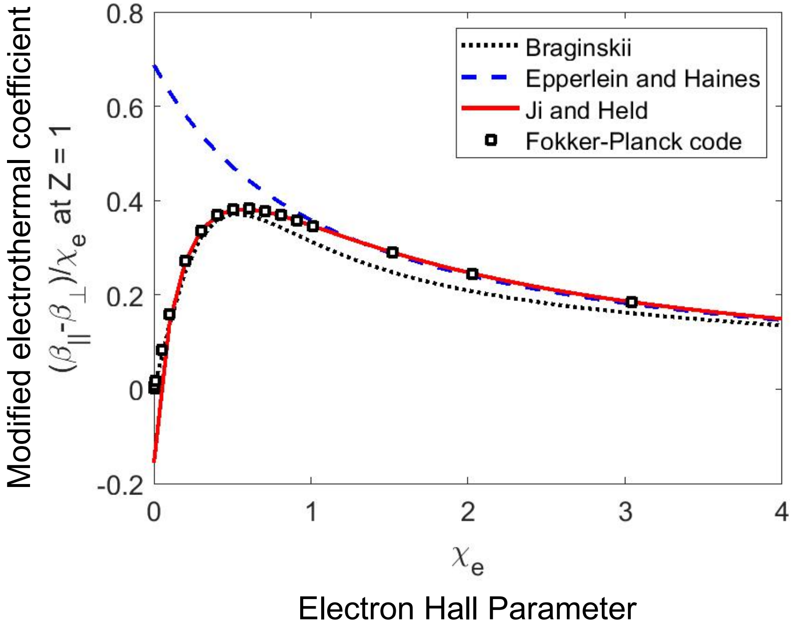
Transport Coefficients for Magnetic-Field Evolution in Inviscid Magnetohydrodynamics



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Summary

New fits for the resistivity and electrothermal tensors are obtained that give physically accurate results for magnetic field advection

- **Magnetic field advection due to the magnetized resistivity and electrothermal tensors depends on modified transport coefficients**

- **The modified transport coefficients require a reconsideration of existing fits**
  - **Braginskii's fits[1] give significant errors in advection due to the perpendicular resistivity, but are accurate for large Hall parameters $\chi_e \to \infty$**
  - **Epperlein-Haines' fits[2] give non-physical results for advection due to the perpendicular resistivity and perpendicular electrothermal coefficient for Hall parameters $\chi_e < 1$**
  - **Ji-Held's fits[3] give non-physical results for advection due to the perpendicular resistivity and perpendicular electrothermal coefficient for Hall parameters $\chi_e \to 0$**

[1] S. I. Braginskii, *Reviews of Plasma Physics* **1**, 205 (1965)
[2] E. M. Epperlein and M. G. Haines, Phys. Fluids **29**, 1029 (1986)
[3] J.-Y. Ji and E. D. Held, Phys. Plasmas **20**, 042114 (2013)

Collaborators

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The starting point is Braginskii's form of Ohm's law without electron viscosity

$$\vec{E} = -\vec{v} \times \vec{B} + \frac{\vec{j}}{n_{\mathrm{e}} e} \times \vec{B} - \frac{\nabla P_{\mathrm{e}}}{n_{\mathrm{e}} e} + \frac{\eta_{\parallel}}{\varepsilon_0 \omega_{\mathrm{pe}}^2 \tau_{\mathrm{ei}}} \vec{b}(\vec{b} \cdot \vec{j}) + \frac{\eta_{\perp}}{\varepsilon_0 \omega_{\mathrm{pe}}^2 \tau_{\mathrm{ei}}} \vec{b} \times (\vec{j} \times \vec{b}) - \frac{\eta_{\wedge}}{\varepsilon_0 \omega_{\mathrm{pe}}^2 \tau_{\mathrm{ei}}} (\vec{b} \times \vec{j}) - \beta_{\parallel} \vec{b} \left(\vec{b} \cdot \nabla T_{\mathrm{e}}\right) - \beta_{\perp} \vec{b} \times \left(\nabla T_{\mathrm{e}} \times \vec{b}\right) - \beta_{\wedge} \left(\vec{b} \times \nabla T_{\mathrm{e}}\right)$$

Braginskii's resistivity ($\eta_{\|}$, $\eta_{\perp}$, $\eta_{\wedge}$) and electrothermal ($\beta_{\|}$, $\beta_{\perp}$, $\beta_{\wedge}$) coefficients when substituted into the magnetic induction equation can be rearranged into source, advection and diffusion terms

$$\frac{\partial \vec{B}}{\partial t} = \nabla\beta_{\|} \times \nabla T_{\mathrm{e}} + \frac{\nabla P_{\mathrm{e}} \times \nabla n_{\mathrm{e}}}{n_{\mathrm{e}}^2 e} + \nabla \times (\vec{v}_{\mathrm{eff}} \times \vec{B})$$
$$+\vec{v}_{\eta\perp} \times [\vec{b} \times (\nabla \times \vec{B}) \times \vec{b}] + \vec{v}_{\eta\|} \times \vec{b}(\vec{b} \cdot \nabla \times \vec{B})$$
$$-\eta_{\perp}\frac{\delta_{\mathrm{e}}^2}{\tau_{\mathrm{ei}}}\nabla \times [\vec{b} \times (\nabla \times \vec{B}) \times \vec{b}] - \eta_{\|}\frac{\delta_{\mathrm{e}}^2}{\tau_{\mathrm{ei}}}\nabla \times [\vec{b}(\vec{b} \cdot \nabla \times \vec{B})]$$

$$\vec{v}_{\mathrm{eff}} = \vec{v} - \frac{\vec{j}}{n_{\mathrm{e}} e}\left(1 + \frac{\eta_{\wedge}}{\chi_{\mathrm{e}}}\right) - \frac{\beta_{\wedge}}{\chi_{\mathrm{e}}}\frac{e\tau_{\mathrm{ei}}}{m_e}\nabla T_e - \frac{\beta_{\|} - \beta_{\perp}}{\chi_e}\frac{e\tau_{\mathrm{ei}}}{m_{\mathrm{e}}}\vec{b} \times \nabla T_{\mathrm{e}}$$

$$\vec{v}_{\eta} = -\nabla\left(\eta\frac{\delta_e^2}{\tau_{\mathrm{ei}}}\right), \quad \chi_{\mathrm{e}} = \frac{eB\tau_{\mathrm{ei}}}{m_{\mathrm{e}}}, \quad \delta_{\mathrm{e}} = \frac{c}{\omega_{\mathrm{pe}}}$$

Braginskii's resistivity ($\eta_{||}$, $\eta_\perp$, $\eta_\wedge$) and electrothermal ($\beta_{||}$, $\beta_\perp$, $\beta_\wedge$) coefficients when substituted into the magnetic induction equation can be rearranged into source, advection and diffusion terms

$$\frac{\partial \vec{B}}{\partial t} = \boxed{\nabla \beta_{||} \times \nabla T_\mathrm{e}} + \frac{\nabla P_\mathrm{e} \times \nabla n_\mathrm{e}}{n_\mathrm{e}^2 e} + \nabla \times (\vec{v}_\mathrm{eff} \times \vec{B})$$

