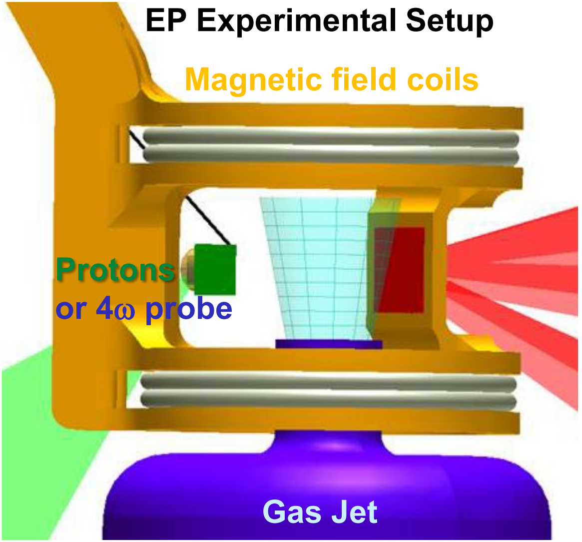
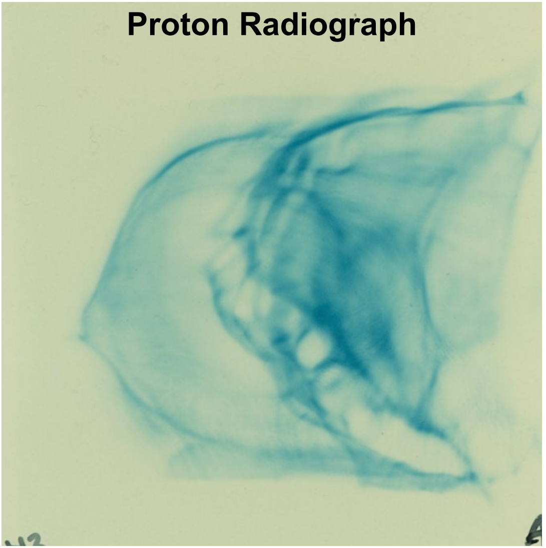
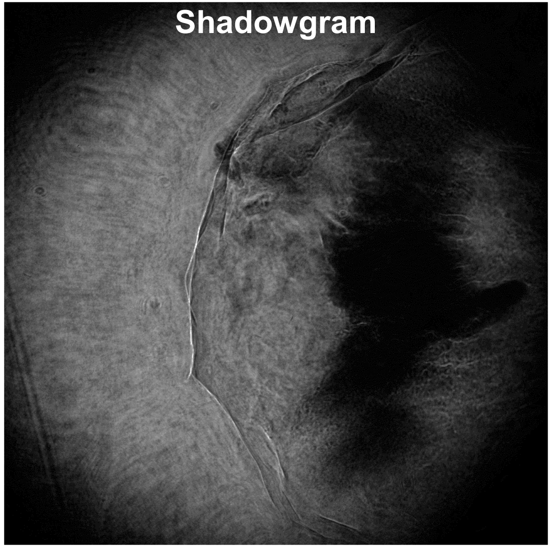
Direct Inversion of Deflectometry Data using an Electrostatic Plasma Model







64th Annual Meeting of the
American Physical Society
Division of Plasma Physics
Spokane, WA
17 – 21 October 2022

J. R. Davies
University of Rochester
Laboratory for Laser Energetics

Summary

A new algorithm to invert charged-particle radiography and shadowgraphy data has been developed based on an electrostatic plasma model

- Treat the source or measured intensity as electrons subject to drag and the other intensity as fixed ions
- Electron displacements in equilibrium give the line-integrated transverse Lorentz force for charged-particle radiography, the line-integrated refractive index gradient for shadowgraphy
- Using a PIC (particle-in-cell) approach the algorithm is robust and could achieve a high parallel efficiency

