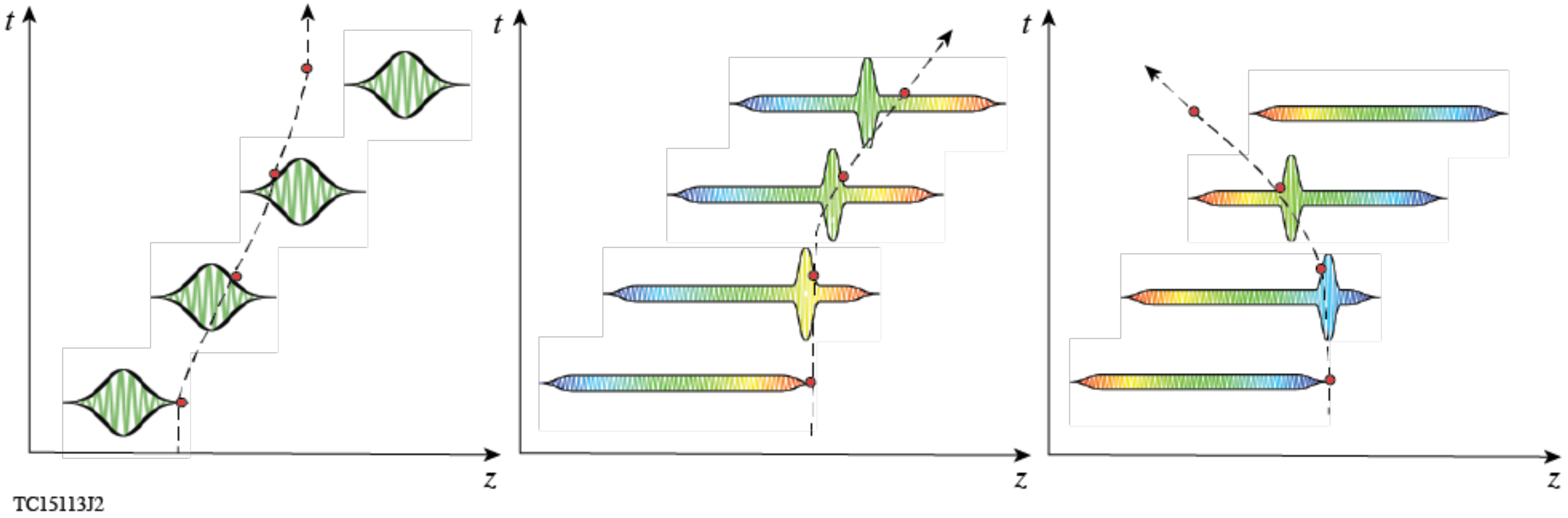
Vacuum Acceleration of Electrons in a Dynamic Laser Pulse



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62nd Annual APS DPP Meeting
Virtual

Collaborators

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The flying focus enables a novel mechanism for vacuum acceleration*

- Typical planar pulses cannot impart net momentum to electrons

- Dynamic planar-like flying focus pulses can accelerate electrons to relativistic momenta (either positive or *negative*)

*Ramsey et al., Phys. Rev. E **102**, 43207 (2020)

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  - In an accelerated intensity peak the electron can indefinitely remain in the rising edge of the intensity peak

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Lawson-Woodward Theorem (LWT): The net energy gain for an electron in a laser pulse is zero

For net energy gain, one of the assumptions of the LWT needs to be exploited:

1. Highly relativistic electron
2. No boundaries
3. No static fields
4. Infinite interaction region
5. No non-linear effects (ponderomotive)

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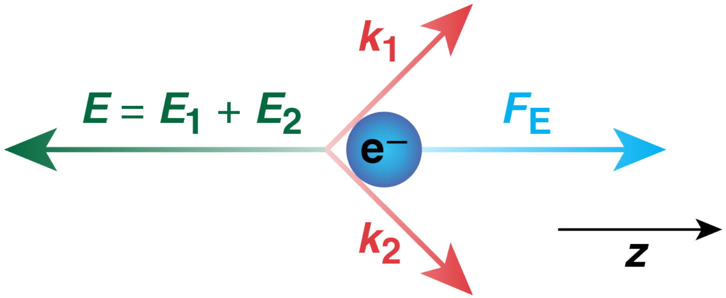

Crossed Laser-Beam Acceleration,
C.M. Halland, Opt. Commun. **114**, 280 (1995)

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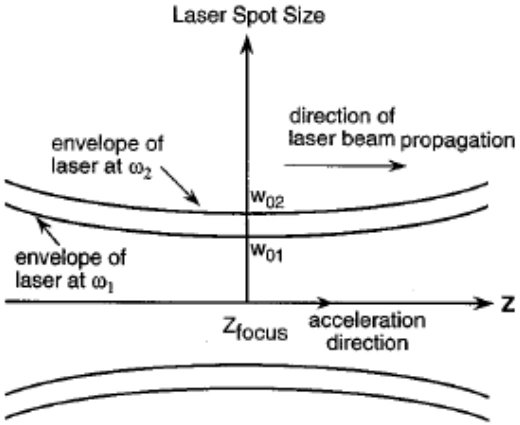

Vacuum Beatwave Accelerator,
Esarey et al., Phys. Rev. E **52**, 5443 (1995)

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The nonlinear ponderomotive force of a plane wave is not sufficient to break the LWT

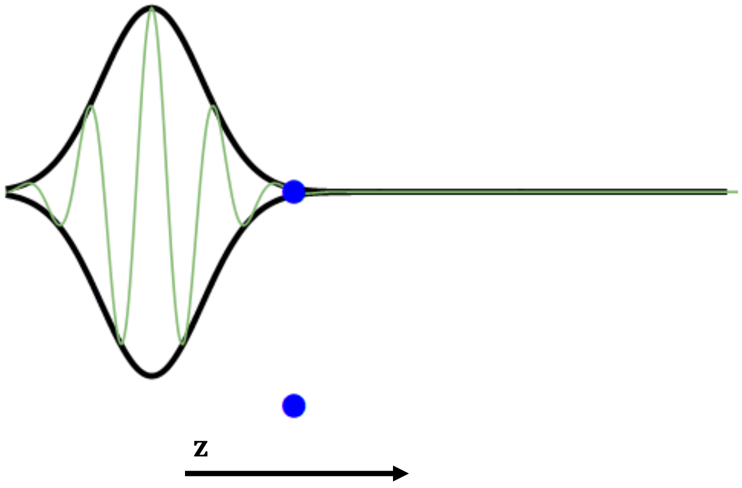
An electron in the electromagnetic field of the pulse satisfies the conservation relation

$$\frac{d}{dt}\left(\gamma - \frac{p_z}{m_e c}\right) = 0$$

From the constant of motion:

$$\frac{p_z}{m_e c} = \frac{1}{2} a^2 (z, t)$$

Once the pulse has outrun the electron,
the electron has transferred its energy back to the pulse



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The electron can gain net energy when the velocity of the intensity peak is less than the speed of light

An electron in the electromagnetic field of the pulse satisfies the conservation relation