Electrothermal source term

$$+\vec{v}_{\eta\perp} \times [\vec{b} \times (\nabla \times \vec{B}) \times \vec{b}] + \vec{v}_{\eta||} \times \vec{b}(\vec{b} \cdot \nabla \times \vec{B})$$

$$-\eta_\perp \frac{\delta_\mathrm{e}^2}{\tau_\mathrm{ei}} \nabla \times [\vec{b} \times (\nabla \times \vec{B}) \times \vec{b}] - \eta_{||} \frac{\delta_e^2}{\tau_\mathrm{ei}} \nabla \times [\vec{b}(\vec{b} \cdot \nabla \times \vec{B})]$$

$$\vec{v}_\mathrm{eff} = \vec{v} - \frac{\vec{j}}{n_\mathrm{e} e}\left(1 + \frac{\eta_\wedge}{\chi_\mathrm{e}}\right) - \frac{\beta_\wedge}{\chi_\mathrm{e}} \frac{e\tau_\mathrm{ei}}{m_e} \nabla T_e - \frac{\beta_{||} - \beta_\perp}{\chi_e} \frac{e\tau_\mathrm{ei}}{m_\mathrm{e}} \vec{b} \times \nabla T_\mathrm{e}$$

$$\vec{v}_\eta = -\nabla\left(\eta \frac{\delta_e^2}{\tau_\mathrm{ei}}\right), \quad \chi_\mathrm{e} = \frac{eB\tau_\mathrm{ei}}{m_\mathrm{e}}, \quad \delta_\mathrm{e} = \frac{c}{\omega_\mathrm{pe}}$$

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$$\frac{\partial \vec{B}}{\partial t} = \nabla \beta_{||} \times \nabla T_{\mathrm{e}} + \frac{\nabla P_{\mathrm{e}} \times \nabla n_{\mathrm{e}}}{n_{\mathrm{e}}^2 e} + \boxed{\nabla \times (\vec{v}_{\mathrm{eff}} \times \vec{B})}$$

Advection

$$+\vec{v}_{\eta\perp} \times [\vec{b} \times (\nabla \times \vec{B}) \times \vec{b}] + \vec{v}_{\eta||} \times \vec{b}(\vec{b} \cdot \nabla \times \vec{B})$$

$$-\eta_\perp \frac{\delta_e^2}{\tau_{\mathrm{ei}}} \nabla \times [\vec{b} \times (\nabla \times \vec{B}) \times \vec{b}] - \eta_{||} \frac{\delta_e^2}{\tau_{\mathrm{ei}}} \nabla \times [\vec{b}(\vec{b} \cdot \nabla \times \vec{B})]$$

$$\boxed{\vec{v}_{\mathrm{eff}} = \vec{v} - \frac{\vec{j}}{n_{\mathrm{e}} e}\left(1 + \frac{\eta_\wedge}{\chi_{\mathrm{e}}}\right) - \frac{\beta_\wedge}{\chi_{\mathrm{e}}} \frac{e \tau_{\mathrm{ei}}}{m_e} \nabla T_e - \frac{\beta_{||} - \beta_\perp}{\chi_e} \frac{e \tau_{\mathrm{ei}}}{m_{\mathrm{e}}} \vec{b} \times \nabla T_{\mathrm{e}}}$$

$$\vec{v}_\eta = -\nabla \left(\eta \frac{\delta_e^2}{\tau_{\mathrm{ei}}}\right), \quad \chi_{\mathrm{e}} = \frac{e B \tau_{\mathrm{ei}}}{m_{\mathrm{e}}}, \quad \delta_{\mathrm{e}} = \frac{c}{\omega_{\mathrm{pe}}}$$

Braginskii's resistivity ($\eta_{||}$, $\eta_\perp$, $\eta_\wedge$) and electrothermal ($\beta_{||}$, $\beta_\perp$, $\beta_\wedge$) coefficients when substituted into the magnetic induction equation can be rearranged into source, advection and diffusion terms

$$\frac{\partial \vec{B}}{\partial t} = \nabla \beta_{||} \times \nabla T_e + \frac{\nabla P_e \times \nabla n_e}{n_e^2 e} + \nabla \times (\vec{v}_{\mathrm{eff}} \times \vec{B})$$

$$+\vec{v}_{\eta\perp} \times [\vec{b} \times (\nabla \times \vec{B}) \times \vec{b}] + \vec{v}_{\eta||} \times \vec{b}(\vec{b} \cdot \nabla \times \vec{B})$$

$$-\eta_\perp \frac{\delta_e^2}{\tau_{ei}} \nabla \times [\vec{b} \times (\nabla \times \vec{B}) \times \vec{b}] - \eta_{||} \frac{\delta_e^2}{\tau_{ei}} \nabla \times [\vec{b}(\vec{b} \cdot \nabla \times \vec{B})]$$

$$\vec{v}_{\mathrm{eff}} = \vec{v} - \frac{\vec{j}}{n_e e}\left(1 + \frac{\eta_\wedge}{\chi_e}\right) \boxed{- \frac{\beta_\wedge}{\chi_e} \frac{e\tau_{ei}}{m_e} \nabla T_e} - \frac{\beta_{||} - \beta_\perp}{\chi_e} \frac{e\tau_{ei}}{m_e} \vec{b} \times \nabla T_e$$

Nernst velocity ~ advection with perpendicular electron heat flux

$$\vec{v}_\eta = -\nabla\left(\eta \frac{\delta_e^2}{\tau_{ei}}\right), \quad \chi_e = \frac{eB\tau_{ei}}{m_e}, \quad \delta_e = \frac{c}{\omega_{pe}}$$

Braginskii's resistivity ($\eta_{||}$, $\eta_{\perp}$, $\eta_{\wedge}$) and electrothermal ($\beta_{||}$, $\beta_{\perp}$, $\beta_{\wedge}$) coefficients when substituted into the magnetic induction equation can be rearranged into source, advection and diffusion terms

$$\frac{\partial \vec{B}}{\partial t} = \nabla \beta_{||} \times \nabla T_e + \frac{\nabla P_e \times \nabla n_e}{n_e^2 e} + \nabla \times (\vec{v}_{\mathrm{eff}} \times \vec{B})$$

$$+\vec{v}_{\eta\perp} \times [\vec{b} \times (\nabla \times \vec{B}) \times \vec{b}] + \vec{v}_{\eta||} \times \vec{b}(\vec{b} \cdot \nabla \times \vec{B})$$

$$-\eta_{\perp} \frac{\delta_e^2}{\tau_{ei}} \nabla \times [\vec{b} \times (\nabla \times \vec{B}) \times \vec{b}] - \eta_{||} \frac{\delta_e^2}{\tau_{ei}} \nabla \times [\vec{b}(\vec{b} \cdot \nabla \times \vec{B})]$$

$$\vec{v}_{\mathrm{eff}} = \vec{v} - \frac{\vec{j}}{n_e e}\left(1 + \frac{\eta_{\wedge}}{\chi_e}\right) - \frac{\beta_{\wedge}}{\chi_e}\frac{e\tau_{ei}}{m_e}\nabla T_e \boxed{- \frac{\beta_{||} - \beta_{\perp}}{\chi_e}\frac{e\tau_{ei}}{m_e}\vec{b} \times \nabla T_e}$$