Collaborators

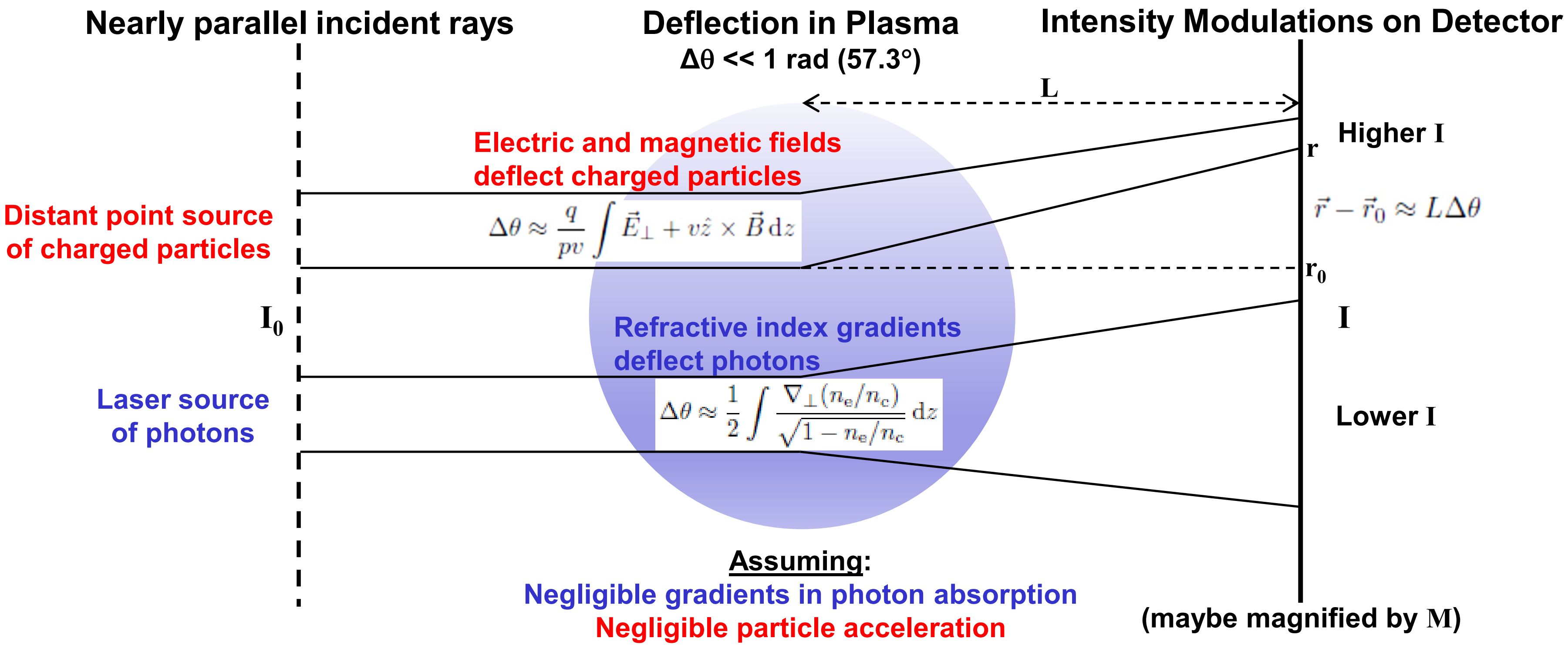
P. V. Heuer

University of Rochester
Laboratory for Laser Energetics

A. F. A. Bott

Oxford University

Charged-particle radiography and shadowgraphy rely on measuring intensity modulations caused by the deflection of rays in an object



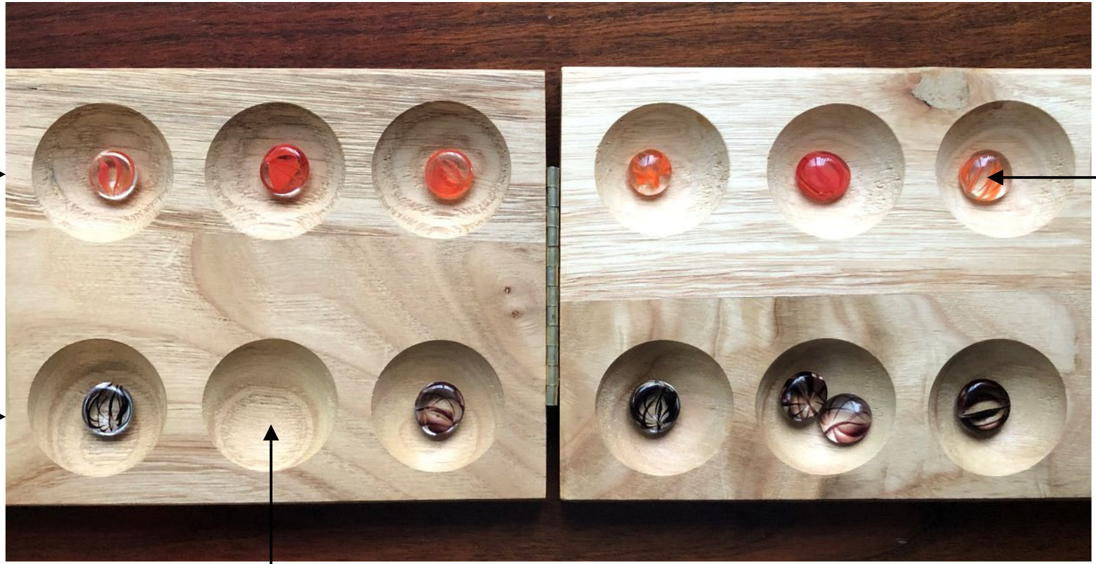
Direct inversion of deflectometry data to obtain the line-integrated transverse Lorentz force or refractive index gradient is possible for small deflection angles

