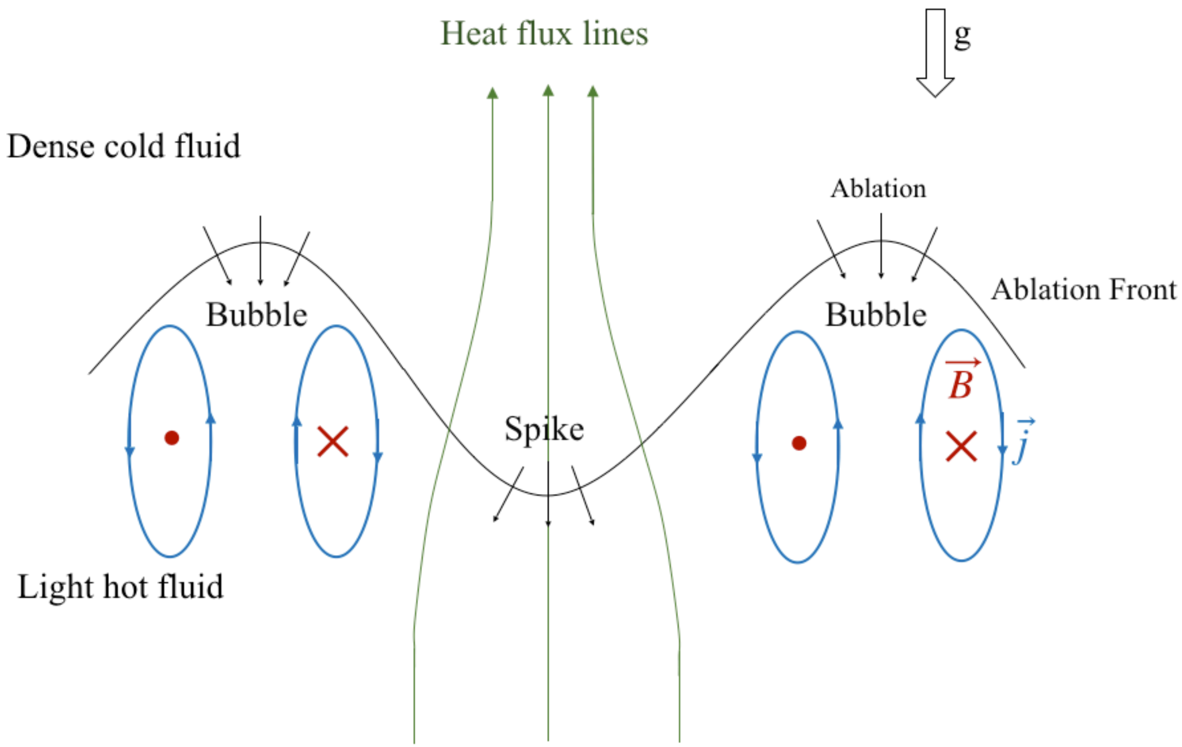
# Magnetic-Field Effect on Rayleigh-Taylor and Darrieus-Landau Instabilities




F. Garcia-Rubio
University of Rochester
Laboratory for Laser Energetics

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# Summary/Conclusions

- **The sharp boundary model (SBM) analysis yields a dispersion relation that is only governed by**
  - **Froude number: Fr $\gg 1$,**
  - **Critical-to-ablation front density ratio: $D_R = n_c/n_a \ll 1$.**

- **The self-generated B field helps to stabilize the RT instability.**

- **The self-generated B field does not affect the DL instability.**

# Collaborators


**Riccardo Betti**
University of Rochester
Laboratory for Laser Energetics
Department of Mechanical Engineering
Department of Physics and Astronomy

**Hussein Aluie**
University of Rochester
Department of Mechanical Engineering

**Javier Sanz Recio**
Universidad Politécnica de Madrid
School of Aerospace Engineering

# The magnetic field is generated during the development of the ARTI, modifying its dynamics

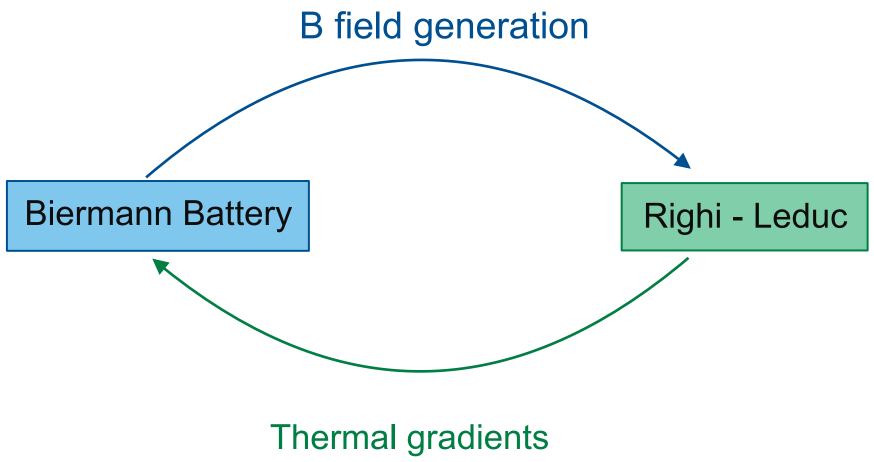
UR LLE

## B-Field generation

- The magnetic field is generated due to the **Biermann-Battery** effect:

$$\frac{\partial \vec{B}}{\partial t} \sim \frac{\nabla p \times \nabla n}{n^2} \propto \frac{\partial \vec{\omega}}{\partial t}$$

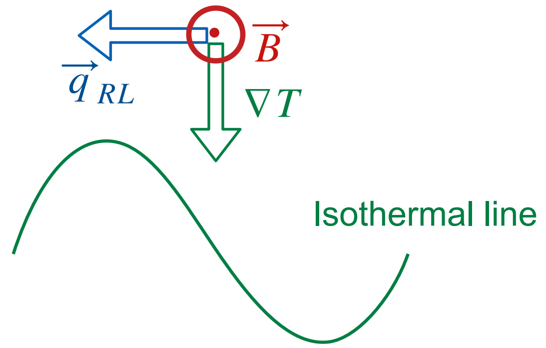
- Its generation comes hand in hand with **vorticity**.
- Most of the B field is generated at the **ablation front** and **convected** towards the hot coronal plasma.