Advection with cross-field (Righi-Leduc) electron heat flux

$$\vec{v}_{\eta} = -\nabla\left(\eta \frac{\delta_e^2}{\tau_{ei}}\right), \quad \chi_e = \frac{eB\tau_{ei}}{m_e}, \quad \delta_e = \frac{c}{\omega_{pe}}$$

Braginskii's resistivity ($\eta_{||}$, $\eta_\perp$, $\eta_\wedge$) and electrothermal ($\beta_{||}$, $\beta_\perp$, $\beta_\wedge$) coefficients when substituted into the magnetic induction equation can be rearranged into source, advection and diffusion terms

$$\frac{\partial \vec{B}}{\partial t} = \nabla \beta_{||} \times \nabla T_e + \frac{\nabla P_e \times \nabla n_e}{n_e^2 e} + \nabla \times (\vec{v}_{\text{eff}} \times \vec{B})$$

$$+ \vec{v}_{\eta\perp} \times [\vec{b} \times (\nabla \times \vec{B}) \times \vec{b}] + \vec{v}_{\eta ||} \times \vec{b}(\vec{b} \cdot \nabla \times \vec{B})$$

$$- \eta_\perp \frac{\delta_e^2}{\tau_{\text{ei}}} \nabla \times [\vec{b} \times (\nabla \times \vec{B}) \times \vec{b}] - \eta_{||} \frac{\delta_e^2}{\tau_{\text{ei}}} \nabla \times [\vec{b}(\vec{b} \cdot \nabla \times \vec{B})]$$

$$\vec{v}_{\text{eff}} = \vec{v} - \boxed{\frac{\vec{j}}{n_e e}\left(1 + \frac{\eta_\wedge}{\chi_e}\right)} - \frac{\beta_\wedge}{\chi_e} \frac{e\tau_{\text{ei}}}{m_e} \nabla T_e - \frac{\beta_{||} - \beta_\perp}{\chi_e} \frac{e\tau_{\text{ei}}}{m_e} \vec{b} \times \nabla T_e$$

Modified Hall term ~ advection with perpendicular electrothermal heat flux -$\beta_\perp$ j T_e

$$\vec{v}_\eta = -\nabla \left(\eta \frac{\delta_e^2}{\tau_{\text{ei}}}\right), \quad \chi_e = \frac{eB\tau_{\text{ei}}}{m_e}, \quad \delta_e = \frac{c}{\omega_{\text{pe}}}$$

Braginskii's resistivity ($\eta_{||}$, $\eta_{\perp}$, $\eta_{\wedge}$) and electrothermal ($\beta_{||}$, $\beta_{\perp}$, $\beta_{\wedge}$) coefficients when substituted into the magnetic induction equation can be rearranged into source, advection and diffusion terms

$$\frac{\partial \vec{B}}{\partial t} = \nabla \beta_{||} \times \nabla T_{\mathrm{e}} + \frac{\nabla P_{\mathrm{e}} \times \nabla n_{\mathrm{e}}}{n_{\mathrm{e}}^2 e} + \nabla \times (\vec{v}_{\mathrm{eff}} \times \vec{B})$$

$$+\vec{v}_{\eta\perp} \times [\vec{b} \times (\nabla \times \vec{B}) \times \vec{b}] + \vec{v}_{\eta||} \times \vec{b}(\vec{b} \cdot \nabla \times \vec{B})$$

Advection to lower resistivity

$$-\eta_{\perp} \frac{\delta_{\mathrm{e}}^2}{\tau_{\mathrm{ei}}} \nabla \times [\vec{b} \times (\nabla \times \vec{B}) \times \vec{b}] - \eta_{||} \frac{\delta_{\mathrm{e}}^2}{\tau_{\mathrm{ei}}} \nabla \times [\vec{b}(\vec{b} \cdot \nabla \times \vec{B})]$$

$$\vec{v}_{\mathrm{eff}} = \vec{v} - \frac{\vec{j}}{n_{\mathrm{e}} e} \left(1 + \frac{\eta_{\wedge}}{\chi_{\mathrm{e}}}\right) - \frac{\beta_{\wedge}}{\chi_{\mathrm{e}}} \frac{e \tau_{\mathrm{ei}}}{m_{\mathrm{e}}} \nabla T_e - \frac{\beta_{||} - \beta_{\perp}}{\chi_e} \frac{e \tau_{\mathrm{ei}}}{m_{\mathrm{e}}} \vec{b} \times \nabla T_{\mathrm{e}}$$

$$\vec{v}_{\eta} = -\nabla \left(\eta \frac{\delta_e^2}{\tau_{\mathrm{ei}}}\right), \quad \chi_{\mathrm{e}} = \frac{e B \tau_{\mathrm{ei}}}{m_{\mathrm{e}}}, \quad \delta_{\mathrm{e}} = \frac{c}{\omega_{\mathrm{pe}}}$$

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$$+\vec{v}_{\eta\perp} \times [\vec{b} \times (\nabla \times \vec{B}) \times \vec{b}] + \vec{v}_{\eta||} \times \vec{b}(\vec{b} \cdot \nabla \times \vec{B})$$

$$\boxed{-\eta_{\perp}\frac{\delta_{\mathrm{e}}^2}{\tau_{\mathrm{ei}}}\nabla \times [\vec{b} \times (\nabla \times \vec{B}) \times \vec{b}] - \eta_{||}\frac{\delta_e^2}{\tau_{\mathrm{ei}}}\nabla \times [\vec{b}(\vec{b} \cdot \nabla \times \vec{B})]}$$

Diffusion

$$\vec{v}_{\mathrm{eff}} = \vec{v} - \frac{\vec{j}}{n_{\mathrm{e}} e}\left(1 + \frac{\eta_{\wedge}}{\chi_{\mathrm{e}}}\right) - \frac{\beta_{\wedge}}{\chi_{\mathrm{e}}}\frac{e\tau_{\mathrm{ei}}}{m_e}\nabla T_e - \frac{\beta_{||} - \beta_{\perp}}{\chi_e}\frac{e\tau_{\mathrm{ei}}}{m_{\mathrm{e}}}\vec{b} \times \nabla T_{\mathrm{e}}$$

$$\vec{v}_{\eta} = -\nabla\left(\eta\frac{\delta_e^2}{\tau_{\mathrm{ei}}}\right), \ \chi_{\mathrm{e}} = \frac{eB\tau_{\mathrm{ei}}}{m_{\mathrm{e}}}, \ \delta_{\mathrm{e}} = \frac{c}{\omega_{\mathrm{pe}}}$$