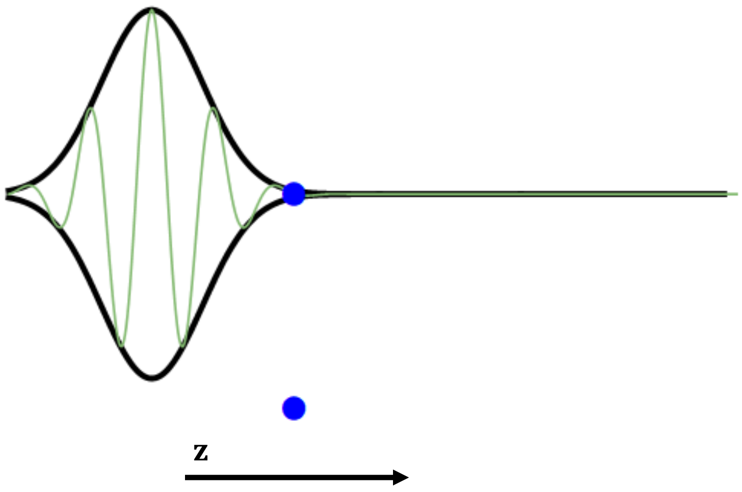
$$\frac{d}{dt}\left(\gamma - \beta_I \frac{p_z}{m_e c}\right) = 0$$

where $\beta_I = \mathrm{v}_I / c$

From the constant of motion:

$$\frac{p_z}{m_e c} = \beta_I \gamma_I^2 \pm \beta_I \gamma_I^2 \left[1 - (\beta_I \gamma_I)^{-2} \langle a^2 \rangle\right]^{1/2}$$

Once the electron has outrun the pulse, the electron retains energy gained in the pulse



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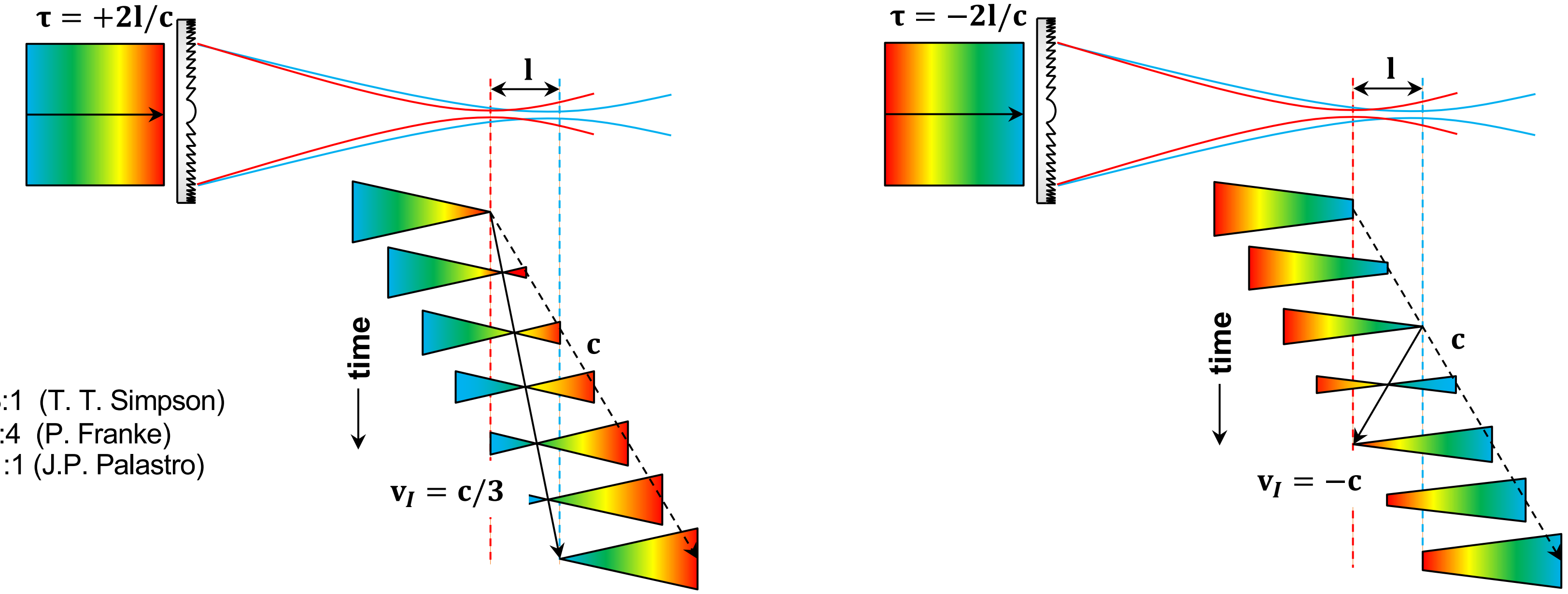
From the constant of motion:

$$\frac{p_z}{m_e c} = 2\beta_I \gamma_I^2$$

Once the electron has outrun the pulse, the electron retains energy gained in the pulse

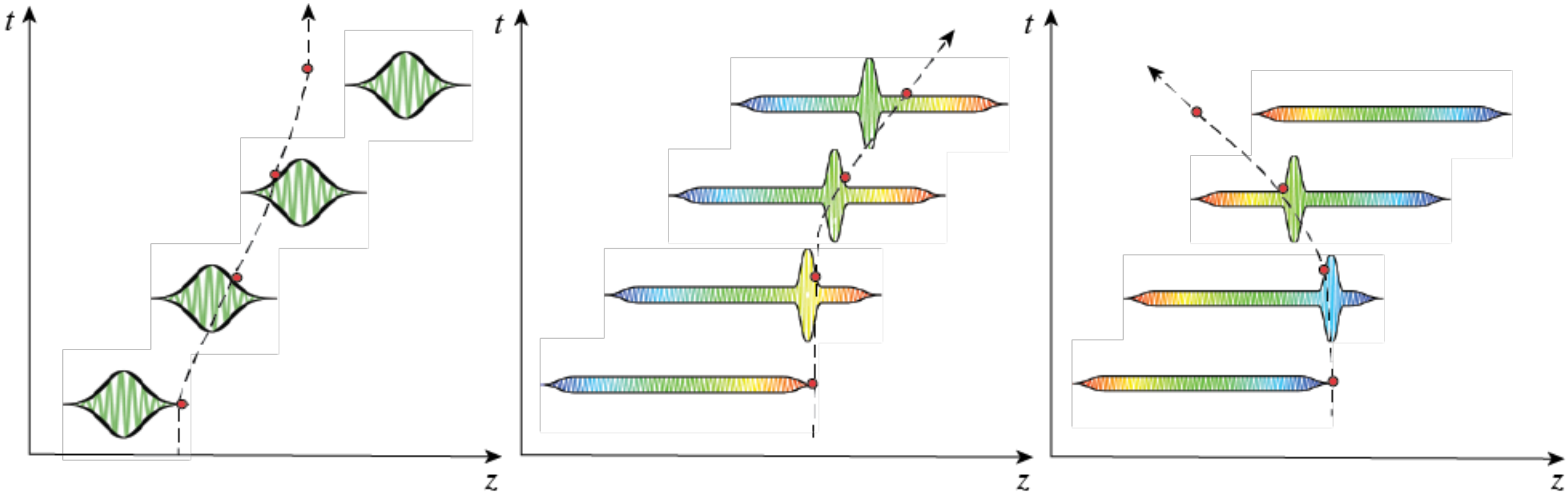
The flying focus combines a chromatic optic with a chirped laser pulse to control the velocity of the intensity peak



NO8:1 (T. T. Simpson)
ZO3:4 (P. Franke)
MR1:1 (J.P. Palastro)

The chromatic optic and chirp determine the focal location and time of each color, respectively, resulting in a peak intensity with a dynamic trajectory

Planar-like flying focus pulses can have subluminal intensity peaks in vacuum, which allow for net energy gain



TC15113J2

In the Lorentz frame of the intensity peak, the energy gain corresponds to a reflection from the ponderomotive potential

In the intensity peak frame, the ponderomotive potential is time-independent, implying the electron energy is conserved: $\gamma_i = \gamma_f$

