Optimization of Beam-Port Configurations to Minimize Low-Mode Perturbations in High-Yield Inertial Confinement Fusion Targets

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Perturbation scaling with beam number

92-beam configuration

2D DRACO simulations at $t_{\text{bang}}$ - 20 ps

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Summary

Candidate beam configurations can be found through charged-particle simulations initialized with icosahedral symmetry and tested in DRACO

- High-yield ICF targets require good symmetry and minimization of low-mode perturbations
- Beam-port geometry is one major contributor to low-mode shell asymmetries
- The best candidate configurations are based on icosahedrons with charged-particle optimization
- Multidimensional radiation-hydrodynamics simulations are underway, 2D (DRACO) and 3D (ASTER)*

ICF: inertial confinement fusion
* I. V. Igumenshchev et al., NO04.00015, this conference.
Collaborators


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High-yield targets require good shell stability, in particular the dynamic shell has long periods of hydrodynamic implosion/expansion.

- High-yield design
  - requires low shell asymmetry

Wetted DT Foam
Adiabat – 1.4
IFAR - 13
CR – 22
\( v_{\text{imp}} \) – 264 km/s
\( E_{\text{Laser}} \) – 1.10 MJ
\( E_{\text{yield}} \) – 145 MJ
Gain – 132

- Dynamic shell target*
  - several stages of hydrodynamic implosion/expansion to consider for stability

* V. N. Goncharov et al., NO04.00012, this conference
Optimization of irradiation uniformity is well studied; a charged-particle optimization technique has been shown to work

- Beam-ports modelled as charged particles fixed to the surface of a sphere
- Historically referred to as the Thomson Problem**
- Many other techniques exist for beam-port optimization†, ‡

\[ F_i = \sum_{j=1(j \neq i)}^{N_B} A \frac{\mathbf{r}_i - \mathbf{r}_j}{|\mathbf{r}_i - \mathbf{r}_j|^3} - B \frac{d\mathbf{r}_i}{dt} \]

\[ E_p = \frac{1}{2} \sum_{i=1}^{N_B} \sum_{j=1(j \neq i)}^{N_B} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} \]

Figures from Murakami et al. *

Charged-Particle Configuration

Low-mode perturbations scale as $N_B^{1/2}$ with a number of dips at specific $N_B$'s

- An analytic laser-absorption model* used to find Legendre mode contributions
- $\sigma_{\text{rms}}$ is the sum of all mode contributions

$$\sigma_{\text{rms}} = \left[ \sum_{\ell=1}^{\infty} \frac{a_{\ell}^2}{2\ell + 1} G_{\ell}^2 \right]^{1/2}$$

Beam factor

$$a_{\ell} = \frac{2l + 1}{2} \int_{-1}^{1} I_\ell(\cos \theta) P_l(\cos \theta) d(\cos \theta)$$

Geometric factor

$$G_{\ell} = \left[ \sum_{j=1}^{N_B} \sum_{k=1}^{N_B} \frac{P_l(\Omega_j \cdot \hat{r}) P_l(\Omega_k \cdot \hat{r})}{I_{\ell}^2} \right] / N_B$$

$r_{\text{beam}} - r$ at 95% integrated beam energy

$$I(r) = I_0 \exp \left( -\left( \frac{r}{r_{\text{beam}}} \right)^n \right)$$

- Significant drop in $\sigma_{\text{rms}}$ observed at $N_B = 72$

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The highest-performing beam-port configurations share icosahedral symmetry

- Class of geometric shapes with icosahedral symmetry
- Formed by subdividing triangular faces and projecting vertices onto the surface of a sphere

Two Classes
- Face-centered points: $N_B = 20, 60, 80, 140, 180…$
- Vertex-centered points: $N_B = 12, 32, 42, 72, 92, 122, 132, 172…$

Images accessed from
The high performance of icosahedral configurations comes from the suppression of the $\ell = 6$ Legendre mode

- Typical charged-particle configurations have small contributions from many modes
- Dip in the single beam factor at $\ell = 6$ significantly reduces $\sigma_{\text{rms}}$
- Decaying single-beam factor means higher modes are less important

$$
\sigma_{\text{rms}} = \left[ \sum_{n=1}^{\infty} \frac{a_n^2}{2n + 1} G_n^2 \right]^{1/2}
$$
Geodesic icosahedral configurations show much lower $\sigma_{\text{rms}}$ than the random charged-particle configurations.

