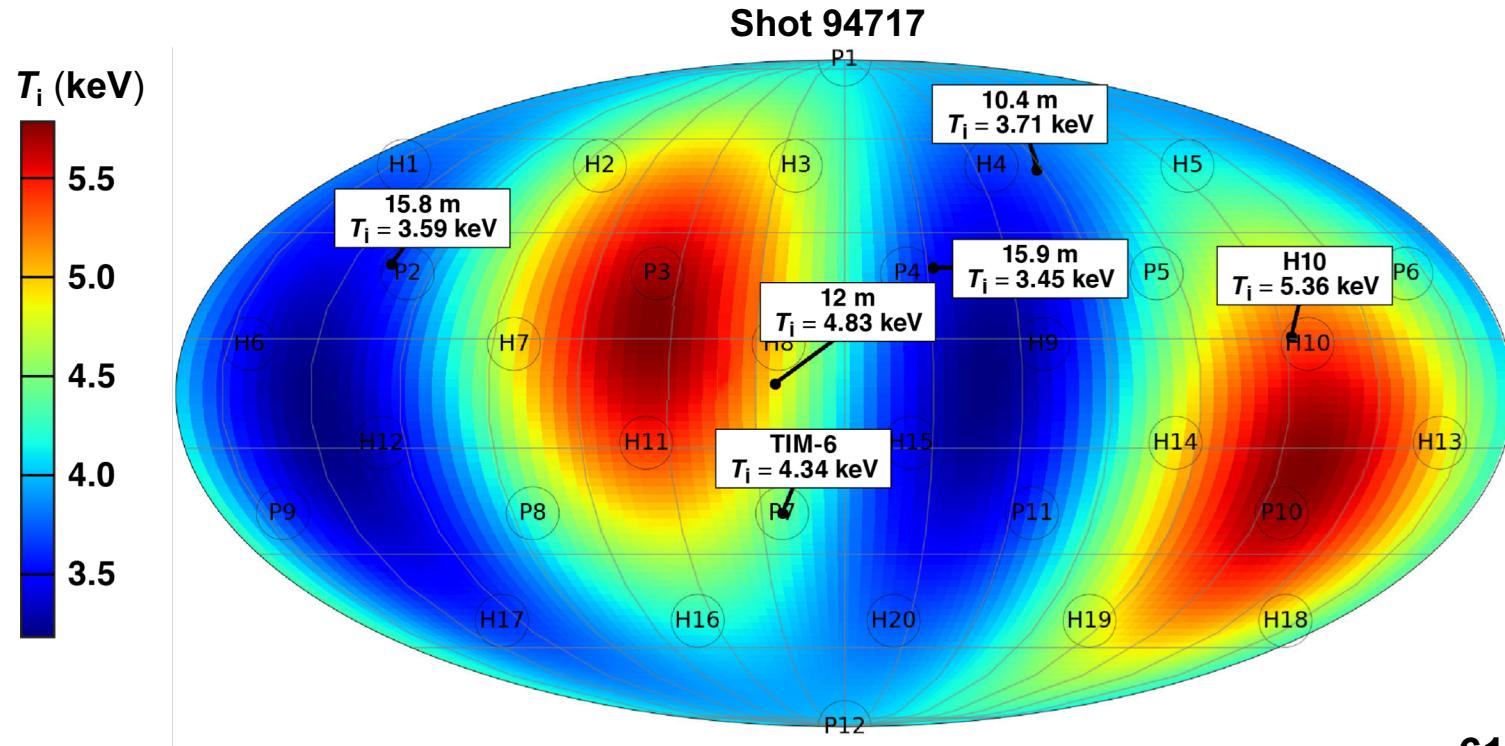


Inferring the Thermal Ion Temperature and Residual Kinetic Energy from Nuclear Measurements in Inertial Confinement Fusion



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Division of Plasma Physics
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A velocity variance analysis was developed to explain the 3-D flow effects on ion-temperature (T_i) measurements and the modal dependence of T_i asymmetries in ICF



- A method using six line-of-sight (LOS) ion-temperature measurements was developed to determine the full temperature map and account for the contribution of anisotropic flows
- The contribution of isotropic flows and the thermal temperature can be determined using both DD and DT ion-temperature measurements
- The minimum of DD neutron-inferred ion temperature was derived from DD and DT ion-temperature measurements along the same single LOS and demonstrated a strong correlation with the experimental yields in the OMEGA implosion database
- Analytical expressions were derived to infer the yield-over-clean in terms of mode-1 ion-temperature asymmetries

Collaborators



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Outline



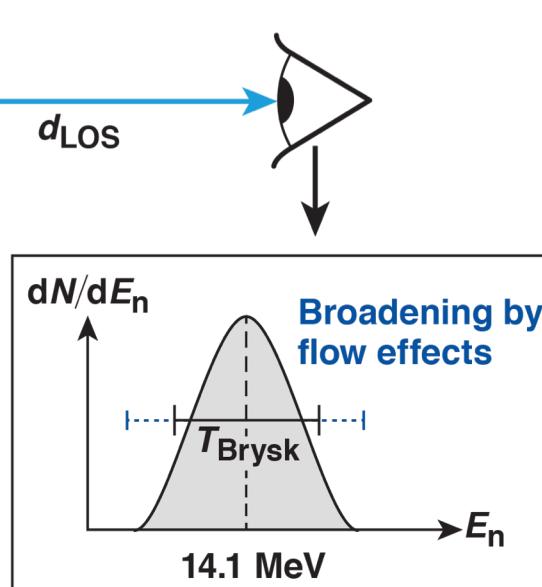
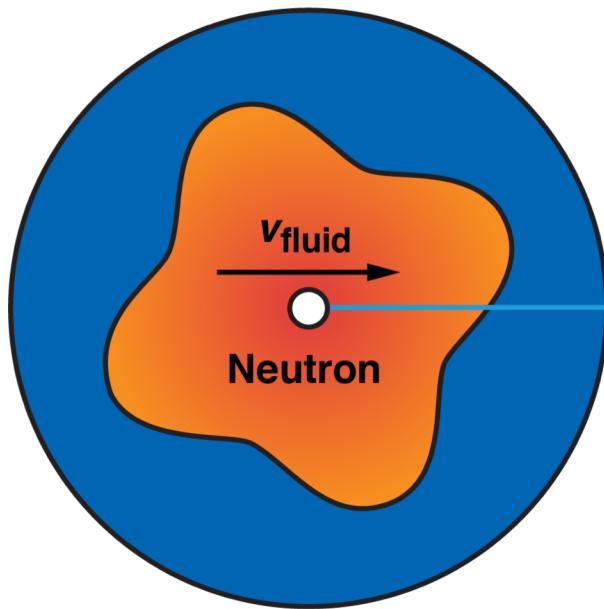
- Introduction to the ion-temperature measurement (T_i) asymmetry
- Impact of anisotropic flows on T_i asymmetry
- Impact of isotropic flows on inferring the minimum DD and thermal ion temperatures

Outline



- **Introduction to the ion-temperature measurement (T_i) asymmetry**
- Impact of anisotropic flows on T_i asymmetry
- Impact of isotropic flows on inferring the minimum DD and thermal ion temperatures

The variation of apparent ion-temperature measurements along different lines of sight (LOS) is uniquely determined by the behavior of the variance^{*,**} of the hot-spot fluid velocities



Isotropic and anisotropic flow effects

$$T_{\text{LOS}} = T_{\text{Brysk}} + M_{\text{DT}} \text{var}[\vec{v}_{\text{fluid}} \cdot \hat{d}]$$

~ Total width of the neutron energy spectrum

$$v_n = v_{14.1 \text{ MeV}} + v_{\text{cm}} + v_{\text{rel}} + v_{\text{fluid}}$$

Signal broadening

v_{cm} : velocity of the DT center-of-mass (CM) frame

v_{rel} : relative velocity of D,T ions in the CM frame

v_{fluid} : fluid velocity in the lab frame

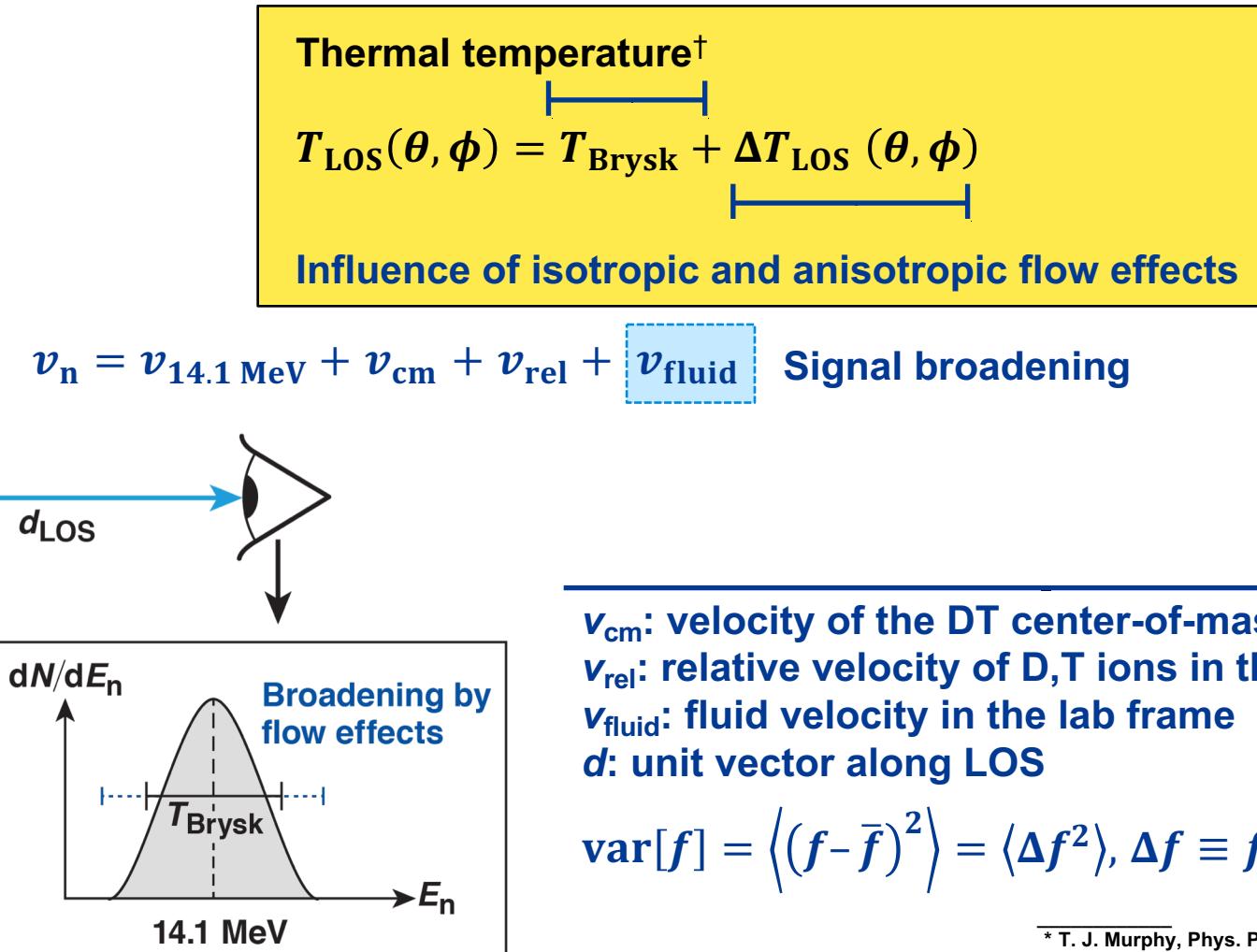
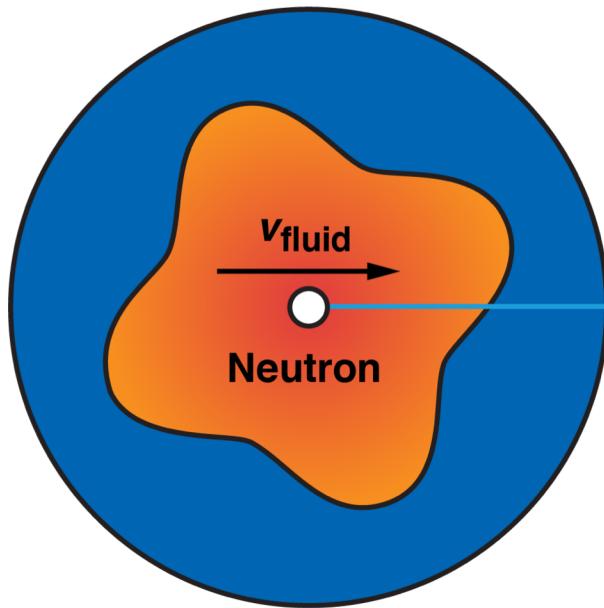
d : unit vector along LOS

$$\text{var}[f] = \langle (f - \bar{f})^2 \rangle = \langle \Delta f^2 \rangle, \Delta f \equiv f - \bar{f}$$

* T. J. Murphy, Phys. Plasmas 21, 072701 (2014).

