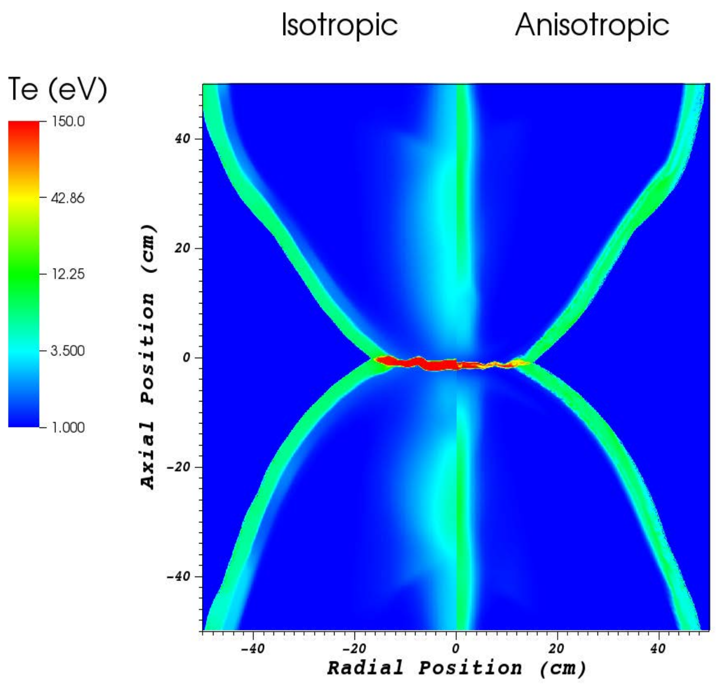
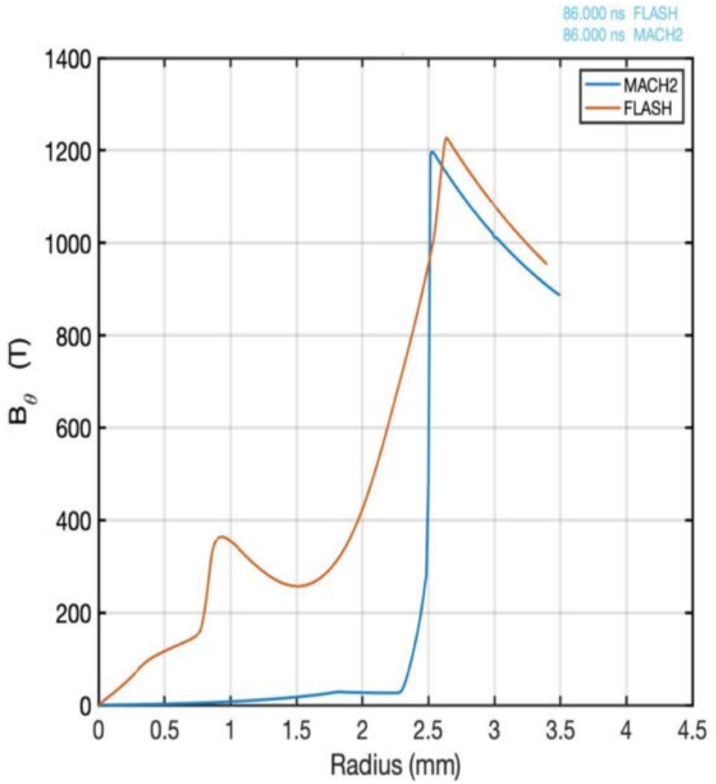
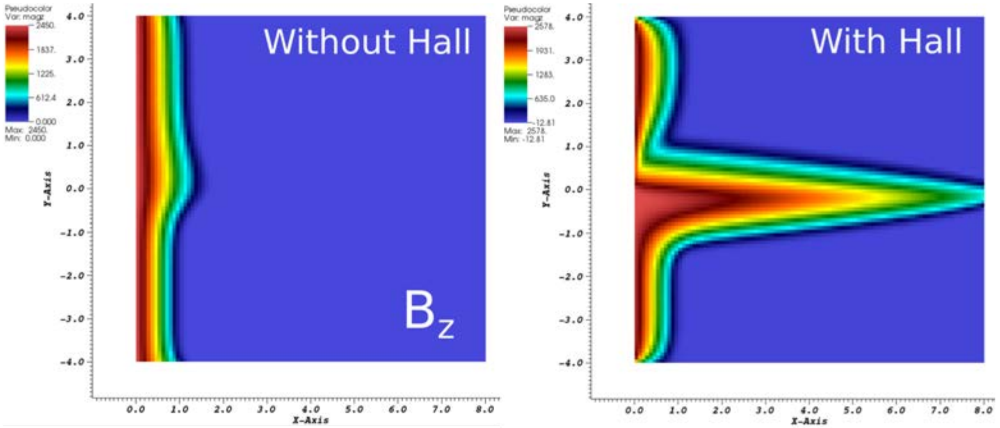
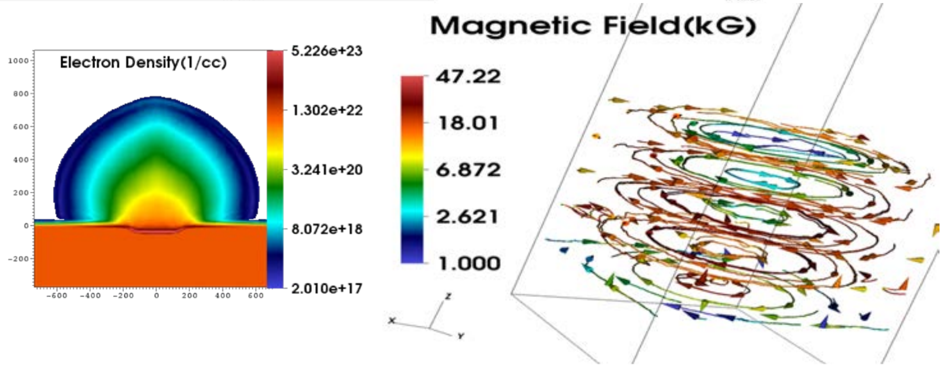
Extended Magnetohydrodynamics in the *FLASH* Code