## B-Field Effect

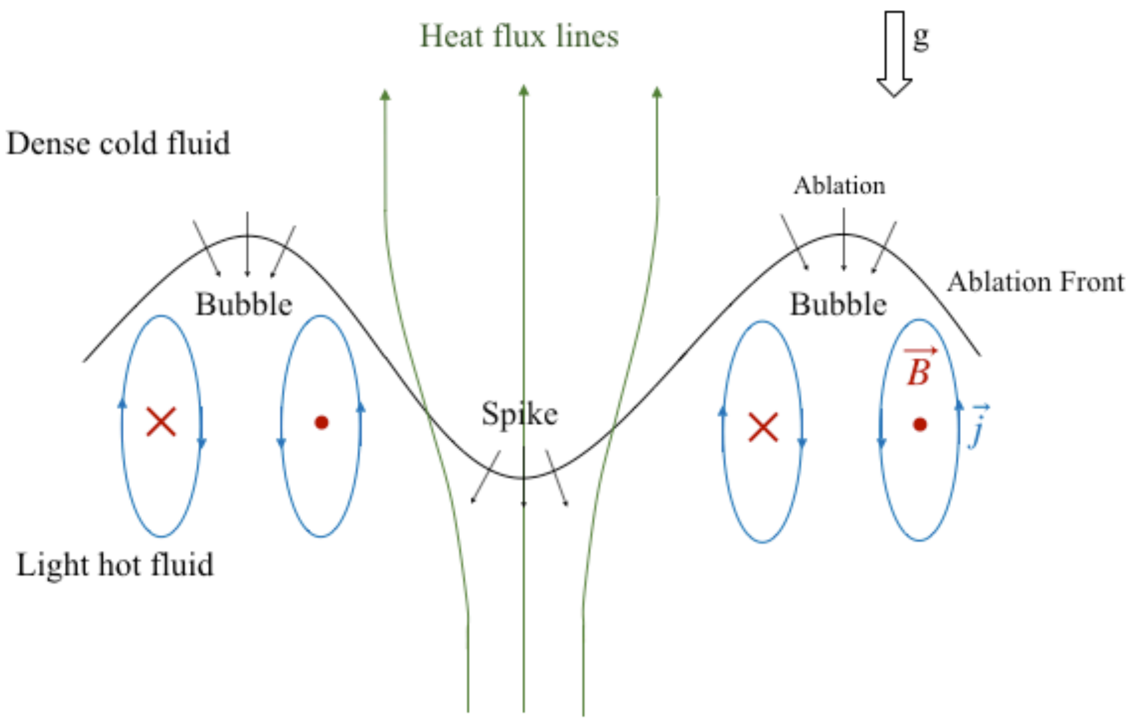
- In the linear regime, the B-field effect on the hydrodynamics lies in the **Righi-Leduc** term:

