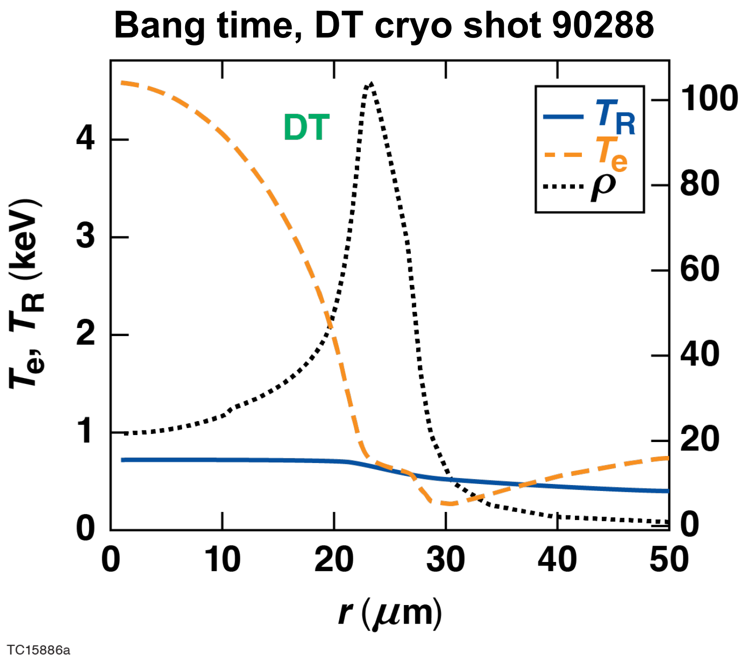
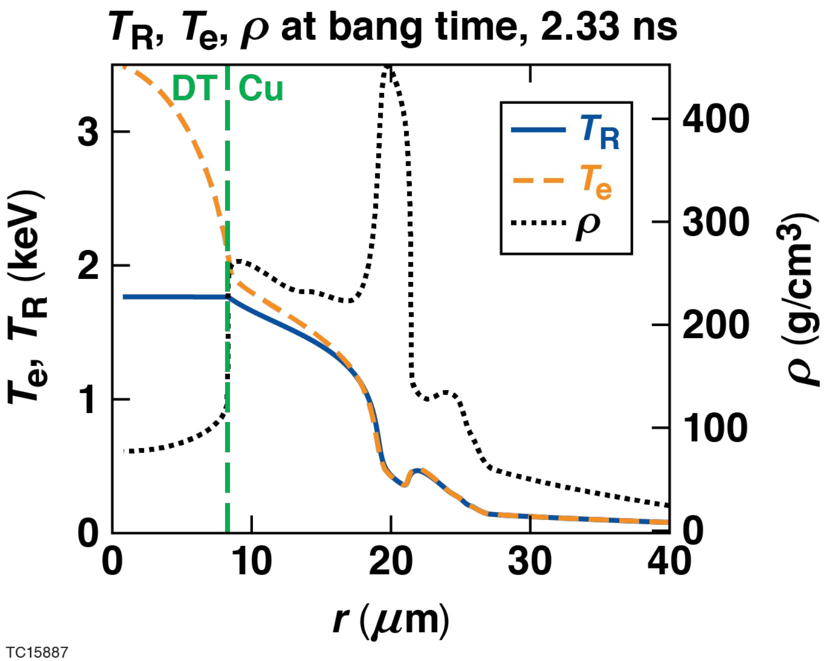
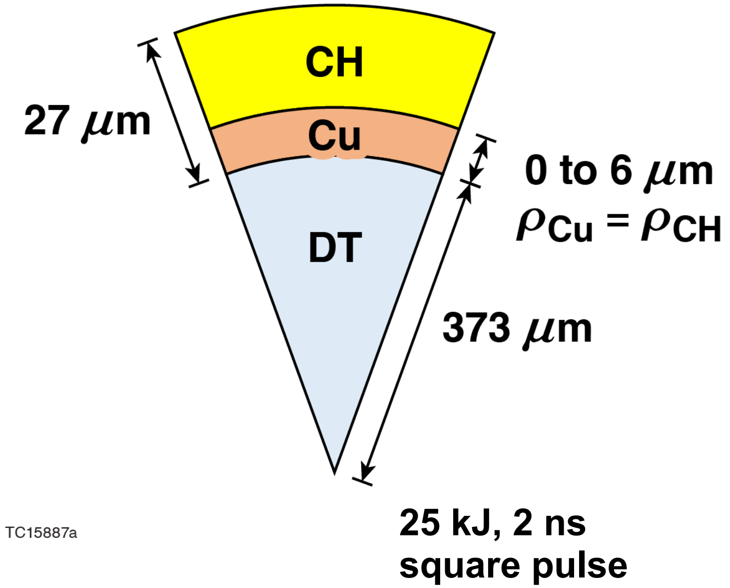
Assessment of Radiation-Trapping in Inertial Confinement Fusion Implosion Experiments Based on Characteristic Quantities of Simple Models







