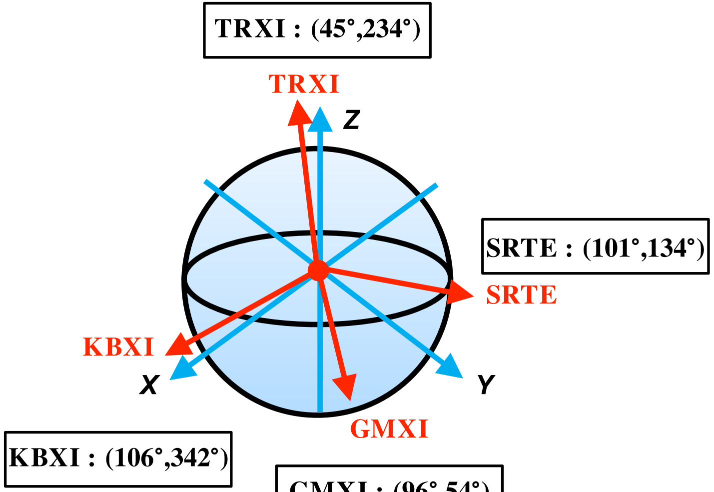
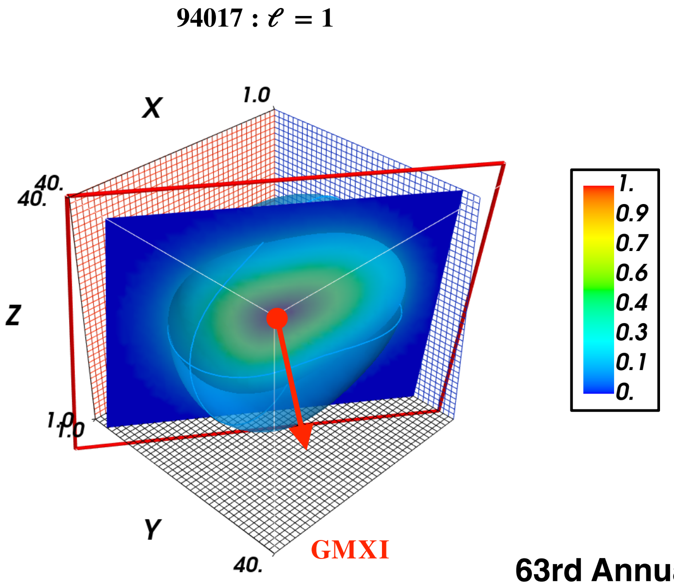
Three-Dimensional Hot-Spot Reconstruction in Inertial Fusion Implosions





K. M. Woo
University of Rochester
Laboratory for Laser Energetics

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Summary

A platform of three-dimensional (3-D) hot-spot reconstruction procedures has been developed to quantify 3-D effects of low modes in ICF implosion experiments

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- A tomography method is developed to reconstruct the 3-D plasma emissivity.
- Procedures of 3-D analysis are developed by integrating 3-D hot-spot shape asymmetries with nuclear measurements including ion-temperature (T_i), flow, and areal-density (ρR) asymmetries.
- Residual kinetic energies (RKE's) are shown to be a driving factor causing low-mode implosion asymmetries.

The 3-D analysis will be applied to minimize the low-mode implosion asymmetry in future work.

See Jim's talk in NO04.00009.
See Kristen's talk in ZO04.00007.

Collaborators

R. Betti, C. Thomas, C. Stoeckl, K. Churnetski, C. Forrest, Z. L. Mohamed, B. Zirps*, S. Regan, T. Collins, W. Theobald, R. Shah, O. Mannion**, D. Patel, D. Cao, J. Knauer, V. Goncharov, R. Bahukutumbi, H. Rinderknecht, R. Epstein, V. Gopalaswamy and F. Marshall

Laboratory for Laser Energetics, University of Rochester

* Graduated Student from University of Rochester
** Currently at Sandia National Laboratories

A 3-D spherical-harmonic Gaussian function is used to reconstruct the 3-D plasma emissivity

3-D hot-spot emissivity ε_ν at a given spectral frequency ν

$$\ln \varepsilon_\nu(r,\theta,\phi) = \sum_{n=0}^{\infty} \sigma_n R^n \left[1 + \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} \sum_{k=0}^{\infty} A_{\ell m k} R^k Y_{\ell m}(\theta,\phi)\right]^n$$

Unfold electron temperature T_e and density n_e

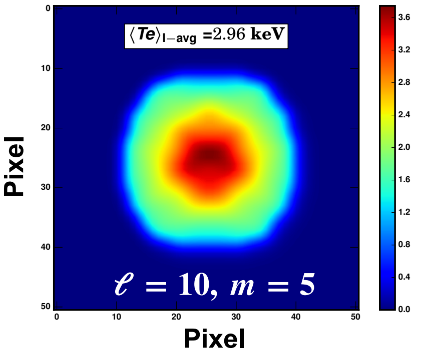
$$T_e = -\varepsilon_\nu / \left[\frac{\partial \varepsilon_\nu}{\partial h\nu}\right] \longrightarrow n_e \propto \sqrt{\varepsilon_\nu / \sqrt{T_e}}$$

For 1-D implosions, the 3-D emissivity model is reduced to a super-Gaussian model with an exponent of 4 and zero mode amplitudes.

