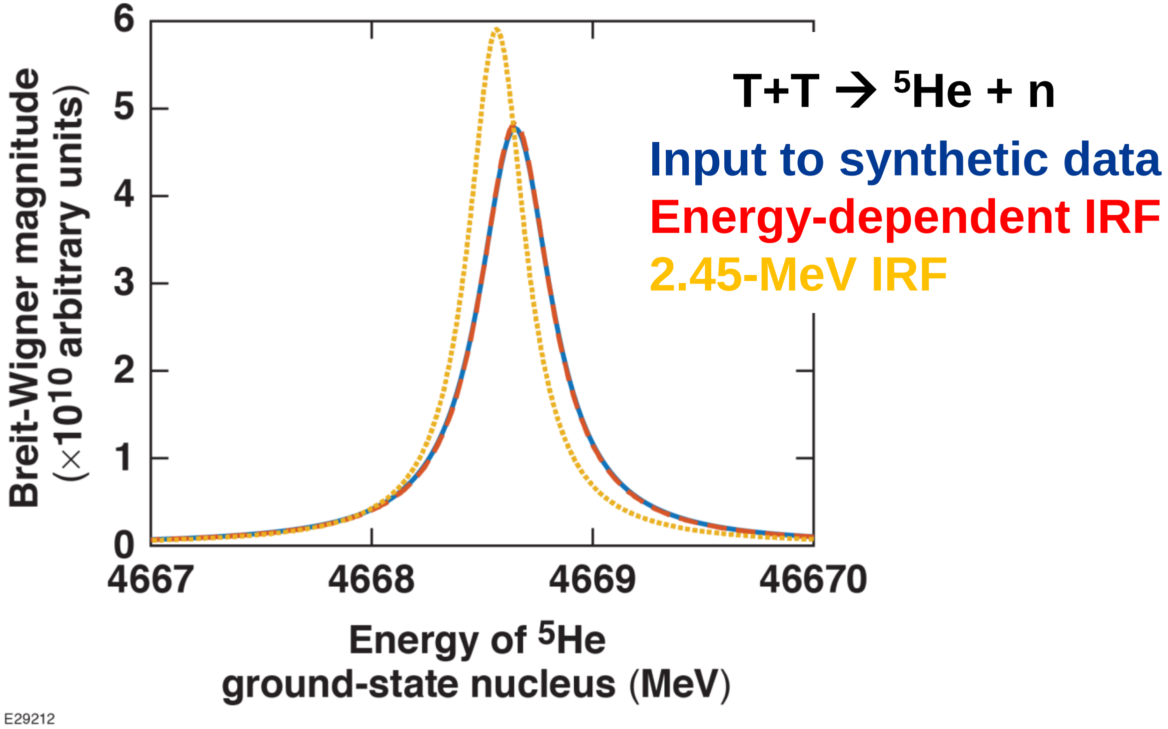
Construction and Implementation of an Energy-Dependent Instrument Response Function for Accurate Analysis of Neutron Time-of-Flight Data

^{5}He ground state inferred from forward fit to TT data



Z. L. Mohamed
University of Rochester
Laboratory for Laser Energetics

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Summary

It is necessary to use an energy-dependent IRF to accurately infer parameters from nTOF data that span a wide range of energies

- The shape of the IRF changes with incident neutron energy
- The matrix representation of the convolution is used to incorporate the energy-dependence of the IRF
- The energy-dependent IRF is needed to infer the correct lifetime and branching ratios of nuclear states (e.g., ^{5}He states inferred from TT data*)

Accurate neutron spectroscopy is now possible over a wide range of energies (e.g., areal density in cryogenic experiments, inelastic reactions involving DT neutrons on D or ^{7}Li).

IRF: instrument response function
nTOF: neutron time of flight
* M. Gatu Johnson *et al.*, Phys. Rev. Lett. 121 042501 (2018).

Collaborators

O. M. Mannion, C. J. Forrest, and J. P. Knauer

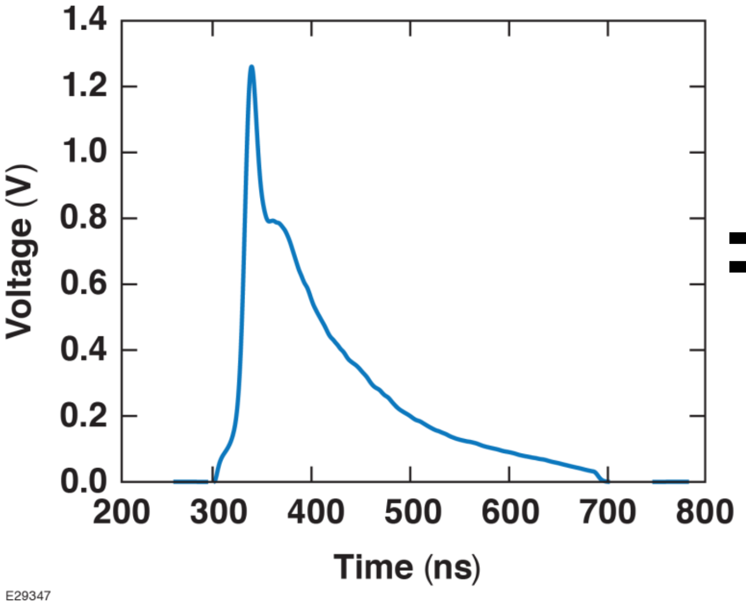
University of Rochester
Laboratory for Laser Energetics