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Collaborators

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Summary

- *FLASH* is a multi-physics magnetohydrodynamics (MHD) code that includes adaptive mesh refinement (AMR), radiation, laser ray-tracing, extended MHD effects, and more. As part of the extended MHD capabilities, Hall and Biermann terms are already in the release version of *FLASH*.

- Recent code development under the auspices of ARPA-E and DOE NNSA has greatly increased *FLASH*'s ability to model a broader range of magnetized plasma regimes.

  - Anisotropic thermal conduction - important for any problem in which heat transport is affected by magnetic fields (typically when Hall parameter $\gtrsim 1$).

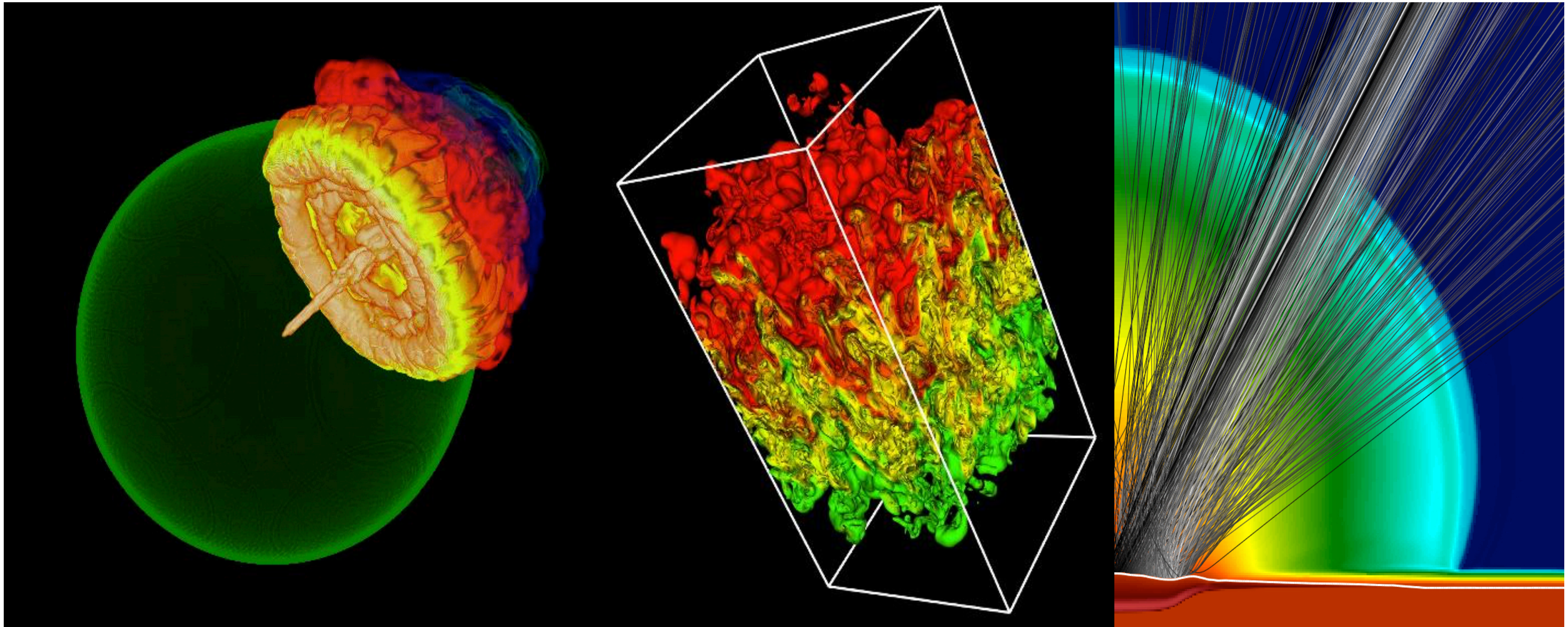
  - Anisotropic magnetic resistivity – the effects of anisotropy are important for strongly-magnetized plasmas, and the implicitly-solved implementation is crucial for studying problems with high resistivity (e.g., Z-pinches and cold plasmas).

  - Thermoelectric effects – includes Seebeck, Righi-Leduc, and Nernst terms. We know these effects can play an important role in magneto-inertial fusion concepts.