The resistivity can be rearranged in the same manner as the electrothermal terms leaving just $\eta_{\|}$ in the advection-diffusion terms and adding a new advection term

$$\frac{\partial \vec{B}}{\partial t} = \nabla \beta_{\|} \times \nabla T_{\mathrm{e}} + \frac{\nabla P_{\mathrm{e}} \times \nabla n_{\mathrm{e}}}{n_{\mathrm{e}}^2 e} + \nabla \times (\vec{v}_{\mathrm{eff}} \times \vec{B}) + \vec{v}_{\eta} \times (\nabla \times \vec{B}) + \eta_{\|} \frac{\delta_e^2}{\tau_{\mathrm{ei}}} \nabla^2 \vec{B}$$

$$\vec{v}_{\mathrm{eff}} = \vec{v} - \frac{\vec{j}}{n_{\mathrm{e}} e}\left(1 + \frac{\eta_{\wedge}}{\chi_{\mathrm{e}}}\right) - \frac{\eta_{\perp} - \eta_{\|}}{\chi_{\mathrm{e}}} \frac{\vec{b} \times \vec{j}}{n_{\mathrm{e}} e} - \frac{\beta_{\wedge}}{\chi_{\mathrm{e}}} \frac{e \tau_{\mathrm{ei}}}{m_e} \nabla T_e - \frac{\beta_{\|} - \beta_{\perp}}{\chi_e} \frac{e \tau_{\mathrm{ei}}}{m_{\mathrm{e}}} \vec{b} \times \nabla T_{\mathrm{e}}$$

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$$\vec{v}_\mathrm{eff} = \vec{v} - \frac{\vec{j}}{n_\mathrm{e} e}\left(1 + \frac{\eta_\wedge}{\chi_\mathrm{e}}\right) \boxed{- \frac{\eta_\perp - \eta_{||}}{\chi_\mathrm{e}} \frac{\vec{b} \times \vec{j}}{n_\mathrm{e} e}} - \frac{\beta_\wedge}{\chi_\mathrm{e}} \frac{e\tau_\mathrm{ei}}{m_e} \nabla T_e - \frac{\beta_{||} - \beta_\perp}{\chi_\mathrm{e}} \frac{e\tau_\mathrm{ei}}{m_\mathrm{e}} \vec{b} \times \nabla T_\mathrm{e}$$

Cross-field Hall term ~ advection with cross-field electrothermal heat flux $-\beta_\wedge$ b×j T_e

$$\vec{v}_\eta = -\nabla \left(\eta_{||} \frac{\delta_e^2}{\tau_\mathrm{ei}}\right), \quad \chi_\mathrm{e} = \frac{eB\tau_\mathrm{ei}}{m_\mathrm{e}}, \quad \delta_\mathrm{e} = \frac{c}{\omega_\mathrm{pe}}$$

Another formulation leaves just $\eta_\perp$ in the advection-diffusion terms and adds a new term with the same modified resistivity coefficient

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$$\frac{\partial \vec{B}}{\partial t} = \nabla \beta_\parallel \times \nabla T_e + \frac{\nabla P_e \times \nabla n_e}{n_e^2 e} + \nabla \times (\vec{v}_{\mathrm{eff}} \times \vec{B}) + \vec{v}_\eta \times (\nabla \times \vec{B}) + \eta_\perp \frac{\delta_e^2}{\tau_{\mathrm{ei}}} \nabla^2 \vec{B}$$
$$+ \nabla \times \left(\frac{\eta_\perp - \eta_\parallel}{\chi_e} \frac{\vec{b} \cdot \vec{j}}{n_e e} \vec{B} \right)$$

$$\vec{v}_{\mathrm{eff}} = \vec{v} - \frac{\vec{j}}{n_e e} \left(1 + \frac{\eta_\wedge}{\chi_e} \right) - \frac{\beta_\wedge}{\chi_e} \frac{e \tau_{\mathrm{ei}}}{m_e} \nabla T_e - \frac{\beta_\parallel - \beta_\perp}{\chi_e} \frac{e \tau_{\mathrm{ei}}}{m_e} \vec{b} \times \nabla T_e$$

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Another formulation leaves just $\eta_\perp$ in the advection-diffusion terms and adds a new term with the same modified resistivity coefficient

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$$+\nabla \times \left(\frac{\eta_\perp - \eta_\parallel}{\chi_\mathrm{e}} \frac{\vec{b} \cdot \vec{j}}{n_\mathrm{e} e} \vec{B} \right)$$

New term with advection and growth/decay effects

$$\vec{v}_\mathrm{eff} = \vec{v} - \frac{\vec{j}}{n_\mathrm{e} e} \left(1 + \frac{\eta_\wedge}{\chi_\mathrm{e}} \right) - \frac{\beta_\wedge}{\chi_\mathrm{e}} \frac{e \tau_\mathrm{ei}}{m_e} \nabla T_e - \frac{\beta_\parallel - \beta_\perp}{\chi_e} \frac{e \tau_\mathrm{ei}}{m_\mathrm{e}} \vec{b} \times \nabla T_\mathrm{e}$$

$$\vec{v}_\eta = -\nabla \left(\eta_\perp \frac{\delta_e^2}{\tau_\mathrm{ei}} \right), \; \chi_\mathrm{e} = \frac{e B \tau_\mathrm{ei}}{m_\mathrm{e}}, \; \delta_\mathrm{e} = \frac{c}{\omega_\mathrm{pe}}$$

All formulations are mathematically equivalent so all forms of the transport coefficients that appear in them must be physically accurate.