- If rays do not cross direct inversion gives *the* solution
- If rays cross direct inversion gives a solution that minimizes deflection
- Five direct-inversion codes are available on GitHub:
  - invert_shadowgraphy: github.com/mfkasim1/invert-shadowgraphy [M. F. Kasim *et al.* Phys. Rev. E 95, 023306 (2017)]
  - PROBLEM: github.com/flash-center/PROBLEM [A. F. A. Bott *et al.* J. Plasma Physics 83, 905830614 (2017)]
  - fast_invert_shadowgraphy: github.com/mfkasim1/invert-shadowgraphy/tree/fast-inverse
  - PRNS: github.com/OxfordHED/proton-radiography-no-source [M. F. Kasim *et al.* Phys. Rev. E 100, 033208 (2019)]
  - PRaLine: github.com/flash-center/PRaLine [C. Graziani *et al.* Rev. Sci. Instr. 88, 123507 (2017)]

In terms of the data direct inversion involves determining the movement of counts in detector bins that map source intensity I_0 to measured intensity I

A simple representation of deflectometry data
(aka mancala)

I_0



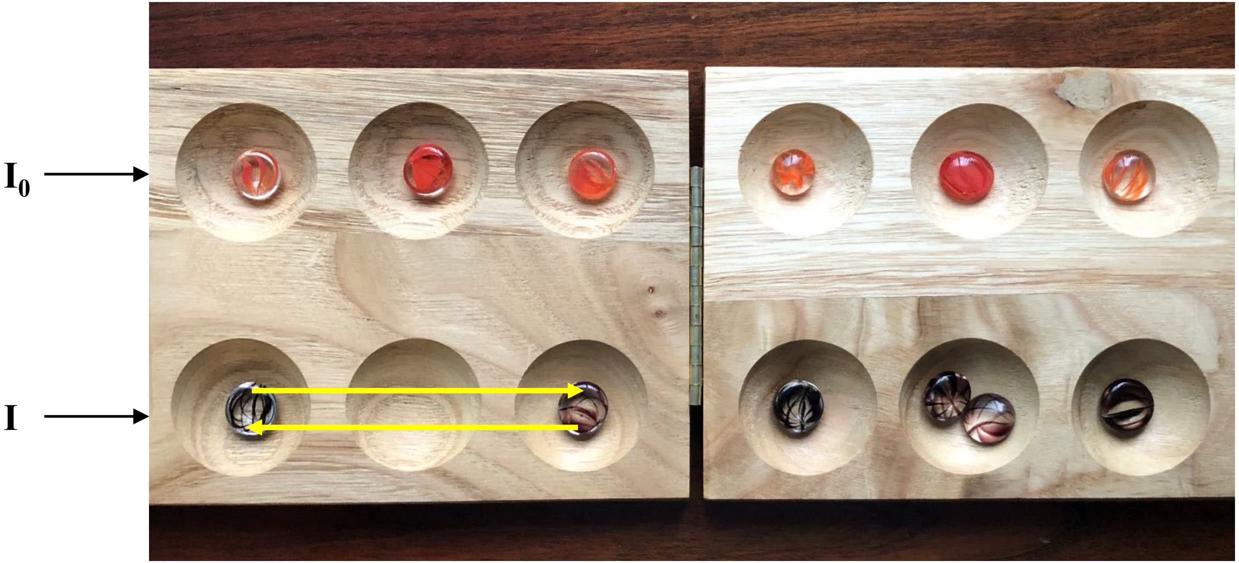
I

Counts

Detector bin

If both rows have an identical distribution and someone rearranges the bottom row without you looking can you work out the moves they made?