The FLASH code

- *FLASH* is a publicly available, high performance computing (HPC), adaptive mesh refinement (AMR), finite-volume, hydro and MHD code with extended physics capabilities. Supported primarily by the U.S. DOE NNSA.
- *FLASH* is professionally managed software in continuous development for 20 years: coding standards; version control; daily automated regression testing; extensive documentation; user support; integration of extensive code contributions from external users.

> 3,500 users world wide **>1,200 papers published with FLASH**



From ideal to extended MHD

- Ideal MHD uses a relatively simple Ohm's Law in which magnetic field is "frozen-in" and must follow motion of plasma.

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B}$$

- Extended MHD uses a much more complex generalized Ohm's Law, which takes on this form in *FLASH* with logical switches to turn on/off each term:

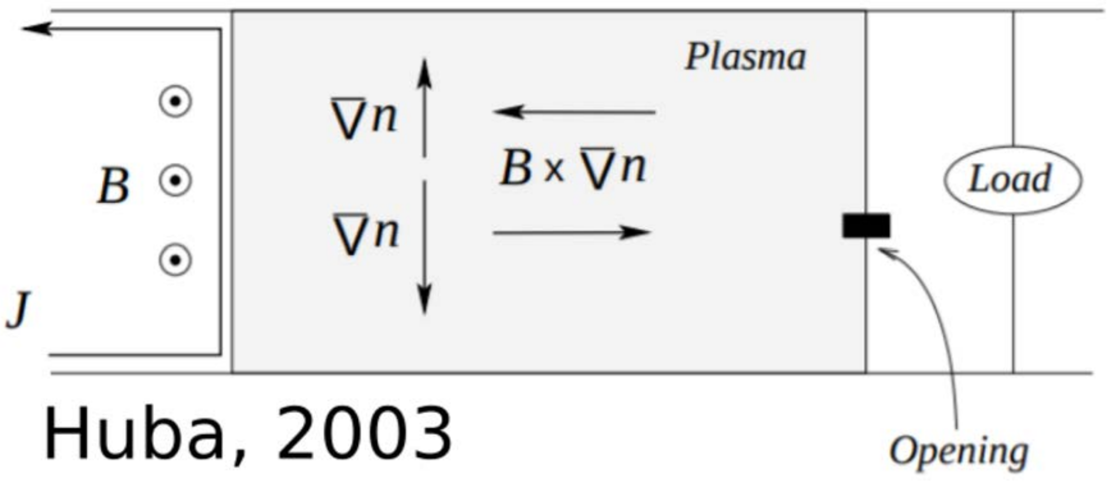
$$\begin{aligned}
\mathbf{E} &= -\mathbf{u} \times \mathbf{B} + \frac{\mathbf{J}}{n_e e} \times \mathbf{B} - \frac{\nabla P_e}{n_e e} \\
&+ \frac{\eta_{||}}{\epsilon_0 \omega_{pe}^2 \tau_{ei}} \mathbf{b}(\mathbf{b} \cdot \mathbf{J}) + \frac{\eta_{\perp}}{\epsilon_0 \omega_{pe}^2 \tau_{ei}} \mathbf{b} \times (\mathbf{J} \times \mathbf{b}) - \frac{\eta_{\wedge}}{\epsilon_0 \omega_{pe}^2 \tau_{ei}} (\mathbf{b} \times \mathbf{J}) \\
&- \beta_{||} \mathbf{b}(\mathbf{b} \cdot \nabla T_e) - \beta_{\perp} \mathbf{b} \times (\nabla T_e) \times \mathbf{b}) - \beta_{\wedge} (\mathbf{b} \times \nabla T_e)
\end{aligned}$$