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Radiation trapping by an ICF pusher layer can be characterized in terms of a Marshak wave model

- Radiation trapping is most apparent in simulations through a characteristic Marshak waveform, where radiation and electron temperatures are equal ($T_R = T_e$), indicating atomic-radiative LTE
- The Marshak wave model describes radiation trapping in pusher layers in terms of useful characteristic quantities
- The classic Marshak wave model is extended to cylindrical and spherical geometries and to a uniformly compressing pusher layer, preserving its self-similar analytic form

Volume-ignition capsule designs rely on radiation trapping.

Collaborators

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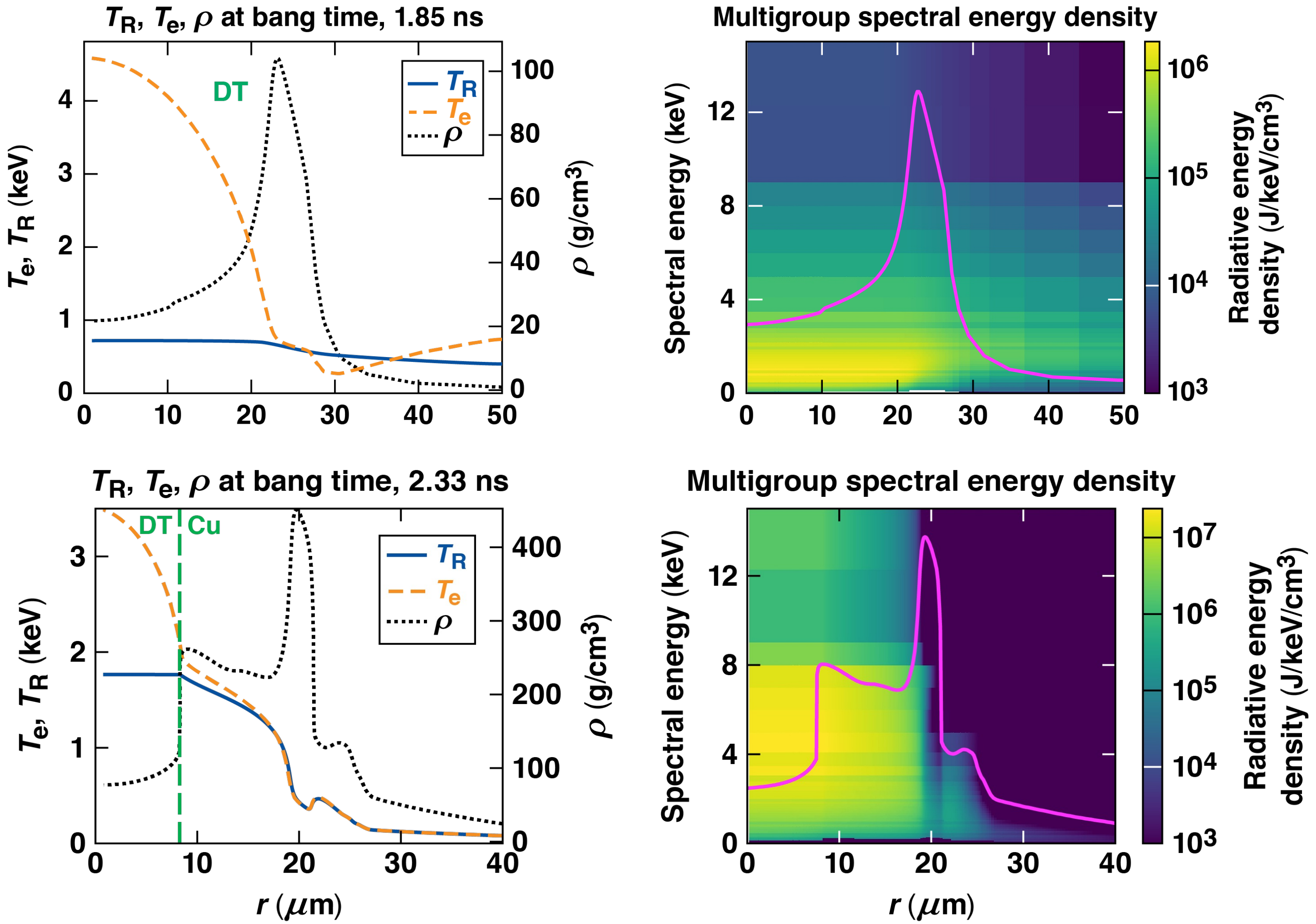
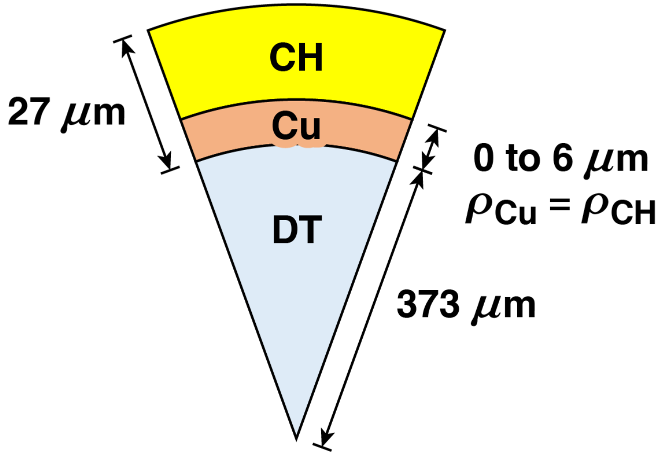
Pure-CH OMEGA-scale imploded shells do not trap radiation, while a 6 μm Cu inner-pusher layer traps radiation, as seen in LILAC simulations