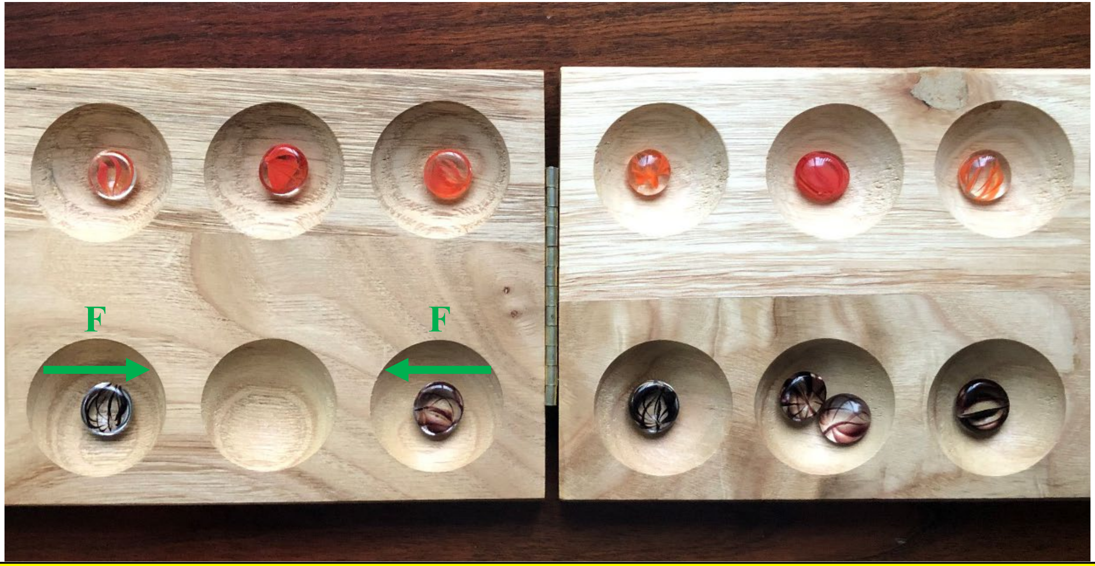
NO (except by luck)
There is no way of knowing if they simply swapped counts between bins



NO (except by luck)
There is no way of knowing if they simply swapped counts between bins

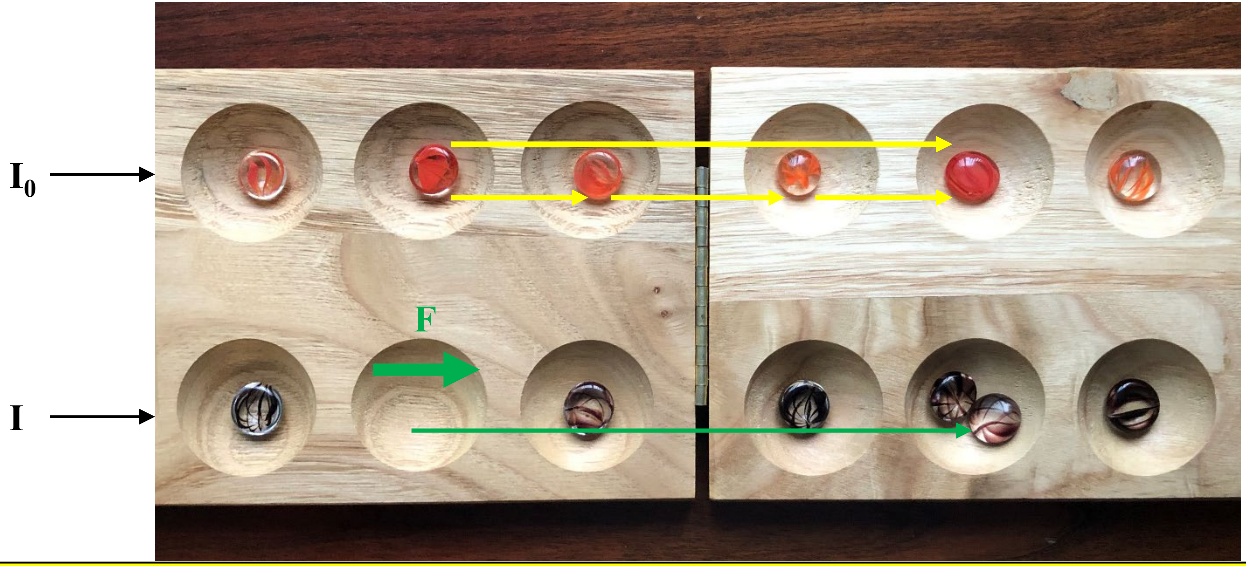
NO (except by luck)
There is no way of knowing if they simply swapped counts between bins