E. P. Hartouni

Lawrence Livermore National Laboratory

The measured signal from an nTOF detector can be written as a function of time of arrival at the detector

Fit function



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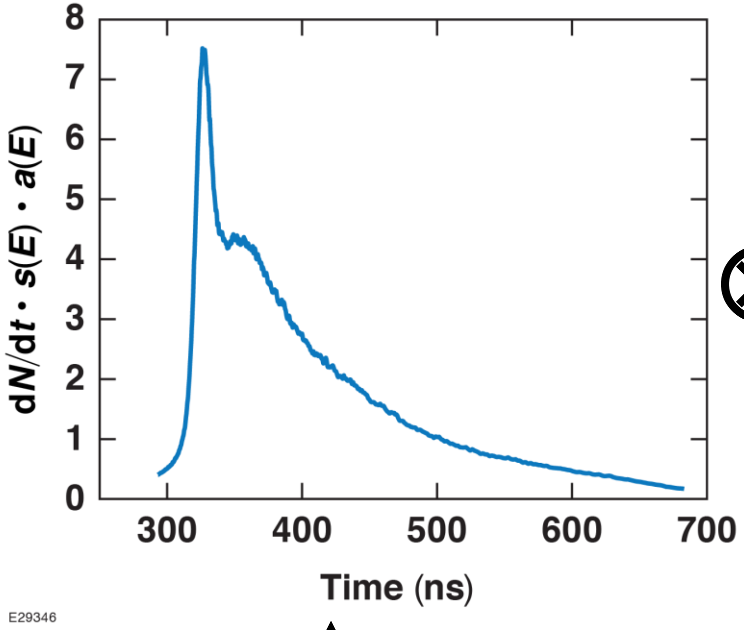
TT model



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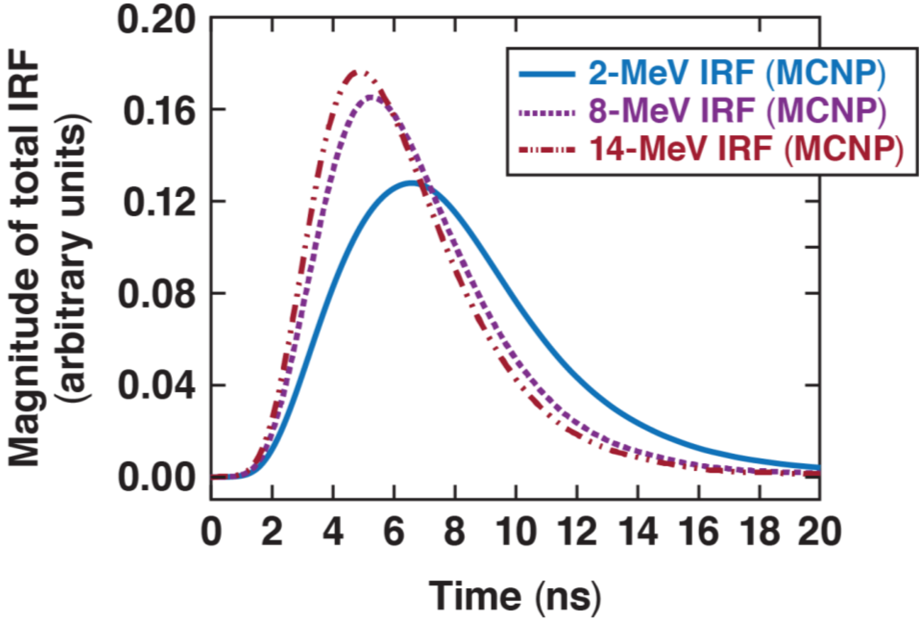
Total IRF



$$\mathrm{m}(t) = C\int_0^{t'} \underbrace{\frac{dN}{dE}[E(t')]\frac{dE}{dt'}[E(t')]}\, R[E(t'_i), t_k - t'_i]dt'$$

