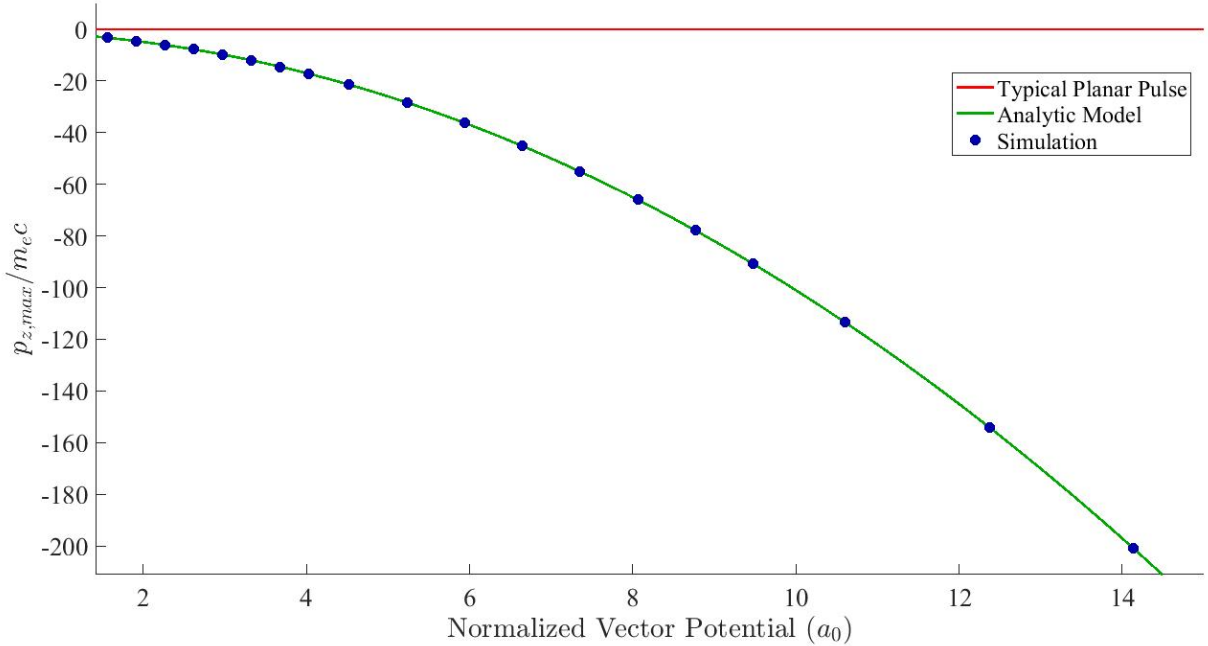
Vacuum acceleration in a flying focus



D.W. Ramsey, D.H. Froula, and J.P. Palastro

61st Annual APS DPP Meeting
Fort Lauderdale, FL Oct 23rd 2019

Summary

The flying focus enables a novel mechanism for vacuum acceleration

UR LLE

- Typical planar pulses cannot impart net momentum to electrons

  The energy gained during ponderomotive acceleration in the leading edge of the pulse is lost during ponderomotive deceleration in the trailing edge

- Planar-like flying focus pulses can accelerate electrons to relativistic momenta (either positive or *negative*)

  The ponderomotive force propagates at a subluminal velocity

  The electron gains enough energy during ponderomotive acceleration in the leading edge of the pulse that it outruns the pulse and never experiences the trailing edge

Lawson-Woodward Theorem (LWT): The net energy gain for an electron in a laser pulse is zero

For net energy gain, one of the assumptions of the LWT needs to be exploited:

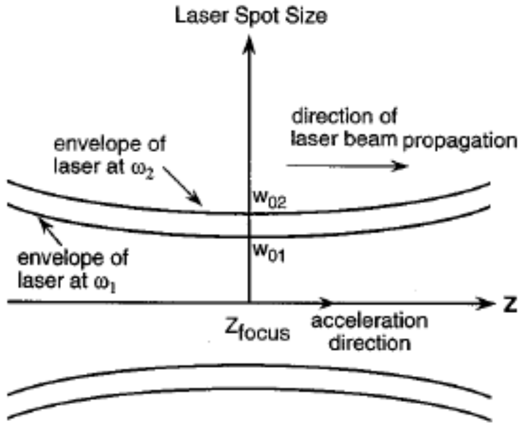
1. Highly relativistic electron
2. No boundaries
3. No static fields
4. No non-linear effects (ponderomotive)
5. Infinite interaction region

Lawson-Woodward Theorem (LWT): The net energy gain for an electron in a laser pulse is zero

For net energy gain, one of the assumptions of the LWT needs to be exploited:

1. Highly relativistic electron
2. No boundaries
3. No static fields
4. No non-linear effects (ponderomotive)
5. Infinite interaction region

Examples:



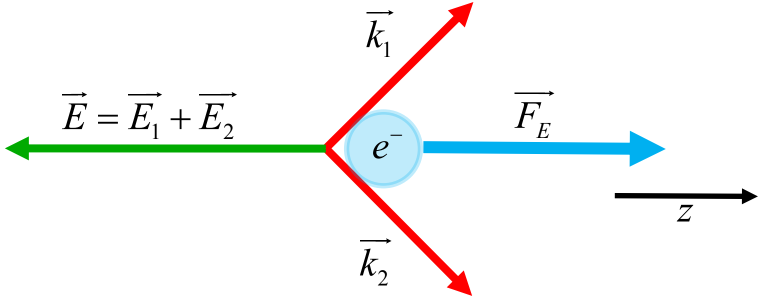
Vacuum Beatwave Accelerator,
Esarey et al., Phys. Rev. E 52, 5443 (1995)