$$\vec{q}_{RL} \sim -\frac{T^4}{n}\vec{B} \times \nabla T$$

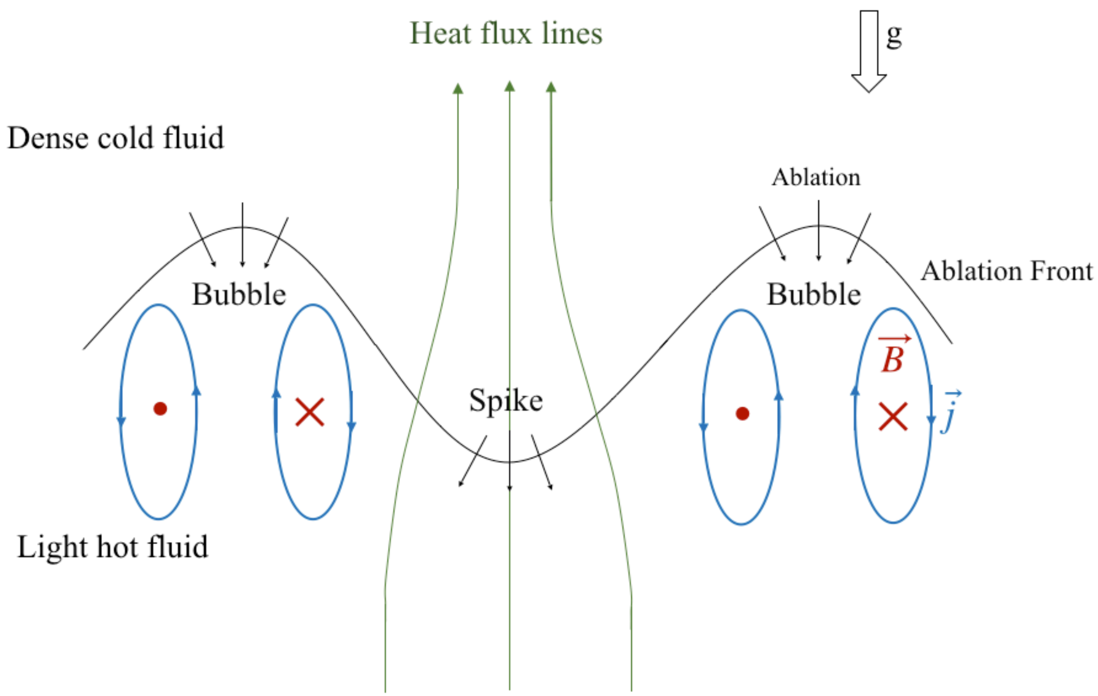


# The B field modifies the ablation rate. Depending on its sign, it can be enhanced or diminished

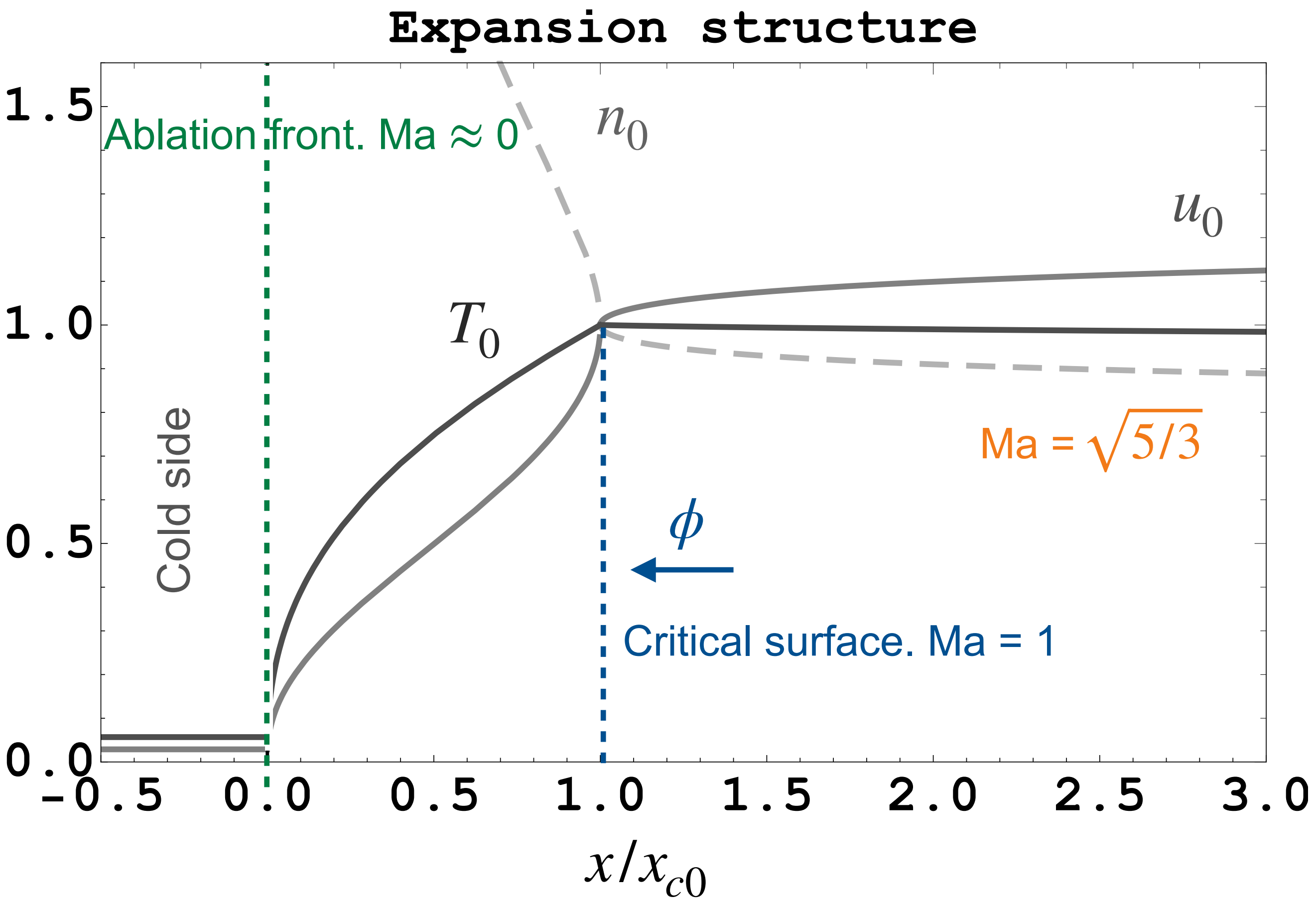
Destabilizing configuration: Ablation is diminished




Stabilizing configuration: Ablation is enhanced



# We consider a full expansion structure that undergoes RT and DL instabilities




- Instabilities:
  - **Rayleigh-Taylor**: Short wavelengths.
  - **Darrieus-Landau**: Long wavelengths.
- Perturbed **mass** and **momentum** fluxes at the ablation front

$$n_0 u_1 + n_1 u_0 = f k \quad \text{Mass}$$

$$2u_1 + n_1 u_0^2 + p_1 = q k^{3/5} \quad \text{Momentum}$$

**Dispersion relation**

$$\gamma^2 + \gamma k u_a (1 + f) - k^2 u_a^2 f - kg \left(1 - q k^{3/5} \frac{u_a^2}{g L_a^{2/5}}\right) = 0$$

# We exploit the fact that the hot plasma is quasi-stationary to derive the stabilization mechanisms

Quasistationary — Nonsteady

Mass $$f = f_1 + \gamma \frac{f_2}{k^{3/5}} + O\left(\gamma^2\right)$$

Momentum $$q = q_1 + \gamma \frac{q_2}{k^{3/5}} + O\left(\gamma^2\right)$$

$$\boxed{\frac{\gamma L_a}{u_a} = \sqrt{\underbrace{\frac{kL_a}{\mathrm{Fr}}}_{\text{RT}} - \underbrace{q_1 \left(kL_a\right)^{8/5}}_{\text{Dyn. pressure}}} - \underbrace{\frac{1 + f_1 + q_2}{2} kL_a}_{\text{Conv. stabilization}}}$$