TC15887

- High-yield shot 90288
- Radiation source $r < 22$ μm
- Near-free escape of radiation through the DT shell



TC15887a

- 25 kJ, 2 ns square pulse
- $T_R = T_e$ indicates optically thick LTE
- LTE and a Planck spectrum, are key Marshak wave assumptions

The Marshak wave is based on a simple balance of electron thermal energy and radiative heating near local thermodynamic equilibrium (LTE)

Begin with radiation spectral energy density: $\frac{1}{c}\frac{\partial \phi_\nu}{\partial t} = \frac{\varepsilon_\nu}{c} - \kappa_\nu \phi_\nu + \frac{\partial}{\partial x}\left(\frac{1}{3\kappa_\nu}\frac{\partial \phi_\nu}{\partial x}\right)$ **and thermal energy:** $C_V \frac{\partial T_e}{\partial t} = \int\left(-\frac{\varepsilon_\nu}{c} + \kappa_\nu \phi_\nu\right) d\nu$

Assume quasi-static radiation: $\frac{1}{c}\frac{\partial \phi_\nu}{\partial t} \approx 0$

Define the radiation temperature: $E_R = \frac{4\sigma_{SB}}{c} T_R^{\,4} = \int \varphi_\nu \, d\nu$

Assume detailed balance: $\varepsilon_\nu \approx 4\pi\kappa_\nu B_\nu\left(T_e\right)$ **and LTE:** $\varphi_\nu \approx \frac{4\pi}{c} B_\nu\left(T_e\right)$

which gives: $\sigma_{SB} T_e^{\,4} = \pi \int B_\nu\left(T_e\right) d\nu$ **or** $T_R \approx T_e \equiv T$

Obtain the Marshak wave equation:

$$C_V \frac{\partial T}{\partial t} \approx \frac{\partial}{\partial x}\left(\frac{c}{3\kappa_R}\frac{\partial}{\partial x}\left(\frac{4\sigma_{SB}}{c} T^4\right)\right)$$

Define the Rosseland mean opacity κ_R: $\frac{1}{\kappa_R}\int \frac{\partial B_\nu(T)}{\partial T} d\nu \equiv \int \frac{1}{\kappa_\nu + n_e \sigma_{Th}} \frac{\partial B_\nu(T)}{\partial T} d\nu$

The Planck mean opacity κ_R does not appear: $\kappa_P \int B_\nu(T) d\nu \equiv \int \kappa_\nu B_\nu(T) d\nu$

The appearance of LTE, e.g., $T_e = T_R$, is a sign of "trapped" radiation.