Unlikely to occur in a system of physical interest



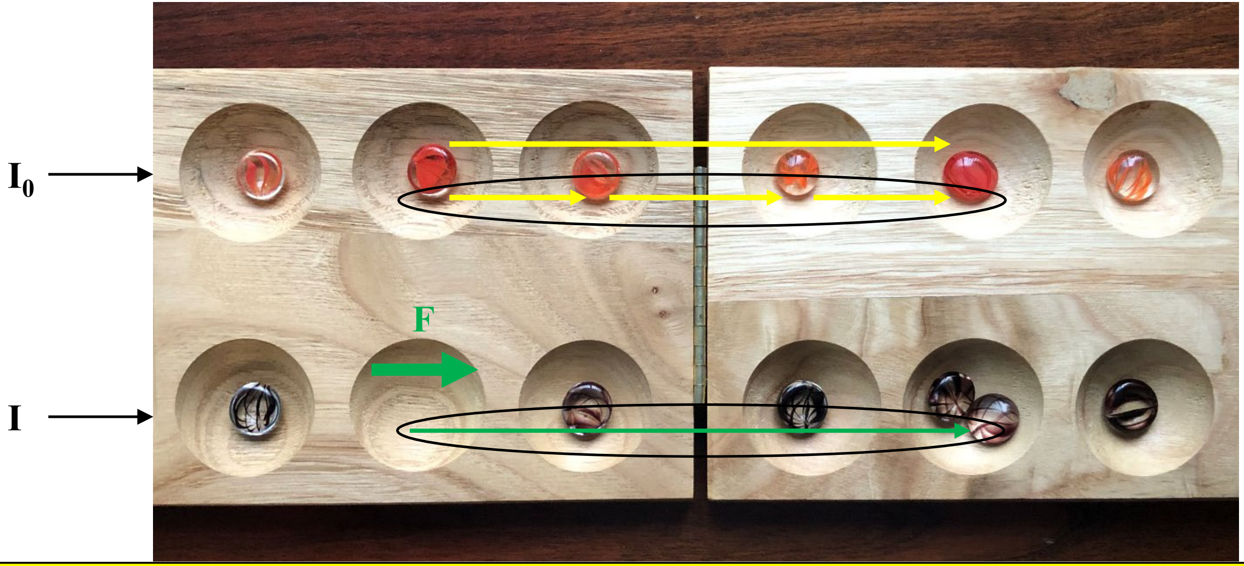
Rule 1: reproduce the bottom row with the minimum number of moves possible

There are still multiple solutions with a total move of three bins



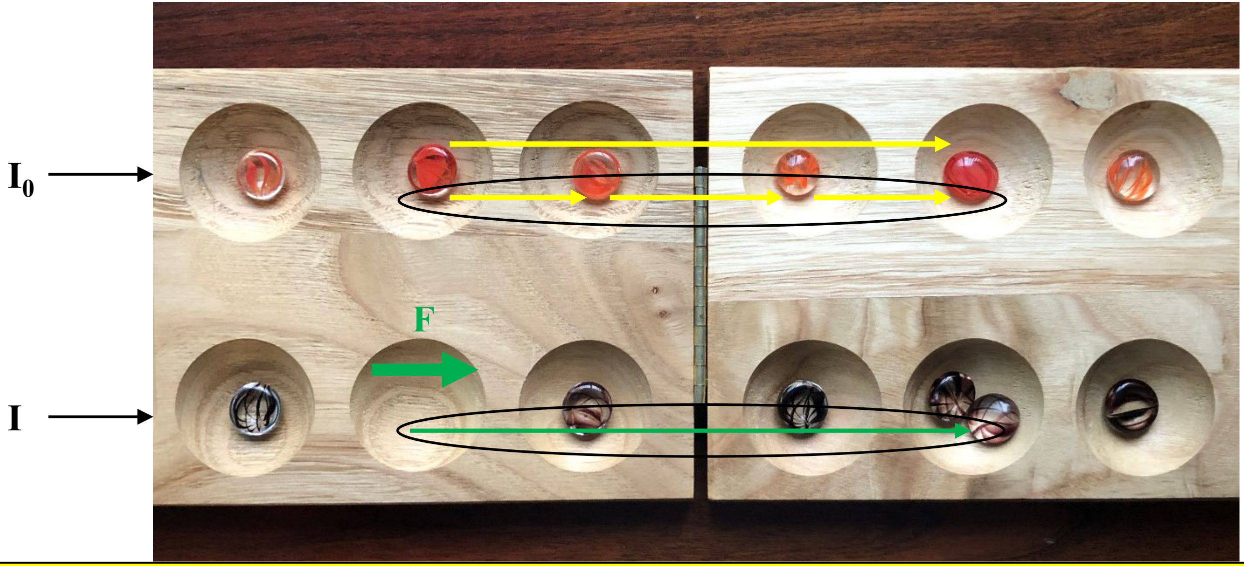
Rule 1: reproduce the bottom row with the minimum number of moves possible

A simple rule valid for many physical systems leaves only one solution



Rule 1: reproduce the bottom row with the minimum number of moves possible
Rule 2: you cannot move counts through one another

In deflectometry rays can cross so direct inversion only gives *the* solution if trajectories do not cross, one of many possible solutions if they do



Rule 1: reproduce the bottom row with the minimum number of moves possible
Rule 2: you cannot move counts through one another

If one set of counts were electrons and the other fixed ions then plasma physics would win the game for you