There are two ways this can occur

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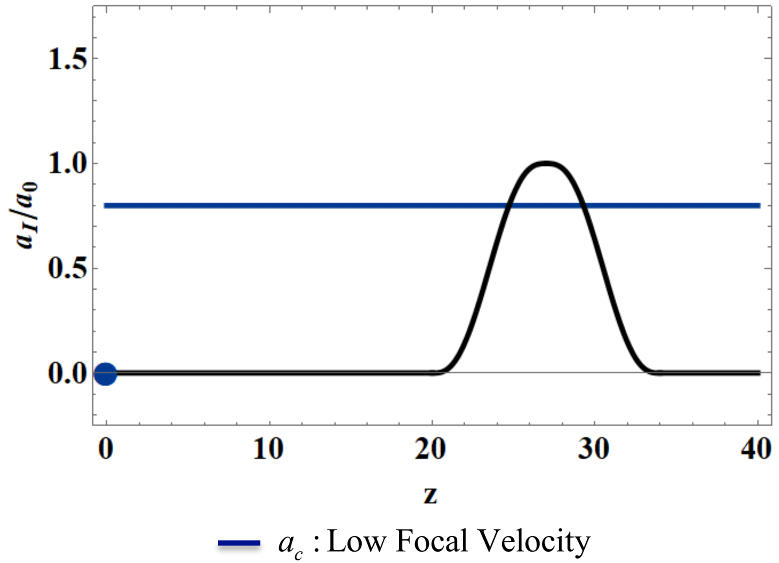
There are two ways this can occur

1. $v_f = -v_i$

   The initial kinetic energy of the electron is insufficient to overcome the ponderomotive potential hill



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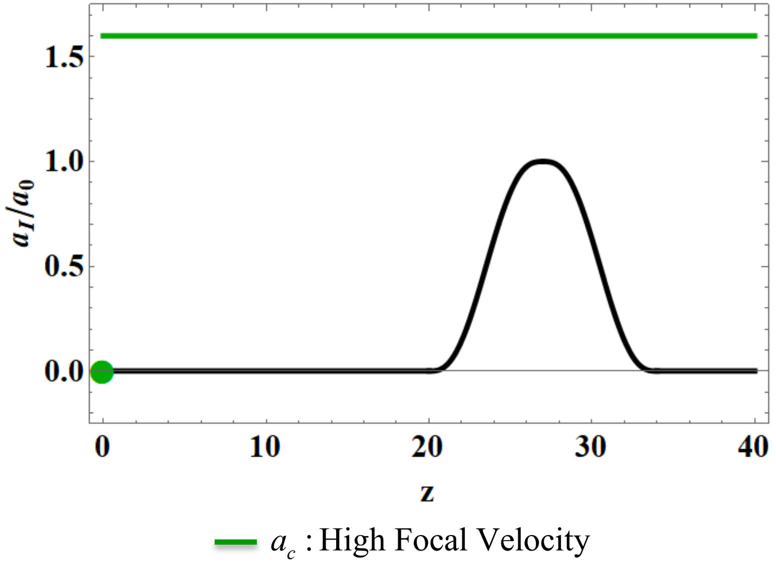
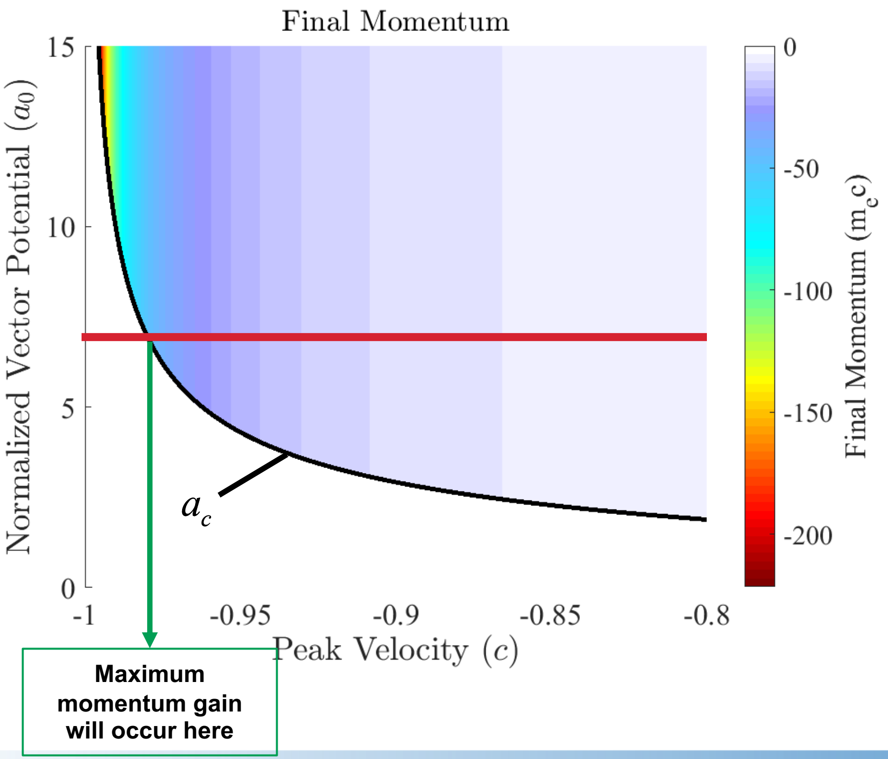


For a particular a_0, there is an absolute limit to the energy gain



To achieve the maximum energy, the peak velocity is set such that a_c is slightly less than a_0

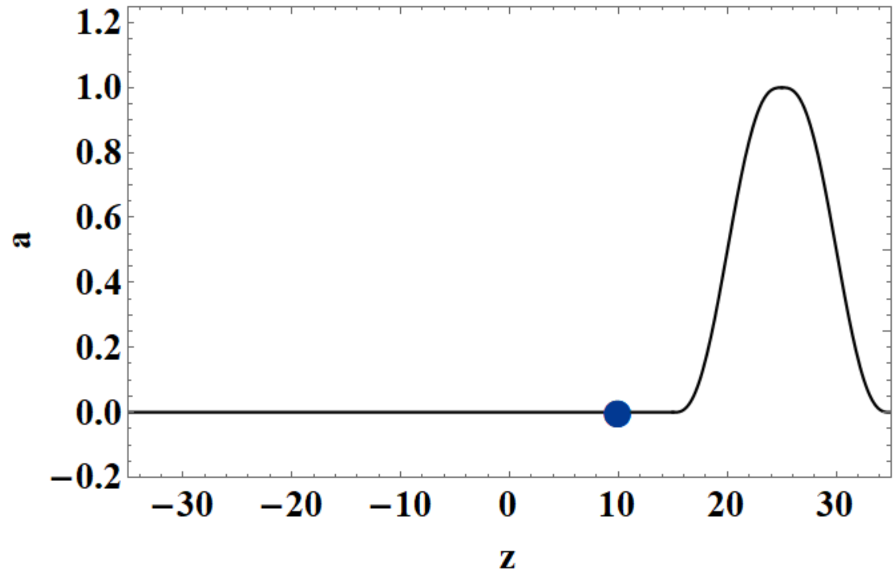
This leaves **no remaining free parameters** to further increase the energy gain

Furthermore, any **unused interaction length** in a single reflection is **wasted**

Can an **accelerated** intensity peak further increase energy gains?