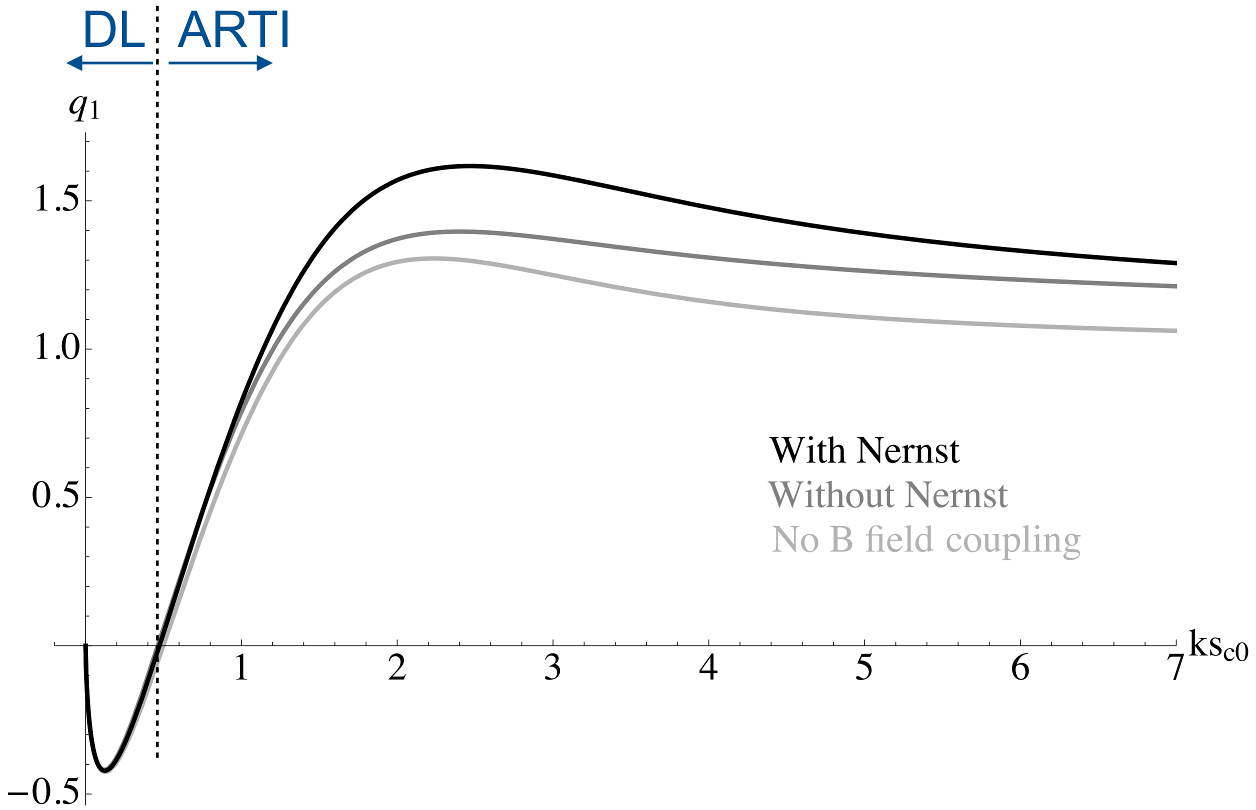
**Stabilization mechanisms**

- Stabilization by dynamic pressure:
  - Conservative restoring force.
  - Stationary momentum flux $q_1 \sim p_1/k^{3/5} \sim \omega/k^{8/5}$.
  - Cutoff $k_c L_a \sim \mathrm{Fr}^{-5/3}$.
- Stabilization by convection:
  - Non-conservative damping.
  - Stationary mass flux $f_1$ and nonsteady momentum flux $q_2$.
  - Cutoff $k_c L_a \sim \mathrm{Fr}^{-1/2}$.

**Dynamic pressure becomes the main stabilization mechanism**

# Analysis of the dynamic pressure term

$$\frac{\gamma L_a}{u_a} = \sqrt{\underbrace{\frac{kL_a}{\mathrm{Fr}}}_{\text{RT}} - \underbrace{q_1 \left(kL_a\right)^{8/5}}_{\text{Dyn. pressure}}} - \underbrace{\frac{1+f_1+q_2}{2} kL_a}_{\text{Conv. stabilization}}$$




- The **eigenvalues** $q_1, q_2, f_1$ depend on: $\boxed{ks_{c0} \sim \frac{kL_a}{\left(n_c/n_a\right)^{5/2}}}$
- The momentum flux / pressure / vorticity **changes sign** at

$$\boxed{ks_{c0} \approx 0.5}$$

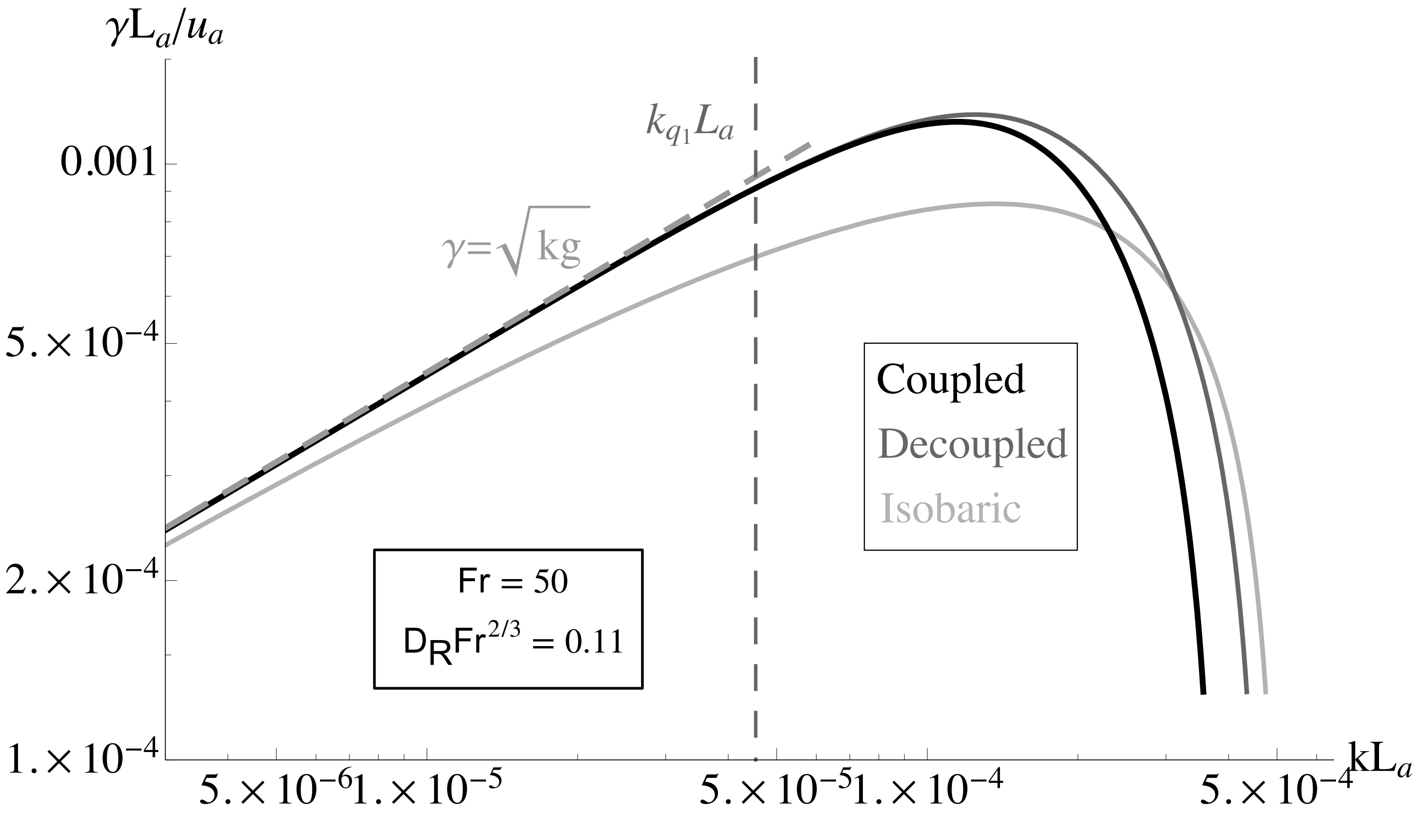
- For $ks_{c0} < 0.5$, **vorticity** is negative and becomes the driving mechanism of the **DL** instability.
- The effect of the **B field** does not depend upon any parameter.
- It **enhances** the rocket effect increasing the perturbed pressure.