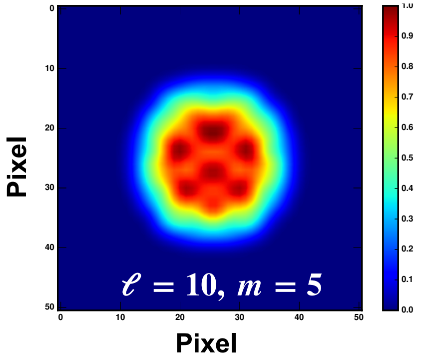
$$\ln I_{1D} = \sum_{n=0}^{4} \sigma_n R^n \rightarrow I_{1D} = I_0 e^{-(R/\sigma)^4}$$

$$I_0 = e^{a_0},\ \sigma_4 = -1/\sigma^4$$

Unfolded T_e (keV)

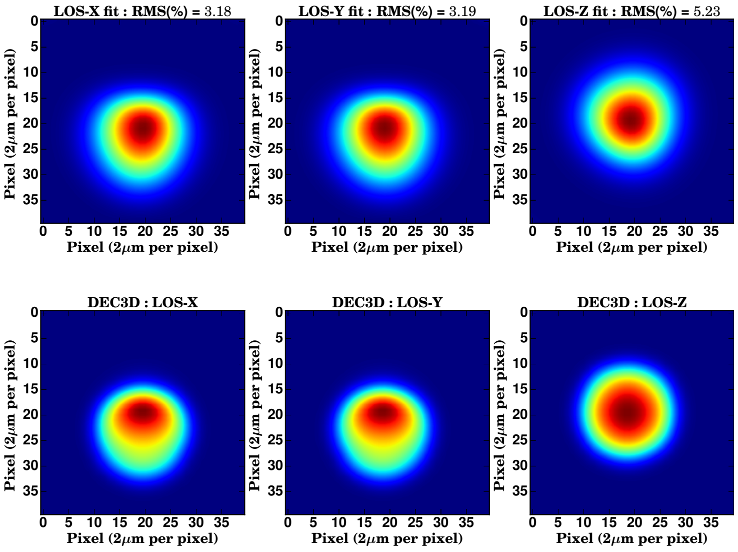


Unfolded n_e (A. U.)

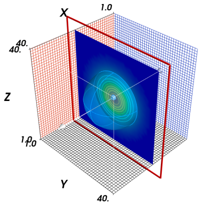


See Ref. [S. Eck et. al. Medical Image Analysis 32, 18-31 (2016)] for applying spherical-harmonic Gaussian functions.
See Ref. [G. Aubert, AIP Advances 3, 062121 (2013)] for rotating spherical harmonics using Wigner d-matrices.
See Kristen's talk in ZO04.00007.

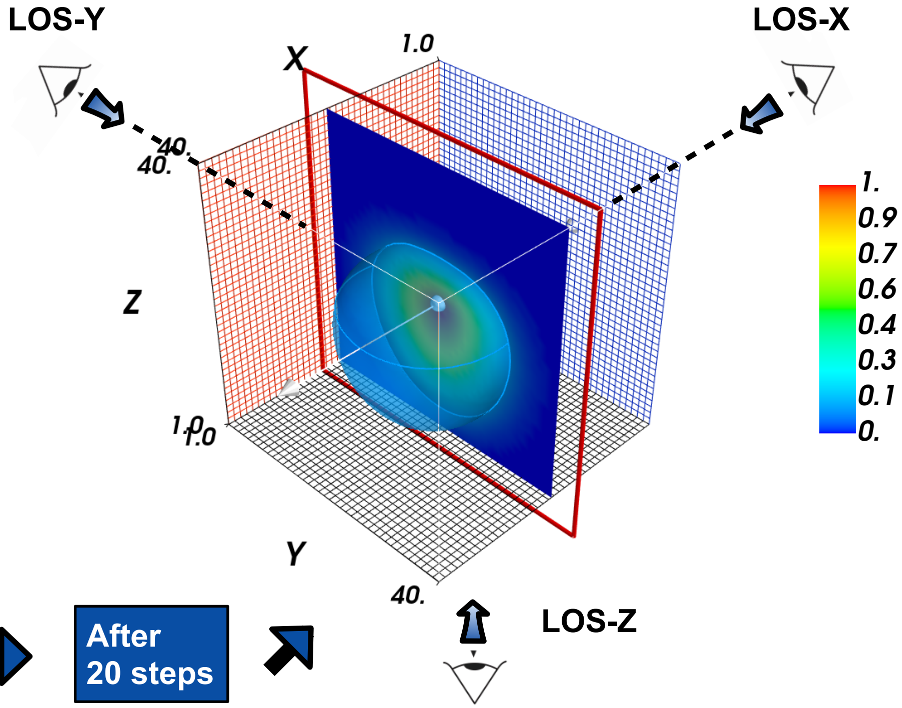
The 3-D plasma emissivity model is optimized by a dynamic learning algorithm* to fit its 2-D projections with x-ray images measured at different lines of sight