The constant-density planar Marshak wave problem has a self-similar temperature profile solution

$$\kappa = \kappa_0 \left(\rho/\rho_0\right)^r \left(T/T_0\right)^{-n}$$

$$C_V = C_0 \left(\rho/\rho_0\right)^s \left(T/T_0\right)^q$$

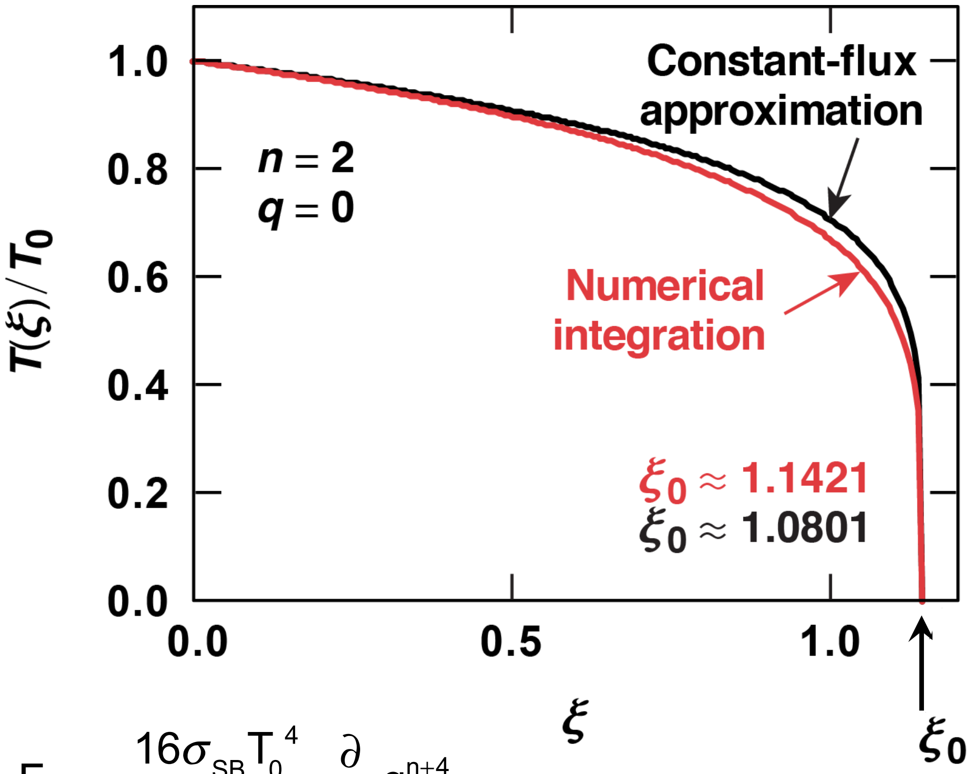
$$T(r,t) = T_0 g(\xi)$$

$$\xi \frac{dg^{q+1}}{d\xi} - \frac{d^2}{d\xi^2} g^{n+4} = 0$$

$$\xi = A \frac{x}{t^{1/2}}$$

$$A^2 = \frac{3}{32} \frac{(n+4)}{(q+1)} \frac{\kappa_0 C_0 \rho}{\sigma_{SB} T_0^3}$$

$$x_0(t) = \xi_0 \frac{t^{1/2}}{A}$$



$$F = -\frac{16\sigma_{SB} T_0^4}{3\kappa_0 (n+4)} \frac{\partial}{\partial x} g^{n+4}$$

Boundary conditions

At $\xi = 0$: conserve energy and $g(0) = 1$

At $\xi = \xi_0$: zero temperature

Constant-flux approximation

$$g(\xi) \approx \left(1 - \xi/\xi_0\right)^{\frac{1}{n+4}}$$

$$\xi_0^2 \approx \frac{5+n+q}{4+n}$$

The "constant-flux" approximation is accurate and gives a useful expression for pretty much every quantity of interest.