# Dispersion relation in an ICF context

- The dispersion relation is governed by **two parameters**:
  - Froude number $\mathrm{Fr} \gg 1$
  - Density ratio $\mathrm{D_R} = n_c/n_a \ll 1$
- A particular **combination** characterizes qualitatively the spectrum:

$$\mathrm{D_R Fr^{2/3}} = 0.015 \frac{\left(\frac{u_a}{1\mu\mathrm{m/ns}}\right)^4 \left(\frac{n_a}{10^{24}\ \mathrm{cm^{-3}}}\right)^{2/3}}{\left(\frac{g}{100\mu\mathrm{m/ns^2}}\right)^{2/3} \left(\frac{T_a}{10\ \mathrm{eV}}\right)^{8/3}}$$

- For $\mathrm{D_R Fr^{2/3}} \ll 1$, the spectrum is **RT dominated**.
- For $\mathrm{D_R Fr^{2/3}} \gg 1$, the **DL instability** becomes important.



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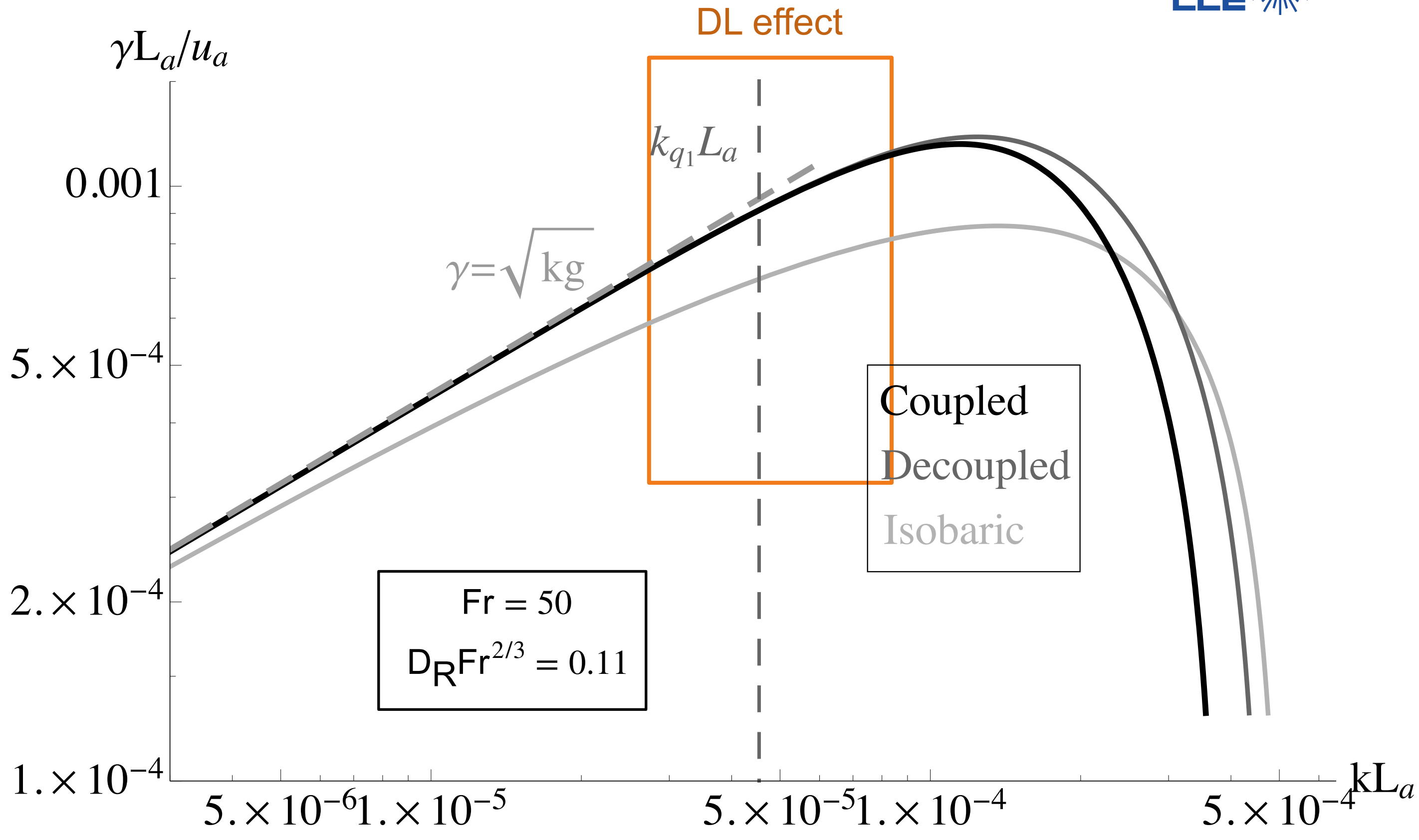