Initial guess

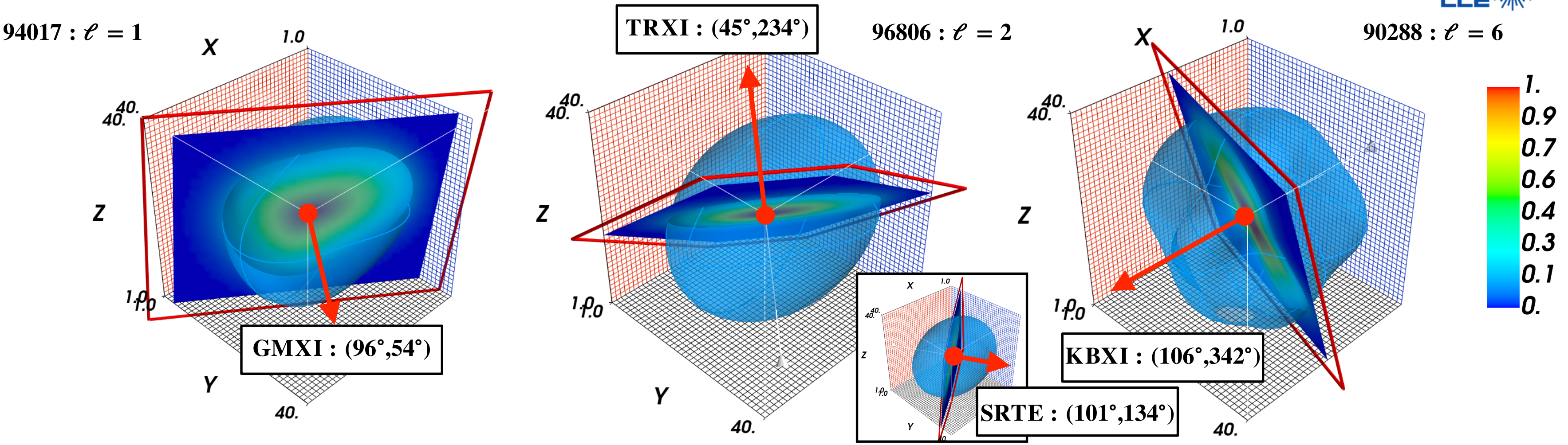


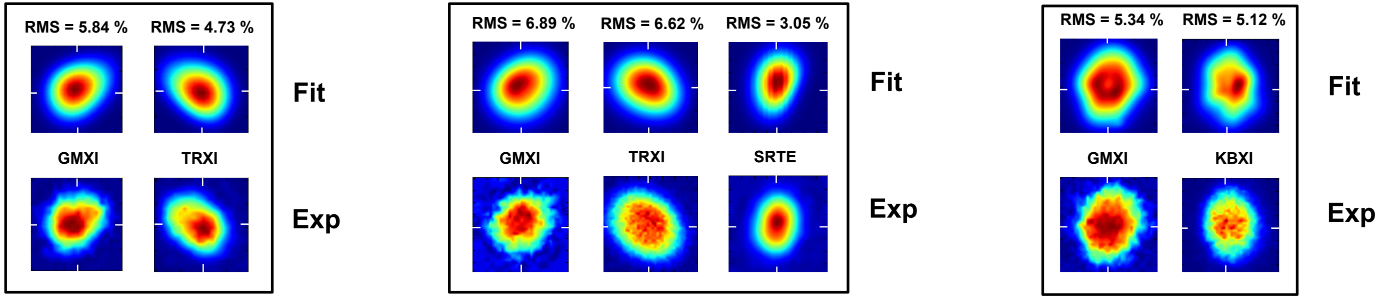
After 20 steps



* Zhenyu Liao et al. arXiv:1805.11917 [stat.ML]

The 3-D plasma emissivity model is used to reconstruct implosions with mode $\ell = 1$, 2, and 6 hot-spot shapes





Definitions

Polar angles : (θ, ϕ)
GMXI : gated-monochromatic X-ray imager
TRXI : temporal-resolved X-ray Imager
SRTE : spatially-resolved X-ray imager
KBXI : Kirkpatrick-Baez X-ray imager

Residual kinetic energies are the driving factor for T_i and hot-spot flow asymmetries

