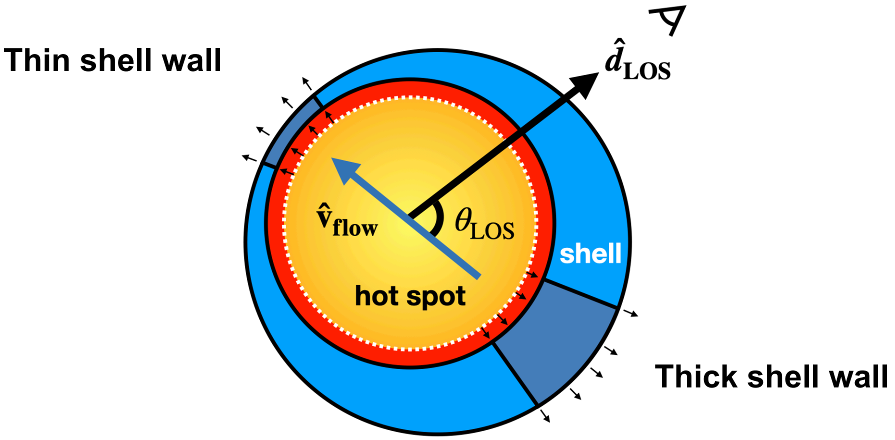
Impact of Low-Mode Areal-Density Asymmetry on the Loss of Confinement for Igniting Capsules



The capsule of mode 1

The 1-D ignition parameter χ (ITFx χ^3)

$$\chi_{1\mathrm{D}} \sim \left(\frac{\mathbf{Yield}}{M_{\mathrm{DT}}}\right)^{1/3} \cdot \left[\rho R_{\mathrm{n,1D}}\right]^{2/3}$$

What is the equivalent ρR in 3-D?

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The impact of areal-density (ρR) asymmetries on the loss of confinement and on the ignition criterion is shown to be interpreted by the harmonic-mean definition of areal densities

- An analytic hot-spot model is generalized to 3-D to include the effects of low modes
- The 3-D ignition threshold is derived including the areal-density modulation from low modes
- The average areal-density degradation can be quantified using the measured ion-temperature asymmetries

Collaborators

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Acknowledgement

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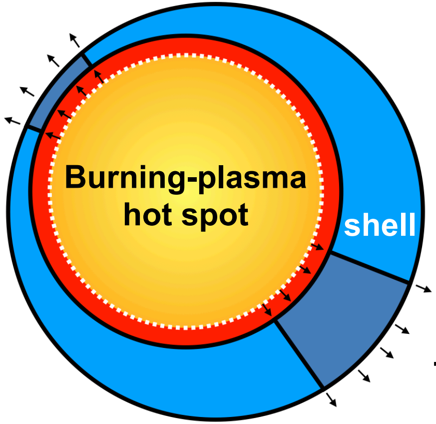
The 3-D hot-spot energy equation is the basis of the ignition model*



Simplest ignition criterion uses the 3-D hot-spot energy equation**

$$\frac{\mathrm{d}}{\mathrm{dt}} \ln \left(P V^{\gamma}\right) = P / S_{\alpha}$$

P = pressure, V = volume, $\gamma = 5/3$

Assumptions

(1) $\langle \sigma v \rangle_{DT} \sim T_i^2 / S_\alpha$, where $6 < T_{i,keV} < 20$

(2) Ignore the radiation loss

(3) Isobaric ($v < c_s$)

* R. Betti *et al.*, J. Phys. 717, 012007 (2016).
** K. M. Woo *et al.*, Phys. Plasma 25, 052704 (2018).

The second-time derivative* of the hot-spot volume at stagnation determines the 3-D confinement time

General solution of hot-spot energy equation starting from the time of stagnation t_s (labeled as subscript "s")

$$\frac{1}{P(t)} = V(t)^{\gamma}\left[\frac{1}{P_s V_s^{\gamma}} - \frac{1}{S_\alpha}\int_{t_s}^{t}\frac{dt'}{V^{\gamma}}\right] \qquad S_\alpha \equiv \frac{24T_i^2}{\langle\sigma v\rangle_{DT}E_\alpha^{3.5MeV}} \sim \text{constant}$$

The bracket vanishes when the hot spot ignites with an explosive pressure $P \rightarrow \infty$ (ignition)

$$\frac{1}{P_s} = \frac{1}{S_\alpha}\int_{t_s}^{t}\left(\frac{V_s}{V}\right)^{\gamma} dt'$$

Use steepest descent to find ignition criteria and confinement time

Expand near stagnation: $V \approx V_s + \frac{1}{2}\ddot{V}_s(t - t_s)^2$

Lawson criteria: $\frac{P_s\tau_c}{S_\alpha} = 1$, where $\tau_c = \sqrt{V_s/\ddot{V}_s}$

S_α is the minimum $P\tau$ required for ignition.