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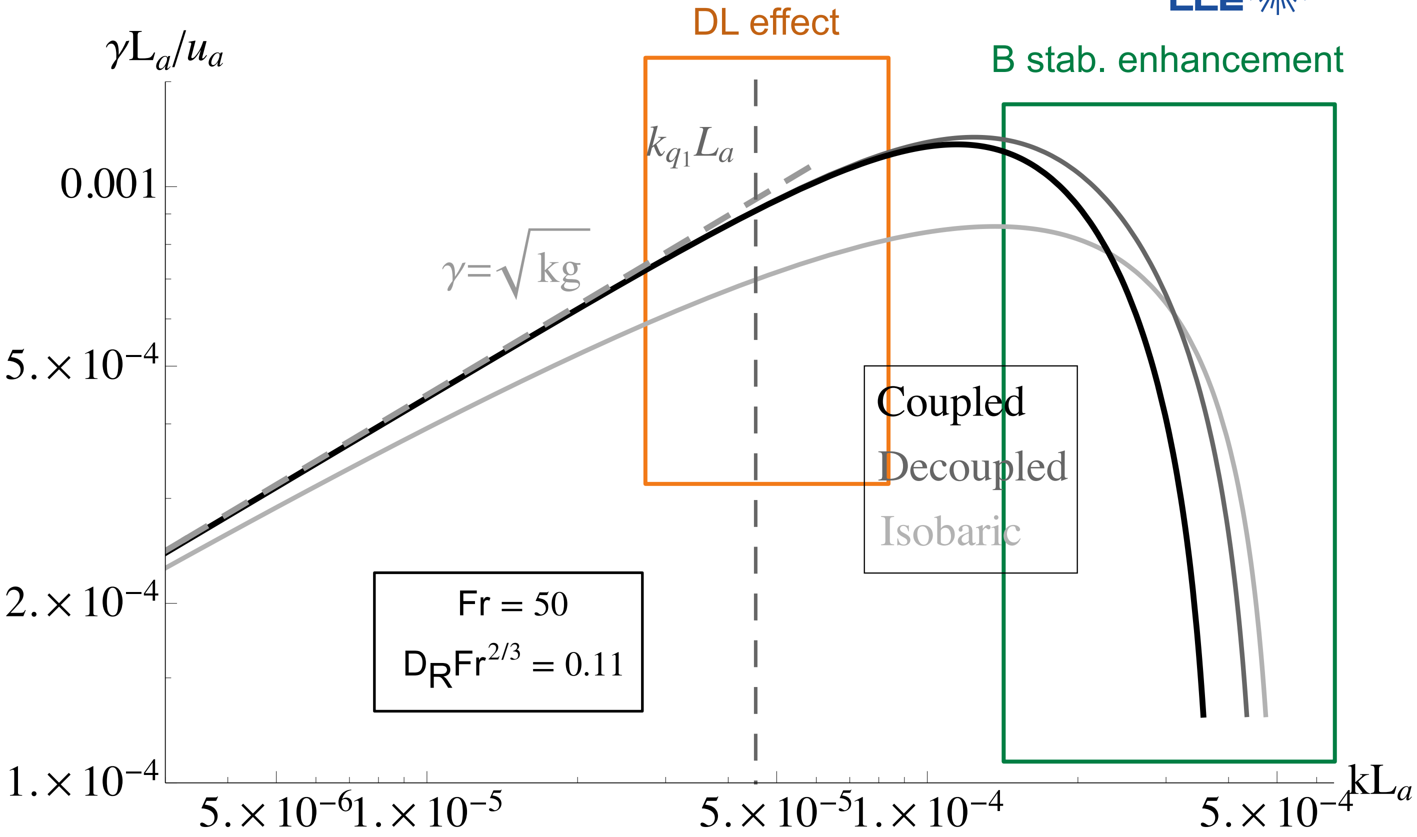
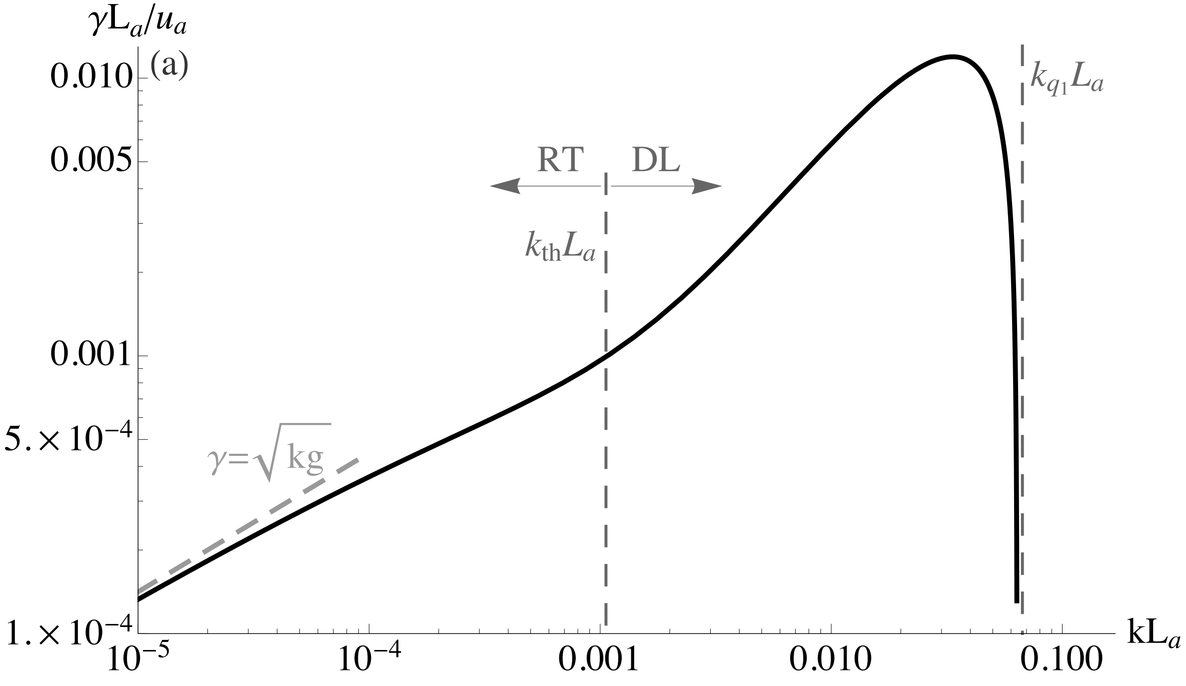
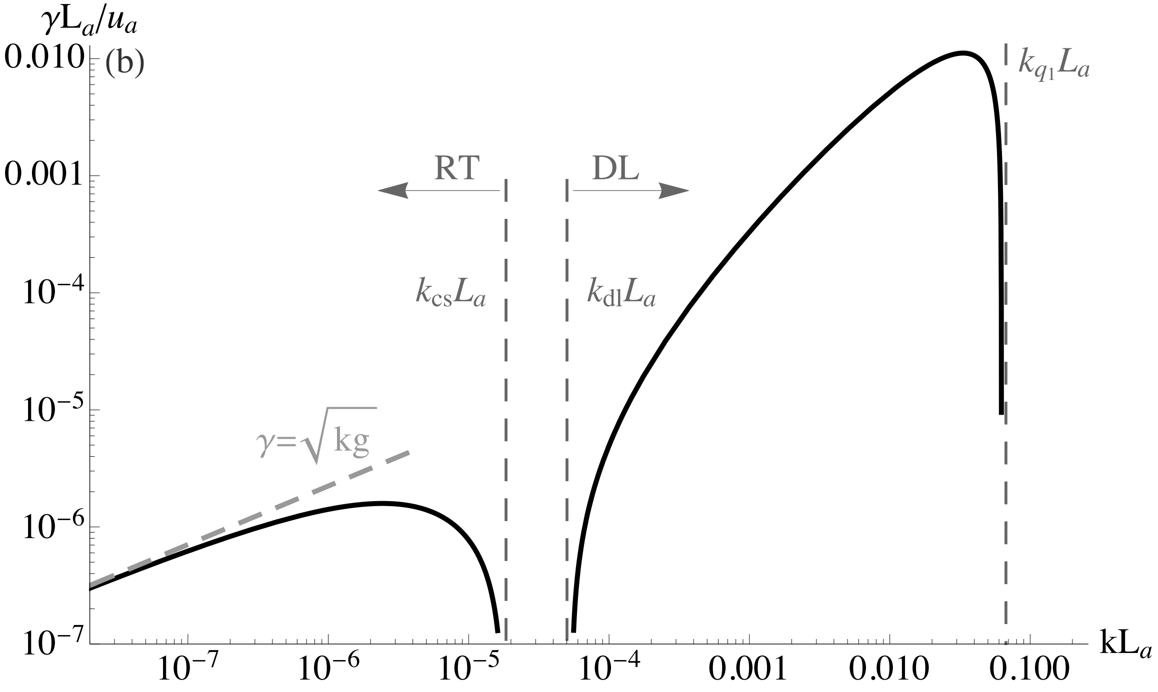