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Apparent ion temperatures

$$T_{\mathrm{apparent}}^{\mathrm{Brysk}**} = T_{\mathrm{th}} + M_0 \cdot \mathrm{Var}\left[\vec{v} \cdot \hat{d}\right]$$

v is the flow velocity in the laboratory frame.
d is the line of sight (LOS) unit vector.
M_0 is the total nuclear reactant mass.
T_{th} is the ion thermal temperature.
$\sigma_{ij} = \langle (v_i - \langle v_i \rangle) \cdot (v_j - \langle v_j \rangle) \rangle$ is the element for $\hat{\sigma} = \mathrm{Var}[\vec{v} \cdot \hat{d}]$

Since the matrix elements $\sigma_{ij} = \sigma_{ji}$ commute, the velocity-variance matrix* is Hermitian; hence, it is diagonalizable.

$$T_{\mathrm{apparent}}^{\mathrm{Brysk}} = T_{\mathrm{th}} + M_0 \cdot \left(\sigma'_{xx} \sin^2\theta' \cos^2\phi' + \sigma'_{yy} \sin^2\theta' \sin^2\phi' + \sigma'_{zz} \cos^2\theta'\right)$$

The eigenvalues σ' are hot-spot residual kinetic energies ($\mathrm{RKE_{HS}}$) along three rotated orthogonal axes.

Definition

$$\mathrm{RKE_{HS}} = M_{\mathrm{HS}}\sigma'_{xx}/2 + M_{\mathrm{HS}}\sigma'_{yy}/2 + M_{\mathrm{HS}}\sigma'_{zz}/2$$

* K. M. Woo et. al., Phys. Plasmas 25, 102710 (2018).
** H. Brysk, Plasma Physics 15 611 (1973); the velocity variance term can be obtained by removing the isotropic flow assumption in Brysk's analysis.

Ion-temperature asymmetries in OMEGA experiments are mostly driven by mode 1

$\mathrm{RKE}_{\mathrm{HS}}^{\perp}$ $\mathrm{RKE}_{\mathrm{HS}}^{\mathrm{Jet}}$

Mode $\ell = 1 : \sigma'_{xx} = \sigma'_{yy} \ll \sigma'_{zz}$

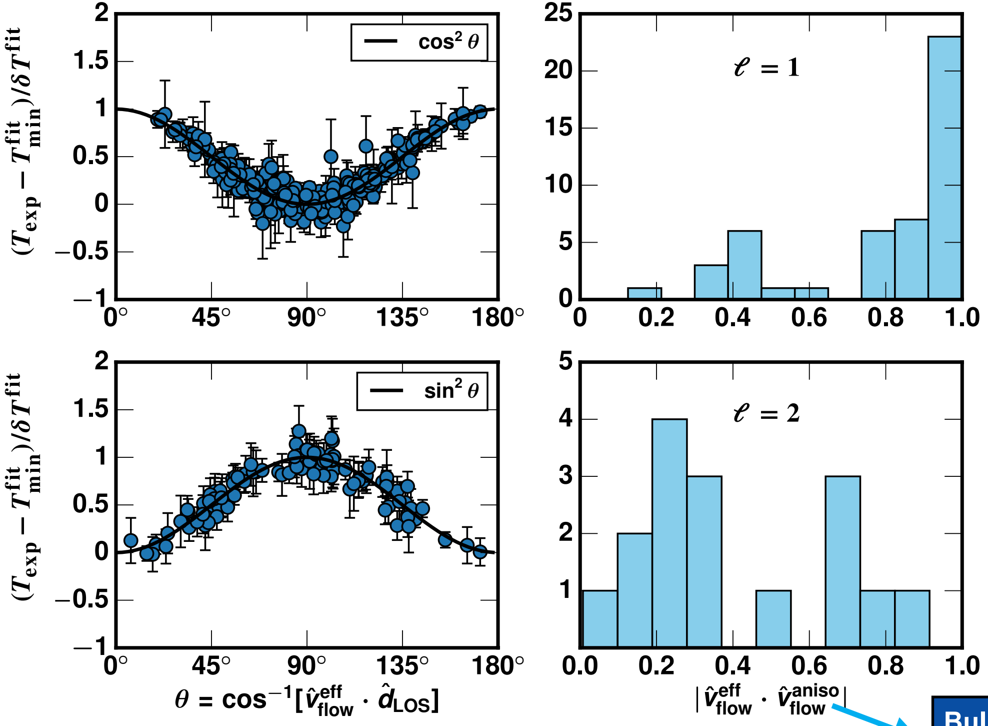
$$T_{\mathrm{i}} = T_{\mathrm{th}} + M_0 \cdot \left(\sigma_{\mathrm{iso}} + \sigma'_{zz} \cos^2 \theta'\right)$$

$\mathrm{RKE}_{\mathrm{HS}}^{\mathrm{Bubble/Spike}}$ $\mathrm{RKE}_{\mathrm{HS}}^{\mathrm{Spike/Bubble}}$

