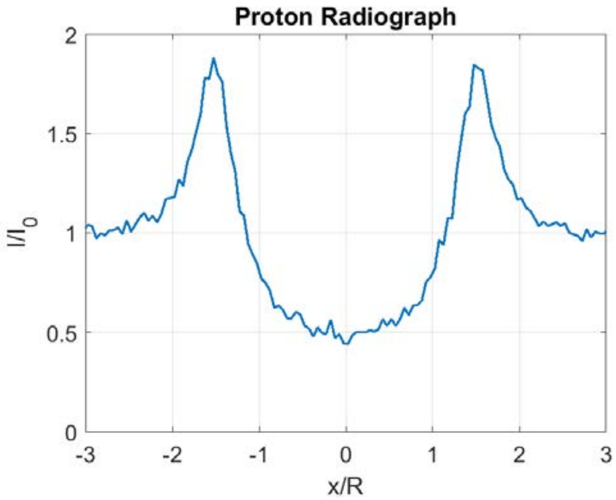
Evaluation of Direct Inversion of Proton Radiographs in the Context of Cylindrical Implosions



github.com
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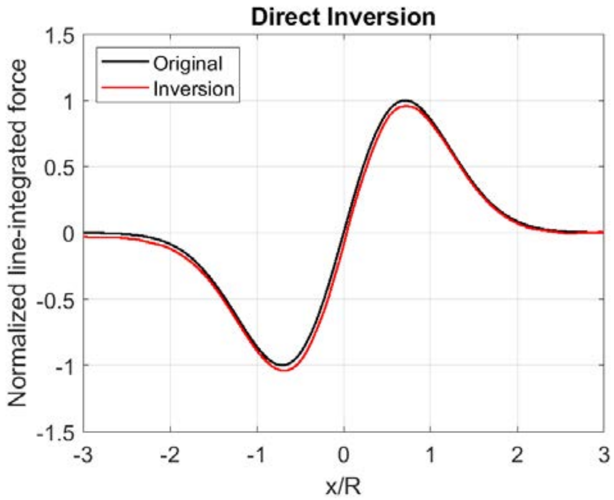

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Summary

Routines to obtain the line-integrated transverse Lorentz force directly from proton radiographs are publicly available*,**

- **If field gradients are sufficiently shallow proton trajectories do not intersect and a unique solution exists for the line-integrated transverse Lorentz force making direct inversion straightforward and ideal**
- **If proton trajectories intersect there does not exist a unique solution, but we found one algorithm* that can still find *a* solution that minimizes proton deflection provided all deflected protons are detected**
- **Direct inversion to obtain a minimum deflection solution allowed us to put a lower bound on the self-generated azimuthal magnetic field in the corona of a cylindrical implosion**

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Collaborators

D. H. Barnak, E. C. Hansen, P. V. Heuer, L. S. Leal, and J. L. Peebles

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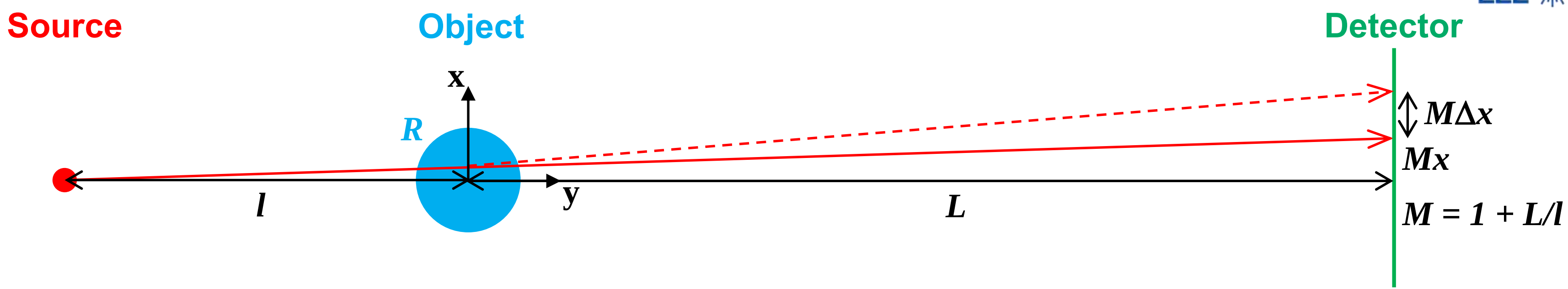
The relation between line-integrated transverse Lorentz force and proton intensity modulations can be demonstrated with a 1-D paraxial model