Previous work assumes that the IRF varies slowly with incident neutron energy

- **If the IRF varies slowly with neutron energy*,**, $R(E, t - t') \approx R(E = E_0, t - t') = R_0(t - t')$, so the fit function becomes**

$$\mathbf{m}(t) = C \int_0^{t'} \frac{dN}{dE}[E(t')] \frac{dE}{dt'}[E(t')] R_0(t - t') dt'$$

- **The integral can be written as a matrix multiplication**

$$m(t_k) = \sum_{i=1}^{k} P[E(t'_i)] R_0(t_k - t'_i) \rightarrow \vec{m} = \overline{T}\ \vec{P}$$

where T is a Toeplitz matrix of the response vector with shape N_m x N_p

and C $\frac{dN}{dE}[E(t')] \frac{dE}{dt'}[E(t')] \Delta t'_i = P[E(t'_i)] \mathrm{T}$

$$\mathbf{T} = \begin{bmatrix} r_0 & 0 & \cdots & 0 & 0 \\ r_1 & r_0 & \cdots & \cdots & \cdots \\ r_2 & r_1 & \cdots & 0 & 0 \\ \cdots & r_2 & \cdots & r_0 & 0 \\ r_{N_r-1} & \cdots & \cdots & r_1 & r_0 \\ r_{N_r} & r_{N_r-1} & \cdots & \cdots & r_1 \\ 0 & r_{N_r} & \cdots & r_{N_r-2} & \cdots \\ 0 & 0 & \cdots & r_{N_r-1} & r_{N_r-2} \\ \cdots & \cdots & \cdots & r_{N_r} & r_{N_r-1} \\ 0 & 0 & 0 & \cdots & r_{N_r} \end{bmatrix}.$$

* R. Hatarik *et al.*, Journal of Applied Physics 118 184502 (2015).
**E. P. Hartouni *et al*, Review of Scientific Instruments 87 11D841 (2016).

The energy-dependent convolution replaces the matrix elements with rows that are energy dependent*

$$\mathbf{m}(\boldsymbol{t}) = \boldsymbol{C}\int_0^{\boldsymbol{t'}} \frac{\boldsymbol{dN}}{\boldsymbol{dE}}[\boldsymbol{E}(\boldsymbol{t'})]\frac{\boldsymbol{dE}}{\boldsymbol{dt'}}[\boldsymbol{E}(\boldsymbol{t'})]\,\boldsymbol{R}[\boldsymbol{E}(\boldsymbol{t'}_i),\boldsymbol{t}_k-\boldsymbol{t'}_i]\boldsymbol{dt'} \rightarrow \boldsymbol{m}(\boldsymbol{t}_k) = \sum_{i=1}^{k}\boldsymbol{P}[\boldsymbol{E}(\boldsymbol{t'}_i)]\boldsymbol{R}[\boldsymbol{E}(\boldsymbol{t'}_i),\boldsymbol{t}_k-\boldsymbol{t'}_i] \rightarrow \vec{\boldsymbol{m}} = \overline{\boldsymbol{T}}\,\vec{\boldsymbol{P}}$$

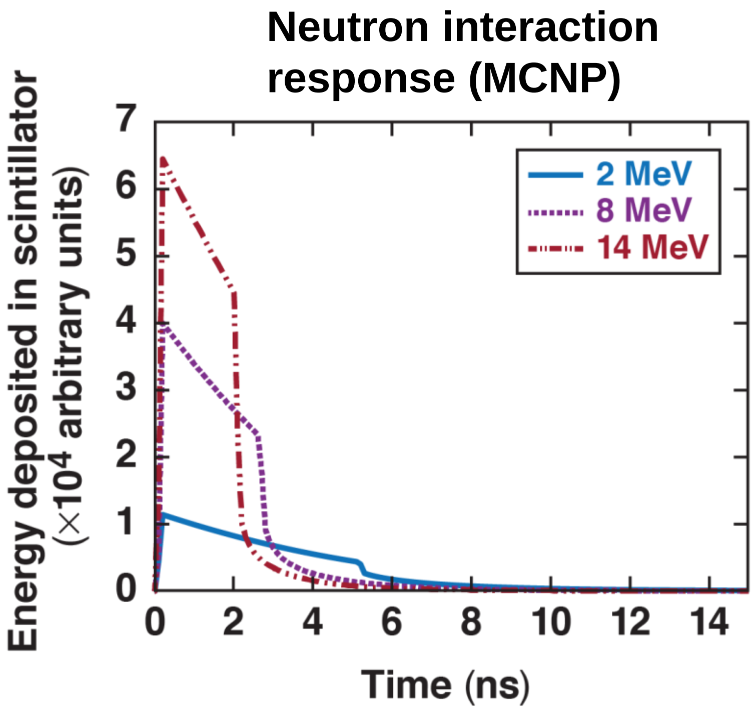
- **Each column in the matrix represents a monoenergetic IRF**

