Implementation and Verification of Braginskii Viscosity in the FLASH Code

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Abstract

The inclusion of ion viscosity when solving a radiative magnetohydrodynamic ansatz to simulate high-energy-density physics (HEDP) laser-driven plasma experiments is key to correctly capturing momentum diffusion and the flow's fluid Reynolds number (Re). When the diffusion time scale becomes comparable to the advection timescale, ion viscosity will impact flow dynamics and transport properties. Here we present the numerical implementation of Braginskii[1] ion viscosity in the FLASH code. We detail the numerical implementation that couples with the flux-based viscosity module in the code and show results from an array of verification benchmarks. The inclusion of a realistic ion viscosity significantly broadens the range of plasma regimes where FLASH can be applied and enhances the fidelity of HEDP simulations where Re ≅ 1.

The FLASH Code

The Flash Center for Computational Science at the Department of Physics and Astronomy at the University of Rochester is developing capabilities of the FLASH code (flash.rochester.edu). FLASH[3] is a multi-physics, parallel, adaptive mesh refinement (AMR), finite-volume Eulerian hydrodynamics and magneto-hydrodynamics (MHD) code. To date, FLASH has been downloaded over 3,500 times, and more than 1,200 papers have been published in a wide range of disciplines, including, HEDP, fluid dynamics, and astrophysics. Extensive HEDP capabilities[4] have been added to the FLASH code under the guidance of U.S. DOE NNSA, enabling the accurate modeling of laser-driven plasma experiments. The FLASH code and its new capabilities have been validated through extensive benchmarks, code-to-code comparisons, and direct application for the design, execution, and interpretation of many laboratory experiments around the world.

Implementation of the Braginskii Viscosity

The thermodynamic irreversibility that results from particle collisions within a plasma give rise to an internal friction force[5]. Braginskii viscosity results after the closure of the magnetohydrodynamic (MHD) equations for a collisional quasi-neutral plasma via the electrons and ions collision times. We neglect the election contribution to the kinematic viscosity for values of Z less than six[6]. Thus we write, for the HEDP regime, the kinematic viscosity as:

$$\nu = 1.92 \times 10^{19} \frac{T_i^{5/2}}{M^{1/2} Z^4 n_i \log \Lambda}$$

where T_i is the ion temperature, M the average mass, Z the atomic charge, n_i the ion density, and $\log \Lambda$ the Coulomb Logarithm.

Quantity	Symbol	Value
Ion Temperature	T_i	$5.2 \cdot 10^6$ K
Average atomic weight	M	8.5 a.m.u.
Average ion charge	Z	4.4
Ion density	n_i	$\sim 10^{20}$
Coulomb Logarithm	$\log \Lambda$	~7
Gaussian Width	σ^2	1/128
Density	ρ	$3.2 \cdot 10^{-3}$ g cm^{-3}
Dynamic Viscosity	ν	$1.40 \cdot 10^6$ cm^2 s^{-1}

Verification set-up

Using FLASH's unsplit hydrodynamic solver, in the absence of external forces, the equations evolved are

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0$$
$$\partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) + \nabla P = \nabla \cdot \tau$$
$$\partial_t (\rho E) + \nabla \cdot \mathbf{u} (\rho E) = \nabla \cdot (\mathbf{u} \cdot \tau)$$