UR LLE

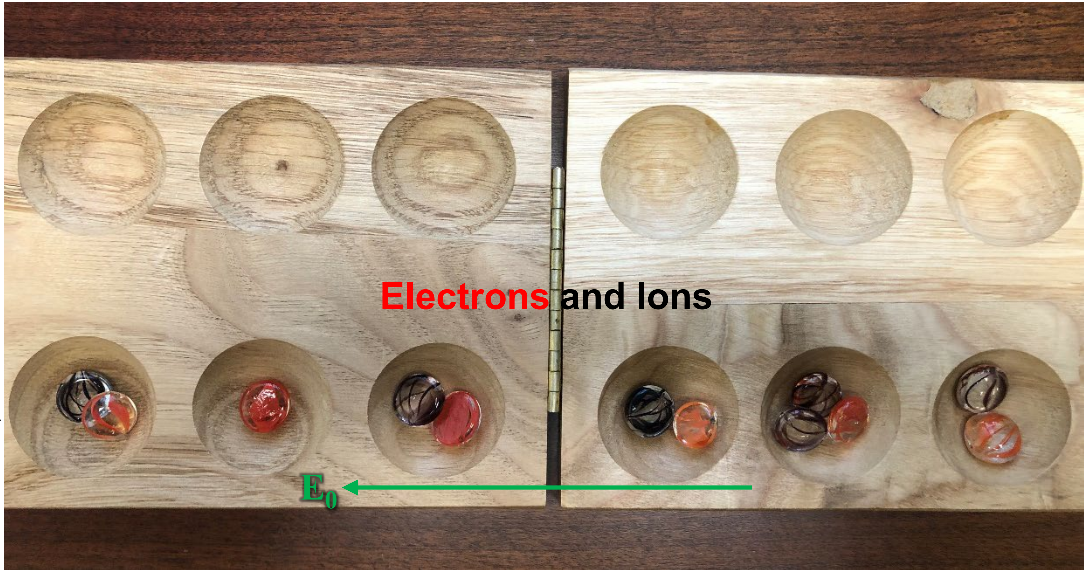
Electrons →

Ions →

UNIVERSITY of ROCHESTER

Direct inversion using a 2-D electrostatic PIC* code

PIC grid



*Particle in cell

Direct inversion using a 2-D electrostatic PIC* code

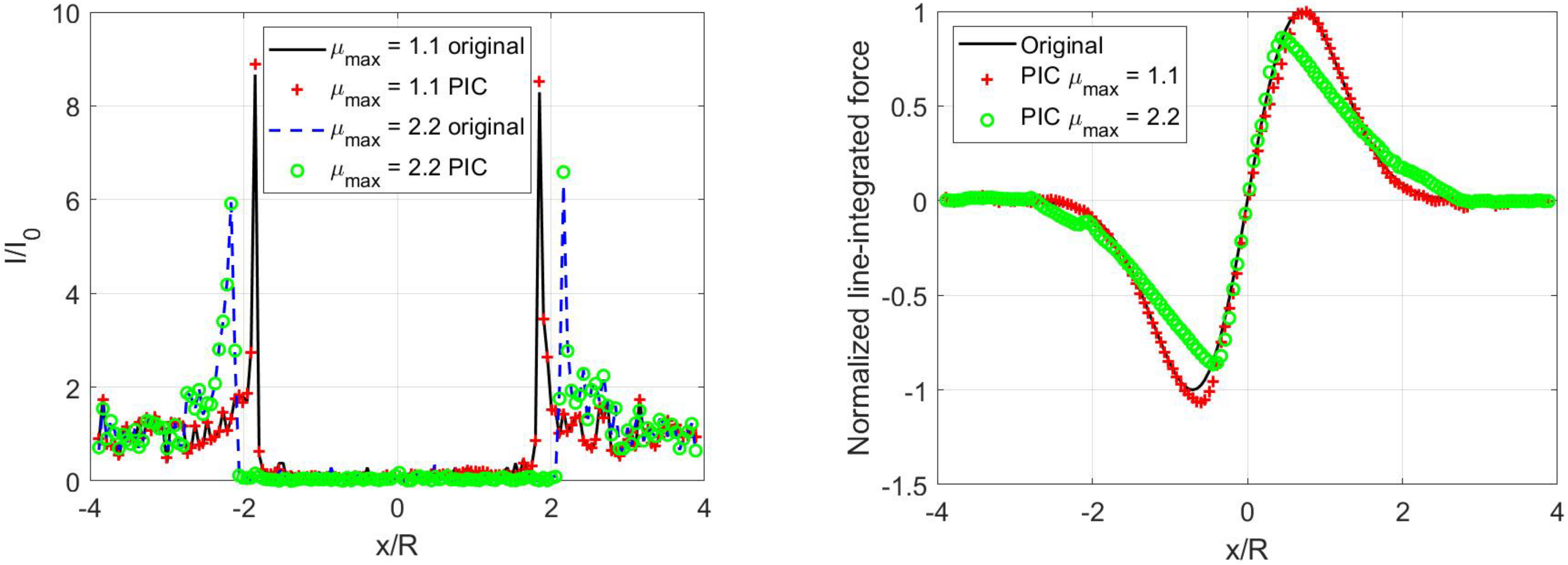
UR LLE

- Well-established numerical technique in plasma physics that is robust and efficient
- Oscillations are damped by adding electron drag
  - In a harmonic oscillator a collision frequency of twice the resonant frequency prevents oscillation
  - Tests showed that setting the electron collision frequency to twice the plasma frequency determined from the local ion density led to the fastest convergence
- Kinetic plus electrostatic energy will decay steadily reaching zero in equilibrium giving a simple convergence criterion