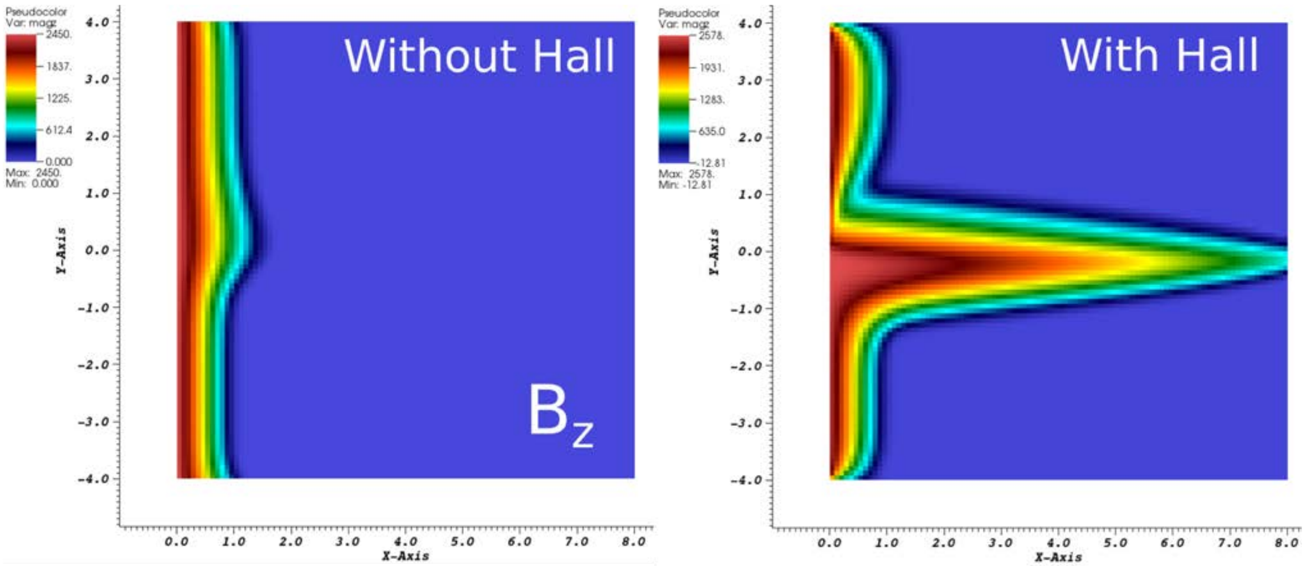
Hall term

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B} + \frac{\mathbf{J}}{n_e e} \times \mathbf{B}$$

- Hall term is implemented via an explicit flux-based method, and the velocity used to determine the computational time step must now include the current.
  - Dispersive Whistler waves and Hall drift waves are introduced (wave speeds can also affect time step on scales smaller than ion inertial length).

- Plasma opening switch shows how including the Hall term leads to drastically different B-field behavior.





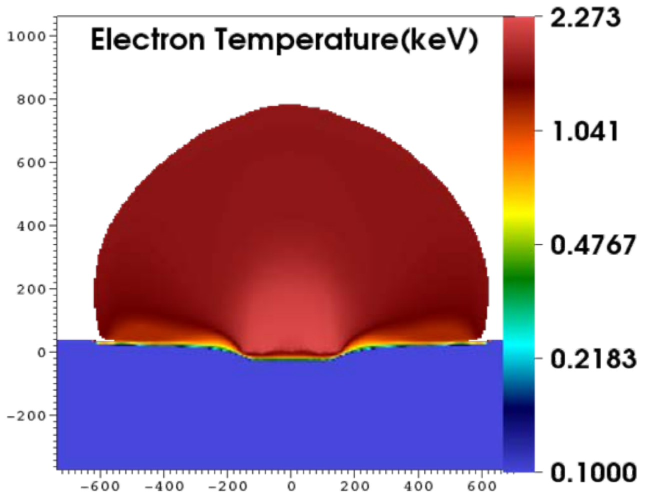
Biermann term

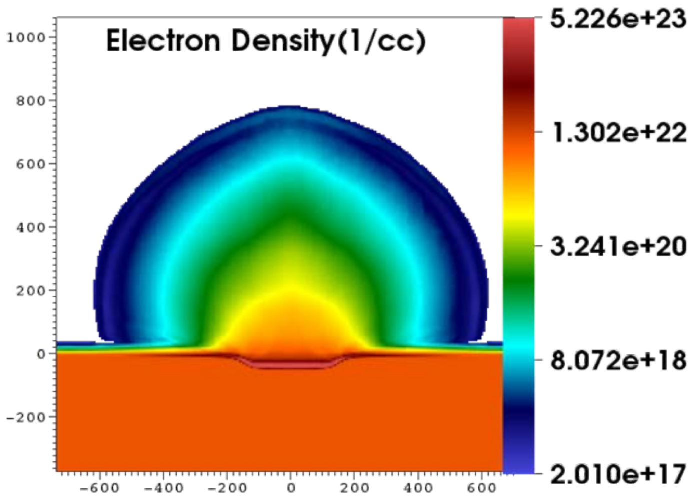
$$\mathbf{E} = -\mathbf{u} \times \mathbf{B} - \frac{\nabla P_e}{n_e e}$$

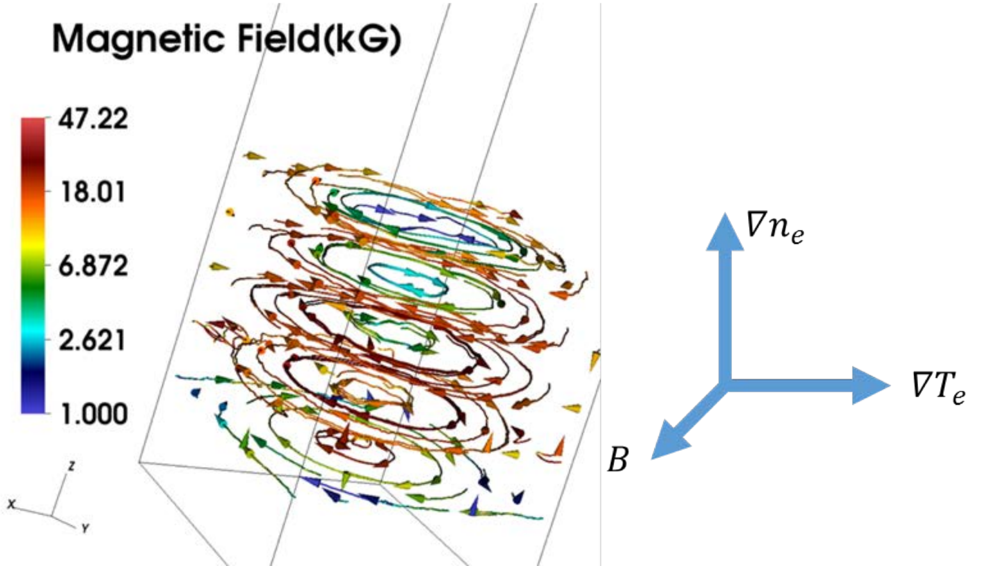
- *FLASH* has two implementations of the Biermann term:
  - An explicit flux-based method similar to the method for the Hall term.
  - A source term method in which the Biermann term is not included as a magnetic flux (appropriate for 1D and 2D cylindrical problems).