** D. H. Munro, Nucl. Fusion 56, 036001 (2016)

The variation of apparent ion-temperature measurements along different lines of sight (LOS) is uniquely determined by the behavior of the variance*,** of the hot-spot fluid velocities



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* T. J. Murphy, Phys. Plasmas 21, 072701 (2014).

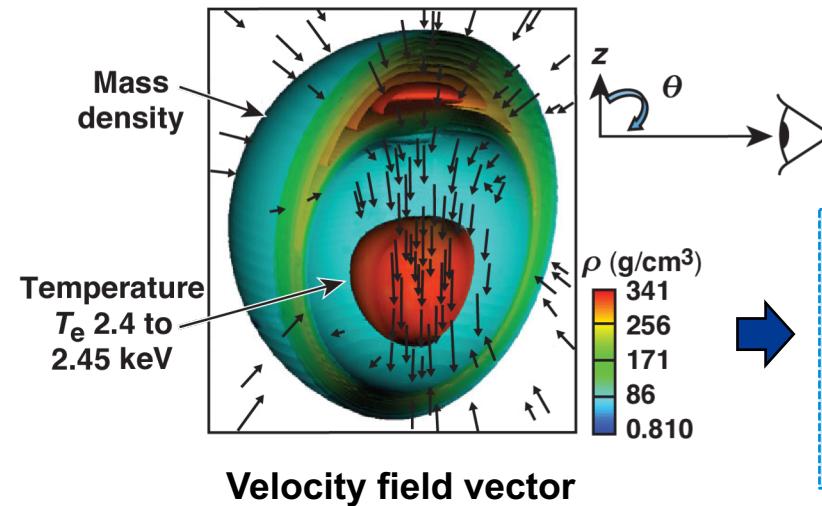
** D. H. Munro, Nucl. Fusion 56, 036001 (2016)

† H. Brysk, Plasma Phys. 15, 611 (1973).

Magnitudes of hot-spot flows in the deceleration-phase simulations are large enough to cause significant variations in ion-temperature measurements



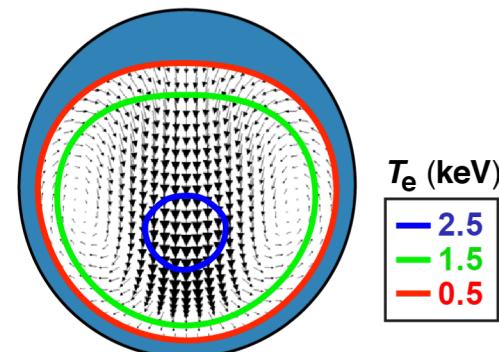
DEC3D* mode-1 simulation



Simulation methods

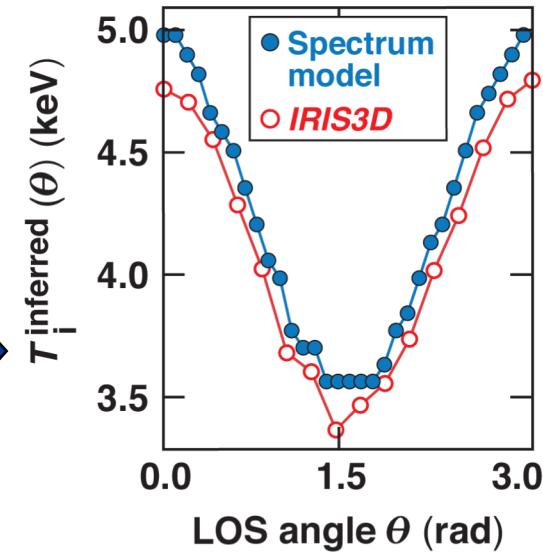
Generate the synthetic neutron energy spectrum^{**,†}

$$f_{\text{LOS}}(E_n) = \sum_{\text{cell}} \frac{Y_{\text{cell}}}{Y_{\text{total}}} e^{-\frac{(E_n - \bar{E})^2}{2\sigma^2}}$$



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Infer T_i at different LOS and compare with IRIS3D

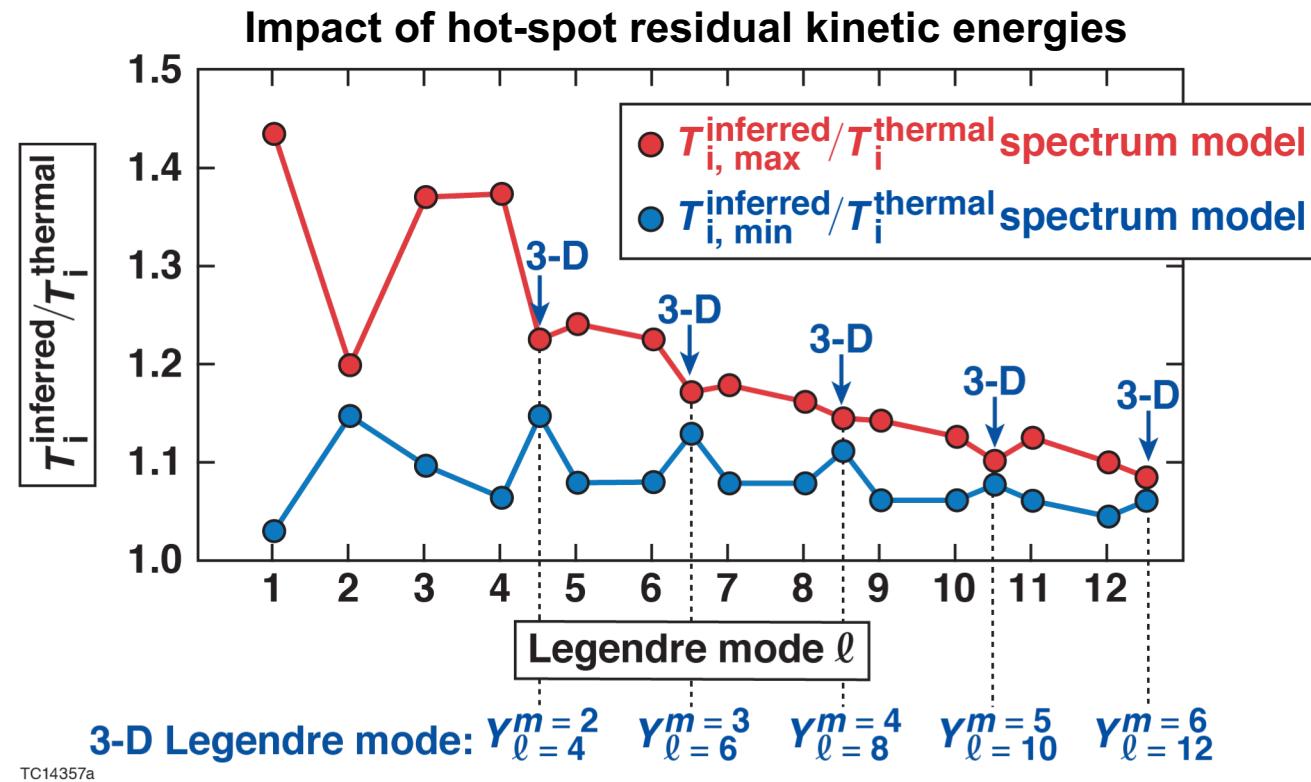


* K. M. Woo, et al. Phys. of Plasmas 25, 052704 (2018)

** F. Weilacher, et al. Phys. of Plasmas 25, 042704 (2018)

† V. N. Goncharov, et al. Phys. of Plasmas 21, 056315 (2014)