  - Typically stop iterating when total energy has fallen below 10^{-3} of the initial value
- The number of computational particles required will depend on the intensity modulations
  - Will never need to be greater than the number of counts on the detector

*Particle in cell

Tests using proton radiographs generated by tracing through specified radial force profiles* show the electrostatic algorithm works

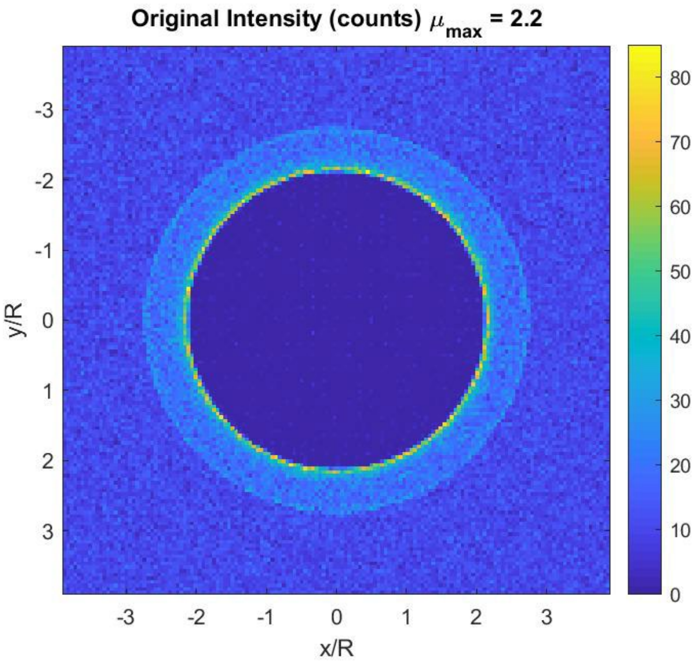
Central lineouts from PIC results with 16 uniformly distributed particles per cell

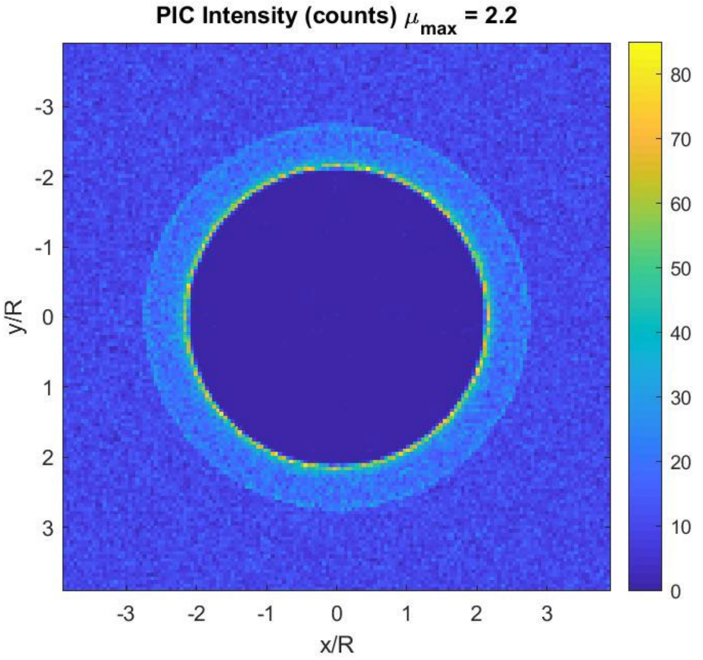


$$\mu = \frac{2LF}{Mpv} \qquad \mu = \frac{L}{M}\frac{\mathrm{d}(n_e/n_c)/\mathrm{d}r}{\sqrt{1 - n_e/n_c}}$$

*J. R. Davies and P. V. Heuer (2022), Synthetic proton radiographs for testing direct inversion algorithms (Version 3) https://doi.org/10.5281/zenodo.6632986