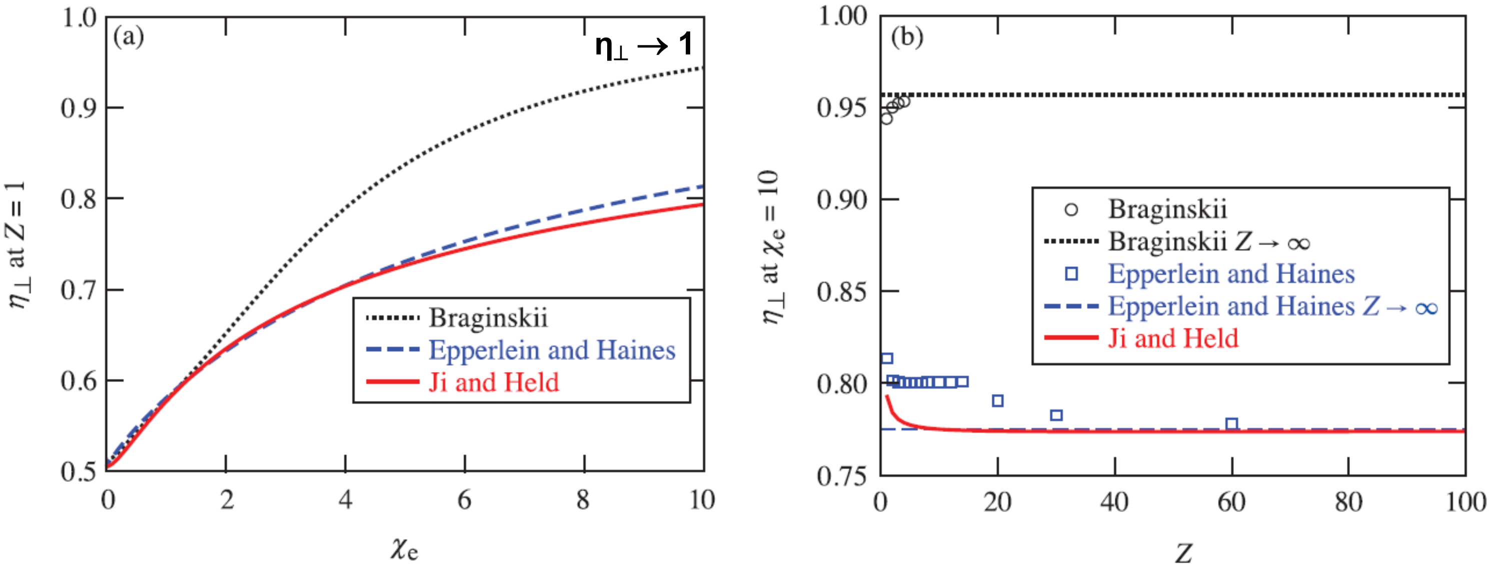
The modified transport coefficients require a reconsideration of well-established fits

- **Braginskii[1] gives fits to a 3-term expansion accurate to ±20%**
- **Epperlein-Haines[2] give fits to a numerical solution accurate to ±15%**
- **Ji-Held[3] give fits to a 160-term expansion accurate to ±1%**

The stated accuracies do not apply to the modified coefficients.

[1] S. I. Braginskii, *Reviews of Plasma Physics* **1**, 205 (1965)
[2] E. M. Epperlein and M. G. Haines, Phys. Fluids **29**, 1029 (1986)
[3] J.-Y. Ji and E. D. Held, Phys. Plasmas **20**, 042114 (2013)

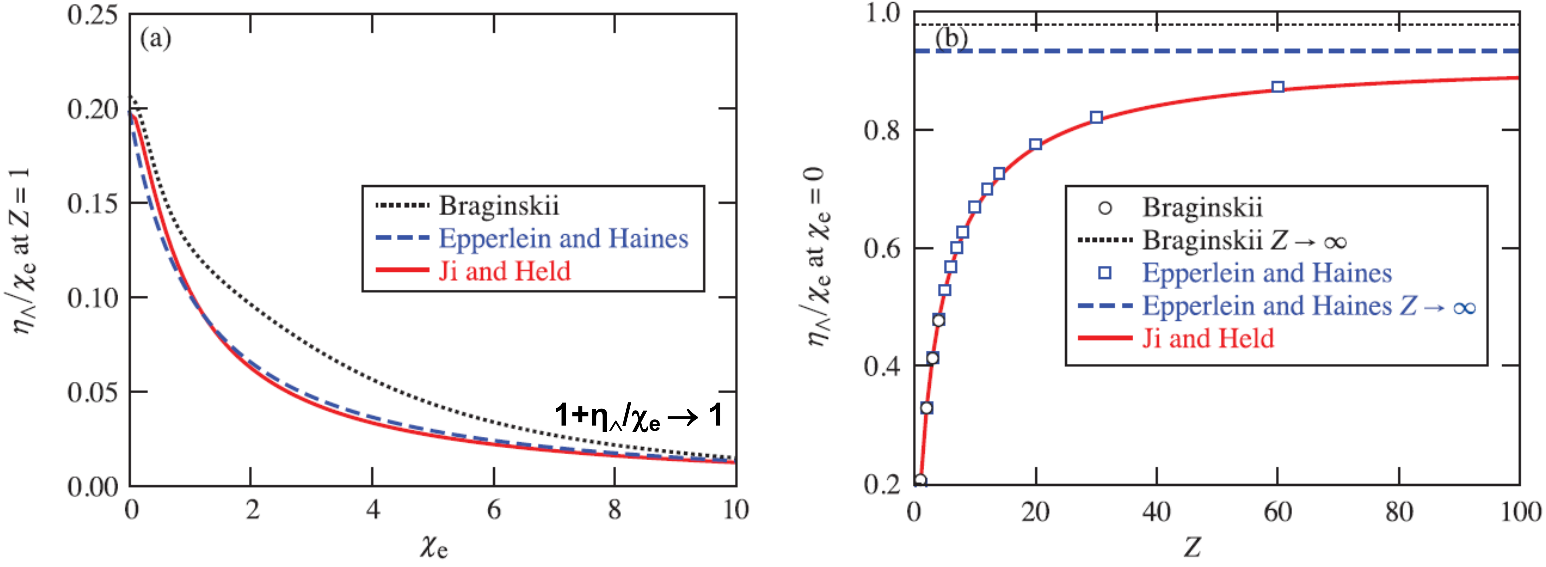
Perpendicular resistivity $\eta_\perp$



Braginskii's faster increase with χ_e can give significant errors in $\nabla\eta_\perp$

Braginskii and Epperlein-Haines can give incorrect values of $\nabla\eta_\perp$ due to variations in Z

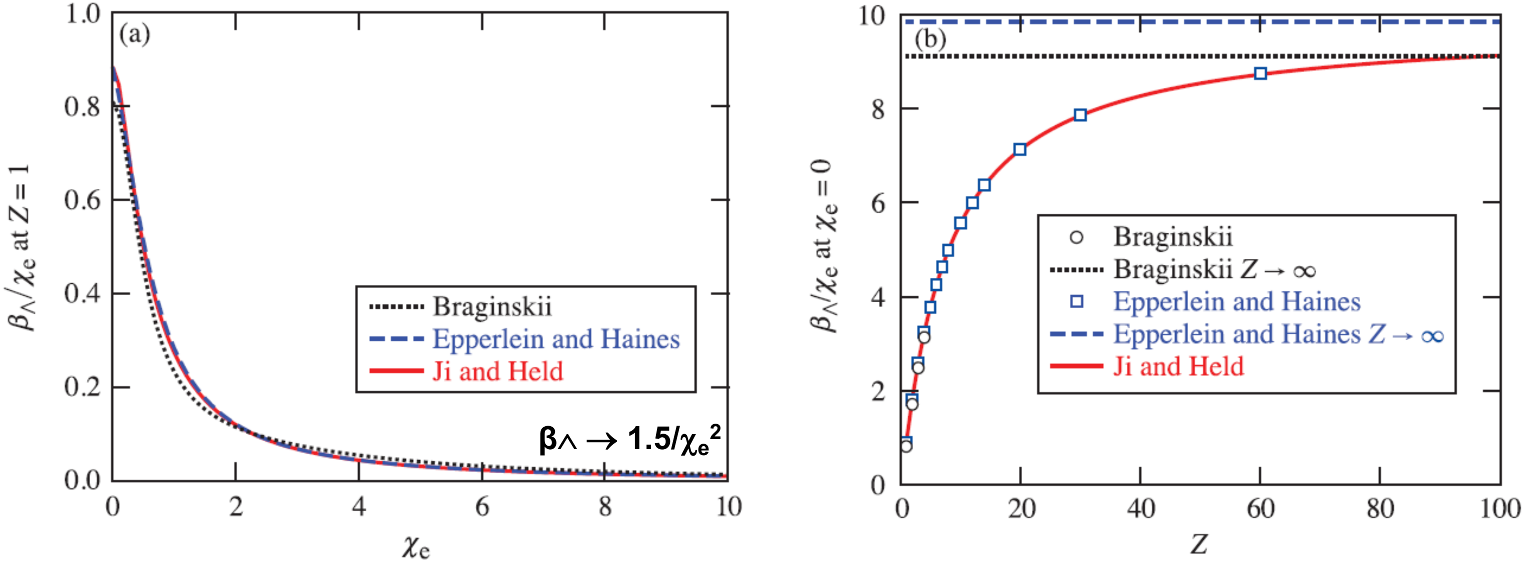
Modified Hall term $1+\eta_\wedge/\chi_e$