The Marshak wave model yields several useful characteristic quantities, particularly $t_{\tau=1}$, the formation time of a one-optical-thickness wave

- **Optical thickness time scale: the time of formation of a $\tau_R = 1$ trapping layer is the key time scale**

$$t_{\tau=1} = \frac{3}{2(n+4)(1+q)\xi_0^2}\frac{C_0\rho}{\sigma_{SB}T_0^3\kappa_0} = \frac{6}{(n+4)\xi_0^2}\frac{E_{Th}}{E_R}\frac{1}{\kappa_0 c}$$

, where

$$E_R = \frac{4\sigma_{SB}T_0^4}{c} \qquad E_{th} = \frac{C_0\rho T_0}{1+q}$$

- **The trapped flux F_R and the trapped energy E_R vary on this time scale**

$$F_R \approx \frac{cE_R}{3}(t_{\tau=1}/t)^{1/2} \qquad E_R \approx 2F_R t \approx \frac{2cE_R}{3}(t_{\tau=1}t)^{1/2}$$

- **The wavefront, defined as $\xi = \xi_0$, decelerates**

$$A^2 = \frac{3(n+4)}{8}\frac{\kappa_0}{c}\frac{E_{th}}{E_R} \qquad x_0(t) = \xi_0\frac{t^{1/2}}{A} = \frac{4}{(n+4)\kappa_0}\left(\frac{t}{t_{\tau=1}}\right)^{1/2}$$

- **Uniform adiabatic compression solved similarly** $\rho(t) = \rho_0\left(t/t_0\right)^{\alpha}$ $\nu = \alpha\left(\frac{(\gamma-1-s)(3+n-q)}{(q+1)} - s - r\right)$

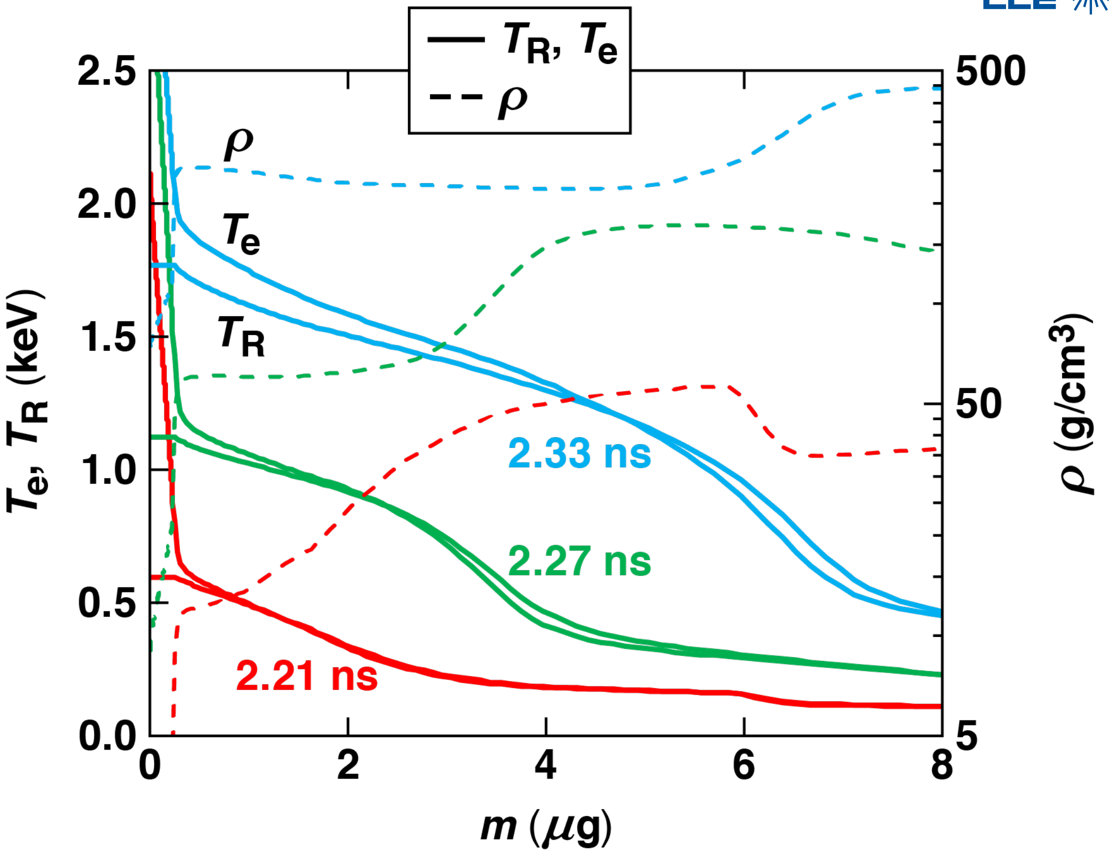
$$m_0(t) = \frac{\xi_0}{(1+\nu)^{1/2}}\frac{t^{1/2}}{A}\left(\frac{t}{t_0}\right)^{\nu/2}$$