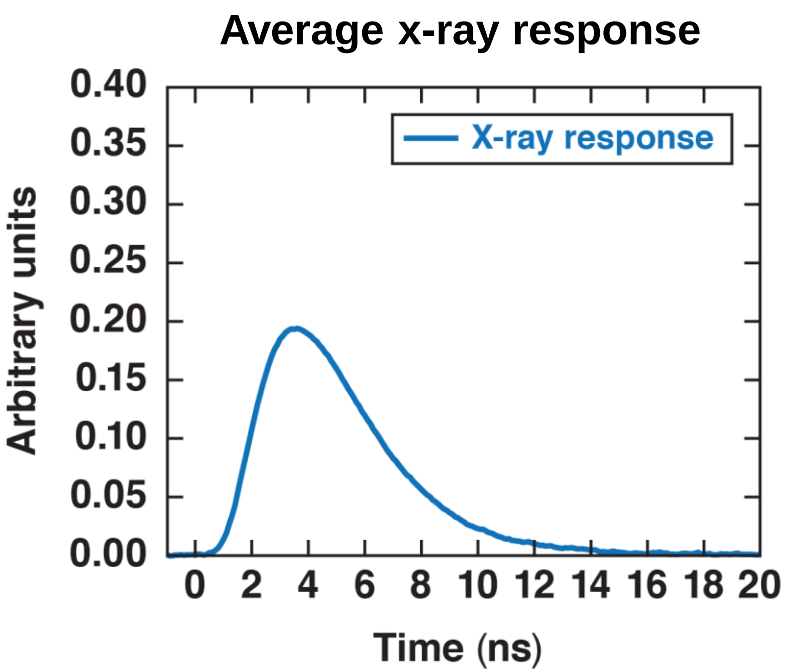
$$\mathbf{T} = \begin{bmatrix} r_{0,0} & 0 & \cdots & 0 & 0 \\ r_{0,1} & r_{1,0} & \cdots & \cdots & \cdots \\ r_{0,2} & r_{1,1} & \cdots & 0 & 0 \\ \cdots & r_{1,2} & \cdots & r_{N_p-1,0} & 0 \\ r_{0,N_r-1} & \cdots & \cdots & r_{N_p-1,1} & r_{N_p,0} \\ r_{0,N_r} & r_{1,N_r-1} & \cdots & \cdots & r_{N_p,1} \\ 0 & r_{1,N_r} & \cdots & r_{N_p-1,N_r-2} & \cdots \\ 0 & 0 & \cdots & r_{N_p-1,N_r-1} & r_{N_p,N_r-2} \\ \cdots & \cdots & \cdots & r_{N_p-1,N_r} & r_{N_p,N_r-1} \\ 0 & 0 & 0 & \cdots & r_{N_p,N_r} \end{bmatrix}.$$

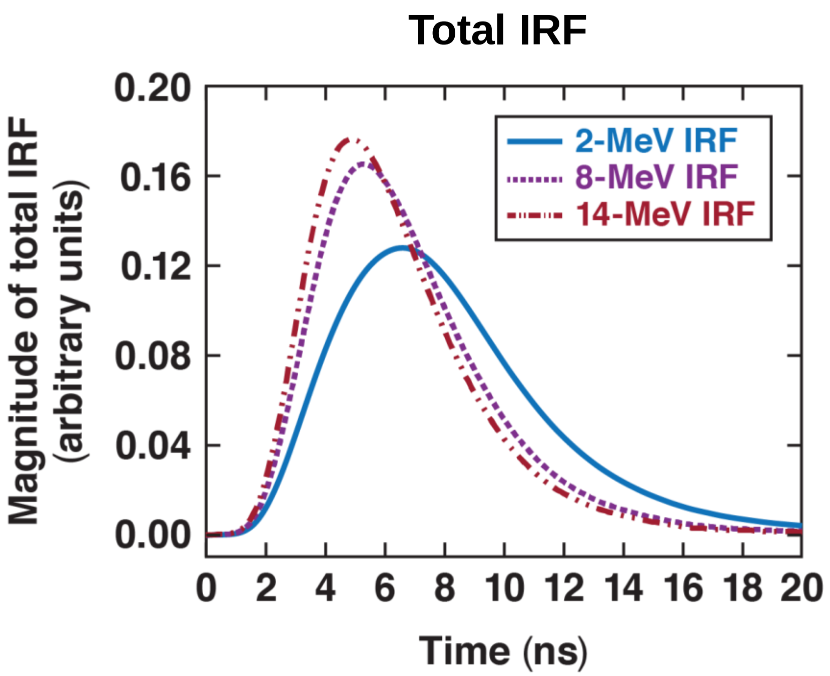
*** Z. L. Mohamed, O. M. Mannion, E. P. Hartouni, J. P. Knauer, and C. J. Forrest, submitted to Journal of Applied Physics**