- Thermal-magnetic waves are introduced (wave speed proportional to sound speed).

- Laser-ablated CH foil shows Biermann-generated fields (credit: Yingchao Lu).







Anisotropic thermal conduction

- Isotropic case is rather simple: $\frac{\partial e_{int}}{\partial t} + \nabla \cdot (-\kappa \nabla T) = 0.$

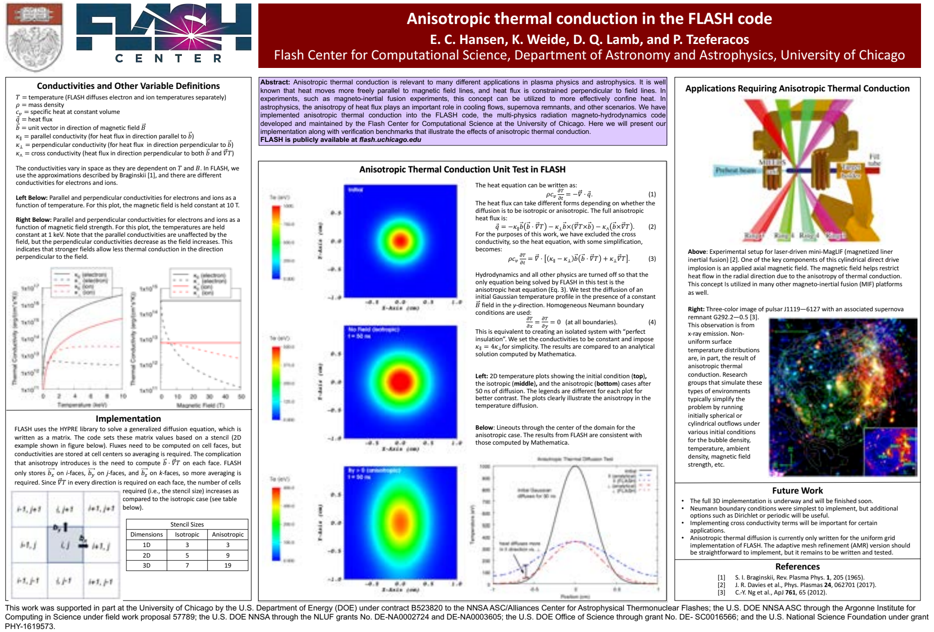
- Anisotropic case can be written in terms of three conductivity coefficients:

$$\frac{\partial e_{int}}{\partial t} + \nabla \cdot (\quad \kappa_{\perp} \nabla T \quad (\kappa_{\|} \quad \kappa_{\perp})\mathbf{b}(\mathbf{b} \cdot \nabla T) \quad \kappa_{\wedge}(\mathbf{b} \times \nabla T)) = 0$$

  - In *FLASH*, this is solved implicitly with an iterative matrix inversion method using LLNL's HYPRE library (Falgout+ 2004, 2005).