* Consistent with P.T. Springer *et al.*, Nucl. Fusion 59, 032009 (2018).

The confinement times depends on the harmonic-mean of the areal density

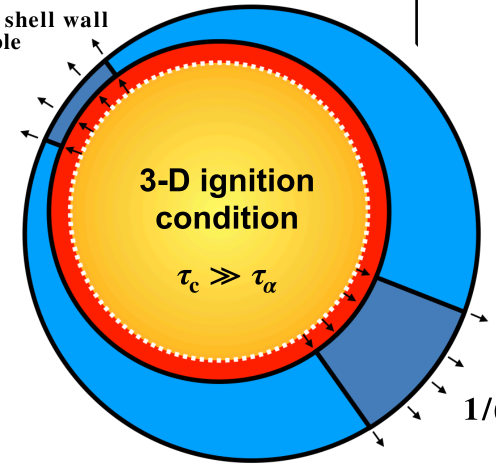
Subsonic flow limit

$$\rho \frac{\partial \vec{v}}{\partial t}\Big|_{x\in A} \approx -\vec{\nabla} P\Big|_{x\in A}$$

Thin shell limit

$$\vec{\nabla} P \approx \frac{-P}{\Delta_n^{\text{shell}}}$$

$$\ddot{V}_s = -\oint \frac{\vec{\nabla} P}{\rho}\Big|_{t_s} \cdot d\vec{A} \simeq P_s A_s \cdot \left\langle \frac{1}{\rho \Delta_n^{\text{shell}}} \right\rangle$$



Assume most ρR from shell

$$\rho\, \Delta_n^{\text{shell}} \approx \rho R_n$$

The confinement time depends on the harmonic-mean areal density:

$$\tau_c \simeq \sqrt{\frac{V_s}{P_s A_s} \underbrace{\langle \rho R_n \rangle_{\text{HM}}}}$$

$$\langle \rho R_n \rangle_{\text{HM}} \equiv \left\langle \frac{1}{\rho R_n} \right\rangle^{-1}$$

The thin spots dominate over the thick regions in setting the confinement time.

The generalized 3-D ignition metric for low modes includes the neutron yield, the hot-spot surface area, and the harmonic mean of the areal density

$$\chi_{3D} \equiv \frac{P_s \tau_c}{S_\alpha} \longrightarrow \chi_{3D}^3 = \left(\frac{P_s}{S_\alpha}\right)^3 \cdot \tau_c^2 \cdot \tau_c$$

The generalized 3-D ignition criteria in terms of *Y*, *A*, ρR_{HM}

$$\chi_{3D} \sim \left(\frac{Y}{A_s}\langle \rho R_n \rangle_{HM}\right)^{1/3}$$

From confinement time in the previous slide

$$\tau_c^2 = \frac{V_s}{P_s A_s}\left\langle \frac{1}{\rho R_n}\right\rangle^{-1}$$

From the neutron yield

$$Y \sim P_s^2 V_s \tau_c \longrightarrow \tau_c \sim \frac{Y}{P_s^2 V_s}$$

The arithmetic mean of the areal density enters through the hot-spot surface area

Define the arithmetic mean ρR

$$\langle \rho \Delta_n \rangle \equiv \frac{1}{A_s} \oint (\rho \Delta r_n)\, \mathrm{d}A \approx M_s / A_s$$

The hot-spot surface area is replaced by the shell mass and the arithmetic mean areal density

$$A_s \approx \frac{M_s}{\langle \rho R_n \rangle} \longrightarrow \chi_{3D} \sim \left(\frac{Y}{A_s} \langle \rho R_n \rangle_{HM} \right)^{1/3}$$

The generalized 3-D ignition criteria in terms of *Y*, *M*, ρR, ρR_{HM}

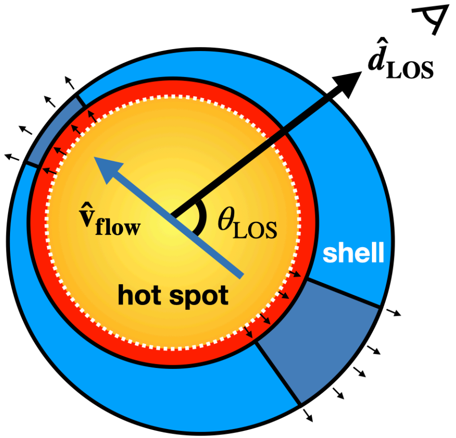
$$\chi_{3D} \sim \left(\frac{Y}{M_s} \right)^{1/3} \cdot \left[\langle \rho R_n \rangle_{HM} \langle \rho R_n \rangle \right]^{1/3}$$

The 3-D ignition criterion for low modes depends on both the harmonic and the arithmetic mean of the areal density.

