Implementation and Verification of LC Circuit for Z-pinch FLASH Simulations

K. Moczulski[1], A. Reyes[1], M.B.P. Adams[1], A. Armstrong[1], P. Farmakis[1], E. Hansen[1], Y. Lu[1], D. Michta[1], D.Q. Lamb[2], P. Tzeferacos[1]

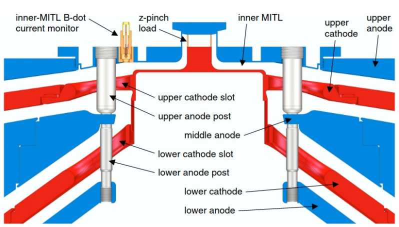
[1]University of Rochester, [2]University of Chicago

Abstract:

High-fidelity simulations of pulsed-power-driven high energy density physics (HEDP) experiments frequently necessitate the modeling of the electrical interplay of current between the vacuum-insulator stack and the load (i.e., the plasma generated across the anode-cathode gap). Recent additions to the FLASH code, the high-performance computing, radiation magneto-hydrodynamics (MHD) code developed by the Flash Center for Computational Science, have enabled it to model Z-pinch experiments. Here we discuss the implementation in FLASH of the circuit model presented in in McBride et al[1]. who proposed a simple LC model to be used as a drive circuit in refurbished Z accelerator simulations. We outline the numerical implementation and show results from select verification benchmarks.

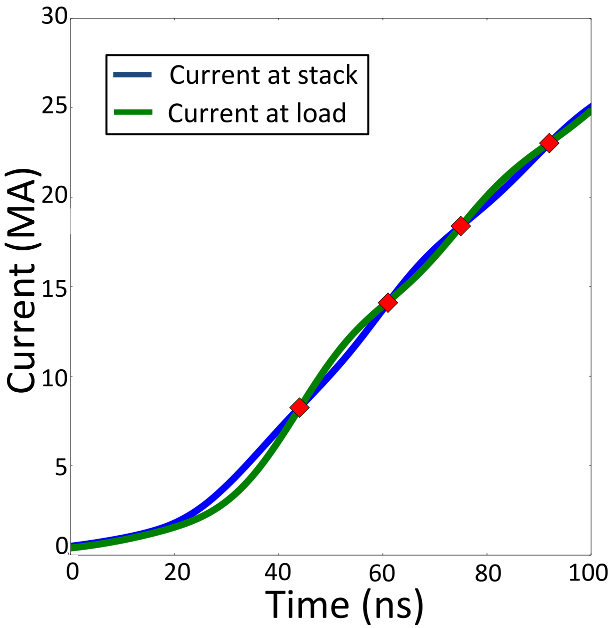
Z-Machine:

The refurbished Z accelerator can deliver pulses of approximately 4MV and 26MA that last between 100-600ns. To achieve said power requirements, the current and voltage are passed through four magnetically insulated transmission lines (or MITLs). The voltage and current are measured at the start of the outer MITLs (this location is called the *stack*) which are then connected through a double post-hole convolute. In the Z-machine the double post-hole convolute connects the outer MITLs to inner MITLs which deliver power to the load where the current is measured right before the load. When comparing the *stack* and *load* currents, there were times that the load current was higher than that of the stack. To understand this discrepancy, a lossless, simplified model was implemented.



The FLASH Code:

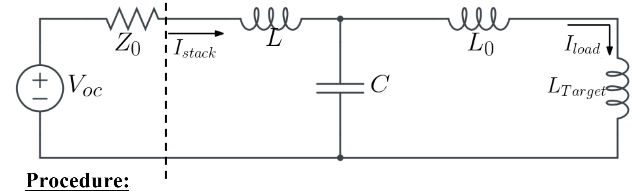
FLASH[3] is a multi-physics, parallel, adaptive mesh refinement (AMR), finite-volume Eulerian hydrodynamics and magneto-hydrodynamics (MHD) code (flash.rochester.edu). The Flash Center for Computational Science at the Department of Physics and Astronomy at the University of Rochester further develops capabilities of the FLASH code for high energy-density physics (HEDP) simulations, such as Z-pinch experiments.



Model:

To model the inner part of the Z machine multiple simplifications are made. The first is that the outer MITL is represented by an inductor and capacitor, L and C respectively. This simplifies the actual geometry that is in fact multiple MITLs in parallel. The second is that the inner MITLs are an inductor L_0 instead of multiple inner MITLs that connect to the target. The target is also considered to be an inductor, L_{target}. Finally, the model is considered lossless resulting in no real resistive losses included.

To calculate the current delivered to the load by an input open circuit voltage three equations are used. These result in the following equations, where V_t is the voltage across the target, I_{stack} is the current through the stack, and I_{load} is the current delivered to the load. To calculate V_t Faraday's Law was used. To calculate magnetic flux from the B_ϕ component of the magnetic field FLASH was used. Once the equations are solved the axial current that is determined from the circuit model is used with Ampere's Law to define the azimuthal magnetic field on the upper-r domain boundary. This field is used as a boundary condition for the implicit magnetic resistivity (diffusion) solver.



Procedure:

The procedure that is implemented of this model in FLASH goes as the following:

1. Set all the old values and V_t=0 at t=0 and apply voltage input
2. Solve for V_C^{new}, I_{stack}^{new}, and I_{load}^{new} at t=0
3. Move values from "new" variables to "old" variables
4. Apply solution as boundary condition to FLASH simulation
5. Use FLASH to advance Z-pinch simulations
6. Calculate newly generated magnetic flux and apply Faraday's Law to calculate V_t at new time
7. Repeat this process using newly calculated V_t, V_c^{old}, I_{stack}^{old}, and, I_{load}^{old}

$$\begin{pmatrix} \frac{V_c^{new} - V_c^{old}}{dt} \\ \frac{I_{stack}^{new} - I_{stack}^{old}}{dt} \\ \frac{I_{load}^{new} - I_{load}^{old}}{dt} \end{pmatrix} = \begin{pmatrix} \frac{I_{stack}^{new} - I_{load}^{new}}{C} \\ \frac{-V_c^{new} - Z_o I_{stack}^{new} + V_{oc}}{L} \\ \frac{V_c^{new} - V_t}{L_o} \end{pmatrix}$$

Results:

The resulting plot shows the stack and load current with an open voltage source that has a rise time of approximately 100ns. As seen in the plot there are multiple instances where the stack and load current cross, resulting in a higher current applied to the load. This replicates the issue seen at the Z accelerator. The capacitor acts to store up the electrical energy in the system until the input voltage slows, resulting in the MITL (portrayed as the capacitor) discharging. This allows a higher current applied to the load than is coming into the stack. The simplistic differencing method allows for a quick correction to the applied current. The result of this is a computationally inexpensive correction to the applied current. These results mirror the work done by McBride et al[1] as seen by the multiple positions that the stack and load current cross. This model is currently being applied to Z-pinch simulations done in collaboration with scientists at MIFTI.

References:

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