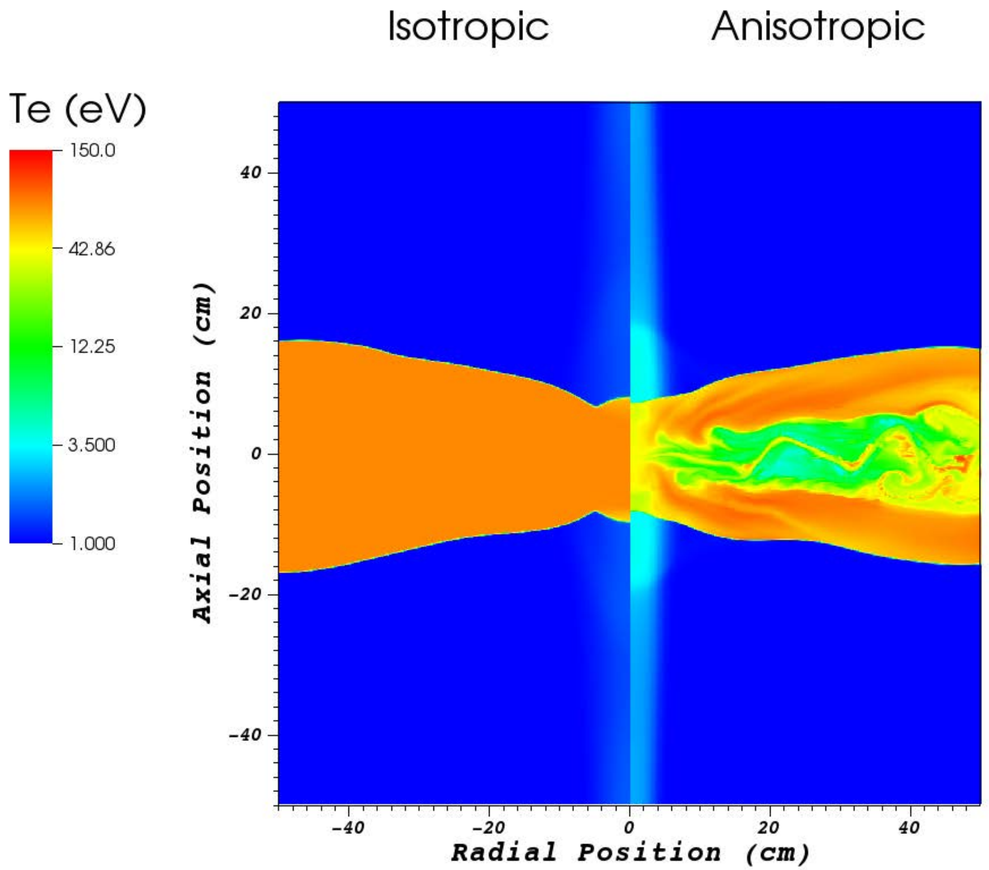
$\kappa_{\perp} < \kappa_{\|}$ implies thermal diffusion is hindered perpendicular to magnetic fields.

Poster from 2019 APS DPP meeting showcases analytic test, verifying the code's implementation.



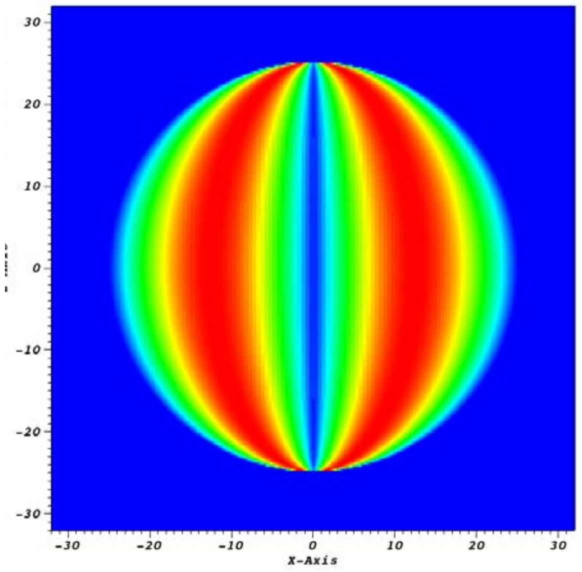
Plasma Liner Experiment (PLX)

- In the experiment, magnetized plasma jets collide to form a target region and additional, heavier jets form a liner that compresses the target.
- In these 2D cylindrical simulations, two target-forming jets collide, and we observe different temperature profiles when using isotropic vs anisotropic thermal conduction.



- Azimuthal B-field peaks at accretion shocks that surround target, so when modeling with anisotropic thermal conduction, this creates a hot shell.

B_ϕ (x-z)



- Testing how thermal conduction behaves in different field configurations during target compression phase.

See Chuang Ren's talk Wed. for more details about our PLX collaboration.

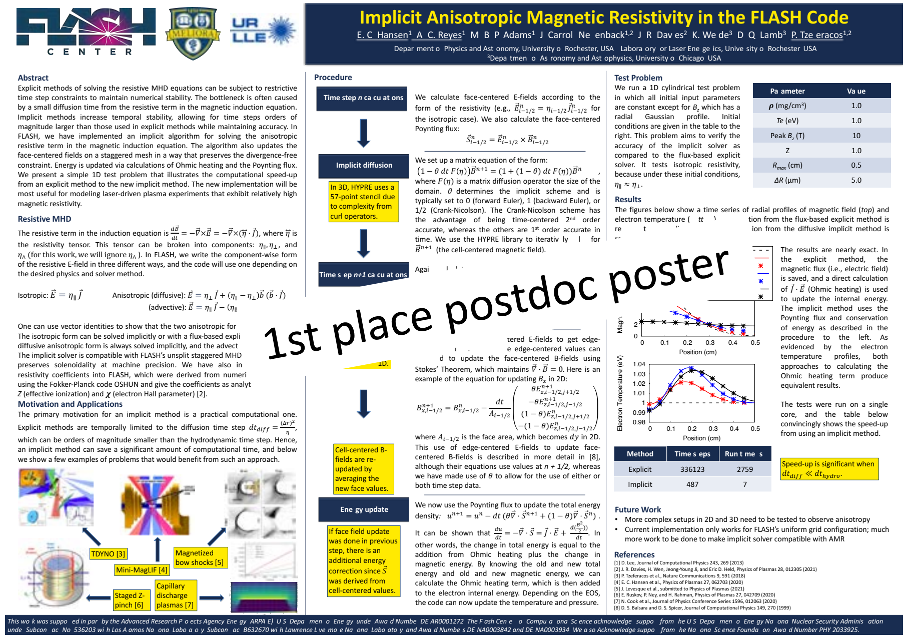
Anisotropic magnetic resistivity

$$\mathbf{E} = -\mathbf{u}\times\mathbf{B} + \frac{\eta_{\parallel}}{\epsilon_0\omega_{pe}^2\tau_{ei}}\mathbf{b}(\mathbf{b}\cdot\mathbf{J}) + \frac{\eta_{\perp}}{\epsilon_0\omega_{pe}^2\tau_{ei}}\mathbf{b}\times(\mathbf{J}\times\mathbf{b}) - \frac{\eta_{\wedge}}{\epsilon_0\omega_{pe}^2\tau_{ei}}(\mathbf{b}\times\mathbf{J})$$