Mode $\ell = 2 : \sigma'_{xx} = \sigma'_{yy} \gg \sigma'_{zz}$

$$T_{\mathrm{i}} = T_{\mathrm{th}} + M_0 \cdot \left(\sigma_{\mathrm{iso}} + \Delta\sigma'_{xx} \sin^2 \theta'\right)$$

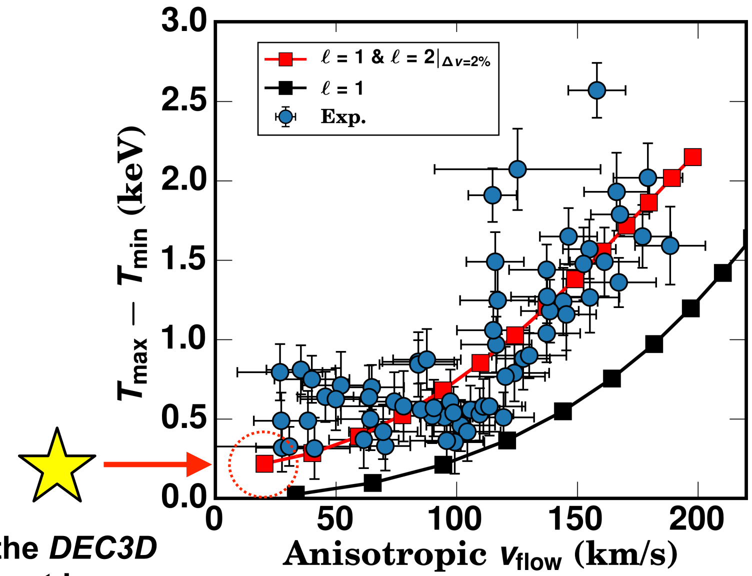
$$\Delta\sigma'_{xx} = \sigma'_{xx} - \sigma_{\mathrm{iso}}$$



Bulk flow

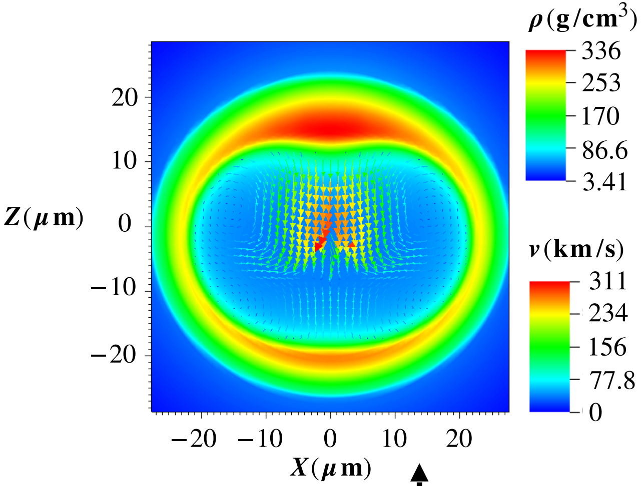
Definition : $\sigma_{\mathrm{iso}} = \min[\sigma'_{xx}, \sigma'_{yy}, \sigma'_{zz}]$

The presence of quasi-isotropic flows from even-*L* modes provides additional ion-temperature asymmetries



See the *DEC3D* hot-spot image on the right.

Indication of mode $\ell = 2$



The flow structure in a mode-1 and mode-2 *DEC3D* simulation.

The *v.d* term in the mode-1 areal-density model* captures the ρR asymmetry**

Mode-1 ρR Model* : $\rho R_{\mathrm{LOS}}^{\mathrm{apparent}} = \langle \rho R_{\mathrm{3D}} \rangle_{\mathrm{AM}} - \Delta\rho R(\hat{v}_{\mathrm{flow}}^{\mathrm{aniso}} \cdot \hat{d}_{\mathrm{LOS}})$

The arithmetic-mean (AM) and harmonic-mean (HM) ρR are related.

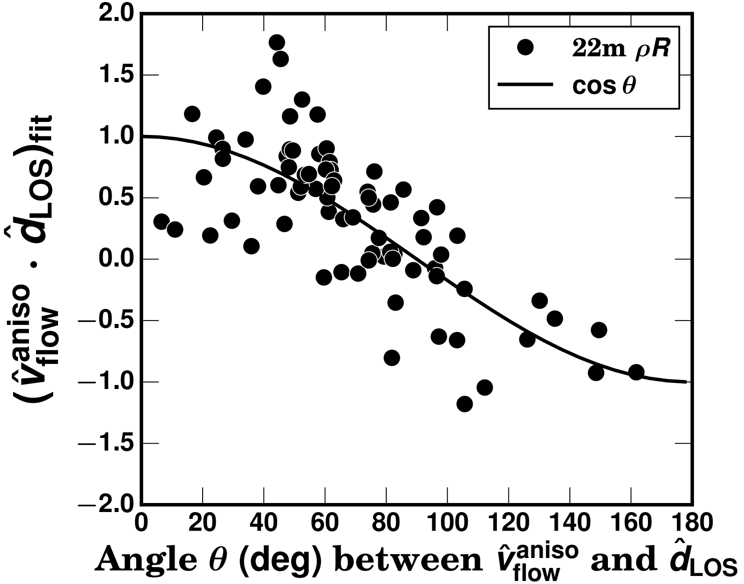
$$\langle \rho R \rangle_{\mathrm{HM}} = \sqrt{\langle \rho R_{\mathrm{3D}} \rangle_{\mathrm{AM}}^2 - (\Delta\rho R)^2}$$