where

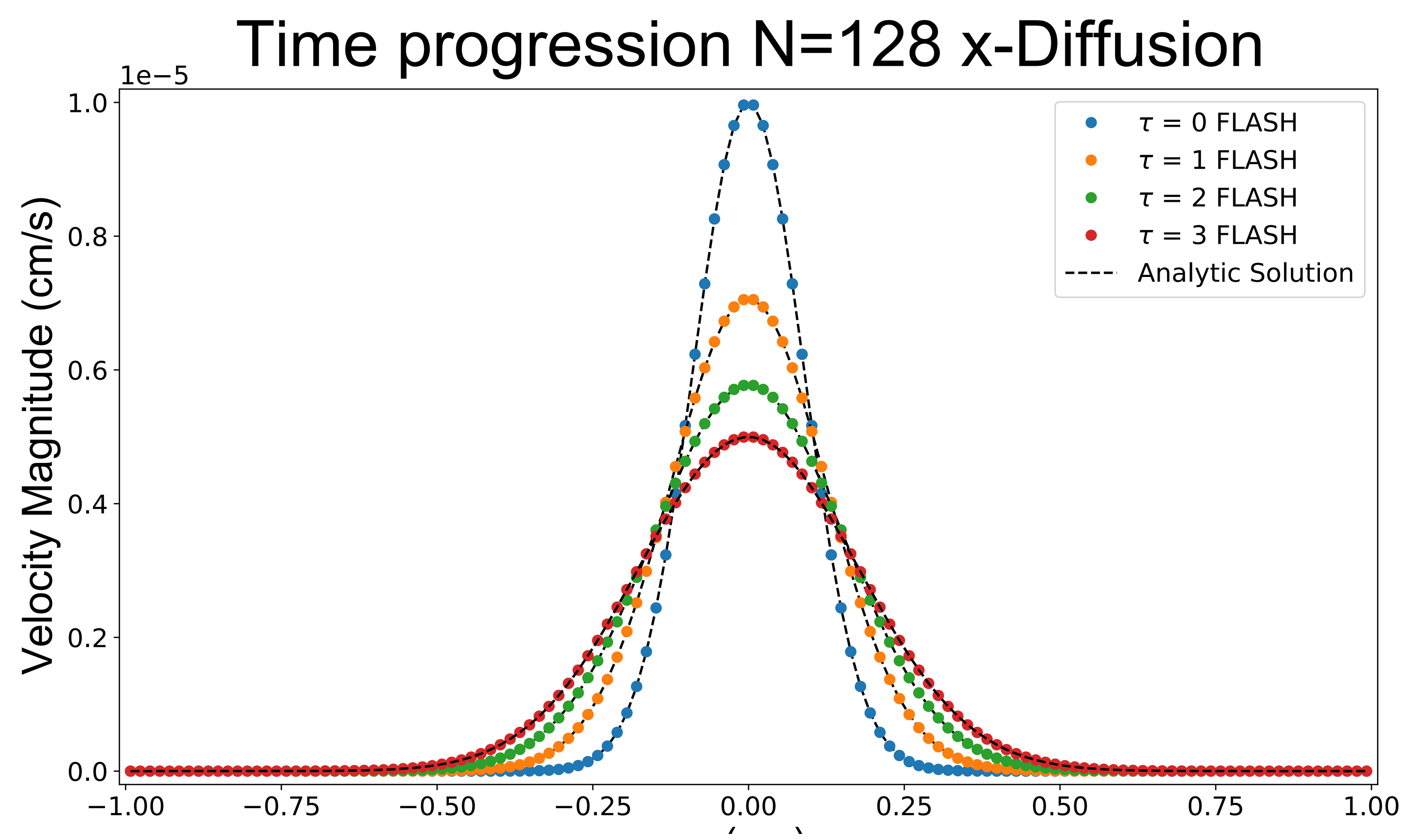
$$E = \frac{1}{2} u^2 + \epsilon$$
$$\tau = \mu \Big((\nabla \mathbf{u}) + (\nabla \mathbf{u})^{\mathrm{T}} - \frac{2}{3} (\nabla \cdot \mathbf{u}) \mathbf{I} \Big)$$

are the specific total energy and viscous stress tensor. With the further simplifications of isotropic pressure, constant density, and a simple gaussian initial condition, this reduces the Navier-Stokes equation to the one-dimensional heat equation.

$$\partial_t u = \nu \triangle u$$

The direction of diffusion is always perpendicular to the direction of the flow. The analytical solutions are given as, and the initial condition can be recovered at $t = 0$.