- For $l >> R$ and small deflections $\Delta v_x << v$ proton trajectories through the object are approximately straight lines

$$\frac{\Delta v_x}{v} \approx \frac{1}{2\mathrm{E}} \int F_x dy = \frac{\mathcal{F}_x}{2\mathrm{E}} \ll 1$$

  - E is proton energy, F is Lorentz force in the object, and $\mathcal{F}$ is line-integrated force

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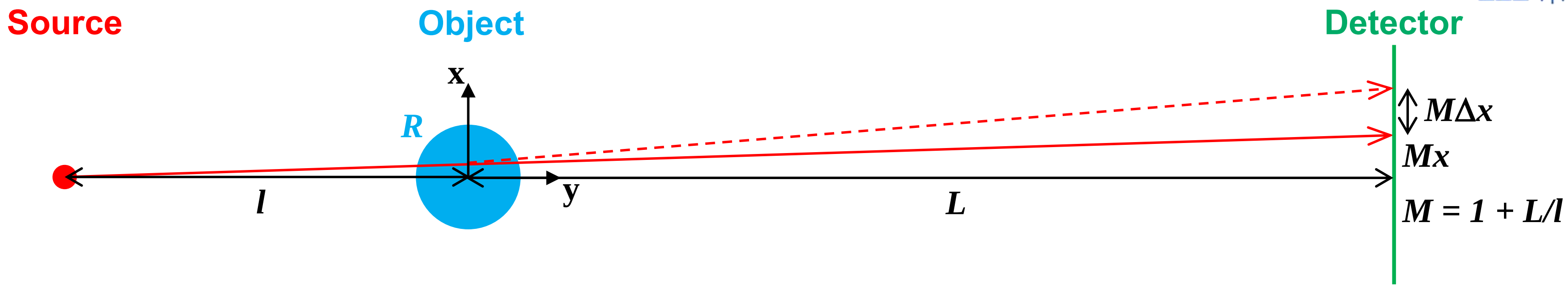
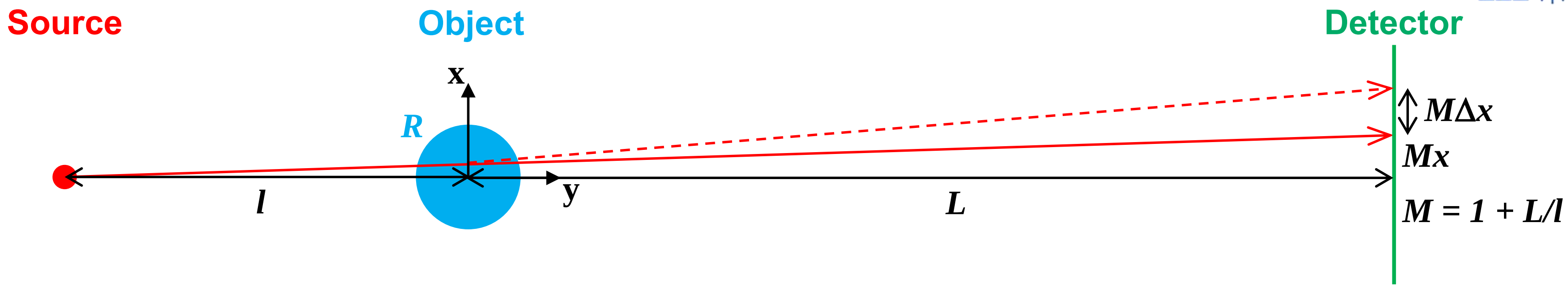


- For $L >> R$ proton deflection can be considered to occur at a distance L from the detector, giving a deflection at the detector of

$$\Delta x \approx \frac{L\mathcal{F}_x}{2M\mathrm{E}}$$

  – In object plane equivalent distance

The relation between line-integrated transverse Lorentz force and proton intensity modulations can be demonstrated with a 1-D paraxial model