$$\Delta\rho R = (\rho R_{\max} - \rho R_{\min})/2$$

$$\langle \rho R_{\mathrm{3D}} \rangle_{\mathrm{AM}} = \rho R_{\mathrm{1D}} R_T^{\alpha}, \quad \langle \rho R_{\mathrm{3D}} \rangle_{\mathrm{HM}} = \rho R_{\mathrm{1D}} R_T^{\beta}$$

$$R_{\mathrm{T}} = T_{\max}/T_{\min}, \quad \alpha = -0.3, \quad \beta = -0.47$$

$$\rho R_{\mathrm{model}}(\theta, \phi, R_T) = \rho R_{\mathrm{1D}} \cdot \left[R_T^{\alpha} - \sqrt{R_T^{2\alpha} - R_T^{2\beta}} \times \hat{v}_{\mathrm{flow}}^{\mathrm{head}} \cdot \hat{d}_{\mathrm{LOS}}(\theta, \phi) \right]$$

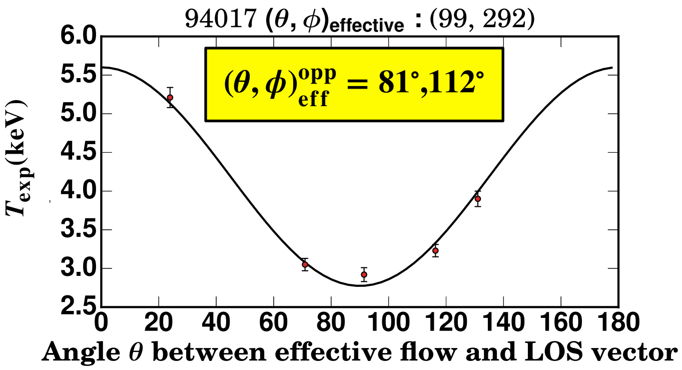


* K. M. Woo *et al.*, Phys. Plasmas 28, 054503 (2021)
** Z. L. Mohamed *et al.* Rev. Sci. Instrum. 92, 043546 (2021)

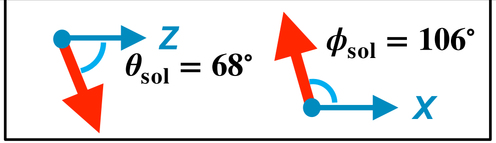
See Jim's talk in NO04.00009.

The mode information is quantified by T_i and hot-spot shape asymmetries

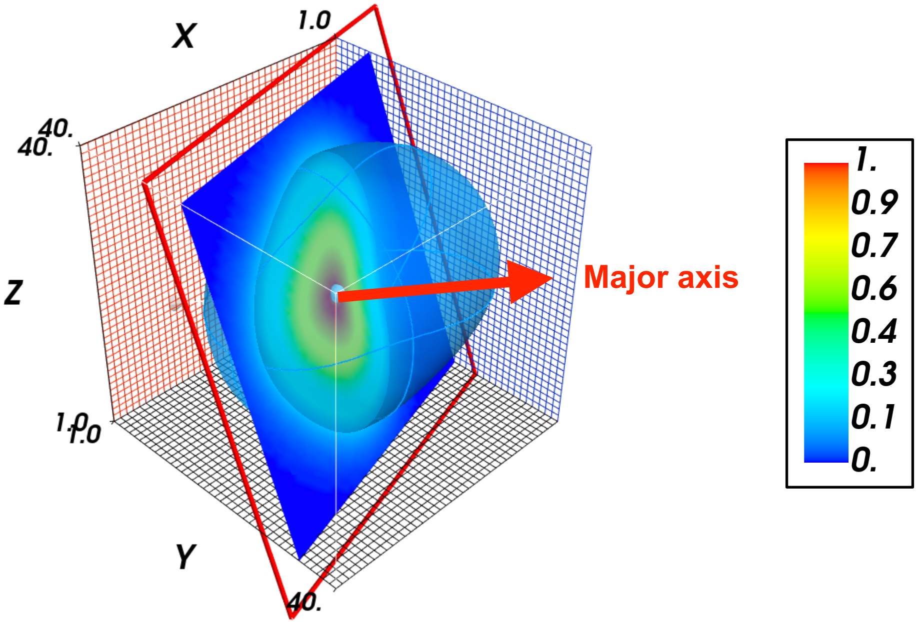
Effective flow reconstruction



Major axis (explained in appendix)



3-D hot-spot reconstruction



A platform of three-dimensional (3-D) hot-spot reconstruction procedures has been developed to quantify 3-D effects of low modes in ICF implosion experiments

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The 3-D analysis will be applied to minimize the low-mode implosion asymmetry in our future work.