$$\mathbf{u}(r,t) = \frac{e^{-\frac{x^2}{4t\nu + 2\sigma^2}}}{\sqrt{1 + \frac{2t\nu}{\sigma^2}}} \hat{\mathbf{y}} \qquad \mathbf{u}(r,t) = \frac{e^{-\frac{y^2}{4t\nu + 2\sigma^2}}}{\sqrt{1 + \frac{2t\nu}{\sigma^2}}} \hat{\mathbf{x}} \qquad \mathbf{u}(r,t) = \frac{e^{-\frac{r^2}{4t\nu + 2\sigma^2}}}{1 + \frac{2t\nu}{\sigma^2}} \hat{\mathbf{z}}$$

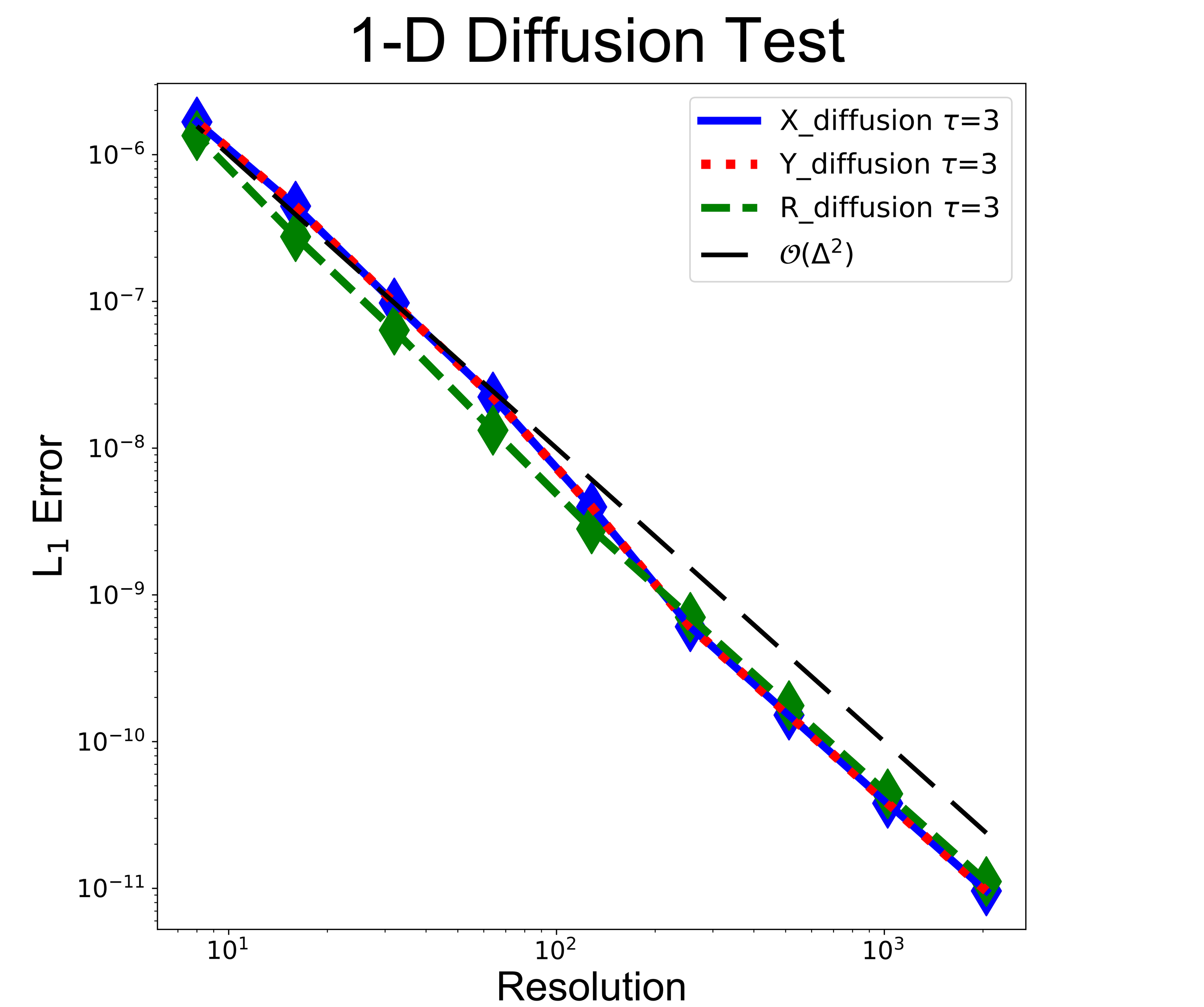


Knowing the analytical solution to the one-dimensional heat equation allows us to directly compare the numerical solution and evolution to exact, analytical value.

Quantity	Symbol	Value
Renyolds Number	Re	$\sim 1 \cdot 10^{-11}$
Viscous Time Scale	$\tau_{\text{visc}} = \sigma^2 / 2\nu$	$\sim 2.8 \cdot 10^{-9}$ s

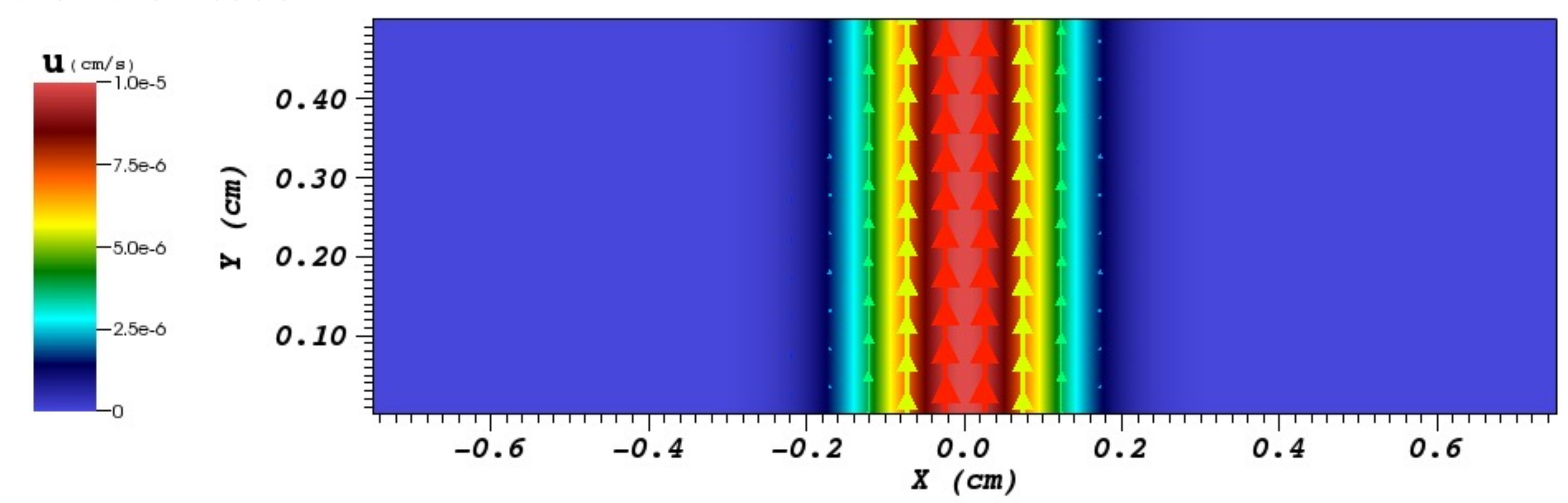
Results

FLASH uses second order face centered finite differencing to solve for the viscous fluxes in order to advance Euler's equation. As seen by the log-log plot below, the relationship between the $\mathcal{L}_1$ error and resolution is approximately second order. This accurately reflects the finite differencing implementation for explicit viscous fluxes in the hydro solver within the FLASH code.



Conclusion

The new implementation of a Braginskii viscosity will allow more fidelity in simulations within new HEDP ranges. The roughly second order error trend of the error-resolution $\mathcal{L}_1$ plot agree with the hydrodynamic solver implementation within the FLASH code.



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