A monoenergetic IRF can be constructed by convolving a measured x-ray response with a calculated neutron interaction response

- The x-ray response → impulse response
- Neutron interaction response → neutron transport through the detector
  - IRF shape is a function of neutron energy



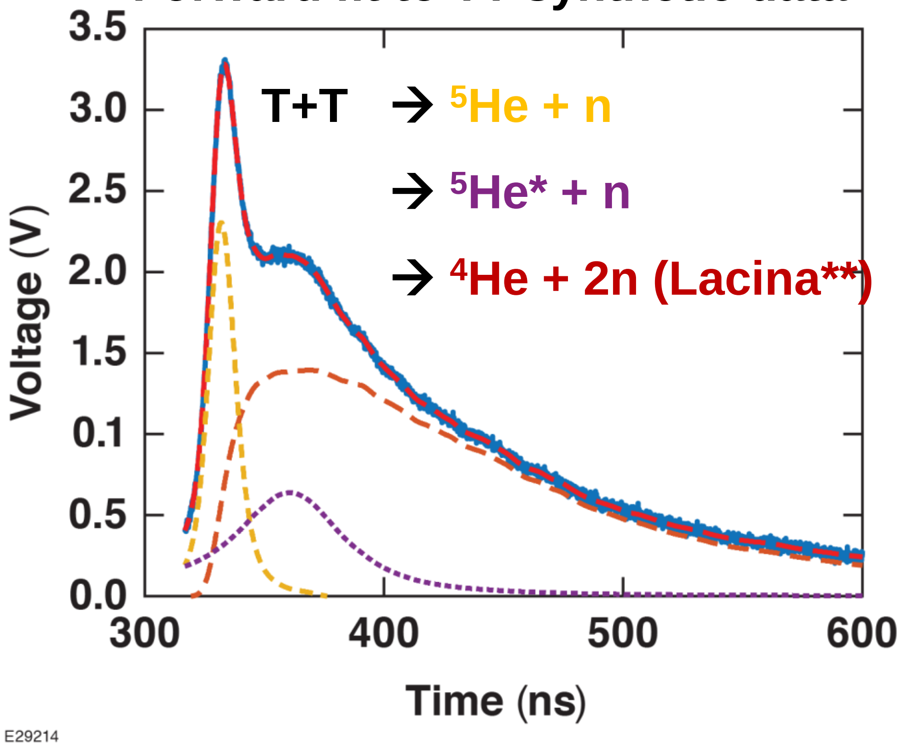




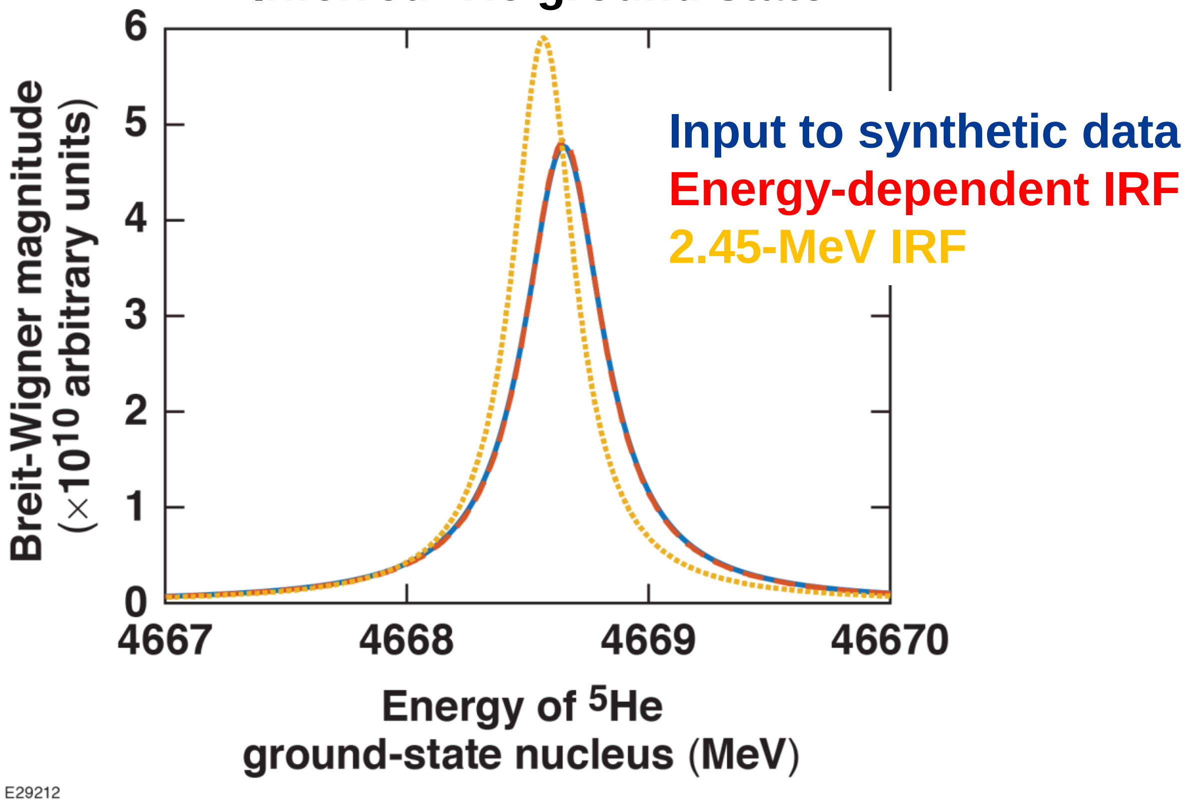
E29211a

Use of the energy-dependent IRF accurately infers the widths and masses of the ^{5}He states from TT nTOF data*

Forward fit to TT synthetic data



Inferred ^{5}He ground state



** M. Gatu Johnson *et al.*, Phys. Rev. Lett. 121 042501 (2018).
** B. Lacina, J. Ingley, and D. W. Dorn, Lawrence Livermore National Laboratory, Report UCRL-7769 (1965).

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It is necessary to use an energy-dependent IRF to accurately infer parameters from nTOF data that span a wide range of energies

- **The shape of the IRF changes with incident neutron energy**
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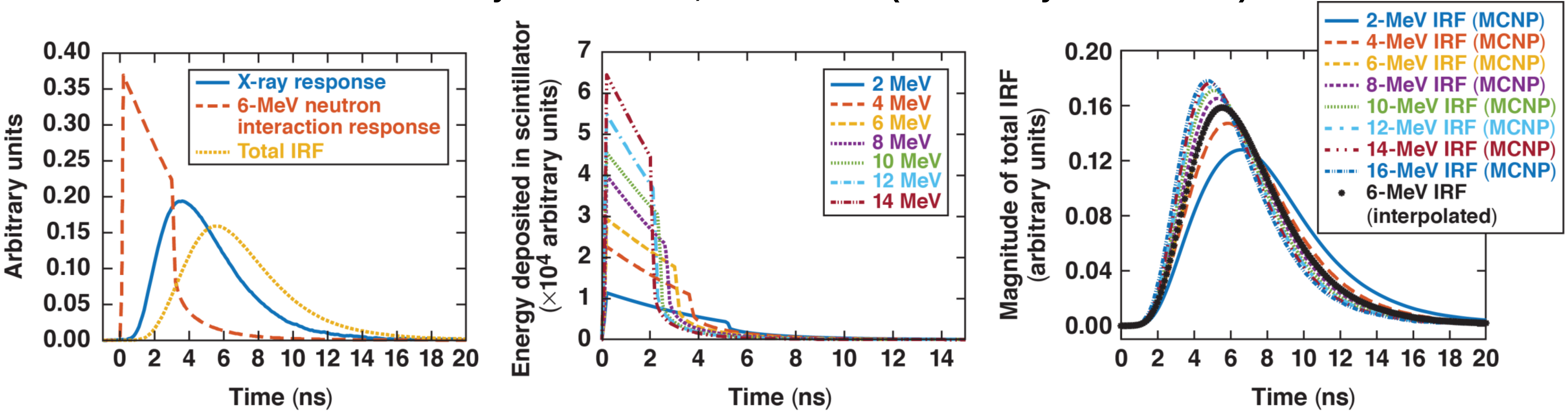
Backup

Interpolation over the total IRF is more accurate than attempting to directly interpolate over the neutron interaction responses

- The sharp features in the neutron IRF make direct linear interpolation difficult
  - Generate a representative set of neutron interaction responses, then interpolate over the total IRF
- The x-ray response and neutron interaction responses must be start at t=0 and be area normalized to conserve correct timing and yields in the forward fit
- Uncertainty in the total IRF arises mainly from noise on x-ray IRF*
  - This is an uncertainty in the model, not the data (address by Monte Carlo)



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Mohamed et al., submitted to Journal of Applied Physics

The measured signal from an nTOF detector can be written as a function of time of arrival at the detector

- $$\mathrm{m}(t) = C\int_0^t \frac{dN}{dE}[E(t)]\,\frac{dE}{dt}[E(t)]\,R(E,t)dt$$

m(t) = the measured nTOF signal

t = time scale recorded by oscilloscope

C = calibration constant

dN/dE = neutron energy spectrum

dE/dt = Jacobian

R(E,t) = total IRF

- This can be rewritten as a function of the neutron's time of arrival at the detector (t')