The electron begins at rest and the peak propagates at a constant velocity

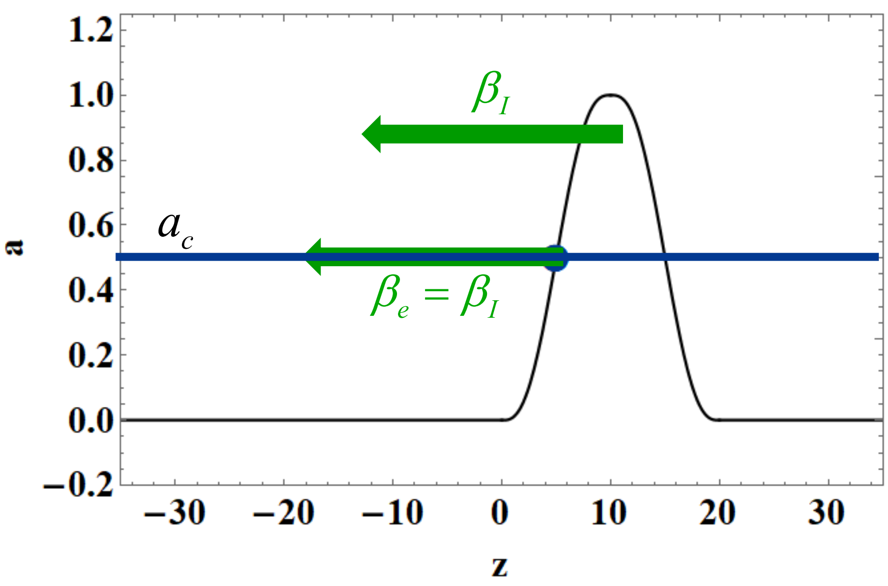
Lab Frame



Once the electron is moving at the intensity peak velocity, the peak is accelerated

When the electron reaches a_c, the peak velocity and electron velocity are equal

Lab Frame



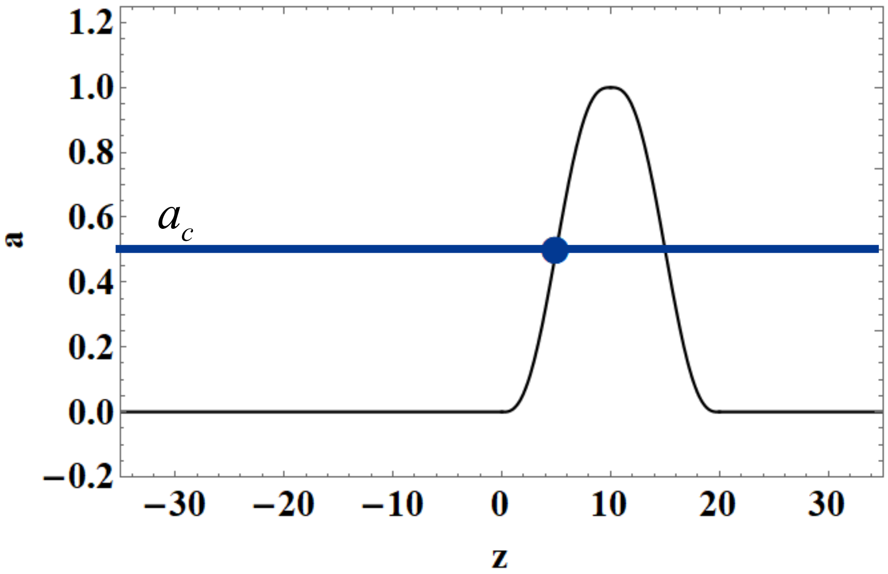
Once the electron is moving at the intensity peak velocity, the peak is accelerated

When the electron reaches a_c, the peak velocity and electron velocity are equal

The peak is then continuously accelerated to match the increasing velocity of the electron

This locks the electron to a fixed location within the pulse

Lab Frame



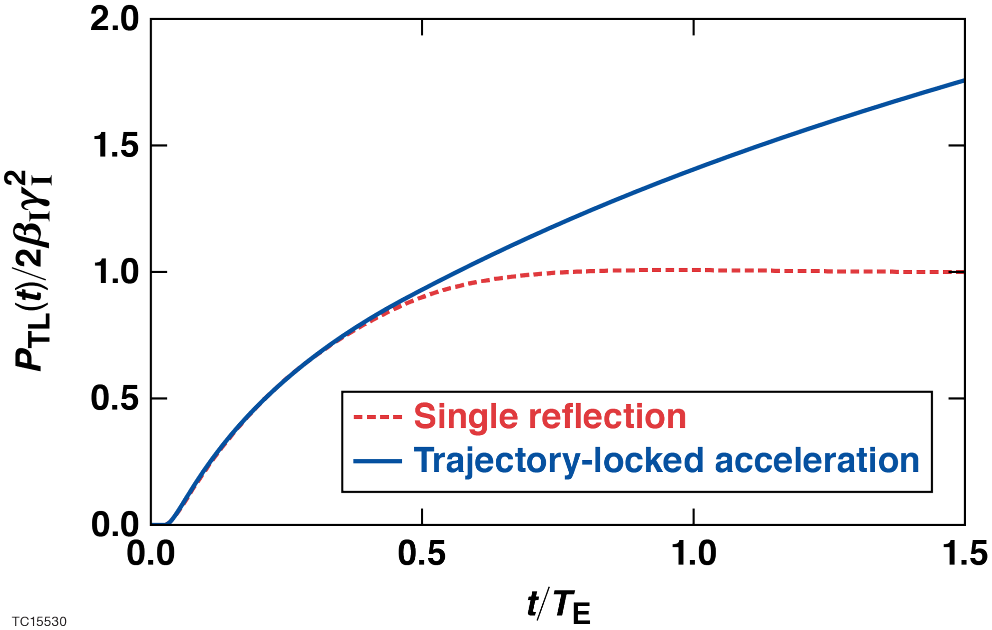
The trajectory-locked peak continually accelerates the electron

The equation of motion

$$\frac{dP_z}{dt} = -\frac{1}{2\gamma}\nabla a^2$$

can be integrated to find the momentum gain of the electron

$$\langle P_z \rangle \approx \sqrt{(\beta_I \gamma_I^2) + \tfrac{1}{2}(t - t_c)\nabla\langle a^2\rangle\Big|_{a=a_c}}$$