$$\sigma_{\text{rms}} = \left[ \sum_{n=1}^{\infty} \frac{a_n^2}{2n + 1} G_n^2 \right]^{1/2}$$

Geodesic icosahedral configurations
Minimization of $\ell = 6$

Geodesic icosahedral configurations are further improved with charged-particle optimization

92-beam configuration with charged-particle distortion

Changes to the spectral structure of the modes

Minimization of $\ell = 6$

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Changes to the spectral structure of the modes
Geodesic icosahedral configurations with charged particle optimization show highest performance

\[ \sigma_{\text{rms}} = \left[ \sum_{n=1}^{\infty} \frac{a_n^2}{2n + 1} G_n^2 \right]^{1/2} \]

Geodesic icosahedral configurations

Geodesic icosahedral + charged-particle configurations
The analytic laser deposition model is not sufficient, 2D radiation hydrodynamics modelling was carried out with DRACO for 4 configurations.
Two-Dimensional Radiation Hydrodynamics

Some of the suppression of the $\ell=6$ mode is lost but crucially the contributions from lowest modes is smallest for the icosahedral configurations.

**Graphs:**
- **80-beam cp**
  - Mode number vs $\sigma_{\text{rms}}$ ($\mu$m)
  - $\ell < 6$ vs $\ell = 6$ vs $\ell > 6$ Total
  - $\sigma_{\text{rms}}$ ($\mu$m) = 0.12 0.09 0.28 0.32

- **92-beam cp**
  - Mode number vs $\sigma_{\text{rms}}$ ($\mu$m)
  - $\ell < 6$ vs $\ell = 6$ vs $\ell > 6$ Total
  - $\sigma_{\text{rms}}$ ($\mu$m) = 0.08 0.16 0.27 0.32

- **92-beam icosahedral**
  - Mode number vs $\sigma_{\text{rms}}$ ($\mu$m)
  - $\ell < 6$ vs $\ell = 6$ vs $\ell > 6$ Total
  - $\sigma_{\text{rms}}$ ($\mu$m) = 0.03 0.86 0.30 0.91

- **92-beam icosahedral + cp**
  - Mode number vs $\sigma_{\text{rms}}$ ($\mu$m)
  - $\ell < 6$ vs $\ell = 6$ vs $\ell > 6$ Total
  - $\sigma_{\text{rms}}$ ($\mu$m) = 0.02 0.22 0.29 0.36

**Legend:**
- cp: charged particle
- ico: icosahedral
- icocp: icosahedral + charged particle

TC15657

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Summary/Conclusions

Candidate beam configurations can be found through charged-particle simulations initialized with icosahedral symmetry and tested in DRACO

- High-yield ICF targets require good symmetry and minimization of low-mode perturbations
- Beam-port geometry is one major contributor to low-mode shell asymmetries
- The best candidate configurations are based on icosahedrons with charged-particle optimization
- Multidimensional radiation-hydrodynamics simulations are underway, 2D (DRACO) and 3D (ASTER)*

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Additional Slides

Charged Particle Simulation

- Calculates Coulomb Force
- Updates using suvat eqns

\[ F_i = \sum_{j=1}^{NB} A \frac{\hat{r}_i - \hat{r}_j}{|\hat{r}_i - \hat{r}_j|^3} - B \frac{d\hat{r}_i}{dt} \]

\[ E_p = \frac{1}{2} \sum_{i=1}^{NB} \sum_{j=1(j \neq i)}^{NB} \frac{1}{|\hat{r}_i - \hat{r}_j|} \]

Convergence: \( E_{t+1} - E_{t-1} < 10^{-11} \)

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**Analytic rms non-uniformity model**

**Single beam factor** comes from the beam profile, expanding the irradiation profile along the target surface into Legendre polynomials

\[
a_l = \frac{2l + 1}{2} \int_{-1}^{1} I_a(\theta) P_l(\cos \theta) d(\cos \theta)
\]

- \(l\) - mode number
- \(P_n\) – Legendre Polynomial
- \(\theta\) – Angle from beam axis

**Geometric factor** comes from the relative pointings and powers of each beam

\[
G_l = \frac{\sum_{j=1}^{NB} \sum_{k=1}^{NB} P_l(\Omega_j \cdot \Omega_k) I_j I_k}{I_T^2} / NB
\]