See Jim's talk in NO04.00009.
See Kristen's talk in ZO04.00007.

The major axis for mode 1 and prolate mode 2 can be reconstructed using its projection measured at two lines of sight

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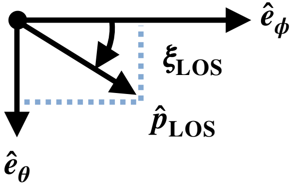
Major axis reconstruction



$$\begin{bmatrix} \hat{p}_1 \cdot \hat{z}\cos(\xi_1) - \hat{e}_{\phi,1} \cdot \hat{z} \\ \hat{p}_2 \cdot \hat{z}\sin(\xi_2) - \hat{e}_{\theta,2} \cdot \hat{z} \end{bmatrix} = \begin{bmatrix} \hat{e}_{\phi,1} \cdot \hat{x} - \hat{p}_1 \cdot \hat{x}\cos(\xi_1) & \hat{e}_{\phi,1} \cdot \hat{y} - \hat{p}_1 \cdot \hat{y}\cos(\xi_1) \\ \hat{e}_{\theta,2} \cdot \hat{x} - \hat{p}_2 \cdot \hat{x}\sin(\xi_2) & \hat{e}_{\theta,2} \cdot \hat{y} - \hat{p}_2 \cdot \hat{y}\sin(\xi_2) \end{bmatrix} \begin{bmatrix} \tan\theta\cos\phi \\ \tan\theta\sin\phi \end{bmatrix}$$

Major axis vector : $\hat{v} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{x} + \cos\theta\,\hat{z}$

Projection vector : $\hat{p}_{\mathrm{LOS}} = \cos\xi_{\mathrm{LOS}}\hat{e}_{\phi,\mathrm{LOS}} + \sin\xi_{\mathrm{LOS}}\hat{e}_{\theta,\mathrm{LOS}}$

Appendix

The *v.d* term in the mode-1 areal-density (ρR) model is shown to capture the ρR variations in OMEGA experiments

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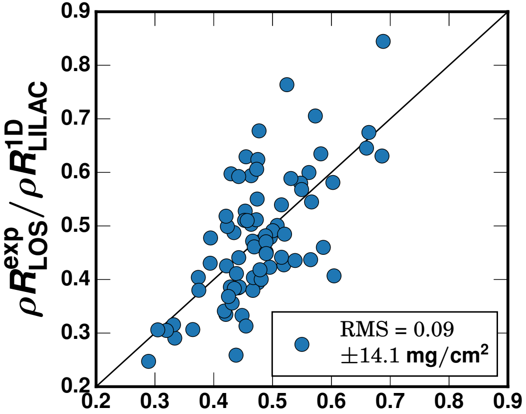
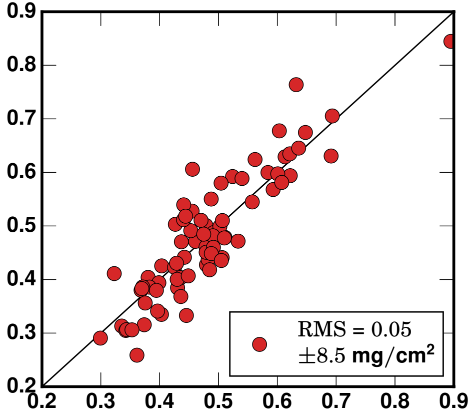
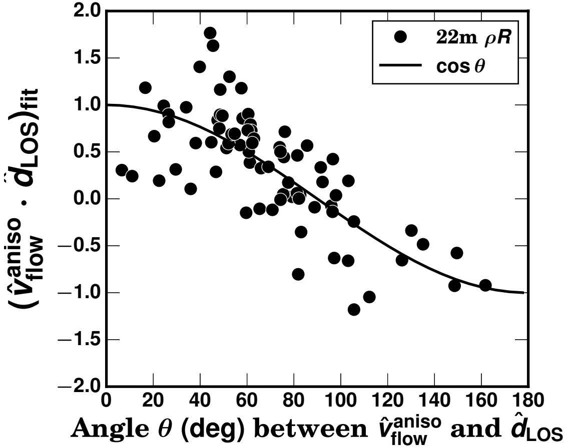
Mode 1 RhoR Model

$$\rho R_{\text{model}}(R_T) = \rho R_{\text{1D}} \cdot \left[R_T^{\alpha} - \sqrt{R_T^{2\alpha} - R_T^{2\beta}} \times \hat{v}_{\text{flow}}^{\text{head}} \cdot \hat{d}_{\text{LOS}}(\theta, \phi) \right]$$

$\alpha = -0.3, \quad \beta = -0.47$







$\text{CR}^{-1.9}\text{IFAR}^{-0.4} R_{\text{B}}^{1.6}$

$\text{CR}^{-1.4}\text{IFAR}^{-0.5} R_{\text{B}}^{0.8} \times$

$$\left(R_T^{\alpha_{\text{fit}}} - \sqrt{R_T^{2\alpha_{\text{fit}}} - R_T^{2\beta}}\, \hat{v}_{\text{aniso}}^{\text{flow}} \cdot \hat{d}_{\text{LOS}} \right)^{\gamma_{\text{fit}}}$$

1-D kernel to account for systematic 1-D coding errors.