TC15530

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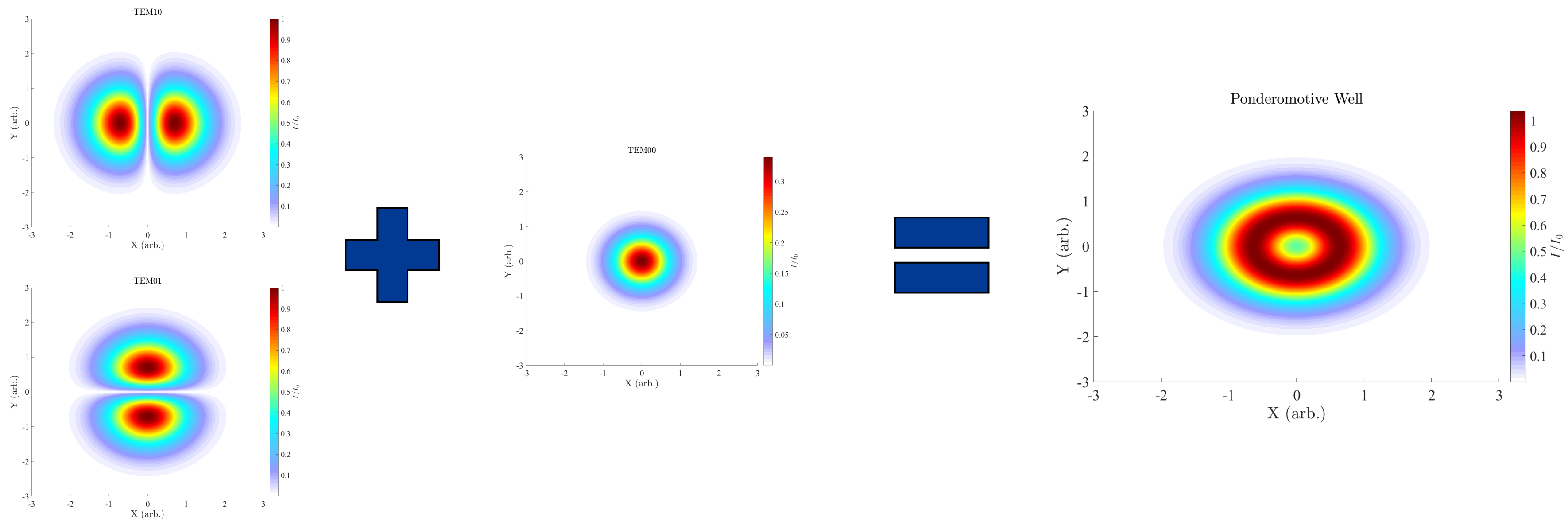
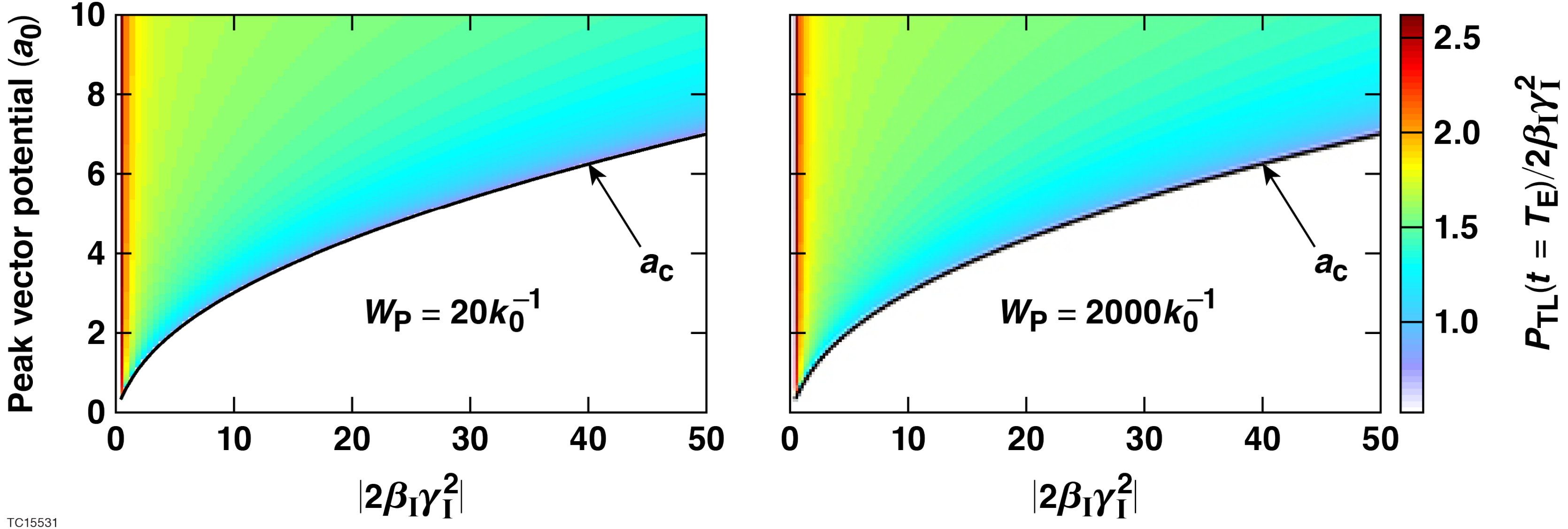
Optical Well/ Guiding Structure



$f_{TEM01} - f_{TEM10} = p / 2$

Off the cutoff curve, momentum gained through trajectory matching at T_E exceeds momentum gained in a single reflection



TC15531

The momentum gained in trajectory matching is **independent** of the duration of the intensity peak

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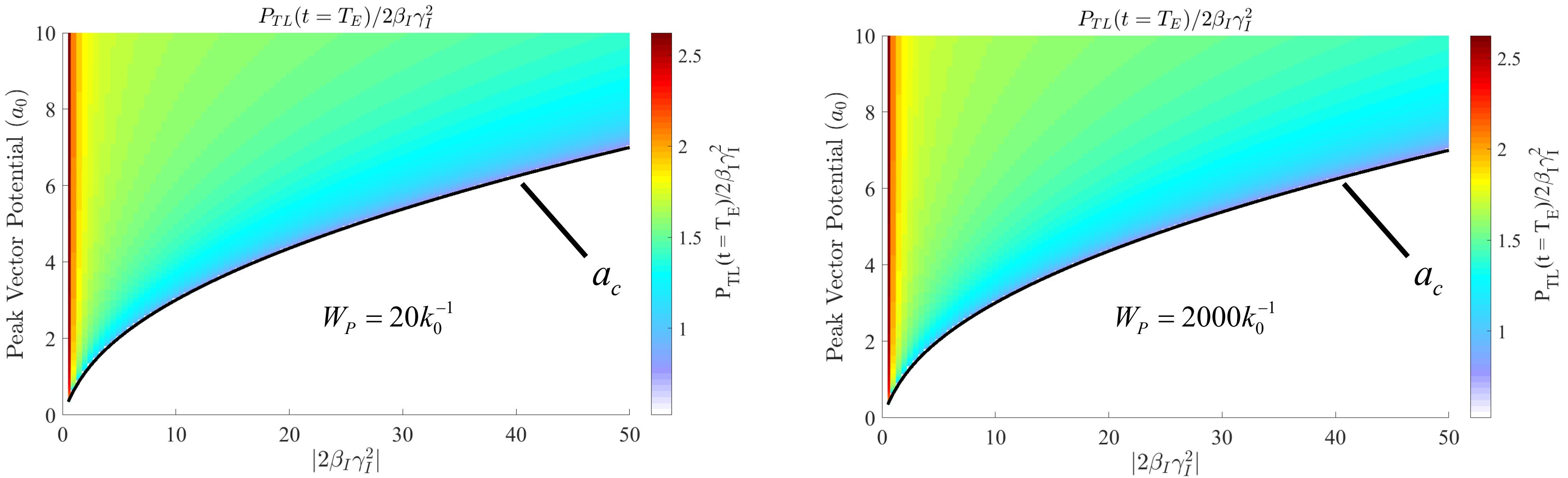

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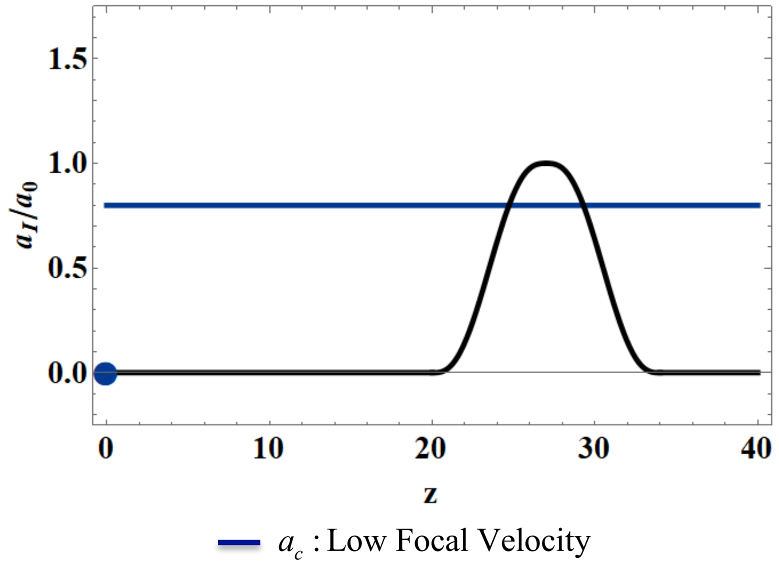