The modal dependence of T_i asymmetries was determined using 3-D simulations

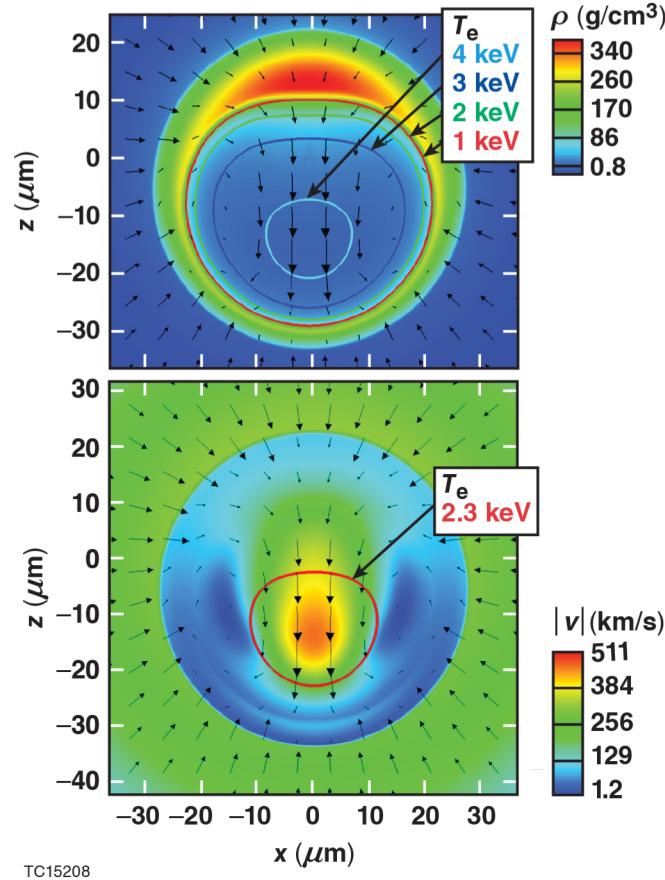


T_i asymmetries are characterized by isotropic and anisotropic flows

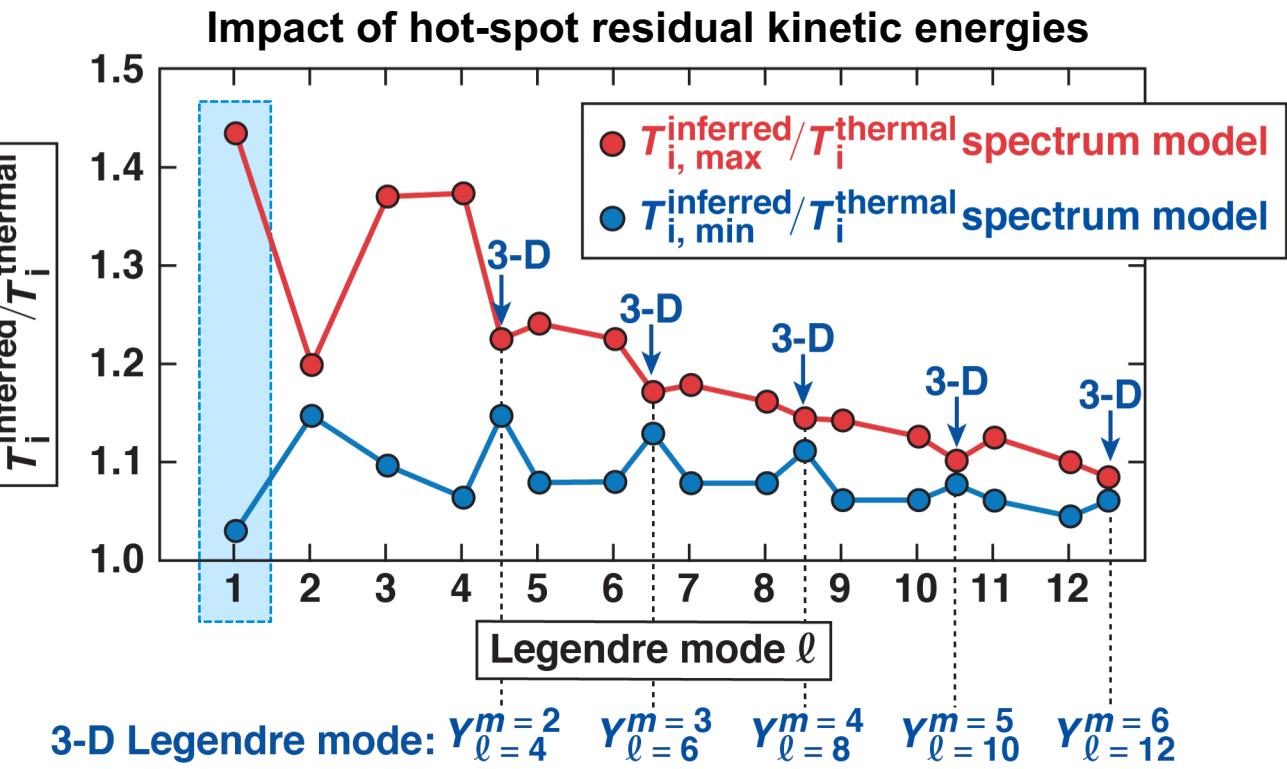
- Low odd-mode numbers give large T_i asymmetries because of anisotropic flows
- Low even-mode numbers give $T_{\min} > T_{\text{thermal}}$ because of isotropic flows
- 3-D modes are more isotropic than 2-D modes

7% dv/v_0 perturbations with YOC ~ 0.4 to 0.6

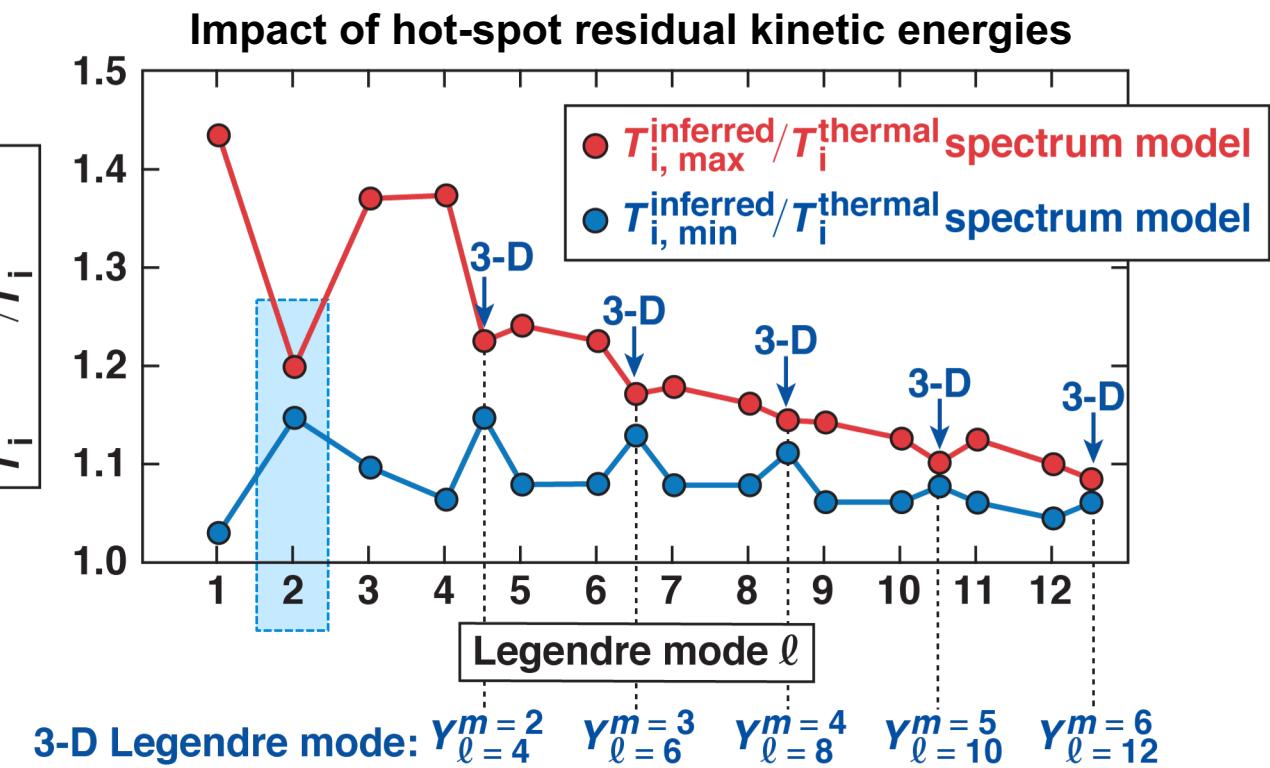
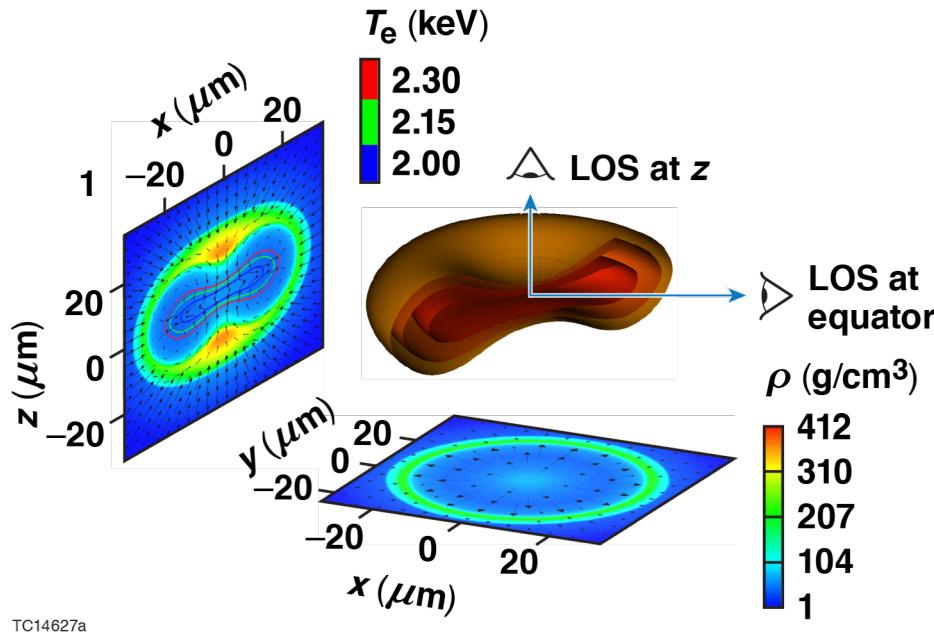
Mode 1 exhibits a large T_i asymmetry and small isotropic flows



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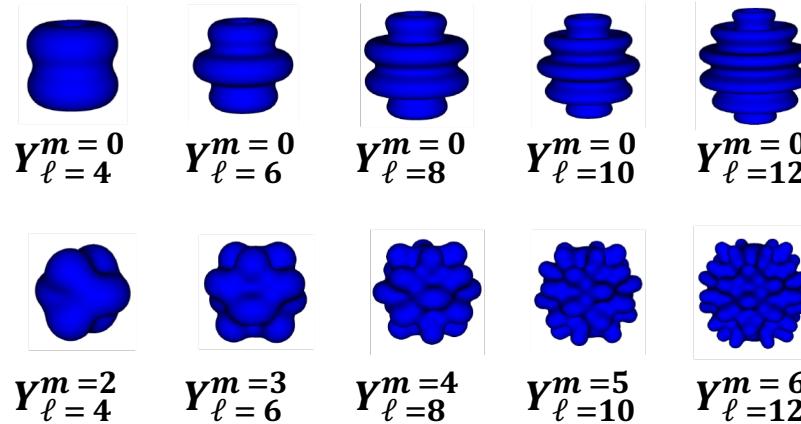