All fits agree to better than 5% for $1+\eta_\wedge/\chi_e$

Braginskii's error in the limiting form of $\eta_\wedge$ as $\chi_e \rightarrow \infty$ is irrelevant

Nernst velocity coefficient $\beta_\wedge/\chi_e$

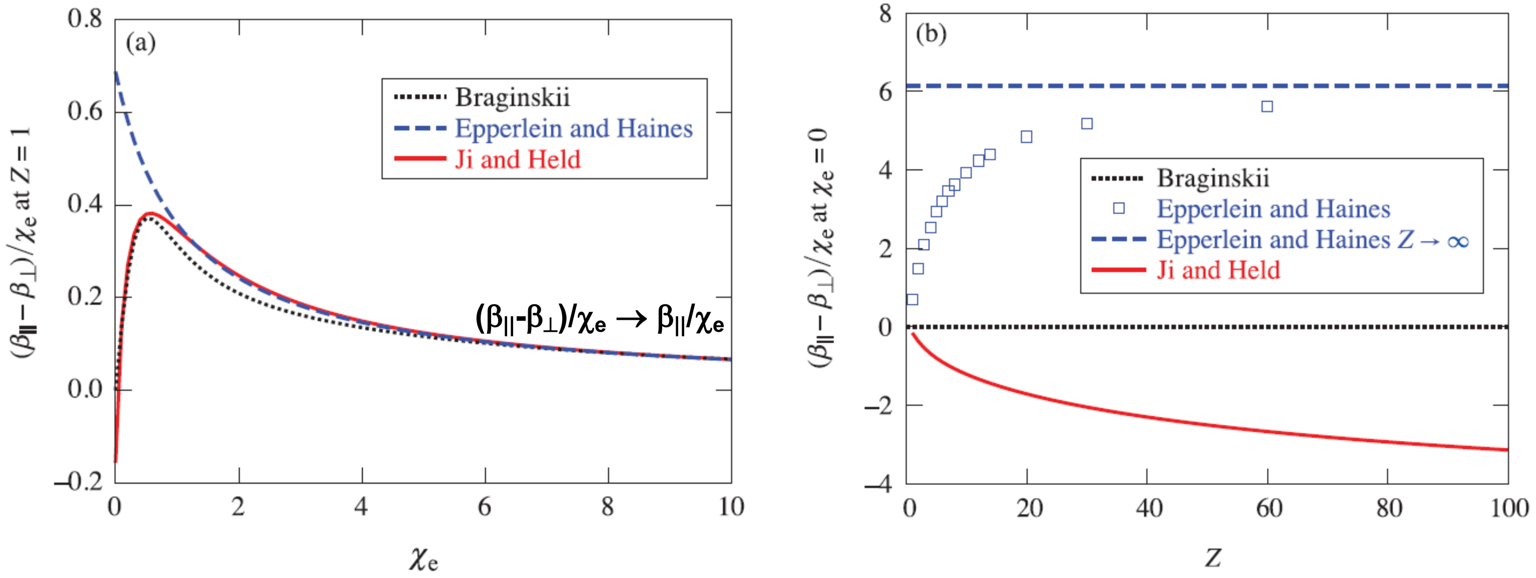


Braginskii's underestimate at $\chi_e < 2$ can be physically significant at Z = 1

Velikovich's* dimensionless Nernst wave velocity *w* at Z = 1 is 0.245 from Braginskii, 0.344 from Epperlein-Haines, and 0.380 from Ji-Held ~ 55% increase from Braginskii

*A. L. Velikovich, J. L. Giuliani, and S. T. Zalesak, Phys. Plasmas **26**, 112702 (2019)

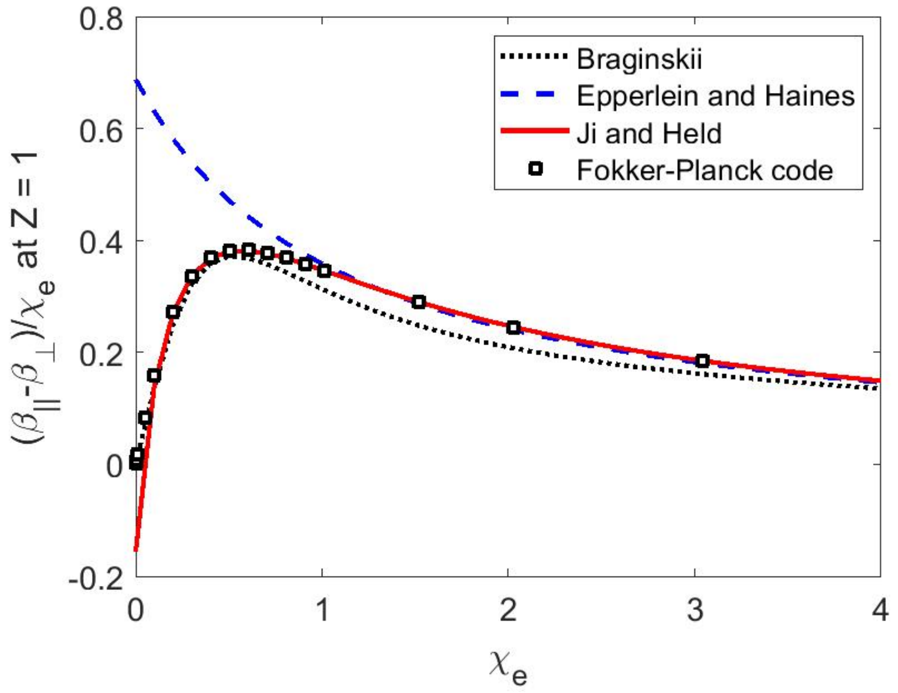
Cross-field velocity coefficient $(\beta_{||}-\beta_{\perp})/\chi_e$