- \(\sigma_{rms}\) - Root Mean Square

\[
\sigma_{rms} = \left( \sum_{l=1}^{\infty} \frac{a_l^2}{2l + 1} \left( G_l^2 + \frac{\sigma_{sys}^2}{NB} \right) \right)^{1/2}
\]
Single beam factor

\[ I(r) = I_0 \exp \left( -\left( \frac{r}{r_0 \alpha} \right)^\beta \right) \]

\[ I_a(\theta) = I_0 \left[ 1 - (1 - \eta)^{\cos^3 \theta} \right] \exp \left[ -(\sin \theta / \alpha)^\beta \right] \cos \theta \]

\[ a_l = \frac{2l + 1}{2} \int_{-1}^{1} I_a(\theta)P_l(\cos \theta) d(\cos \theta) \]
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Geometric Factor

$$G_l = \left[ \sum_{j=1}^{N_B} \sum_{k=1}^{N_B} \frac{P_l(\Omega_j \cdot \Omega_k)I_jI_k}{I_T^2} \right] / N_B$$

$$I_j = I_k, \frac{I_jI_k}{I_T^2} = 1$$

$$P_l(\Omega_j \cdot \Omega_k) = \frac{4\pi}{2l + 1} \sum_{m=-l}^{l} Y_{lm}^*(\Omega_j) Y_{lm}(\Omega_k)$$
Super-Gaussian Parameter Dependence

The performance of a beam configuration varies with the Super-Gaussian spot shape parameters

\[ I(r) = I_0 \exp \left( -\frac{r}{r_{\text{beam}}} \right)^n \]

\[ r_{\text{beam}} - r \] at 95% integrated beam energy

\( \sigma_{\text{rms}} \text{ for } N_B = 92 \)
Scaling with $\sigma_{\text{sys}}$

\[
\sigma_{\text{rms}} = \left[ \sum_{n=l}^{\infty} \frac{a_i^2}{2l + 1} \left( \sigma_i^2 + \frac{\sigma_{\text{sys}}^2}{N_B} \right) \right]^{\frac{1}{2}}
\]

Face Centered

Vertex Centered

80 beams

92 beams
Icosahedral with charged particle configurations have the lowest contribution from modes below dominant mode number.

TC15697

\[ \sigma_{\text{rms}} (\%) \]

\[
\begin{array}{cccc}
\ell < 6 & \ell = 6 & \ell > 6 & \text{Total} \\
0.10 & 0.04 & 0.13 & 0.17 \\
0.07 & 0.06 & 0.08 & 0.12 \\
\end{array}
\]

\[ \sigma_{\text{rms}} (\mu \text{m}) \]

\[
\begin{array}{cccc}
\ell < 6 & \ell = 6 & \ell > 6 & \text{Total} \\
0.02 & 0.33 & 0.11 & 0.35 \\
0.01 & 0.08 & 0.11 & 0.14 \\
\end{array}
\]

cp: charged particle
ico: icosahedral
icocp: icosahedral + charged particle

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Mode structure in DRACO Simulations inner shell position at CR=2

**2D Radiation Hydrodynamics**

80 beam cp

<table>
<thead>
<tr>
<th>$\sigma_{rms}$ (µm)</th>
<th>$\ell &lt; 6$</th>
<th>$\ell = 6$</th>
<th>$\ell &gt; 6$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>0.02</td>
<td>0.02</td>
<td>0.08</td>
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</tbody>
</table>

92 beam cp

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92 beam icosahedral

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<th>Total</th>
</tr>
</thead>
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<tr>
<td>0.02</td>
<td>0.24</td>
<td>0.3</td>
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92 beam icosahedral+cp

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<th>$\sigma_{rms}$ (µm)</th>
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<th>Total</th>
</tr>
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<td>0.01</td>
<td>0.06</td>
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<td>0.07</td>
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Mode structure in DRACO Simulations inner shell position at 2.225ns (bt = 2.45ns)

<table>
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<tr>
<th>Mode Structure</th>
<th>( \ell &lt; 6 )</th>
<th>( \ell = 6 )</th>
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<td>92 beam icosahedral</td>
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<td>0.30</td>
<td>0.91</td>
</tr>
<tr>
<td>92 beam icosahedral+cp</td>
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