Mode 2 exhibits a small T_i asymmetry and large isotropic flows

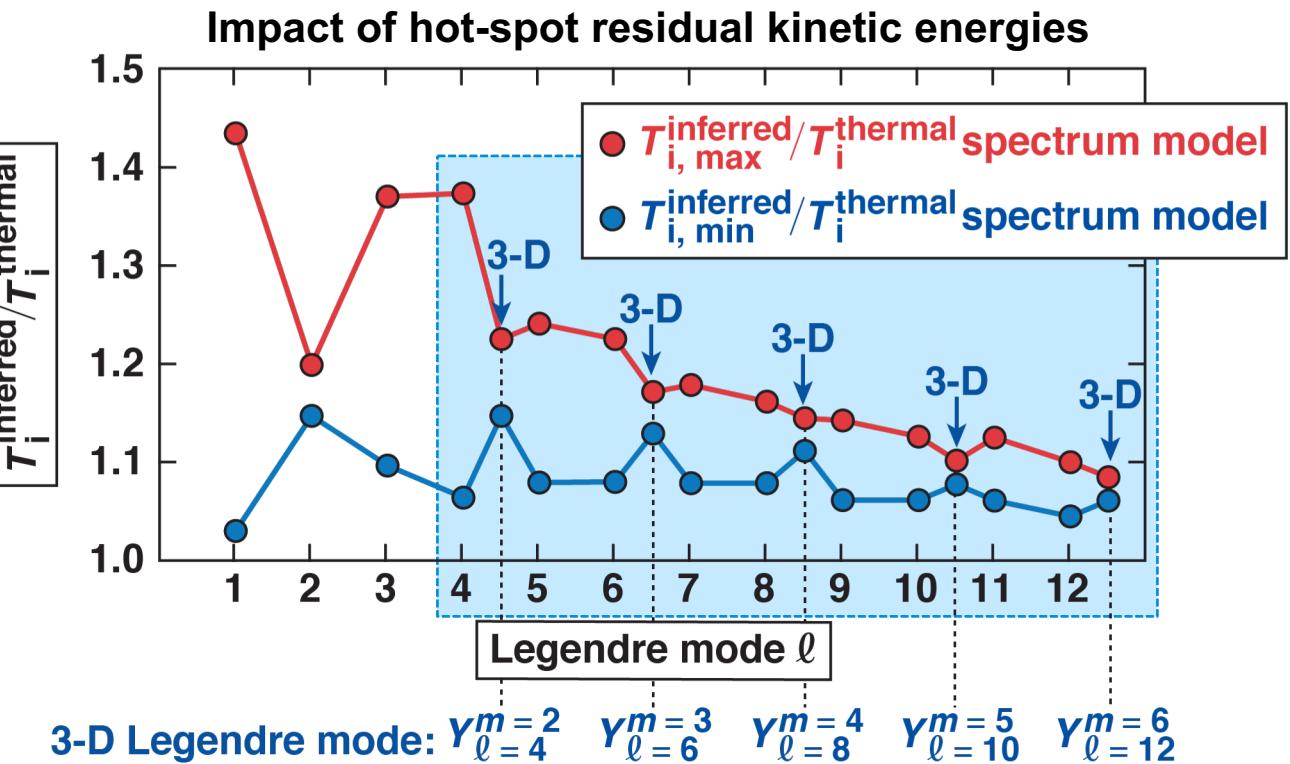


Three-dimensional modes exhibit lower T_i asymmetries than 2-D

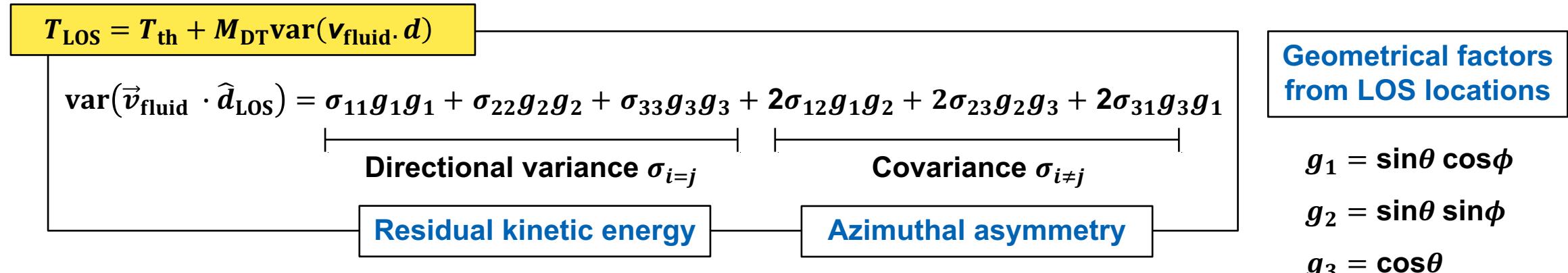
3-D spikes/bubbles along the rotational axis



TC13780a



The six components of the velocity variance determine the full T_i distribution in 4π



Six hot-spot flow parameters: directional-variance and covariance

$$\begin{aligned}\sigma_{ij} &= \langle (v_i - \langle v_i \rangle)(v_j - \langle v_j \rangle) \rangle \\ &= \langle \Delta v_i \Delta v_j \rangle\end{aligned}$$

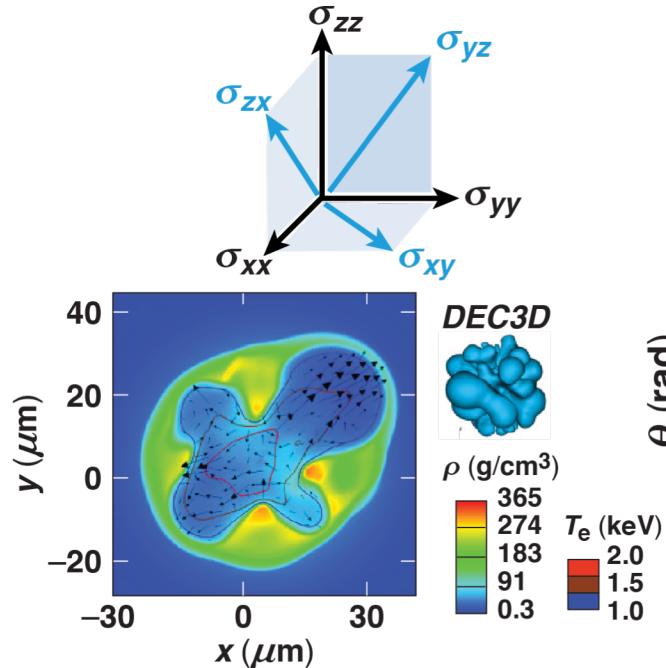
Burn-averaged brackets

Non-translational velocity fluctuations

The 3-D reconstruction of apparent ion temperatures using the six components of velocity variance agrees with the ion temperatures inferred from *IRIS3D*

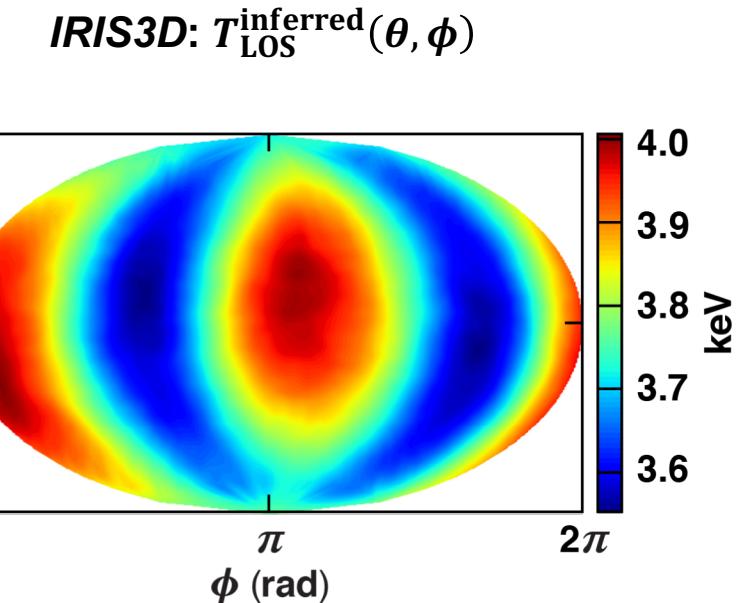
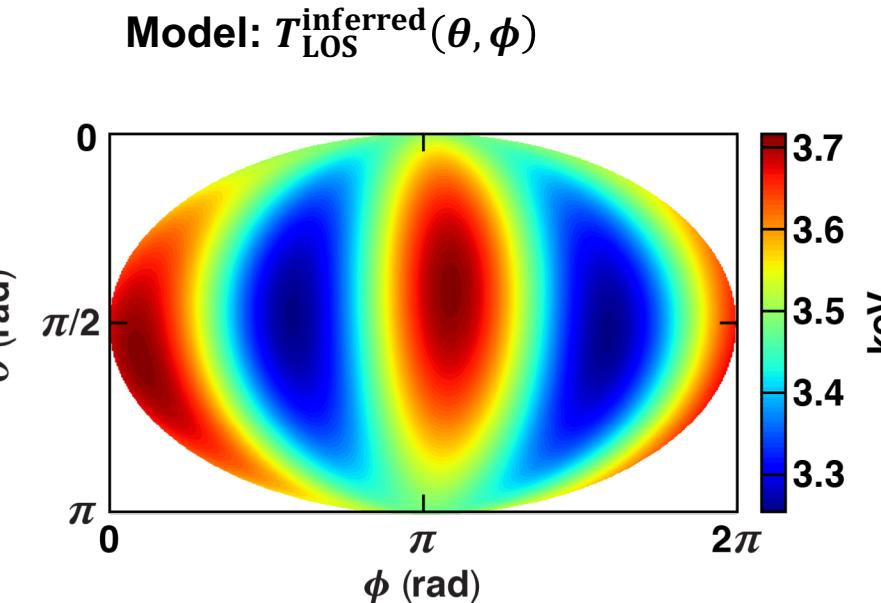


Compute the six hot-spot flow parameters from the *DEC3D* hydrocode



Reconstruct the T_i distribution using
$$T_i^{\text{inferred}} = T_i^{\text{thermal}} + M_{DT} \sigma_{ij} g_i g_j$$

Compare with T_i inferred from synthetic neutron energy spectra



Example of a multimode simulation

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Outline



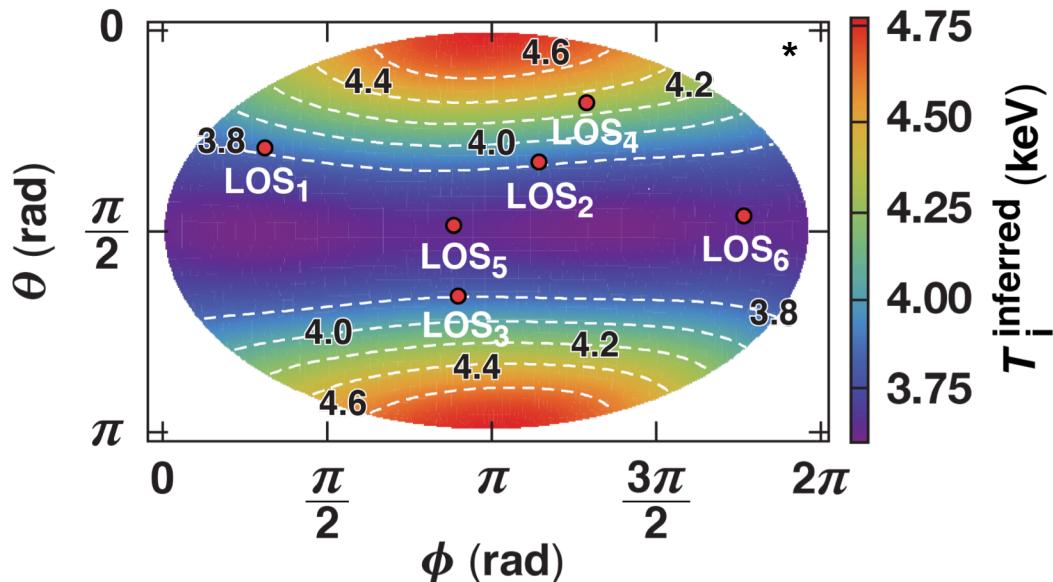
- Introduction to the ion-temperature measurement (T_i) asymmetry
- Impact of anisotropic flows on T_i asymmetry
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The full T_i distribution can be reconstructed using six LOS measurements

$$T_i^{\text{inferred}} = T_i^{\text{thermal}} + M_{DT} \sigma_{ij} g_i g_j$$

T_i distribution from six LOS

$$\vec{T}_p(\theta, \phi) = \hat{M}_p(\theta, \phi) \cdot \hat{M}_{\text{LOS}}^{-1} \cdot \vec{T}_6 + (\hat{I} - \hat{M}_p \cdot \hat{M}_{\text{LOS}}^{-1}) \cdot \vec{T}_{\text{th}}$$