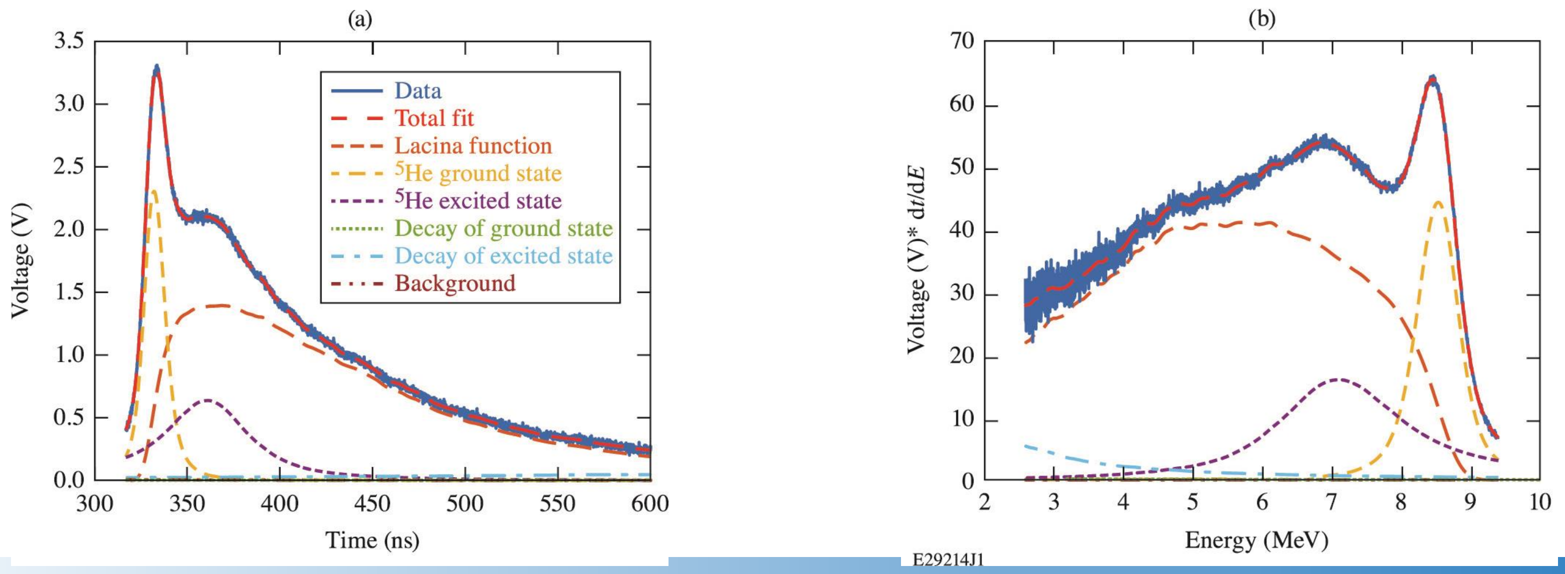
$$\mathrm{m}(t) = C\int_0^{t'} \frac{dN}{dE}[E(t')]\,\frac{dE}{dt'}[E(t')]\,R[E(t'_i),t_k - t'_i]dt'$$

Synthetic TT data (based on real OMEGA TT data) can be used as an example

- **Results:**
  - **Inferred mass isn't affected by choice of IRF (expected because mass~mean neutron energy)**
  - **Use of energy-dependent IRF infers correct width for both states to within 2%**
  - **Use of 2.45-MeV IRF causes ~22% decrease in inferred ^{5}He ground state width, ~10% increase in inferred first excited state width**
    - **Expected because ground state mass ~0.4 MeV and first excited state mass ~2.5 MeV**
  - **Use of 14.03-MeV IRF inferred correct width for both states to within a few percent**
    - **Width of 14.03-MeV IRF was within 300 ps of width of 8.5-MeV IRF (ground state neutron energy)**
    - **This effect only occurs because of this specific detector configuration/material/distance and this specific nuclear data set**
- **More complex and more novel data with many resonances (e.g., n(D,p)2n or ^{7}Li data) requires matrix IRF**