Typically, Cu, Au, etc.: $2\alpha < \nu < 4\alpha$ **Cu:** $\tau(t) \sim t^{\frac{1}{2}+\frac{\alpha}{6}}$ **Au:** $\tau(t) \sim t^{\frac{1}{2}+\frac{\alpha}{3}}$

Marshak wave "$\tau = 1$" formation times for Cu and Au are short relative to the pusher hydro time, but far too long for a pure-CH shell

$$t_{\tau=1} = \frac{3}{2(n+4)(1+q)\xi_0^{\,2}}\,\frac{C_0\rho}{\sigma_{SB}T_0^{\,3}\kappa_0} = \frac{6}{(n+4)\xi_0^{\,2}}\,\frac{E_{Th}}{E_R}\,\frac{1}{\kappa_0 c}$$

	t sim (ns)	T_0 (keV)	ρ (g/cm^3)	Cu *t* (τ = 1) (ps)	Au *t* (τ = 1) (ps)
	2.21	0.6	12.8	5.5	8.5
	2.27	1.12	60.5	3.6	4.1
	2.33	1.77	239	2.7	2.4
CH	1.85	1.12	57	66.8	



TC15889

The "$\tau = 1$" formation time is a parameter that anticipates the effectiveness of radiation trapping in an imploding pusher layer.

Marshak waves have very different $g(\xi) = T/T_0$ profiles in planar, cylindrical, and spherical geometries, but nearly identical wavefronts

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Planar

$$\xi \frac{dg^{q+1}}{d\xi} - \frac{d^2}{d\xi^2} g^{n+4} = 0 \qquad \xi_0^{\;2} \approx \frac{(1+m)}{m}$$

Cylindrical

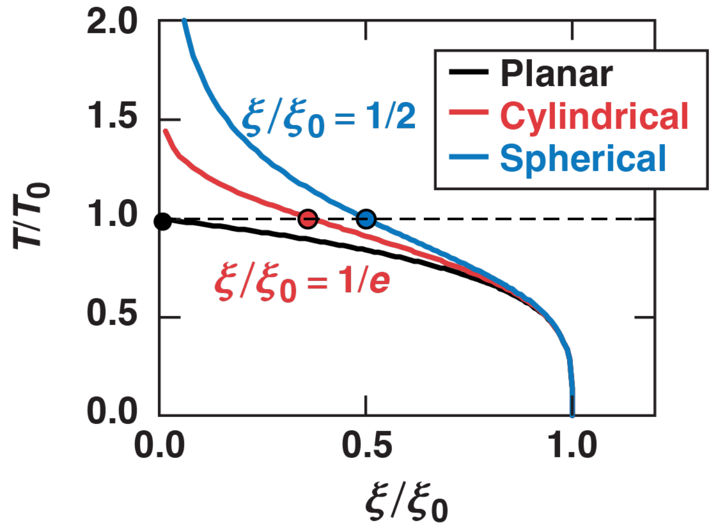
$$\xi \frac{dg^{q+1}}{d\xi} - \frac{1}{\xi}\frac{d}{d\xi}\left(\xi \frac{d^2}{d\xi^2} g^{n+4}\right) = 0 \qquad \xi_0^{\;2} \approx \frac{2^{1/m}}{\Gamma(1+1/m)}$$