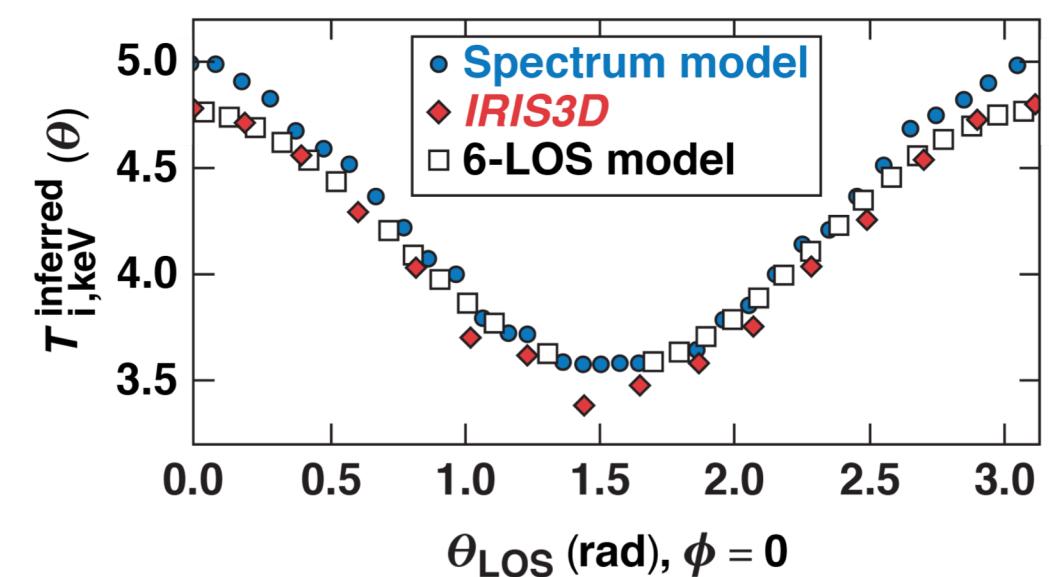


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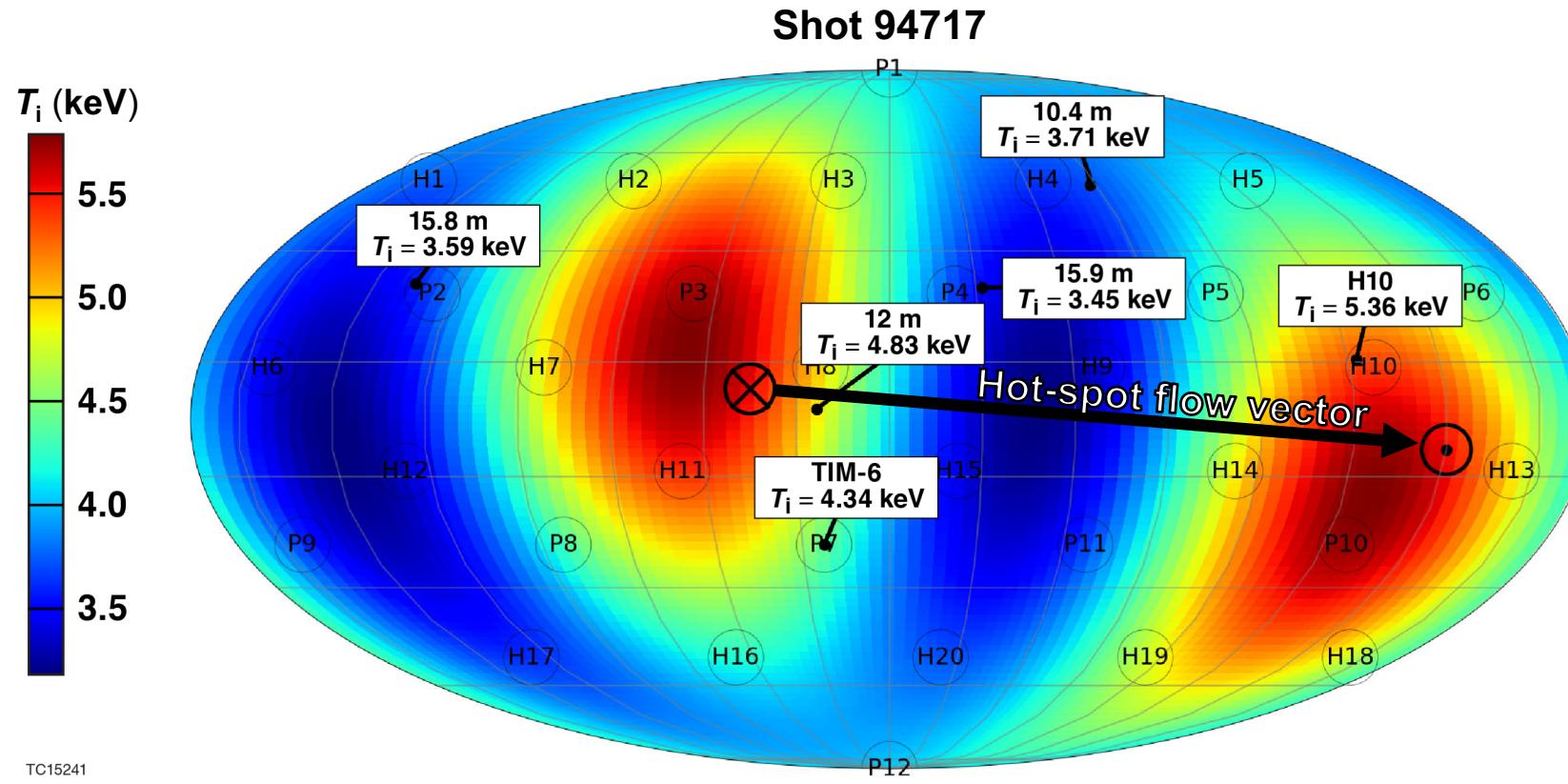
* Mode-1 simulation

The departure matrix is close to zero

$$\ll 1$$



The six-LOS reconstruction map was used on OMEGA implosions and found to be in good agreement with the hot-spot flow measurement



Flow measurement*
from the first moment
of the spectrum

Flow direction (113 km/s)

* Flow measurement by O. Mannion

The T_i asymmetry of mode 1 determines the fusion yield degradation (YOC = Y_{3-D}/Y_{1-D})



T_i asymmetry for mode 1:^{*}

$$\frac{T_{\max}}{T_{\min}} \simeq 1 + 4\overline{RKE}_{\text{tot}} / (1 - \overline{RKE}_{\text{tot}})$$

Total residual kinetic energy:

$$\overline{RKE}_{\text{tot}} = \frac{E_{K,\text{stag}}^{3D} - E_{K,\text{stag}}^{1D}}{E_{K,\text{max}}^{\text{in flight}}}$$

The YOC and RKE_{tot} is related^{**,†} to the T_i asymmetry for mode 1:

$$YOC \simeq (1 - \overline{RKE}_{\text{tot}})^5$$

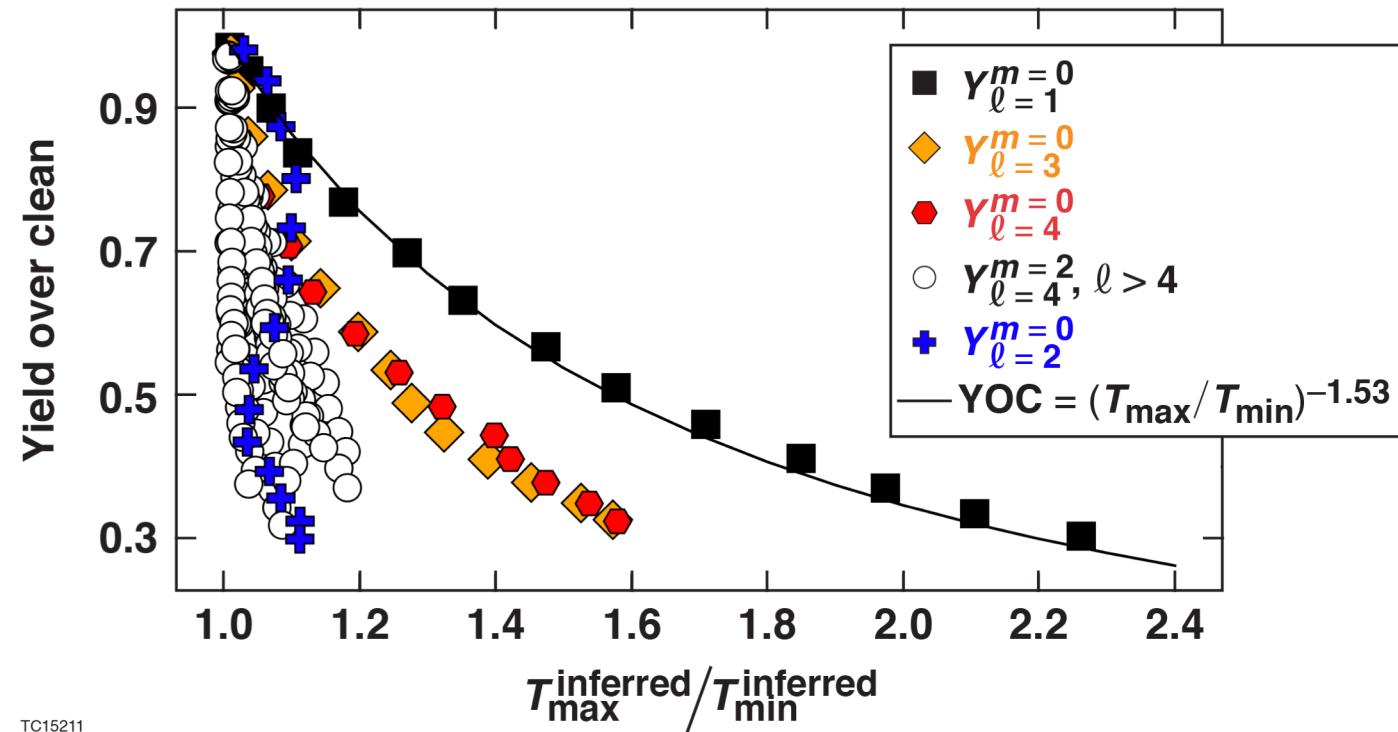
$$YOC \simeq \left[1 - \frac{\frac{1}{4} \left(\frac{T_{\max}}{T_{\min}} - 1 \right)}{\frac{1}{4} \left(\frac{T_{\max}}{T_{\min}} - 1 \right) + 1} \right]^5 \rightarrow \text{fit} \simeq (T_{\max}/T_{\min})^{-1.53}$$

* K. M. Woo, Ph.D. thesis, University of Rochester, 2019.

** A. L. Kitcher, et al. Phys. Plasmas **21**, 042708 (2014).

† K. M. Woo, et al. Phys. Plasmas **25**, 052704 (2018).

The analytic curve shows a good agreement to explain the strong correlation between the yield degradation and mode-1 T_i asymmetries



In experiments, the fitting exponent* is in between 1.25 and 1.43.

* V. Gopalswamy, NO5.00004, this conference.

Outline



- Introduction to the ion-temperature measurement (T_i) asymmetry
- Impact of anisotropic flows on T_i asymmetry
- **Impact of isotropic flows on inferring the minimum DD and thermal ion temperatures**

The isotropic part of the velocity variance is required to find the true thermal temperature from the minimum ion temperature

Minimum $T_i >$ thermal T_i

$$\text{var}[\vec{v}_{\text{fluid}} \cdot \hat{d}_{\text{LOS}}] = \sigma_{\text{iso}} + \sigma_{\text{aniso}}^{\text{LOS}}$$