Tests using proton radiographs generated by tracing through specified radial force profiles* show the electrostatic algorithm works





*J. R. Davies and P. V. Heuer (2022), Synthetic proton radiographs for testing direct inversion algorithms (Version 3) https://doi.org/10.5281/zenodo.6632986

The electrostatic algorithm is more robust than existing algorithms and intermediate in run time without massively parallel processing

- invert_shadowgraphy: github.com/mfkasim1/invert-shadowgraphy [M. F. Kasim *et al.* Phys. Rev. E 95, 023306 (2017)]
- PROBLEM: github.com/flash-center/PROBLEM [A. F. A. Bott *et al.* J. Plasma Physics 83, 905830614 (2017)]
  - A Matlab function to solve the Monge-Ampère equation using the Sulman, Williams and Russell algorithm (Version 1): doi.org/10.5281/zenodo.6685314
- fast_invert_shadowgraphy: github.com/mfkasim1/invert-shadowgraphy/tree/fast-inverse
- PRNS: github.com/OxfordHED/proton-radiography-no-source [M. F. Kasim *et al.* Phys. Rev. E 100, 033208 (2019)]
- PRaLine: github.com/flash-center/PRaLine [C. Graziani *et al.* Rev. Sci. Instr. 88, 123507 (2017)]
- A 2-D electrostatic PIC code for direct inversion of deflectometry data (Version 2): doi.org/10.5281/zenodo.6638811
- InvertDeflectPy a collection of algorithms for inverting deflectometry data: github.com/pheuer/InvertDeflectPy

The electrostatic algorithm is more robust than existing algorithms and intermediate in run time without massively parallel processing

- invert_shadowgraphy: github.com/mfkasim1/invert-shadowgraphy [M. F. Kasim *et al.* Phys. Rev. E 95, 023306 (2017)]
  - ~30× slower than the PIC code
  - Requires considerably more memory
  - Fails for very large peaks in the intensity
  - Gave the best solution for a strongly modulated source intensity

The electrostatic algorithm is more robust than existing algorithms and intermediate in run time without massively parallel processing

- **PROBLEM: github.com/flash-center/PROBLEM [A. F. A. Bott *et al.* J. Plasma Physics 83, 905830614 (2017)]**
  - **A Matlab function to solve the Monge-Ampère equation using the Sulman, Williams and Russell algorithm (Version 1): doi.org/10.5281/zenodo.6685314**
  - **50 to 200× faster than the PIC code (without massively parallel processing)**
  - **Cannot deal with zeros in either the source or measured intensity (numerical fudge)**
  - **Could not fully automate the inversion process (may need to tweak time-step multiplier and tolerance)**
  - **Could not obtain as accurate a solution as the PIC code for strong intensity modulations**

The electrostatic algorithm is more robust than existing algorithms and intermediate in run time without massively parallel processing

- **fast_invert_shadowgraphy: github.com/mfkasim1/invert-shadowgraphy/tree/fast-inverse**
- **PRNS: github.com/OxfordHED/proton-radiography-no-source [M. F. Kasim *et al.* Phys. Rev. E <u>100</u>, 033208 (2019)]**
  - **Use the same algorithm as PROBLEM**
  - **Incorrect boundary conditions**
  - **An adaptive time step that does not work for most cases with caustics**

The electrostatic algorithm is more robust than existing algorithms and intermediate in run time without massively parallel processing

- PRaLine: github.com/flash-center/PRaLine [C. Graziani *et al.* Rev. Sci. Instr. 88, 123507 (2017)]
  - The underlying equation is not as generally applicable as that solved by the other codes

We recommend using the Monge-Ampère code to take a quick first look at data, and the PIC code if that fails or to obtain a subsequent, more accurate inversion

- **A Matlab function to solve the Monge-Ampère equation using the Sulman, Williams and Russell algorithm (Version 1): doi.org/10.5281/zenodo.6685314**
- **A 2-D electrostatic PIC code for direct inversion of deflectometry data (Version 2): doi.org/10.5281/zenodo.6638811**
- **Python versions of both of these Matlab functions are under development: InvertDeflectPy a collection of algorithms for inverting deflectometry data, github.com/pheuer/InvertDeflectPy**

Summary

A new algorithm to invert charged-particle radiography and shadowgraphy data has been developed based on an electrostatic plasma model

- Treat the source or measured intensity as electrons subject to drag and the other intensity as fixed ions
- Electron displacements in equilibrium give the line-integrated transverse Lorentz force for charged-particle radiography, the line-integrated refractive index gradient for shadowgraphy
- Using a PIC (particle-in-cell) approach the algorithm is robust and could achieve a high parallel efficiency