# Dispersion relation in the DL regime $D_R Fr^{2/3} \gg 1$

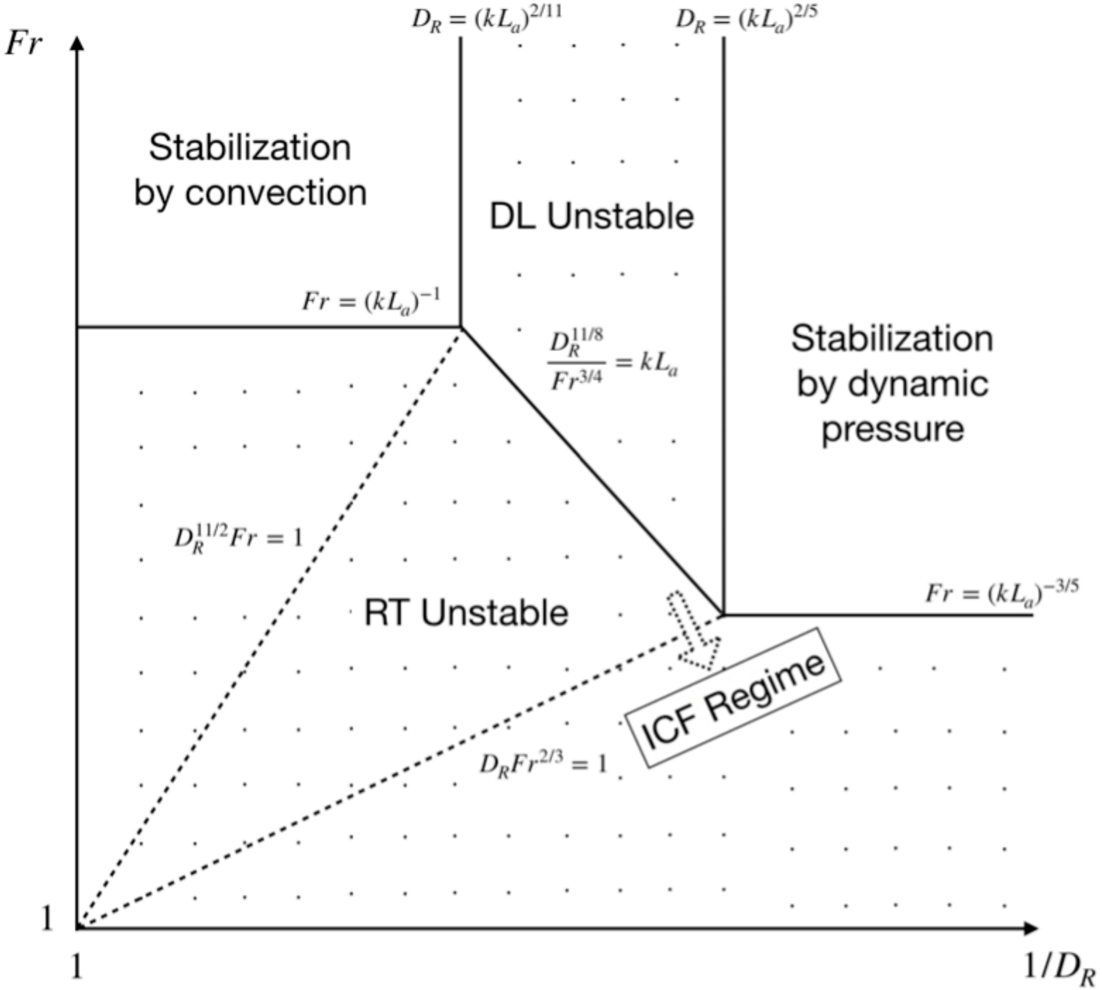
$D_R Fr^{2/3} = 10$




$D_R Fr^{2/3} = 500$



# Summary of regimes for the dispersion relation




F. García-Rubio, R. Betti, J. Sanz, and H. Aluie.

*Self-Consistent Theory of the Darrieus-Landau and Rayleigh-Taylor Instabilities with Self-Generated Magnetic Fields.*

To be published in Physics of Plasmas.