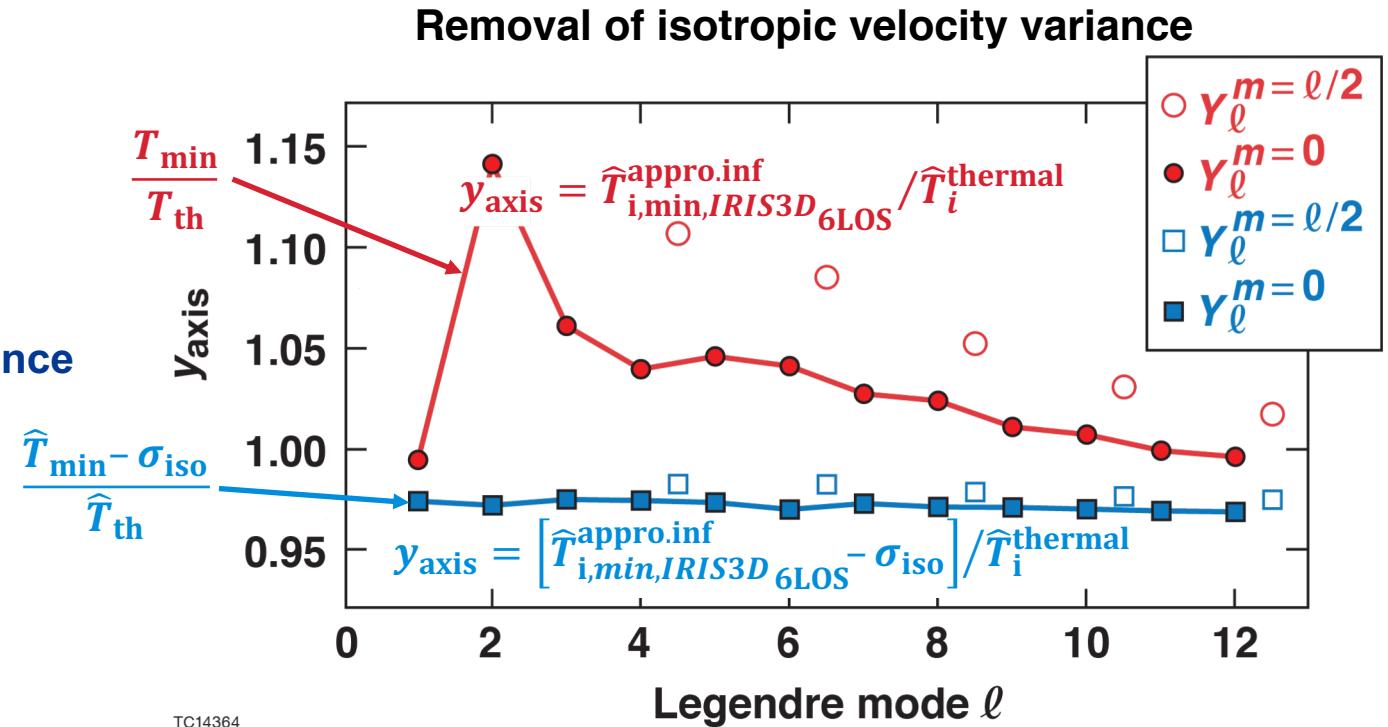
T_i asymmetry

Define the isotropic part of the velocity variance

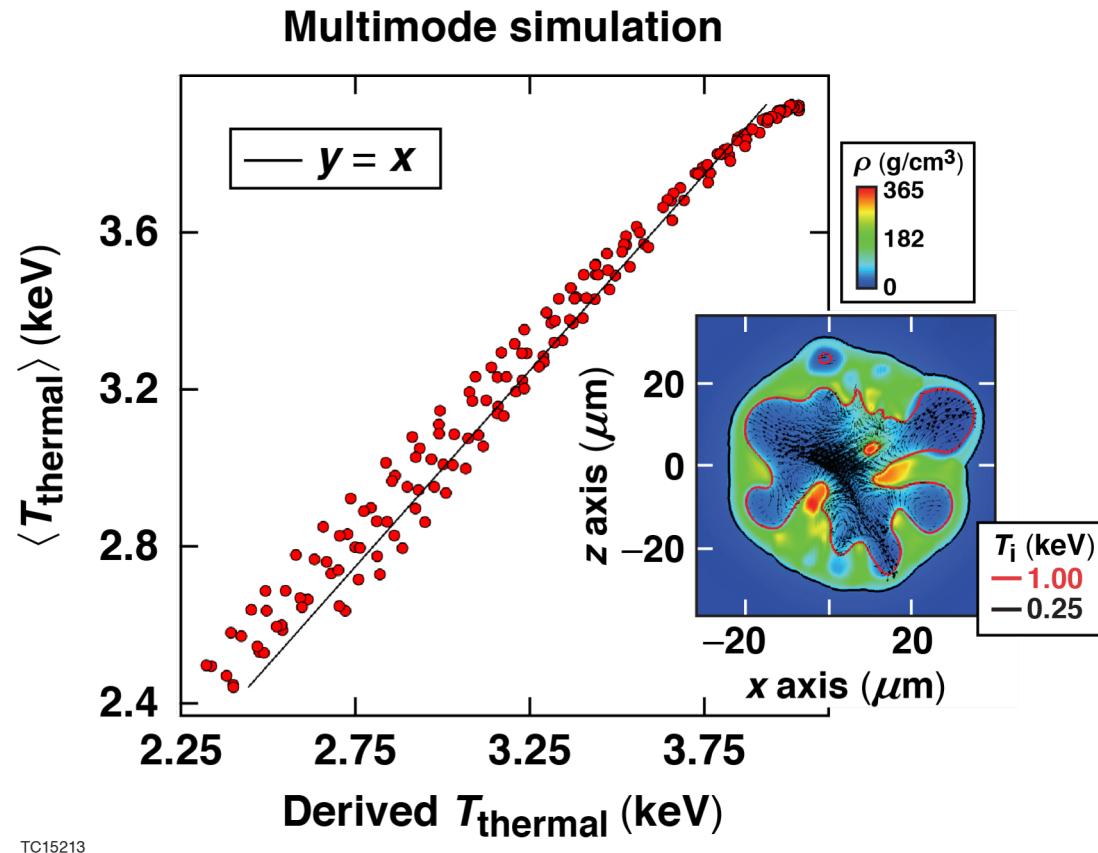
$$\sigma_{\text{iso}} = \text{Min} \left[\text{var}[\vec{v}_{\text{fluid}} \cdot \hat{d}_{\text{LOS}}] \right]_{4\pi}$$

Solve for the thermal temperature by subtracting the isotropic part of velocity variance from the minimum temperature

$$T_{\text{thermal}} = T_{\min} - M_{\text{DT}} \sigma_{\text{iso}}$$



In experiments, measuring the minimum DD temperatures enables the inference of the isotropic velocity variance and the true thermal temperature



Isotropic velocity variance

$$T_{\min}^{\text{DT}} = T_{\text{th}}^{\text{DT}} + M_{\text{DT}} \langle \sigma_{\text{iso}}^{\text{DT}} \rangle$$

$$T_{\min}^{\text{DD}} = T_{\text{th}}^{\text{DD}} + M_{\text{DD}} \langle \sigma_{\text{iso}}^{\text{DD}} \rangle$$

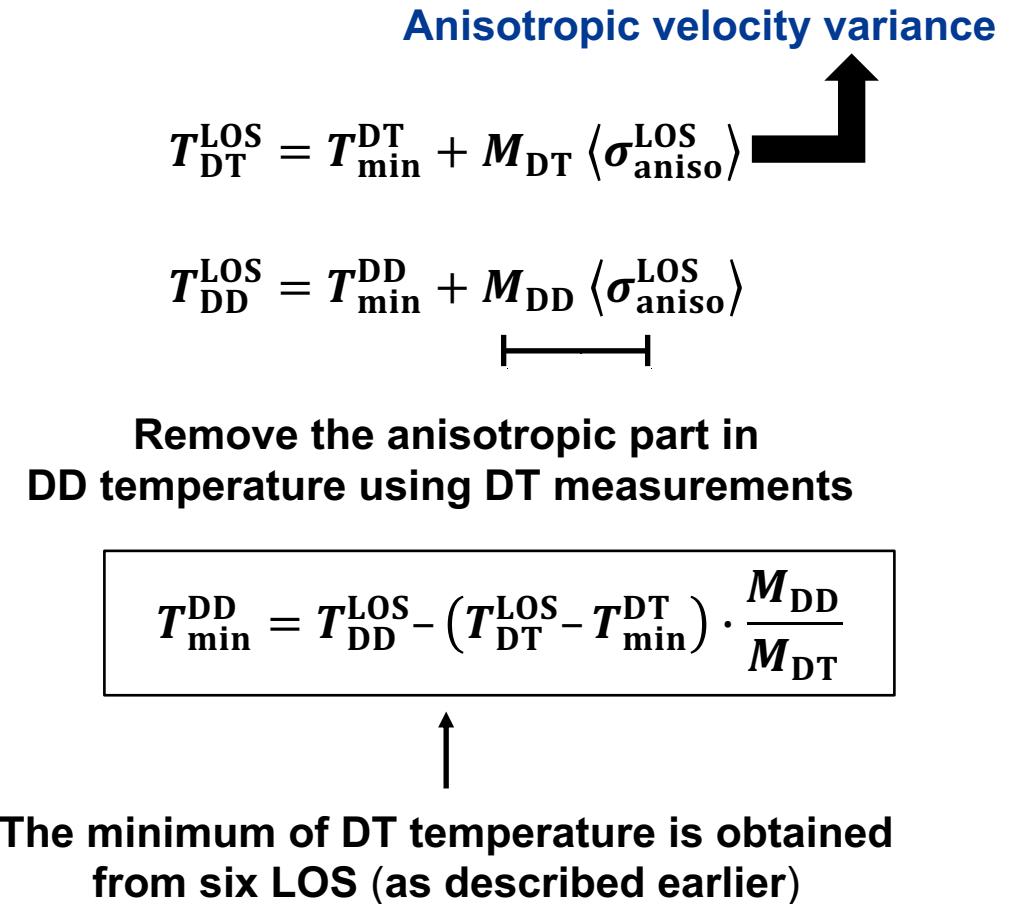
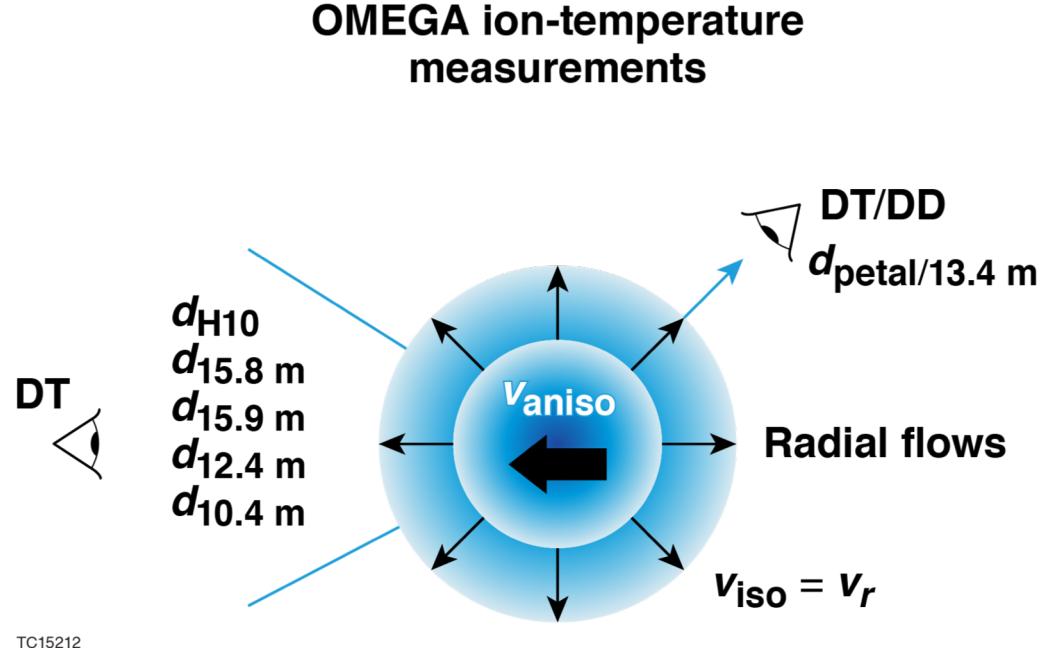
Remove the isotropic part using the DD T_{\min}

$$\langle \sigma_{\text{iso}}^{\text{DD}} \rangle = (T_{\min}^{\text{DD}} - T_{\text{th}}^{\text{DD}}) / M_{\text{DD}}$$

$$\langle T_{\text{th}} \rangle \simeq \left(T_{\min}^{\text{DT}} - \frac{M_{\text{DT}}}{M_{\text{DD}}} \cdot T_{\min}^{\text{DD}} \right) / \left[1 - \frac{M_{\text{DT}}}{M_{\text{DD}}} \right]$$