Only Braginskii is physically accurate; the incorrect limiting form for $\beta_{\perp}$ as $\chi_e \rightarrow \infty$ is irrelevant

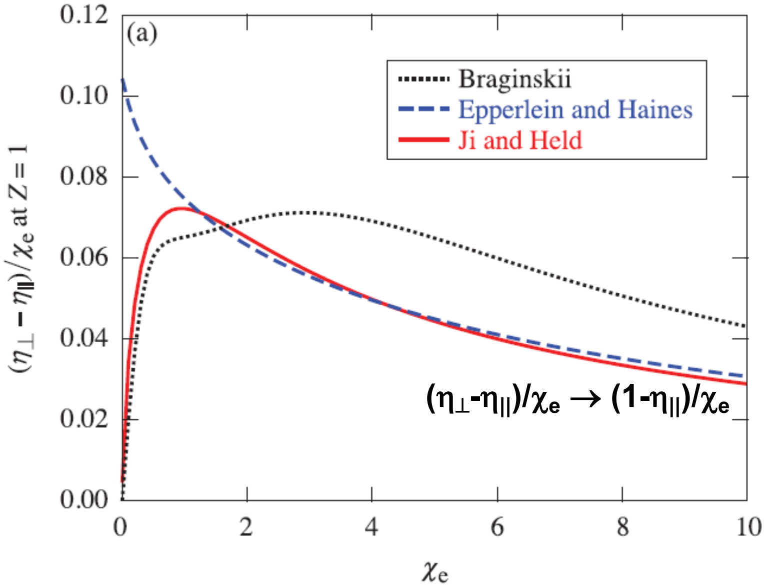
Epperlein-Haines greatly overestimates cross-field advection at $\chi_e < 1$

The Fokker-Planck code OSHUN* shows that Braginskii is accurate at small χ_e and Ji-Held is accurate at higher χ_e

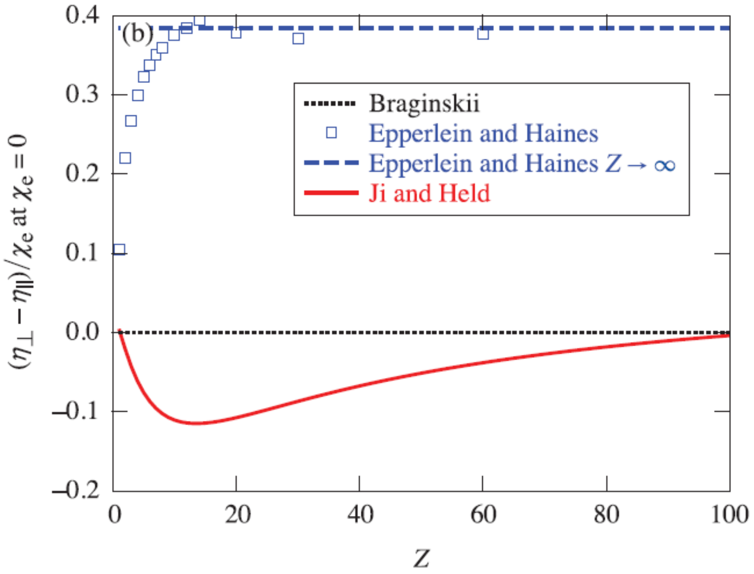


*M. Tzoufras, A. R. Bell, P. A. Norreys and F. S. Tsung, J. Computational Physics **230**, 6475 (2011)
M. Tzoufras, A. Tableman, F. S. Tsung, W. B. Mori and A. R. Bell, Phys. Plasmas **20**, 056303 (2013)

The new resistivity coefficient $(\eta_\perp-\eta_{||})/\chi_e$ mirrors the behavior of the cross-field velocity coefficient $(\beta_{||}-\beta_\perp)/\chi_e$



$(\eta_\perp-\eta_{||})/\chi_e \rightarrow (1-\eta_{||})/\chi_e$



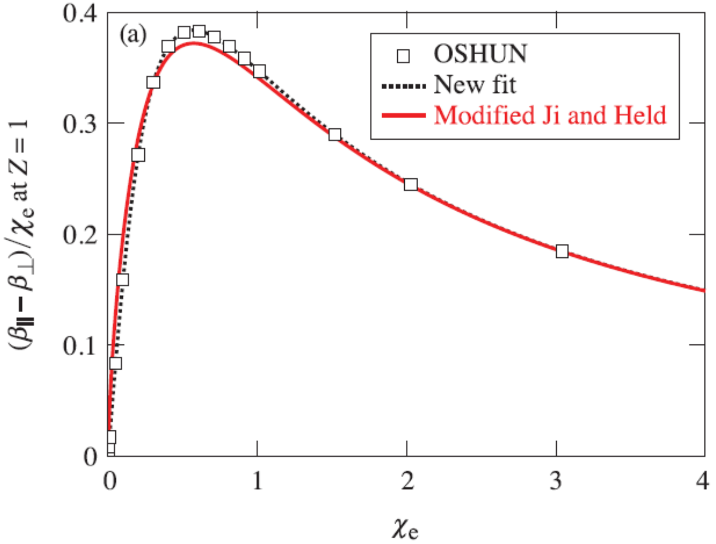
Braginskii has the wrong shape and is a significant overestimate at intermediate χ_e

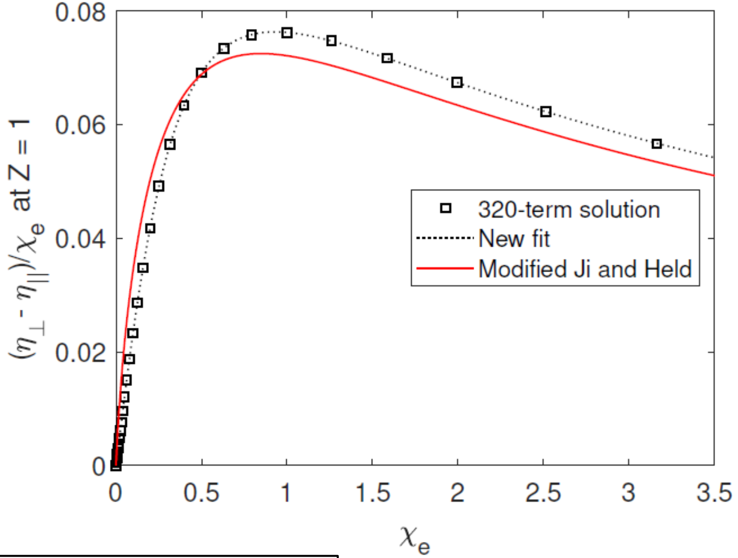
Ji-Held is only incorrect at intermediate *Z*

None of the fits for $\eta_\perp$ are adequate.