The "thin spots" in the shell dominate the loss of confinement as reflected in the harmonic mean of the areal density

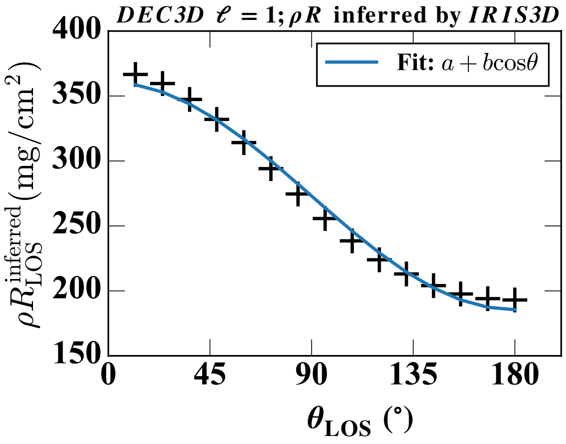
Obtain the harmonic-mean ρR

$$\langle \rho R_{\mathrm{n}} \rangle_{\mathrm{HM}} \approx \left[\frac{1}{N} \sum_{\mathrm{LOS}}^{N} \frac{1}{\rho R_{\mathrm{LOS}}} \right]^{-1}$$



The ρR variation for mode 1 satisfies

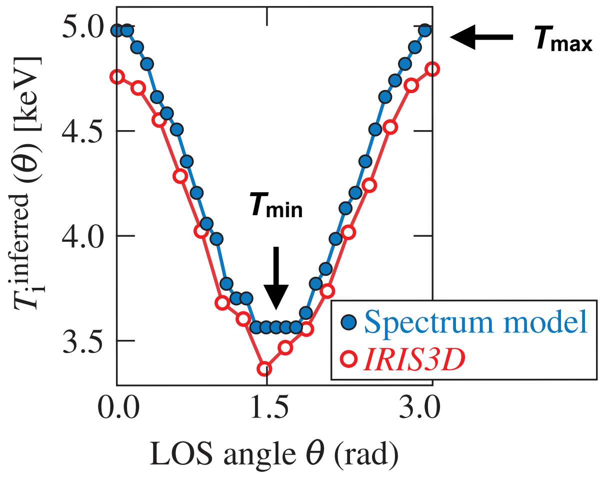
$$\rho R_{\mathrm{LOS}} = \rho R_0 + \hat{\mathbf{v}}_{\mathrm{flow}} \cdot \hat{d}_{\mathrm{LOS}} \Delta \rho R$$



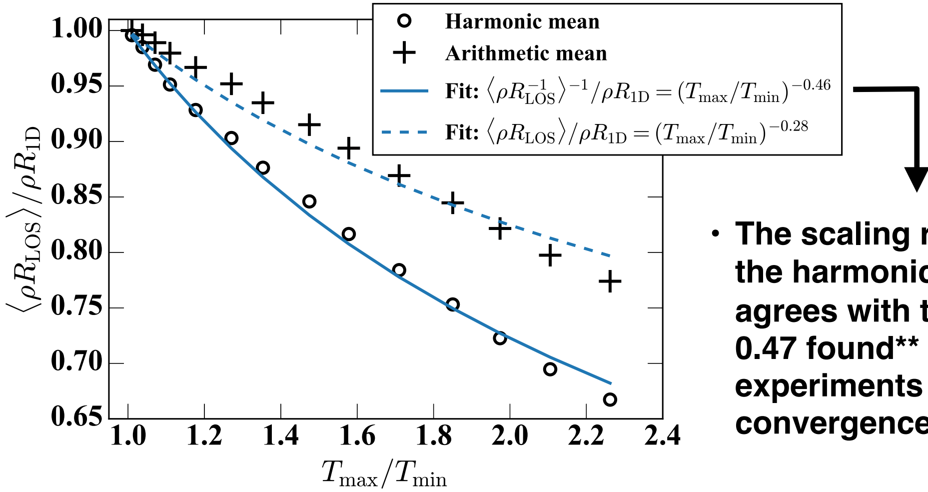
In general agreement with O. A. Hurricane et al., *Nat. Phys*. 12, 800–806 (2016);
DEC3D: K. M. Woo *et al*., Phys. Plasma, 25, 052704 (2018),
IRIS3D: F. Weilacher *et el*., Phys. Plasma, 25, 042704 (2018).

In implosions degraded by $\ell = 1$ modes, the harmonic and arithmetic mean areal densities are related to the T_i asymmetries, enabling the inference of their 1-D values

DEC3D* $\ell = 1$ simulation for shot 77068



DEC3D* $\ell = 1$ simulation for shot 77068



- The scaling relation using the harmonic-mean ρR agrees with the power of -0.47 found** in OMEGA experiments with varying convergence ratio ~11 to 20.

The average areal density degradation from $\ell = 1$ mode can be quantified using measured T_i asymmetries.

K. M. Woo *et al.*, Phys. Plasma, 25, 052704 (2018).
R. Betti *et el.*, BO09.00011, this conference.

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