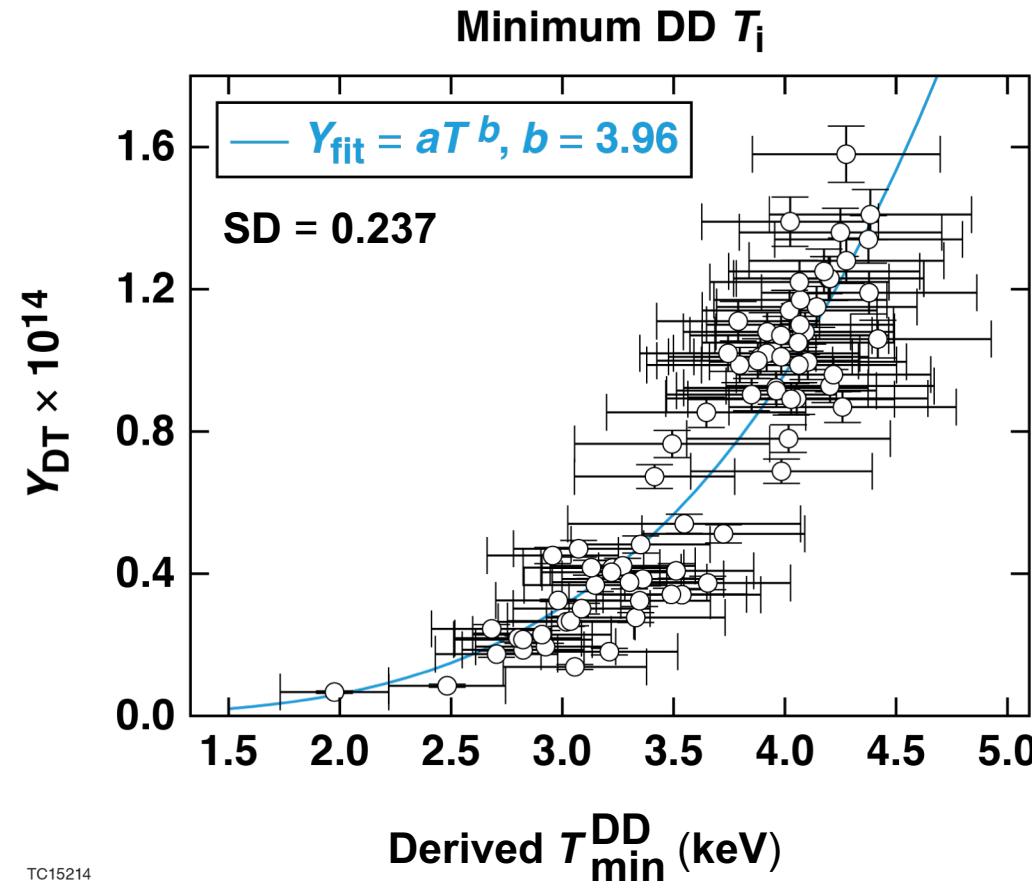
Assumptions: $\langle T_{\text{th}}^{\text{DD}} \rangle \approx \langle T_{\text{th}}^{\text{DT}} \rangle$ and $\langle \sigma_{\text{iso}}^{\text{DD}} \rangle \approx \langle \sigma_{\text{iso}}^{\text{DT}} \rangle$

The minimum DD ion temperature is inferred by performing DD and DT ion-temperature measurements along the same LOS



$$\text{Definition } M_{\text{DD}} = m_n + m_{\text{He}_3} \quad M_{\text{DT}} = m_n + m_\alpha$$

In OMEGA cryogenic implosion experiments, the derived DD minimum ion temperatures are strongly correlated with the yields as $\sim T^4$

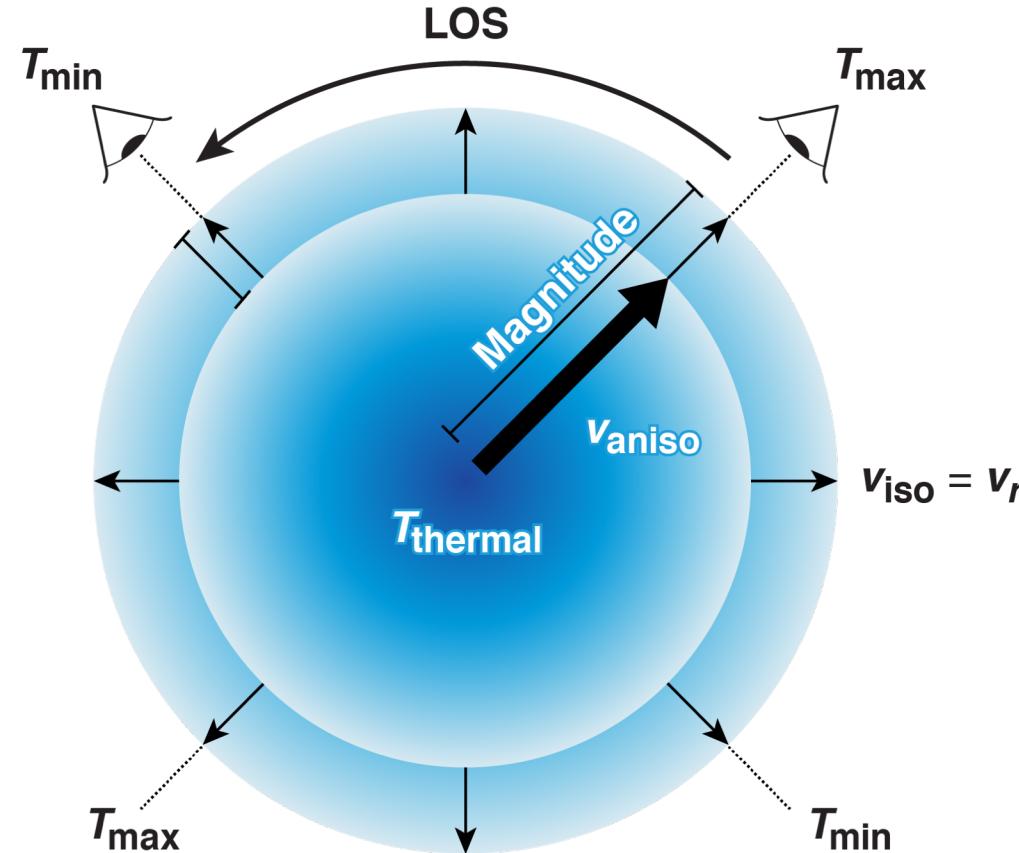


A velocity variance analysis was developed to explain the 3-D flow effects on ion-temperature (T_i) measurements and the modal dependence of T_i asymmetries in ICF



- A method using six line-of-sight (LOS) ion-temperature measurements was developed to determine the full temperature map and account for the contribution of anisotropic flows
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The variation of apparent ion-temperature measurements along different lines of sight (LOS) is uniquely determined by the behavior of the variance*,** of the hot-spot fluid velocities



$$\text{var}[\vec{v}_{\text{fluid}} \cdot \hat{d}] = \langle |\Delta \vec{v}_{\text{fluid}} \cdot \hat{d}|^2 \rangle$$

- Residual kinetic energy
- Square of the velocity fluctuation vector

- Radial flows
- Counter-flowing fluid motion

TC15240

* T. J. Murphy, Phys. Plasmas **21**, 072701 (2014).

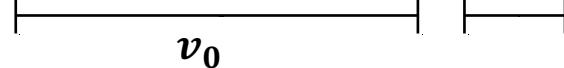
** D. H. Munro, Nucl. Fusion **56**, 036001 (2016)

† H. Brysk, Plasma Phys. **15**, 611 (1973).

The isotropic flow velocity is not measured in the hot-spot flow velocity vector reconstruction

Neutron velocity measured along a given LOS

$$\vec{v}_{\text{LOS}} = v_{14.1 \text{ MeV}} + v_{\text{cm}} + v_{\text{rel}} + \vec{v} \cdot \hat{d}$$

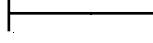


Expand the fluid velocity into isotropic and anisotropic components

$$\vec{v} = \vec{v}_{\text{iso}} + \vec{v}_{\text{aniso}}$$

Burn-averaged neutron velocity

$$\langle \vec{v}_{\text{LOS}} \rangle = v_0 \langle \vec{v}_{\text{iso}} \cdot \hat{d} \rangle + \langle \vec{v}_{\text{aniso}} \cdot \hat{d} \rangle = v_0 + \langle \vec{v}_{\text{aniso}} \cdot \hat{d} \rangle$$



Expand the isotropic flow velocity into parallel and antiparallel components with respect to the LOS unit vector

$$\langle \vec{v}_{\text{iso}} \cdot \hat{d} \rangle = \langle \vec{v}_{\text{iso}}^+ \cdot \hat{d} \rangle + \langle \vec{v}_{\text{iso}}^- \cdot \hat{d} \rangle = \langle \vec{v}_{\text{iso}} \cdot \hat{d} \rangle - \langle \vec{v}_{\text{iso}} \cdot \hat{d} \rangle = 0$$

The general expression for the isotropic velocity variance is derived

Neutron velocity measured along a given LOS

$$\text{var}(\vec{v} \cdot \hat{d}) = \langle (\vec{v} \cdot \hat{d})^2 \rangle + \langle \vec{v} \cdot \hat{d} \rangle^2 = \langle (\vec{v}_{\text{iso}} \cdot \hat{d} + \vec{v}_{\text{aniso}} \cdot \hat{d})^2 \rangle + \langle \vec{v}_{\text{aniso}} \cdot \hat{d} \rangle^2$$

Expand the fluid velocity into isotropic and anisotropic components

$$\begin{aligned} \langle (\vec{v} \cdot \hat{d})^2 \rangle &= \langle (\vec{v}_{\text{iso}} \cdot \hat{d})^2 + (\vec{v}_{\text{aniso}} \cdot \hat{d})^2 + 2(\vec{v}_{\text{iso}} \cdot \hat{d})(\vec{v}_{\text{aniso}} \cdot \hat{d}) \rangle \\ &= \langle v_{\text{iso}}^2 \rangle + \langle (\vec{v}_{\text{aniso}} \cdot \hat{d})^2 \rangle + 2\langle (\vec{v}_{\text{iso}} \cdot \hat{d})(\vec{v}_{\text{aniso}} \cdot \hat{d}) \rangle \end{aligned}$$

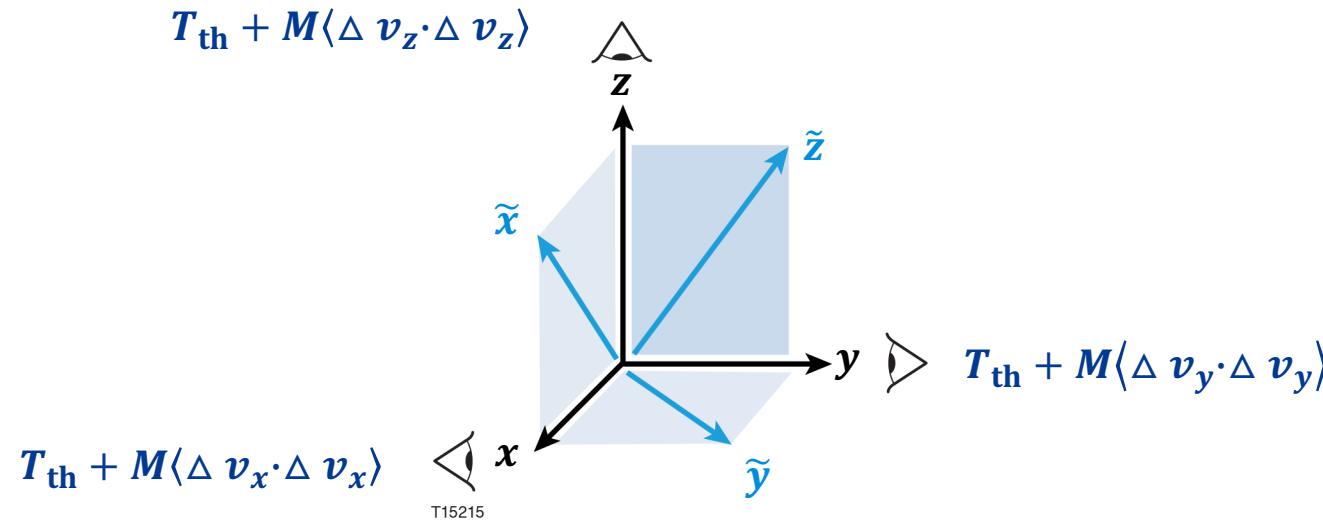
|

Expand the cross term

$$\langle (\vec{v}_{\text{iso}}^+ \cdot \hat{d})(\vec{v}_{\text{aniso}} \cdot \hat{d}) \rangle + \langle (\vec{v}_{\text{iso}}^- \cdot \hat{d})(\vec{v}_{\text{aniso}} \cdot \hat{d}) \rangle = \langle v_{\text{iso}}(\vec{v}_{\text{aniso}} \cdot \hat{d}) \rangle - \langle v_{\text{iso}}(\vec{v}_{\text{aniso}} \cdot \hat{d}) \rangle = 0$$