Only Braginskii's $\beta_\perp$ is physically correct.

Ji-Held's fitting coefficients for $\beta_\perp$ and $\eta_\perp$ have been modified and direct fits found for $(\beta_{||}-\beta_\perp)/\chi_e$ and $(\eta_\perp-\eta_{||})/\chi_e$





$$c_{\text{new}} = \frac{b_0\chi_e}{\chi_e^2 + b_1\chi_e^{4/3} + b_2\chi_e + b_3\chi_e^{2/3} + b_4}$$

Summary

New fits for the resistivity and electrothermal tensors are obtained that give physically accurate results for magnetic field advection

- **Magnetic field advection due to the magnetized resistivity and electrothermal tensors depends on modified transport coefficients**

- **The modified transport coefficients require a reconsideration of existing fits**
  - **Braginskii's fits[1] give significant errors in advection due to the perpendicular resistivity, but are accurate for large Hall parameters $\chi_e \rightarrow \infty$**
  - **Epperlein-Haines' fits[2] give non-physical results for advection due to the perpendicular resistivity and perpendicular electrothermal coefficient for Hall parameters $\chi_e < 1$**
  - **Ji-Held's fits[3] give non-physical results for advection due to the perpendicular resistivity and perpendicular electrothermal coefficient for Hall parameters $\chi_e \rightarrow 0$**

[1] S. I. Braginskii, *Reviews of Plasma Physics* **1**, 205 (1965)
[2] E. M. Epperlein and M. G. Haines, Phys. Fluids **29**, 1029 (1986)
[3] J.-Y. Ji and E. D. Held, Phys. Plasmas **20**, 042114 (2013)

The new fits

$$c_{\text{new}} = \frac{b_0 \chi_e}{\chi_e^2 + b_1 \chi_e^{4/3} + b_2 \chi_e + b_3 \chi_e^{2/3} + b_4},$$

$$b_i = \frac{a_0 + a_1 a_2 (Z-1)^{a_3}}{1 + a_2 (Z-1)^{a_3}}, \quad Z \geq 1,$$

$(\beta_{||}-\beta_{\perp})/\chi_e$

	b_0	b_1	b_2	b_3	b_4
a_0	0.69961	−0.58266	2.1368	−1.0113	0.46702
a_1	1.4982	−0.0054807	0.35425	−0.087198	0.011953
a_2	0.34101	1.0104	1.1035	1.5215	1.9792
a_3	0.92882	0.69114	0.79288	0.81865	1.1120

$b_0 = 1-\eta_{||}$

$b_1 = 1.507$

$(\eta_{\perp}-\eta_{||})/\chi_e$

	b_2	b_3	b_4
a_0	4.4117	−2.5978	2.1494
a_1	1.1344	−0.25567	0.093123
a_2	0.78632	1.0627	1.5738
a_3	0.93091	0.95644	1.0747

The full modified Ji and Held fits; parallel and perpendicular resistivity

$$\eta_{\parallel} = 1 - \frac{Z}{1.42Z - 0.065Z^{2/3} + 0.352Z^{1/3} + 0.32},$$

$$\eta_{\perp} = 1 - \frac{1.46Z^{5/3}\chi_e + a_0(1 - \eta_{\parallel})}{Z^{5/3}\chi_e^{5/3} + a_2\chi_e^{4/3} + a_1\chi_e + a_0},$$

$$a_0 = 0.331Z^{5/3} - 1.24Z^{4/3} + 2.54Z + 0.40,$$

$$a_1 = \frac{1.46Z^{5/3}}{1 - \eta_{\parallel}},$$

$$a_2 = Z^{4/3}(-0.114Z^{1/3} + 0.013).$$

The full modified Ji and Held fits; the modified Hall term

$$\frac{\eta_\wedge}{\chi_\mathrm{e}} = \frac{Z^{5/3}(2.53Z\chi_\mathrm{e} + a_0/a_5)}{Z^{8/3}\chi_\mathrm{e}^{8/3} + a_4\chi_\mathrm{e}^{7/3} + a_3\chi_\mathrm{e}^2 + a_2\chi_\mathrm{e}^{5/3} + a_1\chi_\mathrm{e} + a_0},$$

$$a_0 = 0.0759Z^{8/3} + 0.897Z^2 + 2.06Z + 1.06,$$

$$a_1 = Z(2.18Z^{5/3} + 5.31Z + 3.73),$$

$$a_2 = Z^{5/3}(7.41Z + 1.11Z^{2/3} - 1.17),$$

$$a_3 = Z^2(3.89Z^{2/3} - 4.51Z^{1/3} + 6.76),$$

$$a_4 = Z^{7/3}(2.26Z^{1/3} + 0.281),$$

$$a_5 = 1.18Z^{5/3} - 1.03Z^{4/3} + 3.6Z + 1.32.$$

The full modified Ji and Held fits; parallel and perpendicular electrothermal coefficients

$$\beta_{\parallel} = \frac{1.5Z}{Z - 0.115Z^{2/3} + 0.858Z^{1/3} + 0.401},$$

$$\beta_{\perp} = \frac{6.33Z^{8/3}\chi_e + a_0\beta_{\parallel}}{Z^{8/3}\chi_e^{8/3} + a_4\chi_e^{7/3} + a_3\chi_e^2 + a_2\chi_e^{5/3} + a_1\chi_e + a_0},$$

$$a_0 = 0.288Z^{8/3} + 1.75Z^2 + 5.09Z - 0.322,$$

$$a_1 = 6.33Z^{8/3}/\beta_{\parallel},$$

$$a_2 = Z^{5/3}(9.40Z + 5.42Z^{2/3} - 9.67Z^{1/3} + 3.06),$$

$$a_3 = Z^2(2.62Z^{2/3} + 0.704Z^{1/3} - 0.264),$$

$$a_4 = Z^{7/3}(2.58Z^{1/3} + 0.262).$$

The full modified Ji and Held fits; the Nernst velocity coefficient

$$\frac{\beta_\wedge}{\chi_e} = \frac{Z^2(1.5Z\chi_e + a_0/a_5)}{Z^3\chi_e^3 + a_4\chi_e^{7/3} + a_3\chi_e^2 + a_2\chi_e^{5/3} + a_1\chi_e + a_0},$$

$$a_0 = 0.00687Z^3 + 0.0782Z^2 + 0.623Z + 0.366,$$

$$a_1 = Z(0.134Z^2 + 0.977Z + 0.17),$$

$$a_2 = Z^{5/3}(0.689Z^{4/3} - 0.377Z^{2/3} + 3.94Z^{1/3} + 0.644),$$

$$a_3 = Z^2(-0.109Z + 1.33Z^{2/3} - 3.80Z^{1/3} + 0.289),$$

$$a_4 = Z^{7/3}(2.46Z^{2/3} + 0.522),$$

$$a_5 = 0.102Z^2 + 0.746Z + 0.072Z^{1/3} + 0.211.$$