Evaluation of Direct Inversion of Proton Radiographs in the Context of Cylindrical Implosions

Proton Radiograph

Direct Inversion

github.com
/mfkasim1/invert-shadowgraphy
/flash-center/PRaLine
/flash-center/PROBLEM
/mfkasim1/invert-shadowgraphy/tree/fast-inverse
/OxfordHED/proton-radiography-no-source

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Routines to obtain the line-integrated transverse Lorentz force directly from proton radiographs are publicly available**.

- If field gradients are sufficiently shallow proton trajectories do not intersect and a unique solution exists for the line-integrated transverse Lorentz force making direct inversion straightforward and ideal.
- If proton trajectories intersect there does not exist a unique solution, but we found one algorithm* that can still find a solution that minimizes proton deflection provided all deflected protons are detected.
- Direct inversion to obtain a minimum deflection solution allowed us to put a lower bound on the self-generated azimuthal magnetic field in the corona of a cylindrical implosion.

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Collaborators

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The relation between line-integrated transverse Lorentz force and proton intensity modulations can be demonstrated with a 1-D paraxial model.

- For $l \gg R$ and small deflections $\Delta v_x \ll v$ proton trajectories through the object are approximately straight lines:
  
  \[
  \frac{\Delta v_x}{v} \approx \frac{1}{2E} \int F_x dy = \frac{\mathcal{F}_x}{2E} \ll 1
  \]

  - $E$ is proton energy, $F$ is Lorentz force in the object, and $\mathcal{F}$ is line-integrated force.
The relation between line-integrated transverse Lorentz force and proton intensity modulations can be demonstrated with a 1-D paraxial model.

For \( L \gg R \) proton deflection can be considered to occur at a distance \( L \) from the detector, giving a deflection at the detector of

\[
\Delta x \approx \frac{L F_x}{2 ME}
\]

- In object plane equivalent distance
The relation between line-integrated transverse Lorentz force and proton intensity modulations can be demonstrated with a 1-D paraxial model.

- The distribution of protons on the detector \( I \) can be obtained from the deflection \( \Delta x \) and the distribution in the absence of forces \( I_0 \)
  \[
  \frac{I}{I_0} \approx \left| 1 + \frac{L}{2EM} \frac{dF_x}{dx} \right| \quad \text{(the determinant of the Jacobian of the new positions)}
  \]
  - If \( \Delta x \) is not a differentiable, single-valued function of \( x \) this relation is not valid: proton trajectories intersect and there is no unique solution for the line-integrated transverse Lorentz force, there exists an infinite family of solutions.
The relation between line-integrated transverse Lorentz force and proton intensity modulations can be demonstrated with a 1-D paraxial model

- The distribution of protons on the detector $I$ can be obtained from the deflection $\Delta x$ and the distribution in the absence of forces $I_0$

\[
\frac{I}{I_0} \approx 1 + \frac{L}{2EM} \frac{dF_y}{dx}
\]

Direct inversion algorithms find displacements that map $I_0$ to $I$: the Monge Transport Problem.*

We have found five direct inversion routines on GitHub


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Solves the problem directly by moving random points on a plane to find a solution that minimizes total deflection without moving points through one another
We will refer to it as the power-diagram method after the algorithm it uses
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Solve the Monge-Ampère equation $\det \nabla^2 \Phi = f(x, \nabla \Phi)$ where $\nabla \Phi = -\mathcal{F}$

In the limit of small deflections gives a Poisson equation $\nabla^2 \Phi = I/I_0$
We have found five direct inversion routines on GitHub


Uses probabilistic methods to determine the most probable initial distribution $I_0$

All other routines require an $I_0$ assuming it to be uniform by default

(The “shadowgraphy” routines come with a denoising algorithm to obtain an $I_0$ from $I$)
We have found five direct inversion routines on GitHub


From the Monge-Ampère routines we tested these two, and we will refer to them as simply Monge-Ampère and PRNS, respectively
We chose four field profiles and generated test radiographs for dimensionless radial forces $\mu = (L/M)F_r/E$ with maxima from 1/8 to 4.

**Normalized line-integrated force**

<table>
<thead>
<tr>
<th>Field Profile</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>Radial electric field in an isothermal, cylindrical expansion without the exponential decay in the sheath</td>
</tr>
<tr>
<td>Tophat</td>
<td>Crude model of axial magnetic field in an implosion which is discontinuous at the shell-gas interface</td>
</tr>
<tr>
<td>Gaussian</td>
<td>The most commonly used potential, considered cylindrical and spherical</td>
</tr>
</tbody>
</table>

Setups ensured that $I_0$ was approximately uniform and that all deflected protons were on the radiographs.
The Monge-Ampère based routines failed to reproduce the radiographs whenever trajectories intersected.

**Linear $\mu_{\text{max}} = 0.25$**

**Spherical Gaussian $\mu_{\text{max}} = 0.55$**

**Spherical Gaussian $\mu_{\text{max}} = 1.1$**
The power-diagram routine found a solution for all test cases that matched the original radiograph with lower, broader field profiles.
We applied the power-diagram routine to find a second solution for the azimuthal magnetic field in a cylindrical implosion.

3-D HYDRA simulations reproduced the main bell-shaped feature due to the self-generated azimuthal magnetic field.

Applying the power-diagram routine to a radiograph generated from HYDRA fields gave an azimuthal magnetic field a factor of 1.56 lower than the original value.
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Summary

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Questions from on-demand viewers to jdad@lle.rochester.edu

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