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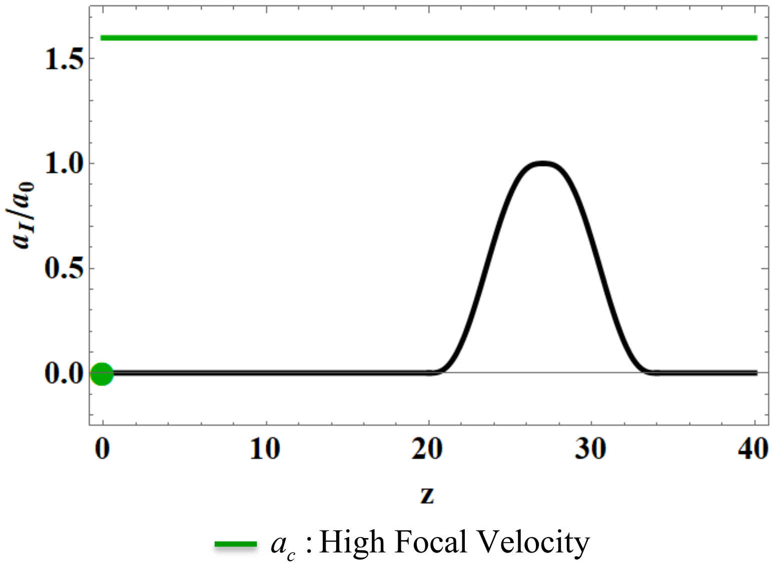
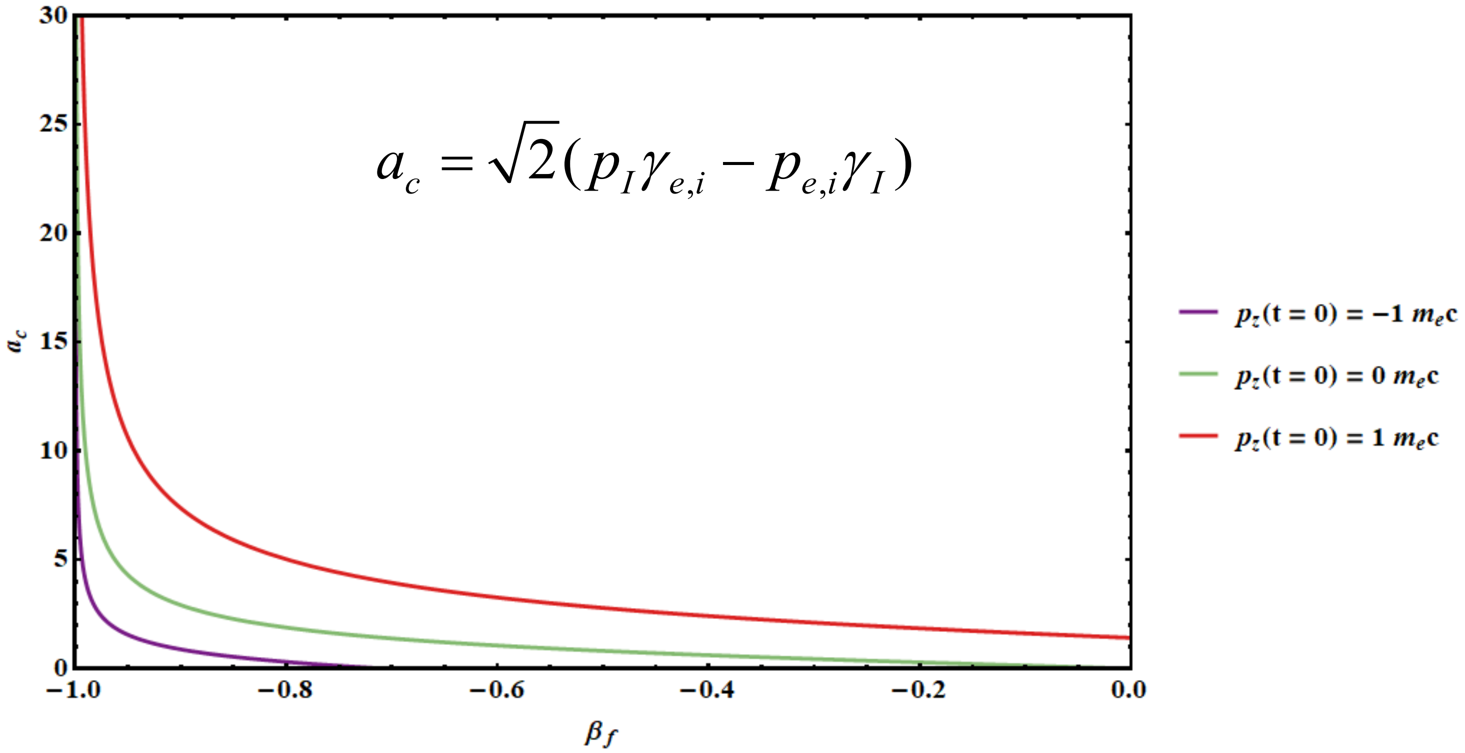
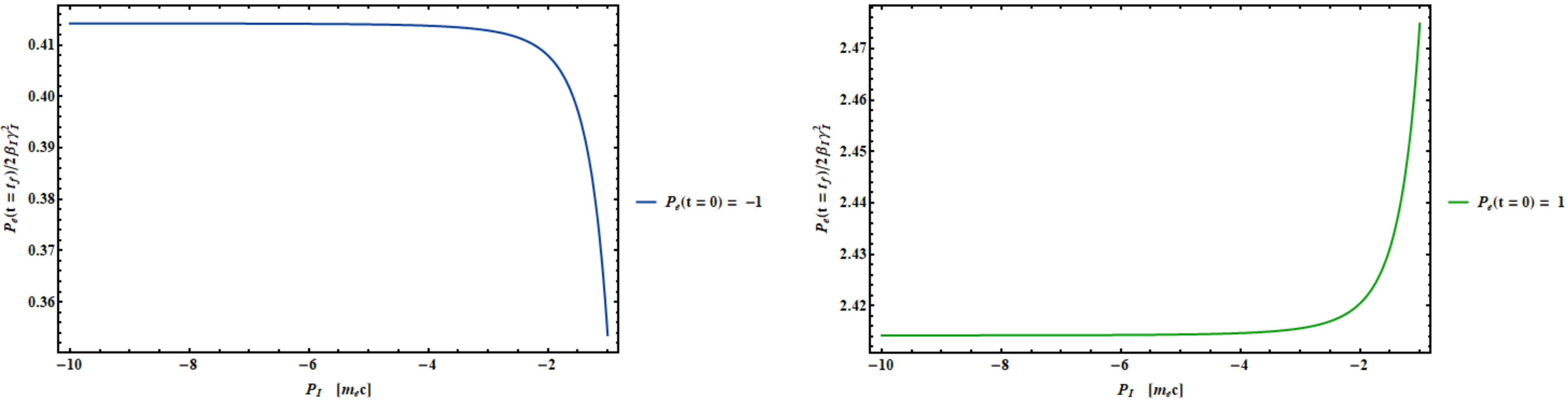


a_c is lowered with non-zero initial co-propagating momentum and raised with non-zero counter-propagating momentum