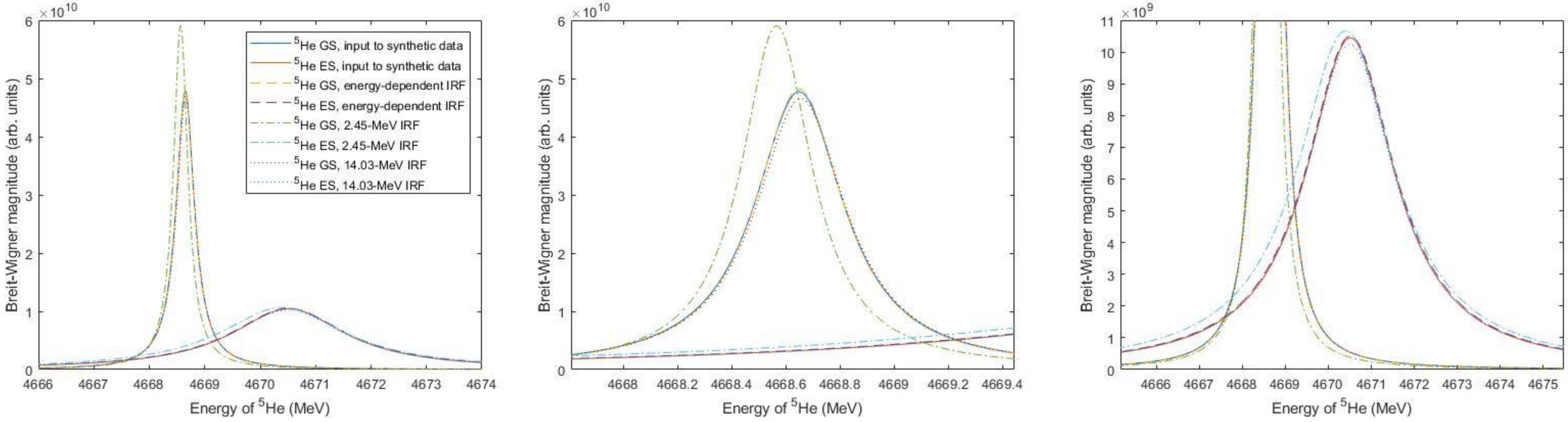
Synthetic TT data (based on real OMEGA TT data) can be used as an example

- The properties (mass and width/lifetime) of the ^{5}He ground state and first excited state can be inferred from a forward fit to TT nTOF data
  - Ground state at ~8.5 MeV neutron energy, 0.4 MeV width, excited state at ~6.5 MeV neutron energy, 2.5 MeV width
- Synthetic data was used to compare inferred values to input used to generate synthetic data
  - Monoenergetic IRF’s at 2.45-MeV (DD) and 14.03-MeV (DT) can be compared to energy-dependent IRF



Synthetic TT data (based on real OMEGA TT data) can be used as an example

- The following results only occur because of this specific detector configuration/material/distance and this specific nuclear data set
- More complex and more novel data with many resonances (e.g., n(D,p)2n or ^{7}Li data) requires matrix IRF



Minimization in the forward fit must include both Gaussian- and Poisson-distributed uncertainties

- Gaussian-distributed uncertainties from digitization noise and Poisson-distributed uncertainties from predicted number of neutrons per time bin
  - Poisson component can only be determined based on the current iteration of the forward fit

- Minimize $\chi^2 = \sum_{i=1}^{x} \frac{(fit_i - data_i)^2}{\sigma_i^2}$ where $\sigma_i^2 = \sigma_{scope}^2 + \sigma_{Vi}^2$
  - First term is Gaussian-distributed digitization noise, second term includes Poisson-distributed uncertainties based on each iteration of the forward fit
  - If fit function has a form resembling $V(t) \sim k\,\{[s(E)a(E)\frac{dN}{dE_{4\pi}}\frac{dE}{dt}] \otimes IRF\}$, then the number of detected neutrons is $n(t) = a(E)\frac{dN}{dE_{4\pi}}\frac{dE}{dt}$
    - $V(t) \sim k\,\{[s(E) \bullet a(E) \bullet n] \otimes IRF\} \rightarrow \sigma_{Vi}^2 = V_i^2\{\frac{\sigma_k^2}{k^2} + \frac{(n \bullet s^2 \bullet dt) \otimes IRF}{[(n \bullet s) \otimes IRF]^2}\}$

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The uncertainty introduced by the IRF must be determined by Monte Carlo

- Minimization in the forward fit must include both Gaussian- and Poisson-distributed uncertainties in the data
- The main source of uncertainty in the total IRF comes from digitization noise on the measured x-ray response
  - I.e., the measured x-ray response is one of an ensemble of responses we could have measured given some unknown "true" x-ray response
  - There are Poisson uncertainties in the calculated neutron interaction responses, but these can be minimized by running simulations with high statistics
- In relation to the forward fit, this is an uncertainty in the model, not an uncertainty in the data
- The most straightforward way to address this type of uncertainty is by Monte Carlo
  - Perturb the x-ray IRF, build a series of new matrix IRFs and histogram the results

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