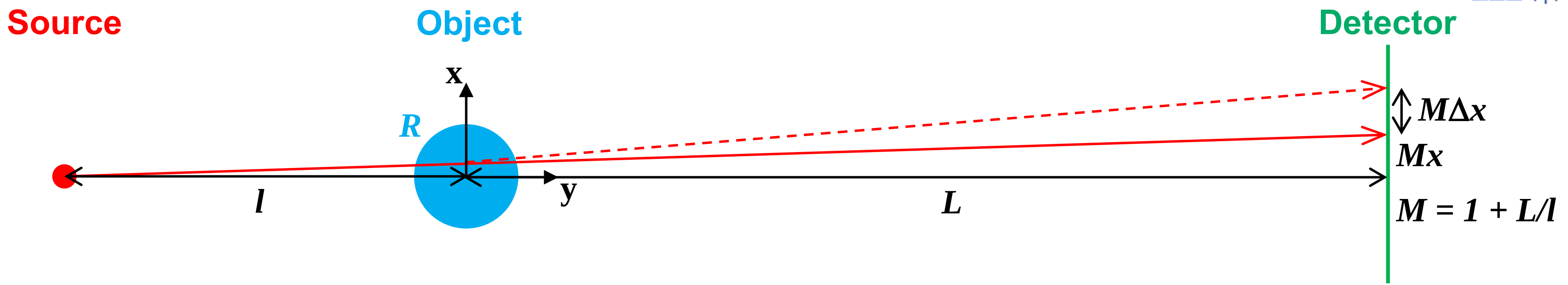


- **The distribution of protons on the detector *I* can be obtained from the deflection Δx and the distribution in the absence of forces I_0**

$$\frac{I}{I_0} \approx \frac{1}{\left|1 + \frac{L}{2EM}\frac{d\mathcal{F}_x}{dx}\right|}$$ (the determinant of the Jacobian of the new positions)

  – **If Δx is not a differentiable, single-valued function of x this relation is not valid: proton trajectories intersect and there is no unique solution for the line-integrated transverse Lorentz force, there exists an infinite family of solutions**

The relation between line-integrated transverse Lorentz force and proton intensity modulations can be demonstrated with a 1-D paraxial model



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$$\frac{I}{I_0} \approx \frac{1}{\left|1 + \frac{L}{2EM}\frac{d\mathcal{F}_x}{dx}\right|}$$

Direct inversion algorithms find displacements that map I_0 to I: the Monge Transport Problem.*

*Gaspard Monge, Mémoire sur la théorie des déblais et des remblais,
Histoire de l'Académie royale des sciences avec les mémoires de mathématique et de physique tirés des registres de cette Académie (1781), 666-705
Commentary and extracts in French: http://images.math.cnrs.fr/Gaspard-Monge,1094.html?lang=fr

We have found five direct inversion routines on GitHub

- **invert_shadowgraphy:** github.com/mfkasim1/invert-shadowgraphy [M. F. Kasim *et al.* Phys. Rev. E 95, 023306 (2017)] (Matlab, works best with quasi-Newton option)
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Solves the problem directly by moving random points on a plane to find a solution that minimizes total deflection without moving points through one another
We will refer to it as the power-diagram method after the algorithm it uses

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Solve the Monge-Ampère equation $\det\nabla^2\Phi = f(\underline{x},\nabla\Phi)$ where $\nabla\Phi = -\mathcal{F}$
In the limit of small deflections gives a Poisson equation $\nabla^2\Phi = I/I_0$

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Uses probabilistic methods to determine the most probable initial distribution I_0
All other routines require an I_0 assuming it to be uniform by default
(The "shadowgraphy" routines come with a denoising algorithm to obtain an I_0 from I)