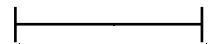
Therefore $\text{var}(\vec{v} \cdot \hat{d}) = \langle v_{\text{iso}}^2 \rangle + \text{var}(\vec{v}_{\text{aniso}} \cdot \hat{d})$

The ability to determine the true minimum ion temperature depends on the number of LOS



The sum of T_i measurements at three orthogonal directions is rotational invariant; this implies the T_i measurements at a given set of LOS is related to another set of LOS by a coordinate transformation

$$3T_{\text{th}} + M\langle \Delta \vec{v} \cdot \vec{v} \rangle = 3T_{\text{th}} + M\langle \Delta \tilde{\vec{v}} \cdot \Delta \tilde{\vec{v}} \rangle \text{ by substituting } \tilde{\vec{v}} = \hat{\mathbf{R}} \cdot \vec{v}$$



Conservation of the total hot-spot fluid residual kinetic energy

$\tilde{x}, \tilde{y}, \tilde{z}$ are rotated axes

The measurements of DT ion temperatures cannot be applied to solve for the thermal temperature because the apparent temperatures contain isotropic flows



The orange parts form \hat{M}_{LOS}

T_1	1	$g^1_x g^1_x$	$g^1_y g^1_y$	$g^1_z g^1_z$	$2g^1_x g^1_y$	$2g^1_y g^1_z$	$2g^1_z g^1_x$	T_{th}	T_{th}
T_2	1	$g^2_x g^2_x$	$g^2_y g^2_y$	$g^2_z g^2_z$	$2g^2_x g^2_y$	$2g^2_y g^2_z$	$2g^2_z g^2_x$	$M_{\text{DT}} \sigma_{xx}$	$M_{\text{DT}} \sigma_{\text{iso}}$
T_3	1	$g^3_x g^3_x$	$g^3_y g^3_y$	$g^3_z g^3_z$	$2g^3_x g^3_y$	$2g^3_y g^3_z$	$2g^3_z g^3_x$	$M_{\text{DT}} \sigma_{yy}$	Turbulence
T_4	1	$g^4_x g^4_x$	$g^4_y g^4_y$	$g^4_z g^4_z$	$2g^4_x g^4_y$	$2g^4_y g^4_z$	$2g^4_z g^4_x$	$M_{\text{DT}} \sigma_{zz}$	$M_{\text{DT}} \sigma_{\text{iso}}$
T_5	1	$g^5_x g^5_x$	$g^5_y g^5_y$	$g^5_z g^5_z$	$2g^5_x g^5_y$	$2g^5_y g^5_z$	$2g^5_z g^5_x$	$M_{\text{DT}} \sigma_{xy}$	0
T_6	1	$g^6_x g^6_x$	$g^6_y g^6_y$	$g^6_z g^6_z$	$2g^6_x g^6_y$	$2g^6_y g^6_z$	$2g^6_z g^6_x$	$M_{\text{DT}} \sigma_{yz}$	0
T_7	1	$g^7_x g^7_x$	$g^7_y g^7_y$	$g^7_z g^7_z$	$2g^7_x g^7_y$	$2g^7_y g^7_z$	$2g^7_z g^7_x$	$M_{\text{DT}} \sigma_{zx}$	0

Apparent ion temperatures
for turbulence at all LOS's

$$T_{\text{LOS}} = T_{\text{th}} + M_{\text{DT}} \sigma_{\text{iso}}$$

The consequence of introducing DD ion-temperature measurements leads to a complete solution for the true thermal temperatures

This 7×7 matrix is invertible with a nonzero determinant

LOS_1^{H10}	T_1^{DT}	1	$g^1_x g^1_x$	$g^1_y g^1_y$	$g^1_z g^1_z$	$2g^1_x g^1_y$	$2g^1_y g^1_z$	$2g^1_z g^1_x$	T_{th}
$\text{LOS}_2^{15.8 \text{ m}}$	T_2^{DT}	1	$g^2_x g^2_x$	$g^2_y g^2_y$	$g^2_z g^2_z$	$2g^2_x g^2_y$	$2g^2_y g^2_z$	$2g^2_z g^2_x$	$M_{\text{DT}} \sigma_{xx}$
$\text{LOS}_3^{15.9 \text{ m}}$	T_3^{DT}	1	$g^3_x g^3_x$	$g^3_y g^3_y$	$g^3_z g^3_z$	$2g^3_x g^3_y$	$2g^3_y g^3_z$	$2g^3_z g^3_x$	$M_{\text{DT}} \sigma_{yy}$
$\text{LOS}_4^{12 \text{ m}}$	T_4^{DT}	1	$g^4_x g^4_x$	$g^4_y g^4_y$	$g^4_z g^4_z$	$2g^4_x g^4_y$	$2g^4_y g^4_z$	$2g^4_z g^4_x$	$M_{\text{DT}} \sigma_{zz}$
$\text{LOS}_5^{10.4 \text{ m}}$	T_5^{DT}	1	$g^5_x g^5_x$	$g^5_y g^5_y$	$g^5_z g^5_z$	$2g^5_x g^5_y$	$2g^5_y g^5_z$	$2g^5_z g^5_x$	$M_{\text{DT}} \sigma_{xy}$
$\text{LOS}_6^{\text{Petal}}$	T_6^{DT}	1	$g^6_x g^6_x$	$g^6_y g^6_y$	$g^6_z g^6_z$	$2g^6_x g^6_y$	$2g^6_y g^6_z$	$2g^6_z g^6_x$	$M_{\text{DT}} \sigma_{yz}$
$\text{LOS}_7^{13.4 \text{ m}}$	T_7^{DD}	R_T	$R_M g^7_x g^7_x$	$R_M g^7_y g^7_y$	$R_M g^7_z g^7_z$	$R_M 2g^7_x g^7_y$	$R_M 2g^7_y g^7_z$	$R_M 2g^7_z g^7_x$	$M_{\text{DT}} \sigma_{zx}$

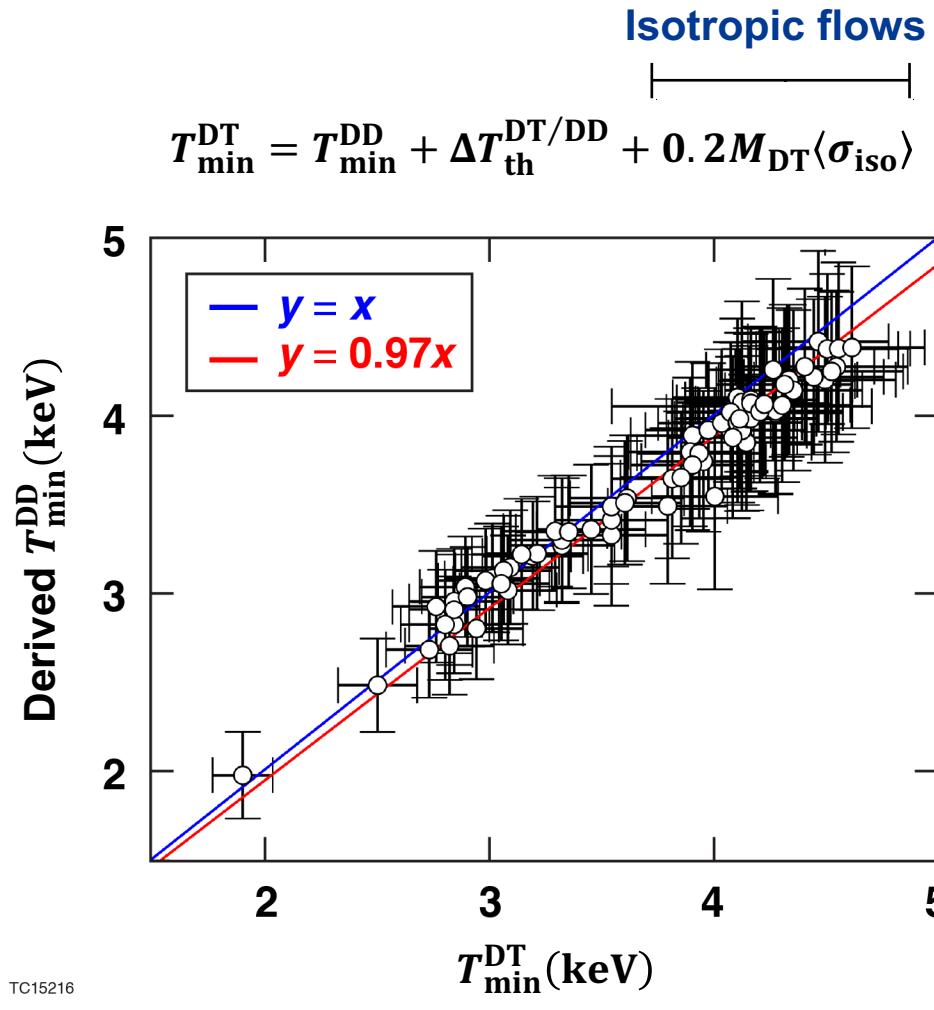


Add more T_{DD} here,
and compute the pseudo inverse

$$R_T = T_{\text{th}}^{\text{DD}} / T_{\text{th}}^{\text{DT}}$$

$$R_M = M_{\text{DD}} / M_{\text{DT}}$$

OMEGA implosions exhibit a small amount of isotropic flows as shown by comparing the minimum DD and DT ion temperatures



Define the residual kinetic energy for isotropic flows

$$T_{\min}^{\text{DD}} = T_{\text{th}} + M_{\text{DD}} \langle \sigma_{\text{iso}} \rangle$$

$$T_{\min}^{\text{DT}} = T_{\text{th}} + M_{\text{DT}} \langle \sigma_{\text{iso}} \rangle = T_{\text{th}} (1 + f_{\text{RKE}})$$

$$f_{\text{RKE}} = \frac{\text{kinetic energy of isotropic fluid motions}}{\text{ion thermal energy}} = \frac{M_{\text{DT}} \langle \sigma_{\text{iso}} \rangle}{T_{\text{th}}}$$

$$\frac{T_{\min}^{\text{DD}}}{T_{\min}^{\text{DT}}} = \frac{1 + (M_{\text{DD}}/M_{\text{DT}})f_{\text{RKE}}}{1 + f_{\text{RKE}}}$$

Definition $\Delta T_{\text{th}}^{\text{DT/DD}} \equiv T_{\text{th}}^{\text{DT}} - T_{\text{th}}^{\text{DD}}$