- Like thermal conduction, this is solved implicitly with HYPRE, but it is a much more complicated system of equations with coupled B-field components.
- Using the Poynting flux and conservation of energy, Ohmic heating can be included in a way that is consistent with the implicit magnetic diffusion.
- $\eta_{\wedge}$ term is added to the explicit Hall flux.

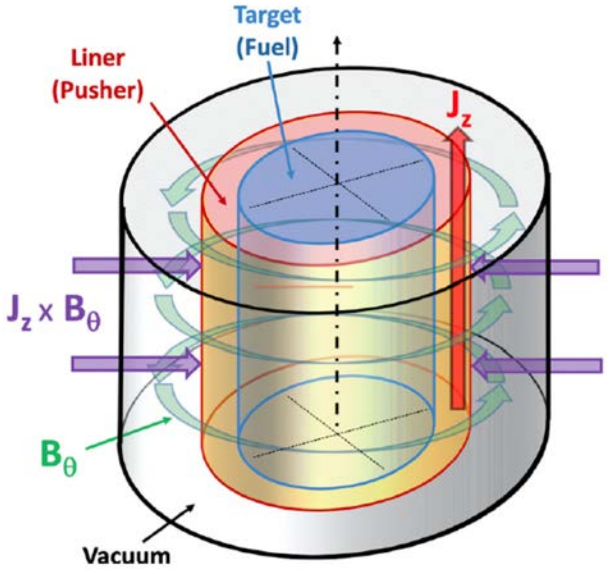
Implicit solve can save orders of magnitude in computational time for problems with high resistivity.

Poster from 2021 NIF/JLF UGM describes the procedure for solving this system implicitly.



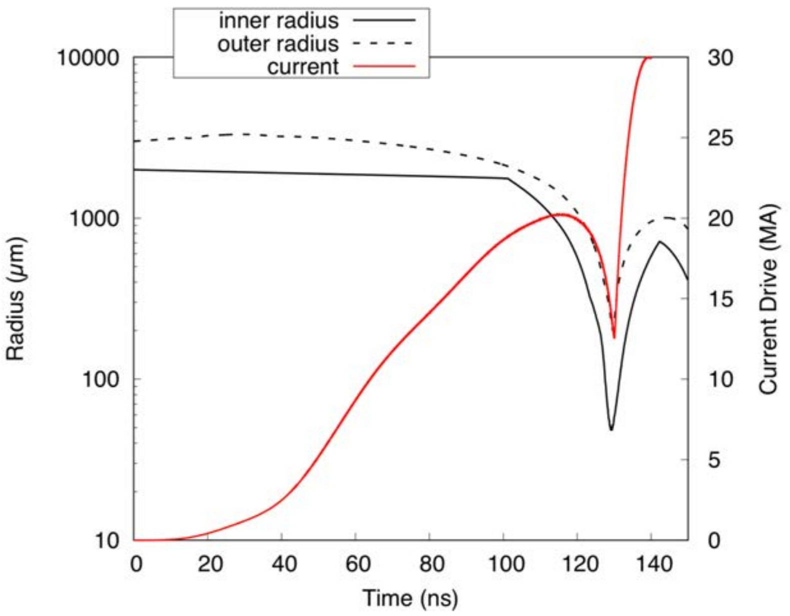
Staged Z-pinch (SZP)

- High-Z liner is imploded with an applied axial current and compresses a DT fuel.
- *FLASH*'s implicit resistivity solver implementation has been verified by comparing 1D cylindrical models with results from the *MACH2* code.