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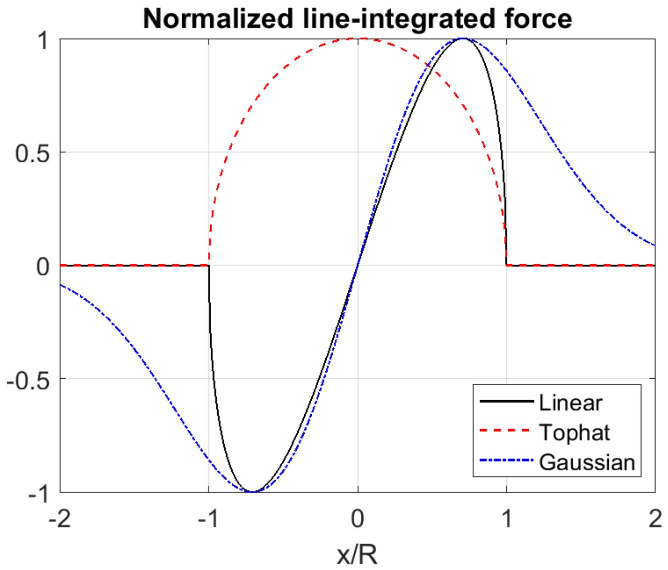
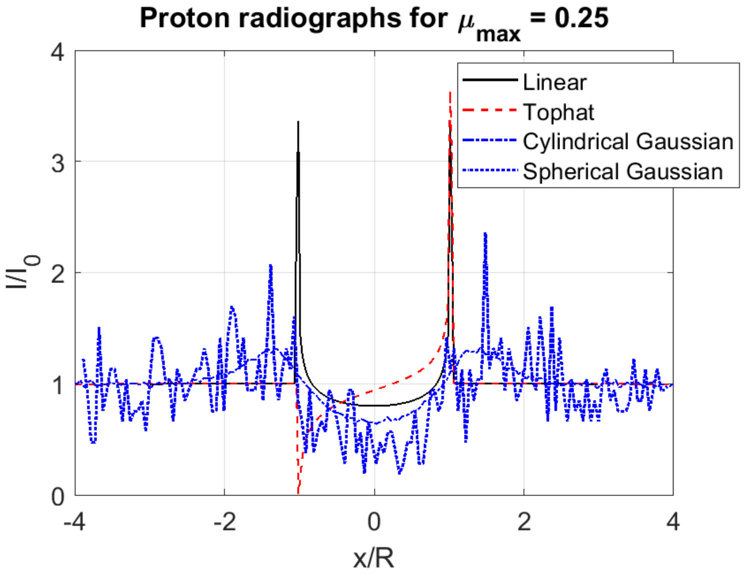
From the Monge-Ampère routines we tested these two, and we will refer to them as simply Monge-Ampère and PRNS, respectively

We chose four field profiles and generated test radiographs for dimensionless radial forces $\mu = (L/M)F_r/E$ with maxima from 1/8 to 4

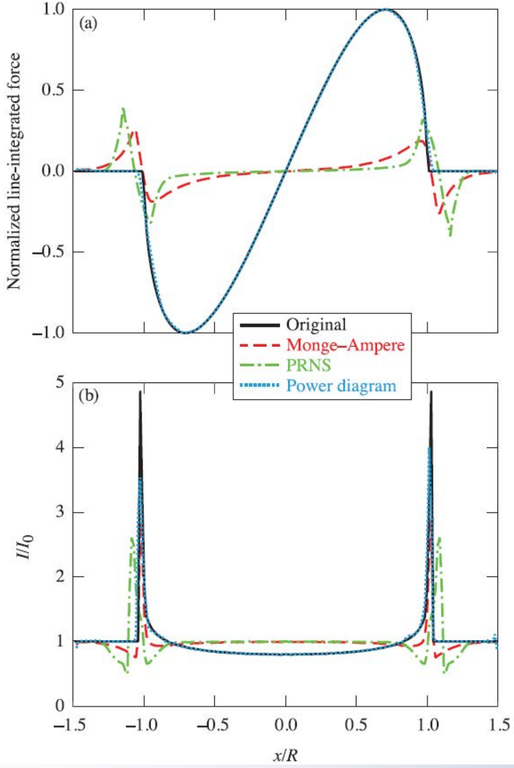




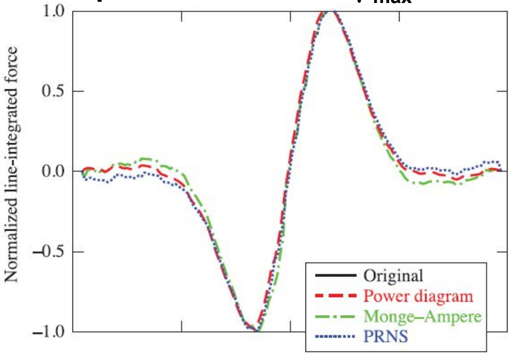
Linear: radial electric field in an isothermal, cylindrical expansion without the exponential decay in the sheath
Tophat: crude model of axial magnetic field in an implosion which is discontinuous at the shell-gas interface
Gaussian: the most commonly used potential, considered cylindrical and spherical
Setups ensured that I_0 was approximately uniform and that all deflected protons were on the radiographs

The Monge-Ampère based routines failed to reproduce the radiographs whenever trajectories intersected

