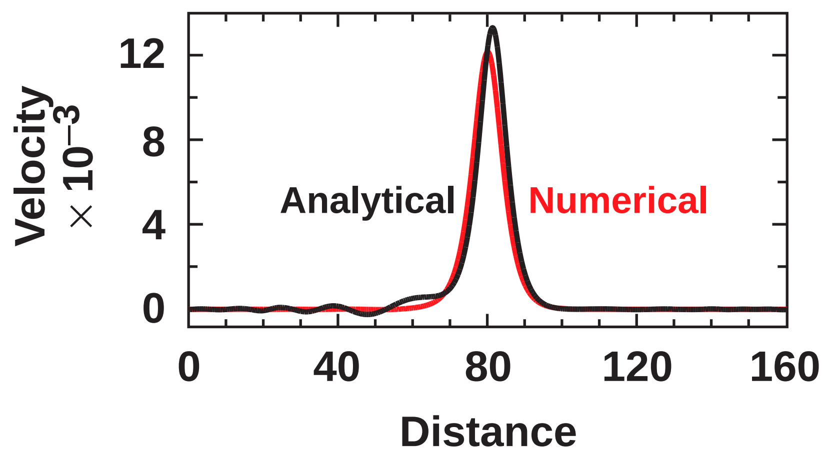
Initial conditions



Velocities of light species
with analytical and numerical IC



Velocities of heavy species
with analytical and numerical IC



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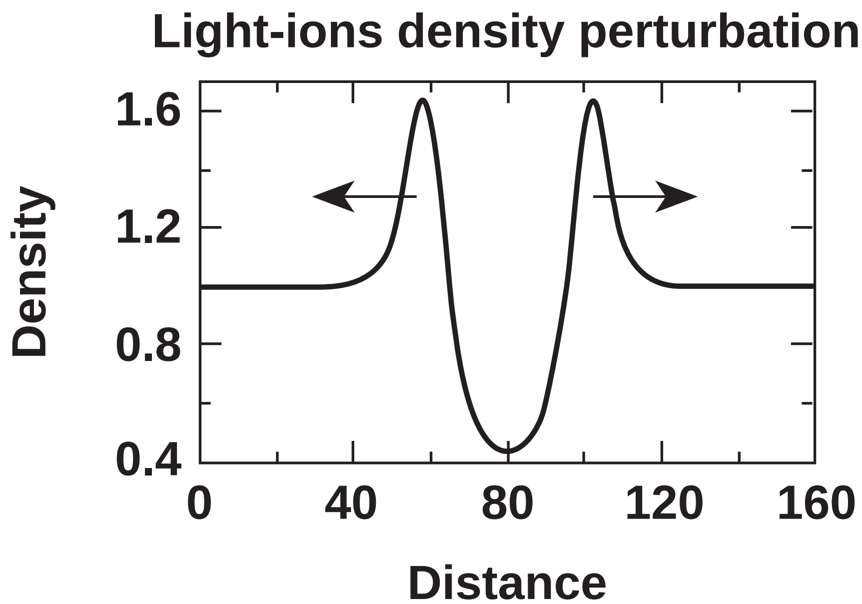
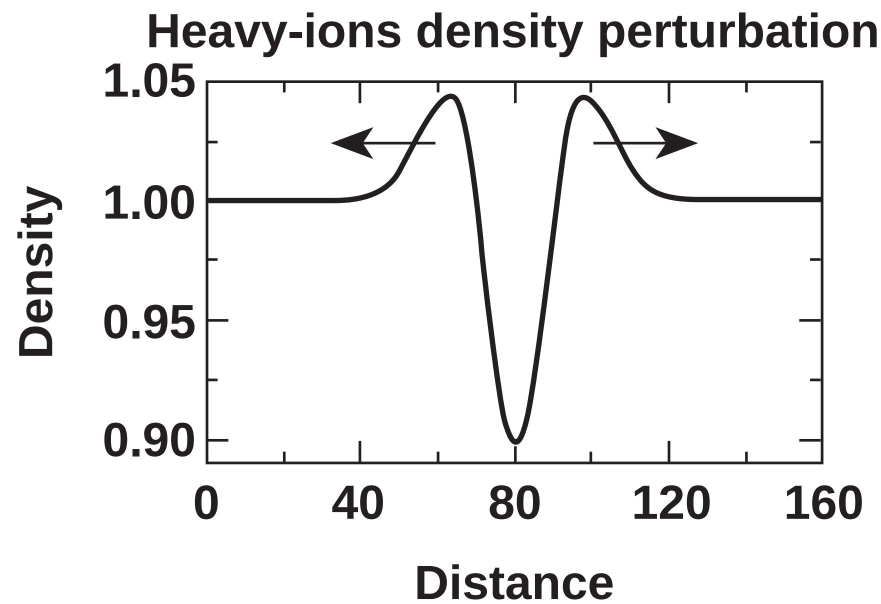
How do waves in one- and two-ion plasmas compare?

- Compare a two-ion plasma to a light-ion plasma of equal electron density.
- The heavy species are less mobile.
- Poisson: $\partial^2_{xx}\phi - e^{\phi} + 1 + \delta n_l + \delta n_h = 0$.
  For a given potential δn_l is . same.
- Since $n_l < 1$, $\delta n_l / n_l$ is larger.
- Since $\delta n_l / n_l \; . \; \left(k^2/\omega^2\right)\delta\phi$, the linear phase speed is lower.
- Nonlinear effects are more important.

In LPI's sound waves are generated by PM force

- The PM generation of a plasma channel by a Gaussian laser beam was simulated by Liu *et al.* The results of our simulations are in qualitative agreement with Liu.





- For a strong PM force the shapes of light- and heavy-ions density perturbations are different. This is the manifestation of nonlinear dynamic effects.

Summary

- We simulated the propagation of linear and nonlinear sound waves in two-ion plasmas.
- For the test problems considered, our code results agree with our theoretical predictions.
- In cold-ion plasmas the light ions dominate the physics of sound waves. Since the heavy ions are immobile, the light ions must maintain charge quasi-neutrality by themselves. If one compares a two-ion plasma to a light-ion plasma of equal electron density, this requirement increases the relative density and velocity perturbations of the light ions, reduces the sound speed, and increases the importance of light-ion fluid nonlinearities.
- In warm-ion plasmas two types of sound waves exist: fast waves ($\omega/k > v_{th_l}$) and slow waves ($v_{th_h} < \omega/k < v_{th_l}$).
- Our preliminary analysis shows that light ions dominate the physics of fast waves, but heavy ions play a significant role in the physics of slow waves.

There are two types of sound waves in “warm” plasma

- **Finite temperature effects modify the momentum conservation equation**

$$\partial_t v_s + v_s \partial_x v_s + \alpha_s \partial_x \phi / \beta_s + 3\theta_s n_s \partial_x n_s / \beta_s Z_l \theta_e = 0$$

- **Dispersion relation**

$$\delta \sum_s \alpha_s^2 \gamma_s k^2 \Big/ \left(\beta_s \omega^2 - \tau_s k^2 \right) = 1$$

- **Relations between amplitudes**

$$n_s = \alpha_s \Big/ \left(\beta_s V_F^2 - \tau_s \right); \quad v_s = V_F \cdot n_s$$

- **Two roots of equation for phase velocity correspond to**
  1. **Fast mode** $\omega/k > V_{th_l}$
  2. **Slow mode** $V_{th_h} < \omega/k < V_{th_l}$

- **For the fast mode $\delta n_h << \delta n_l$, but for the slow mode $\delta n_h \ast \delta n_l$.**

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