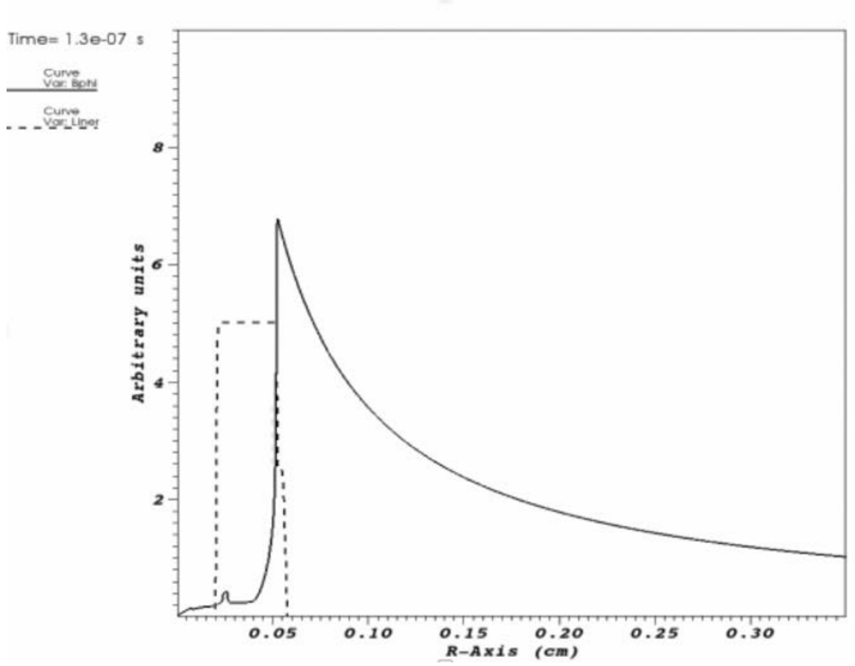


Ruskov+ (Phys. Plasmas 2020)

Shell trajectory from a simplified model without radiation.



B-field profile shows expected diffusion.



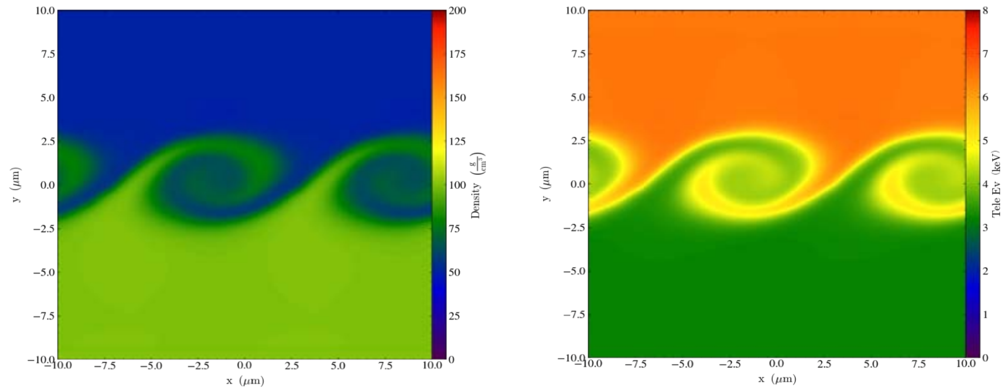
Ohmic heating at liner/vacuum interface decreases resistivity → prevents B-field from diffusing into liner.

Thermoelectric effects

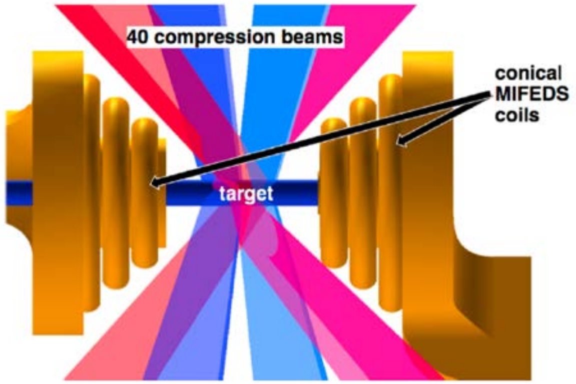
$$\mathbf{E} = -\mathbf{u}\times\mathbf{B} - \beta_{\parallel}\mathbf{b}(\mathbf{b}\cdot\nabla T_e) - \beta_{\perp}\mathbf{b}\times(\nabla T_e)\times\mathbf{b}) - \beta_{\wedge}(\mathbf{b}\times\nabla T_e)$$

- Terms can be rewritten into $\beta_{\parallel}$, $(\beta_{\parallel} - \beta_{\perp})$, and $\beta_{\wedge}$ terms. Nomenclature varies in literature, but in *FLASH*, these terms are called:
  - $\beta_{\parallel}$ = Seebeck
  - $(\beta_{\parallel} - \beta_{\perp})$ = Righi-Leduc. Replaced with β_{new} for $Z \geqslant 1$ (Davies+, Phys. Plasmas 2021)
  - $\beta_{\wedge}$ = Nernst
- Additionally, all terms appear in the electron heat flux: $\boldsymbol{q} = -\frac{T_e}{e}\boldsymbol{\beta}\cdot\boldsymbol{j}$
- All terms are implemented with an explicit flux-based approach.

Currently in the process of testing the Nernst term with a KHI problem (credit: Yingchao Lu).



We know Nernst plays an important role in some MIF concepts like mini-MagLIF (Hansen+, Phys. Plasmas 2020).



New transport coefficients

- We have implemented some of the most up-to-date transport coefficients from the literature:
    - Thermal conductivity coefficients ($\kappa_\parallel$, $\kappa_\perp$, $\kappa_\wedge$) of Ji & Held (Phys. Plasmas 2013).
    - Resistivity coefficients ($\eta_\parallel$, $\eta_\perp$, $\eta_\wedge$) and thermoelectric coefficients ($\beta_\parallel$, $\beta_\perp$, $\beta_\wedge$) of Davies+ (Phys. Plasmas 2021).

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Transport coefficients for magnetic-field evolution in inviscid magnetohydrodynamics

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