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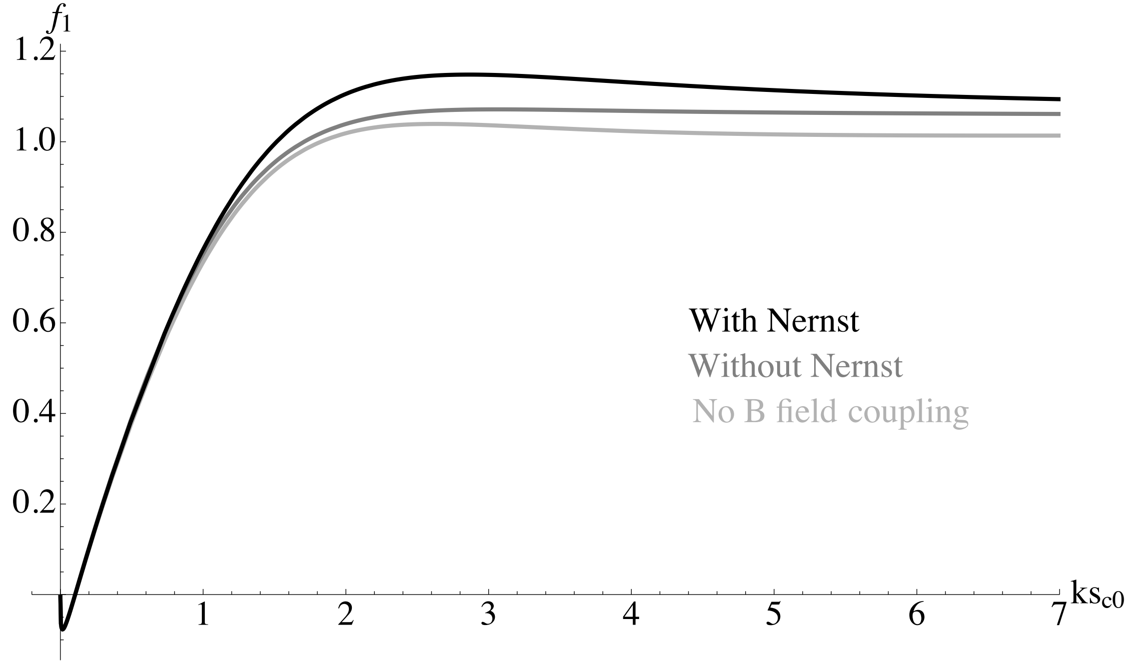
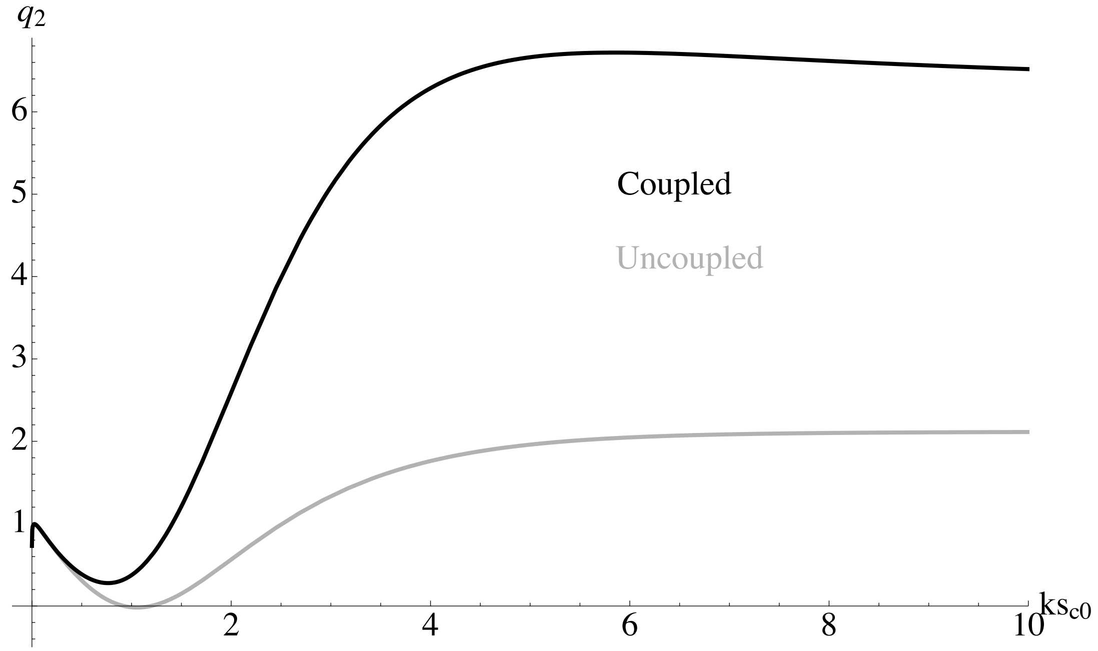
# Analysis of the convective stabilization term

$$\frac{\gamma L_a}{u_a} = \sqrt{\underbrace{\frac{kL_a}{\mathrm{Fr}}}_{\mathrm{RT}} - \underbrace{q_1 \left(kL_a\right)^{8/5}}_{\text{Dyn. pressure}}} - \underbrace{\frac{1+f_1+q_2}{2} kL_a}_{\text{Conv. stabilization}}$$

- **Convective stabilization** is positive for every wavenumber.
- The **B-field** enhances this effect. It is more significant for $q_2$.



# Darrieus-Landau limit $ks_{c0} \ll 1$ considering non-isobaric effects

- The scaling laws in the DL limit with **non-isobaric effects**:

Momentum $q_1 = -5.8\left(ks_{c0}\right)^{11/15}$

Mass $f_1 = -0.49\left(ks_{c0}\right)^{1/3}$

- The **isobaric analysis** performed in Sanz *et al.* PoP 2006:

Momentum $q_1 = -1.4\left(ks_{c0}\right)^{2/5}$

Mass $f_1 = ks_{c0}$

**Taking into account non-isobaric effects modify notably the scaling laws in the DL limit**