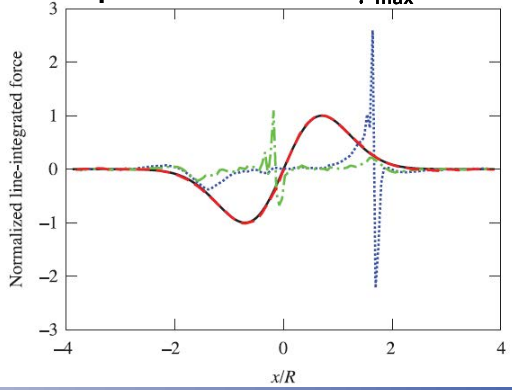
Linear μ_{max} = 0.25



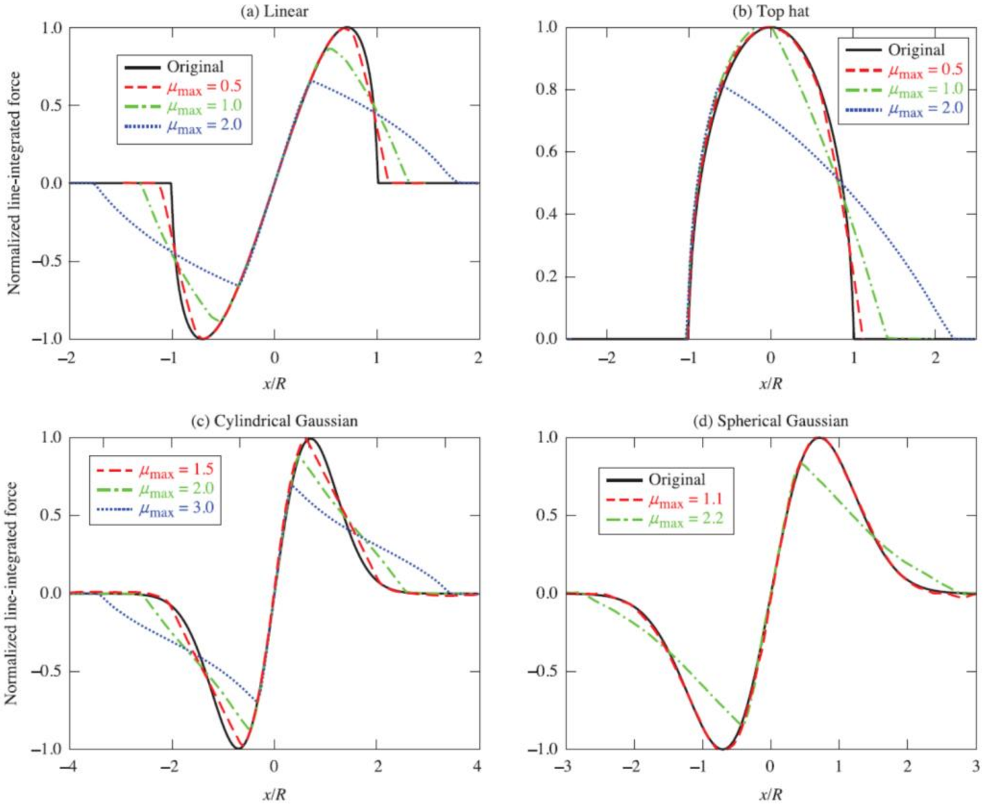
Spherical Gaussian μ_{max} = 0.55



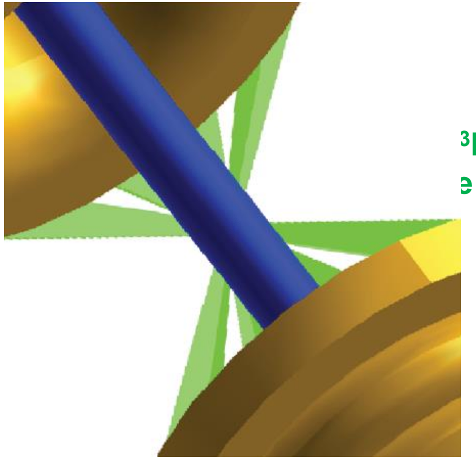
Spherical Gaussian μ_{max} = 1.1

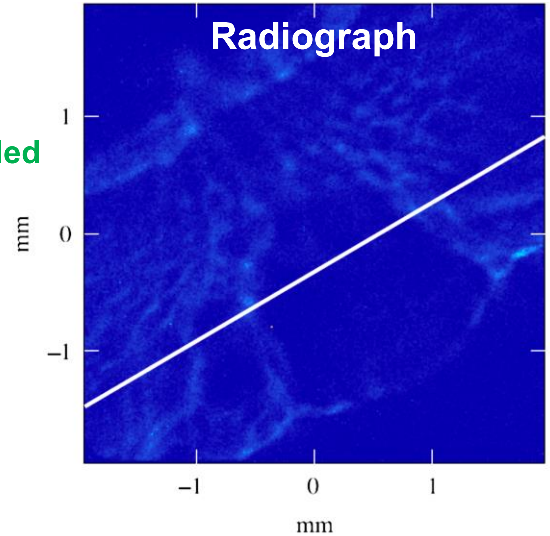


The power-diagram routine found a solution for all test cases that matched the original radiograph with lower, broader field profiles