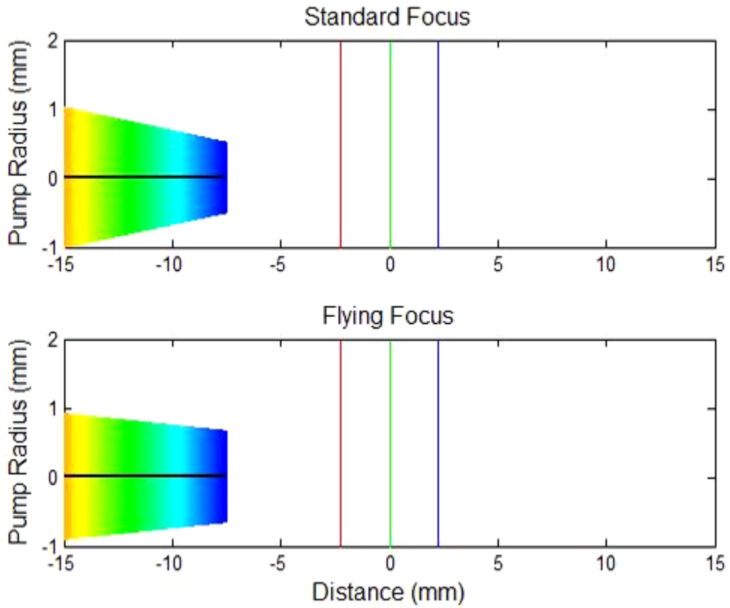


With non-zero initial momentum the final momentum of the electron is altered.

$$P_{e,f} = 2P_I\gamma_I\gamma_{e,i} - P_{e,i}(\gamma_I^2 - P_I^2)$$



Flying Focus Animation: Demonstrates what the "prepulse" is



There are four "phases" to the acceleration in a flying focus pulse

Consider placing an electron such that it first encounters the rising edge of the pulse profile.

1. The electron is accelerated from rest in the forward direction on the rising edge of the pulse.
2. The electron moves out of the rising edge into the constant body of the pulse.
3. The counter-propagating focus intercepts the electron in the pulse causing it to accelerate in the backwards direction.
4. The electron overtakes the focus and as it exits the pulse is further accelerated in the backwards direction on falling edge of the pulse

There are four "phases" to the acceleration in a flying focus pulse

Consider placing an electron such that it first encounters the rising edge of the pulse profile.

1. **The electron is accelerated from rest in the forward direction on the rising edge of pulse.**
2. The electron moves out of the rising edge into the constant body of the pulse.
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4. The electron overtakes the focus and as it exits the pulse is further accelerated in the backwards direction on falling edge of the pulse

$-\nabla\langle a^2\rangle$

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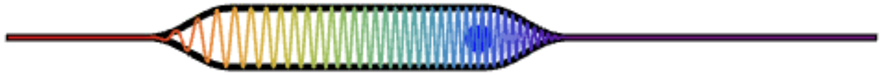
4. The electron overtakes the focus and as it exits the pulse is further accelerated in the backwards direction on falling edge of the pulse



$$-\nabla\langle a^2\rangle = 0$$

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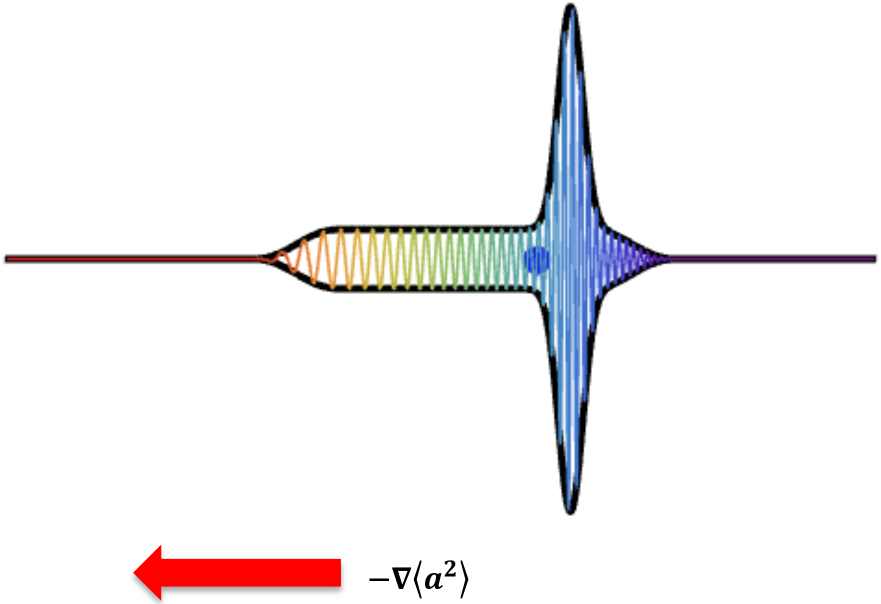

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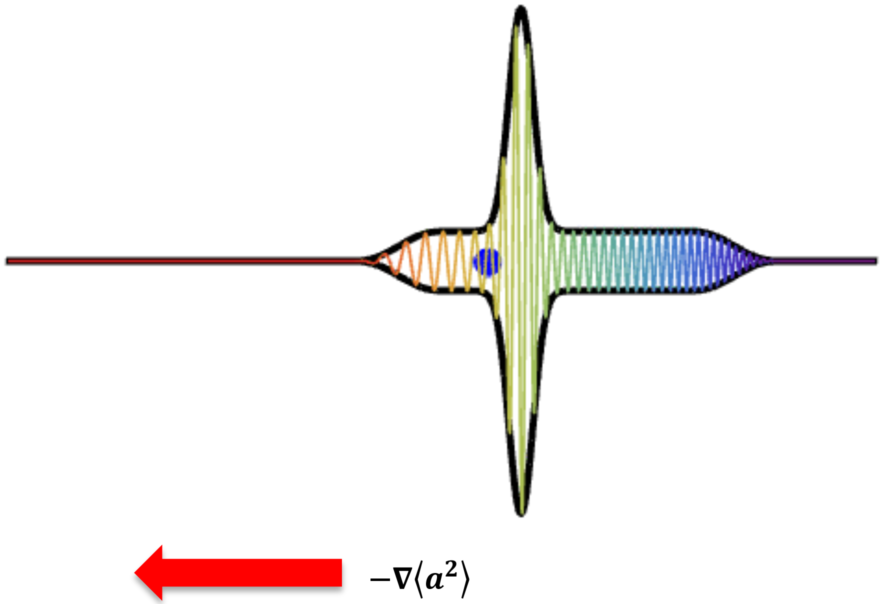
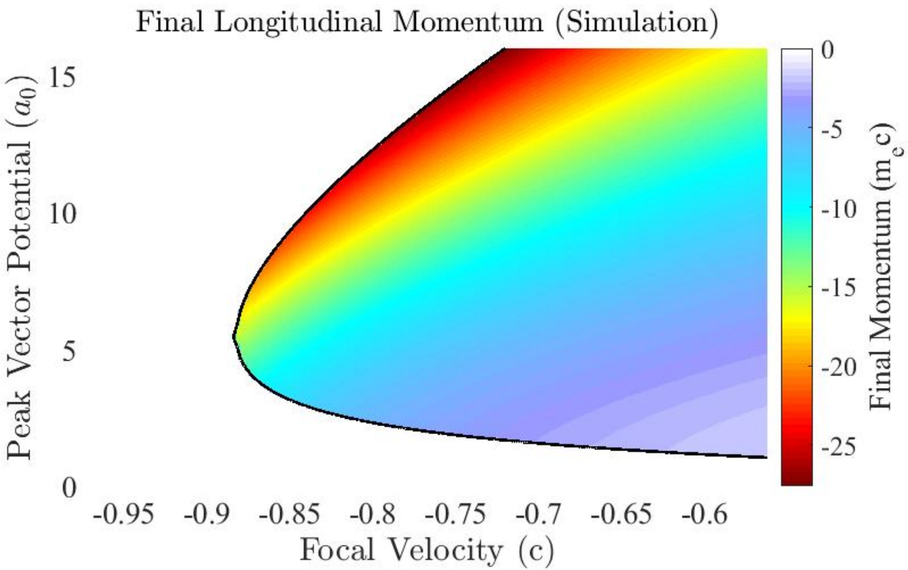
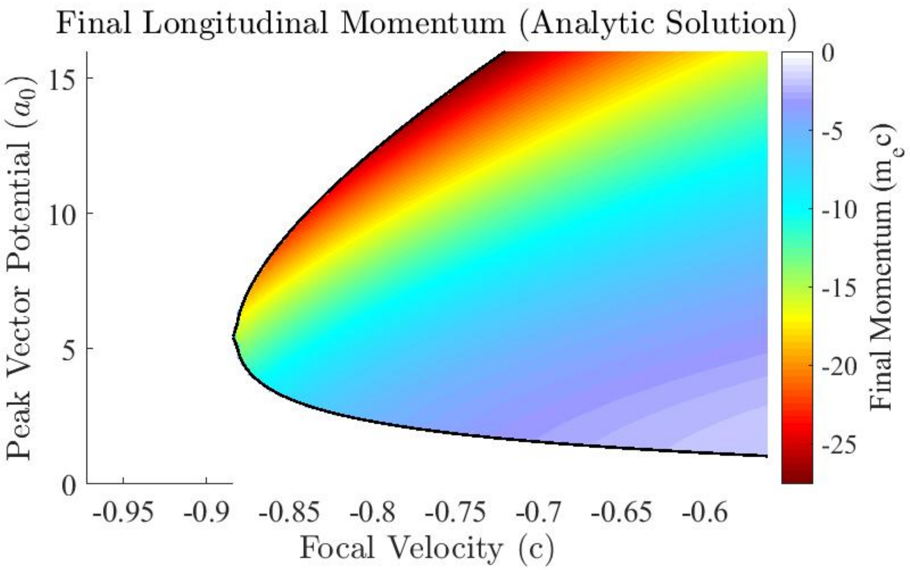