3-D kernel to account for mode-1 RhoR asymmetry.

R_T : $T_{\max}/T_{\min}$
R_B : beam/target radii
$\alpha_{\text{fit}} = -0.37, \quad \gamma_{\text{fit}} = 1.55$

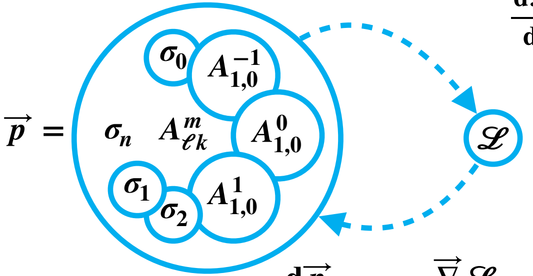
Appendix

The 3-D plasma emissivity model is optimized* by minimizing the fitting error between its 2-D projections and experimental x-ray images

Loss function $\mathscr{L}$

$$\mathscr{L}(\vec{p}) = \frac{1}{N}\sum_{i=1}^{N}\sqrt{\left\langle\left(\mathscr{P}_{\mathrm{LOS}_i}\cdot I^{3\mathrm{D}}(\vec{p}) - I^{\exp}_{\mathrm{LOS}_i}\right)^2\right\rangle}$$

Parameters P

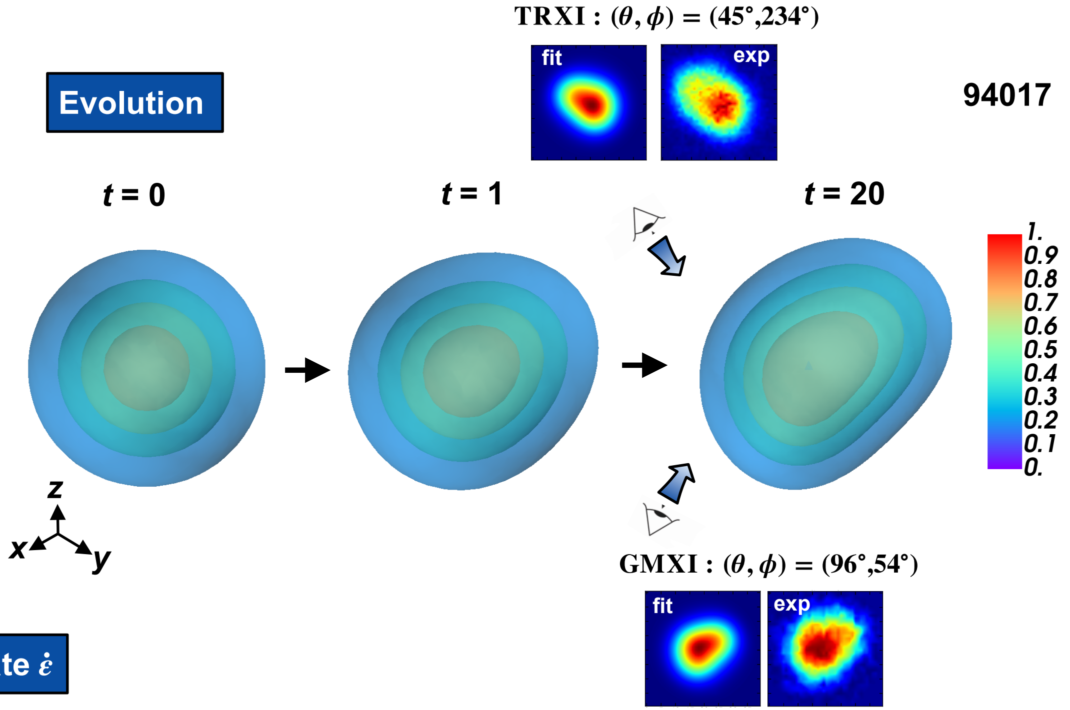


Steepest descent

$$\frac{\mathrm{d}\mathscr{L}}{\mathrm{d}t} = \vec{\nabla}\mathscr{L}\cdot\frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$$

$$\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = -\frac{\vec{\nabla}\mathscr{L}}{|\vec{\nabla}\mathscr{L}|}\dot{\varepsilon}$$

Learning rate $\dot{\varepsilon}$



* Zhenyu Liao *et al.* arXiv:1805.11917