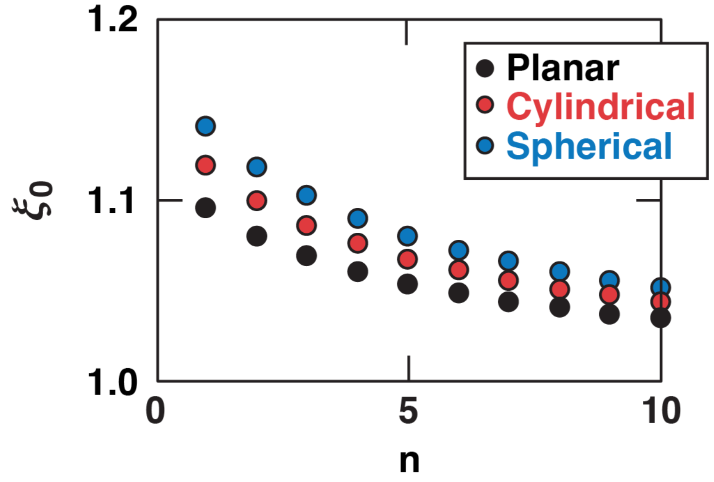
Spherical

$$\xi \frac{dg^{q+1}}{d\xi} - \frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2 \frac{d^2}{d\xi^2} g^{n+4}\right) = 0 \qquad \xi_0^{\;2} \approx \frac{1}{\pi}\frac{2m^3 \sin(\pi/m)}{(2m-1)(m-1)}$$

$$m = \frac{4+n}{1+q}$$

- In cylindrical and spherical geometry, T_0 is not a good fuel/pusher boundary condition parameter; it is fixed at a definite point in ξ, but not in space or time
- A better boundary condition is the total radiated power at or near $\xi = 0$





TC16142

Marshak waves have the same trajectory and flux trapping parameter in all three geometries, all expressed in terms of the optical thickness formation time t_τ

Common to all three geometries:

$$\xi = A\frac{x}{t^{1/2}}$$

$$A^2 = \frac{3(n+4)}{8}\frac{\kappa_0}{c}\frac{E_{Th}}{E_R}$$

$$F = -\frac{16\sigma_{SB}T_0^4}{3\kappa_0(n+4)}\frac{\partial}{\partial x}g^{n+4}$$

Radiated power:

Planar: $$F = \frac{cE_R}{3}\left(\frac{t_\tau}{t}\right)^{1/2}$$

Cylindrical: $$2\pi rF = \frac{8\pi}{(n+4)}\frac{cE_R}{3}\frac{1}{\kappa_0}$$

Spherical: $$4\pi r^2F = \frac{64\pi}{(n+4)^2}\frac{cE_R}{3}\frac{1}{\kappa_0^2}\left(\frac{t}{t_\tau}\right)^{1/2}$$

Flux trapping ratio:

$$F_{SB} = \sigma_{SB}T_0^4 = \frac{cE_R}{4}$$

$$\frac{F}{F_{SB}} = \frac{4}{3}\left(\frac{t_\tau}{t}\right)^{1/2}$$

$$E_R = \frac{4\sigma_{SB}T_0^4}{c} \qquad E_{th} = \frac{C_0\rho T_0}{1+q}$$

Still a key parameter:

$$t_{\tau=1} = \frac{6}{(n+4)\xi_0^2}\frac{E_{th}}{c\kappa_0 E_R}$$

All geometries have the same wavefront trajectory: $x_0(t) = \xi_0\frac{t^{1/2}}{A}$.