Pre-pulse Effects:





For fixed focal velocity, net energy gain requires an a_0 between two bounds

Why use a flying focus at all? Why not just inject electrons into a luminal intensity peak and extract them at peak momentum?

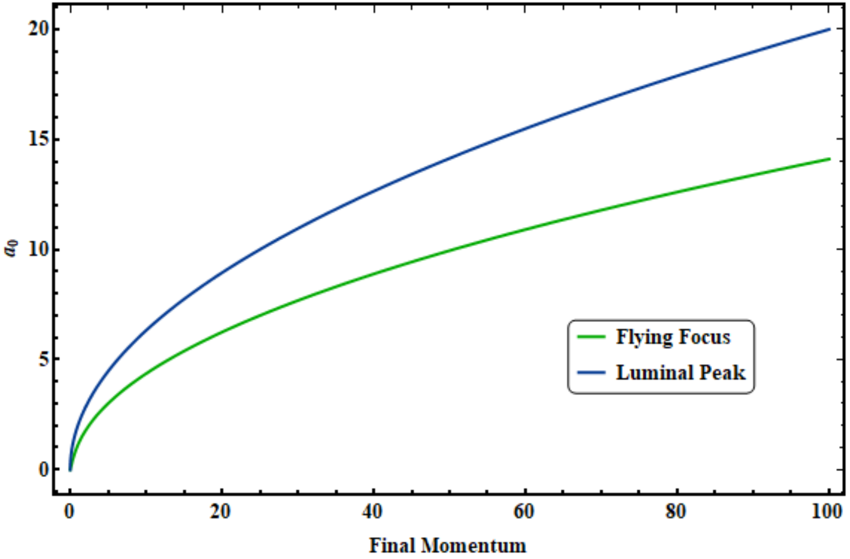
For a luminal intensity peak interacting with an electron at rest, the maximum momentum gain is

$$\langle \Delta P_z \rangle_{Max} = \frac{1}{4} a_0^2$$

For a flying focus peak, moving at a momentum P_I such that $a_0 = a_c$ the maximum momentum gain is

$$\begin{aligned}\langle \Delta P_z \rangle_{Max} &= 2P_I\sqrt{1 + P_I^2} \\ &= \frac{a_0}{2}\sqrt{2 + a_0^2}\end{aligned}$$

$$\boxed{\frac{\langle \Delta P_z \rangle_{Max,P_I}}{\langle \Delta P_z \rangle_{Max,\beta_I=1}} = \frac{2\sqrt{2 + a_0^2}}{a_0} > 2}$$



The intensity "cost" is less using subluminal peaks compared to luminal peaks

Why use a flying focus at all? Why not just inject electrons into a luminal intensity peak and extract them at peak momentum?

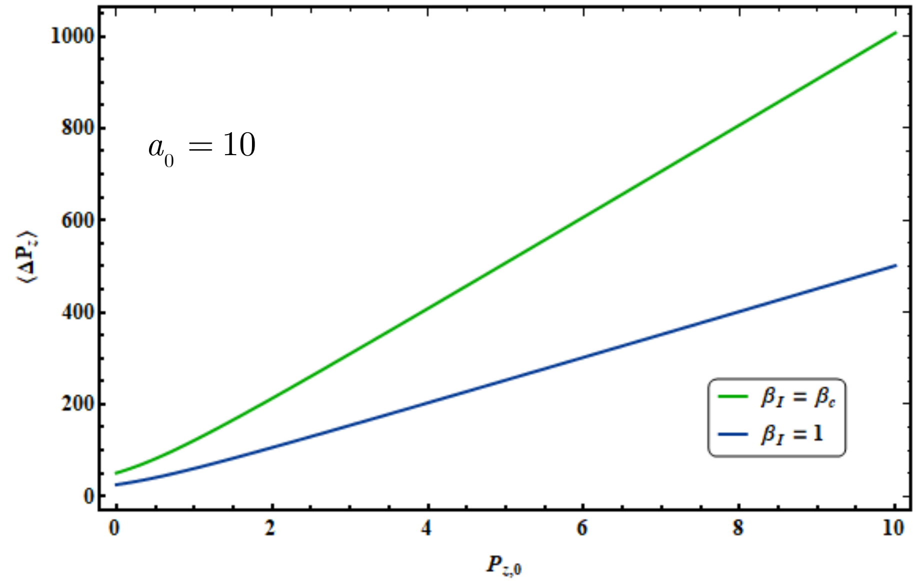
For a luminal intensity peak interacting with an electron with some initial momentum, the maximum momentum gain is

$$\langle \Delta P_z \rangle_{Max} = \tfrac{1}{4} a_0^2 (P_{z,0} + \gamma_0) \approx \tfrac{1}{2} a_0^2 \gamma_0$$

For a flying focus peak, moving at velocity β_c such that $a_0 = a_c$ the maximum momentum gain is

$$\langle \Delta P_z \rangle_{Max} = \tfrac{1}{2} a_0^2 P_{z,0} + \tfrac{1}{2} a_0 \gamma_0 \sqrt{2 + a_0^2}$$
$$\approx (\tfrac{1}{2} a_0^2 + \tfrac{1}{2} a_0 \sqrt{2 + a_0^2}) \gamma_0$$

$$\frac{\langle \Delta P_z \rangle_{Max,\beta_I=\beta_c}}{\langle \Delta P_z \rangle_{Max,\beta_I=1}} = 1 + \sqrt{1 / a_0^2 + 1} \approx 2$$



Subluminal peaks nearly double the momentum gain at maximum as compared to luminal intensity peaks