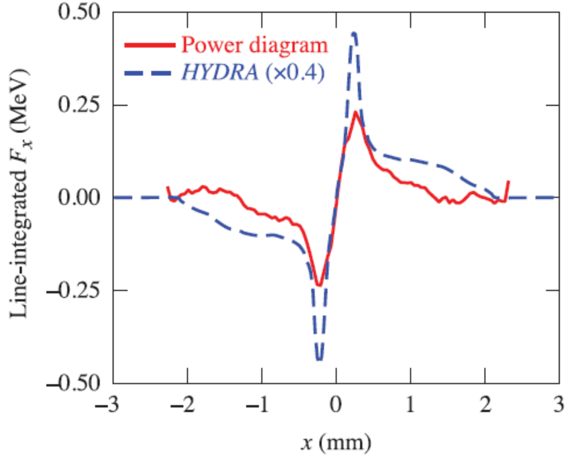


We applied the power-diagram routine to find a second solution for the azimuthal magnetic field in a cylindrical implosion





Solutions for line-integr–– force



3-D *HYDRA* simulations reproduced the main bell-shaped feature due to the self-generated azimuthal magnetic field

Applying the power-diagram routine to a radiograph generated from *HYDRA* fields gave an azimuthal magnetic field a factor of 1.56 lower than the original value

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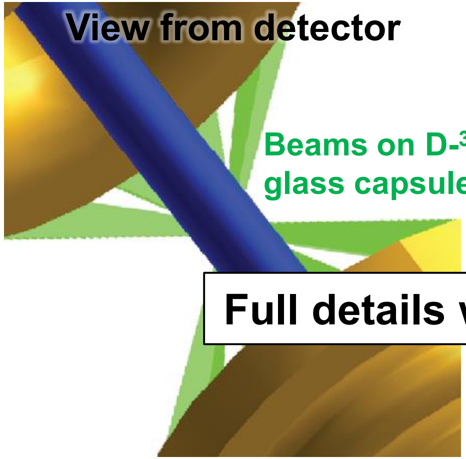

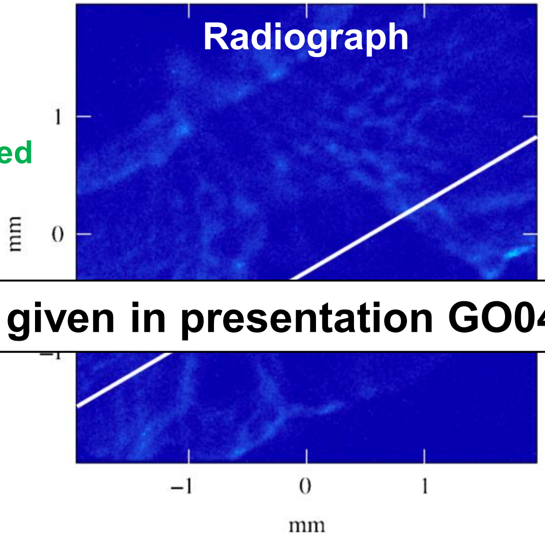

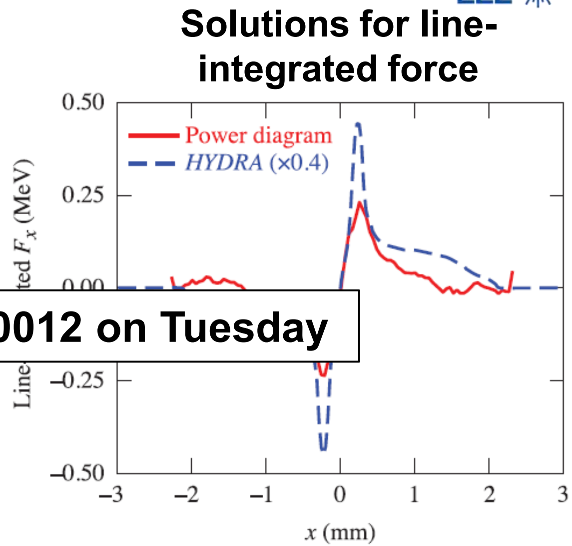

Full details were given in presentation GO04.00012 on Tuesday

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Summary

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Questions from on-demand viewers to jdav@lle.rochester.edu

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