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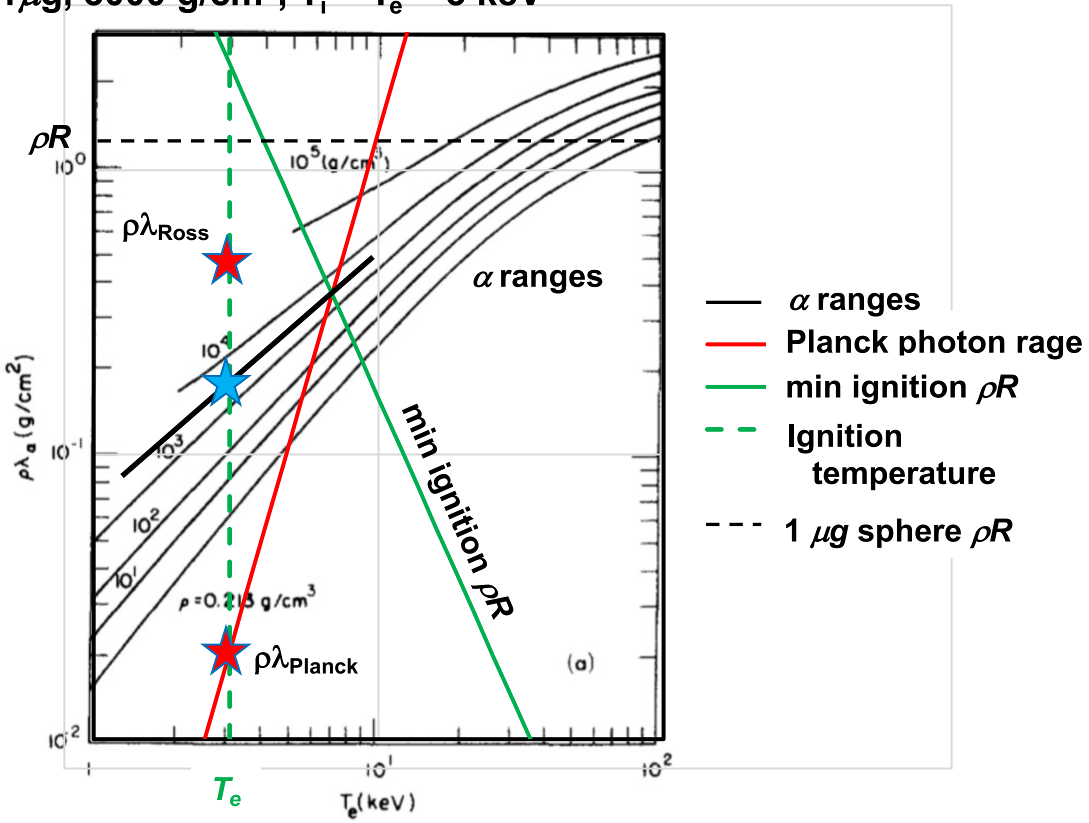
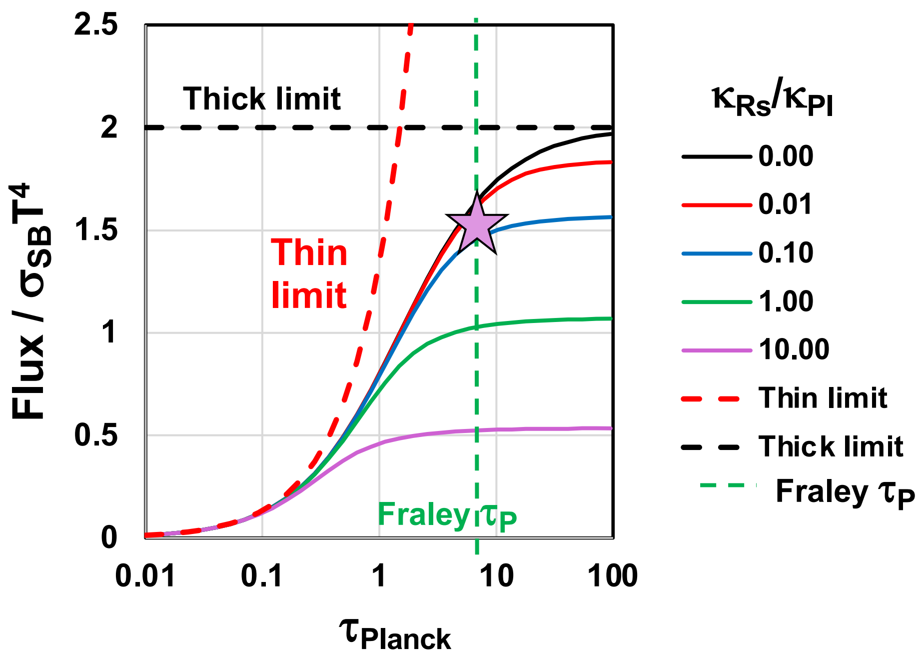
Volume-ignition capsule designs rely on radiation trapping.

Fraley *et al.** characterize radiation trapping in the pre-ignition DT sphere entirely in terms of a photon "mean-free-path"

Alpha and photon ranges in a DT sphere, 1μg, 3000 g/cm^3, $T_i = T_e$ = 3 keV



- "The relative mean-free-path for photons (at the average photon energy) is [~0.017 the sphere radius], so inverse bremsstrahlung retards the photon loss from the microsphere, T_e remains up, and the fuel ignites at at 3 keV."
- Fraley conditions : κ_{Rs}/κ_{Pl} = 0.0438 or $\lambda_{Ross}/\lambda_{Pl}$ = 22.9



*G. S. Fraley, E. J. Linnebur, R. J. Mason, and R. L. Morse, Phys. Fluids 17, 474-489, 1974.