Lawson-Woodward Theorem (LWT): The net energy gain for an electron in a laser pulse is zero

For net energy gain, one of the assumptions of the LWT needs to be exploited:

1. Highly relativistic electron
2. No boundaries
3. No static fields
4. No non-linear effects (ponderomotive)
5. Infinite interaction region

Examples:



Crossed Laser-Beam Acceleration,
C.M. Halland, Opt. Commun. 114, 280 (1995)

Lawson-Woodward Theorem (LWT): The net energy gain for an electron in a laser pulse is zero

For net energy gain, one of the assumptions of the LWT needs to be exploited:

1. Highly relativistic electron
2. No boundaries
3. No static fields
4. No non-linear effects (ponderomotive)
5. Infinite interaction region

The nonlinear ponderomotive force of a plane wave is not sufficient to break the LWT

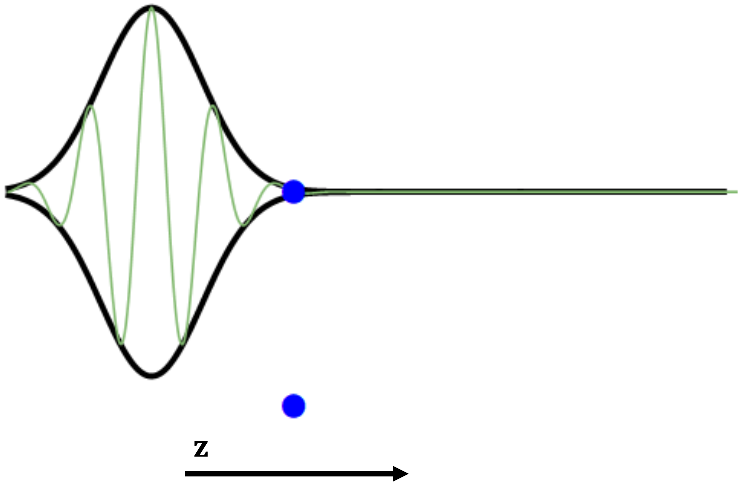
An electron in the electromagnetic field of the pulse satisfies the conservation relation

$$\frac{d}{dt}\left(\gamma - \frac{p_z}{m_e c}\right) = 0$$

From the constant of motion:

$$\frac{p_z}{m_e c} = \frac{1}{2} a_0^2 \left(z,t\right)$$

Once the pulse has outrun the electron,
the electron has transferred its energy back to the pulse



The nonlinear ponderomotive force of a plane wave is not sufficient to break the LWT

An electron in the electromagnetic field of the pulse satisfies the conservation relation

$$\frac{d}{dt}\left(\gamma - \frac{p_z}{m_e c}\right) = 0$$

From the constant of motion:

$$\frac{p_z}{m_e c} = 0$$

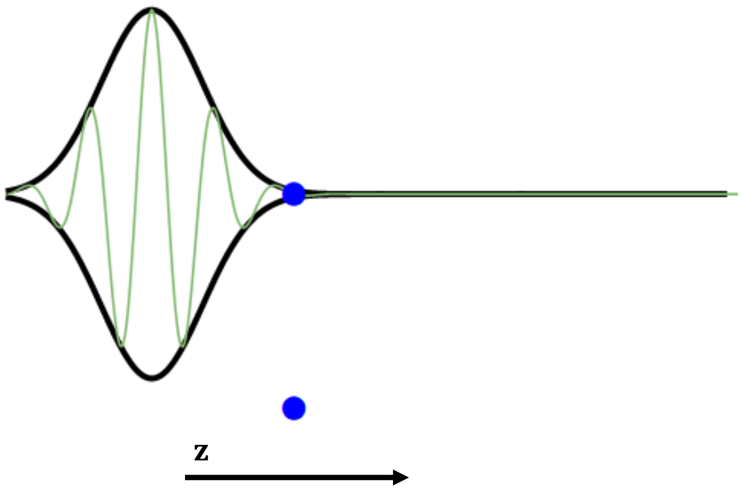
Once the pulse has outrun the electron,
the electron has transferred its energy back to the pulse

The electron can gain net energy when the phase velocity is independent of the velocity of the intensity peak

An electron in the electromagnetic field of the pulse satisfies the conservation relation

$$\frac{d}{dt}\left(\gamma - \beta_I \frac{p_z}{m_e c}\right) = 0$$

where $\beta_I = \mathrm{v}_I / c$



From the constant of motion:

$$\frac{p_z}{m_e c} = \beta_I \gamma_I^2 \pm \beta_I \gamma_I^2 \left[1 + \frac{1}{2}(\beta_I \gamma_I)^{-2} a_0^2\right]^{1/2}$$

Once the electron has outrun the pulse, the electron retains energy gained in the pulse

The electron can gain net energy when the phase velocity is independent of the velocity of the intensity peak

An electron in the electromagnetic field of the pulse satisfies the conservation relation

$$\frac{d}{dt}\left(\gamma - \beta_I \frac{p_z}{m_e c}\right) = 0$$

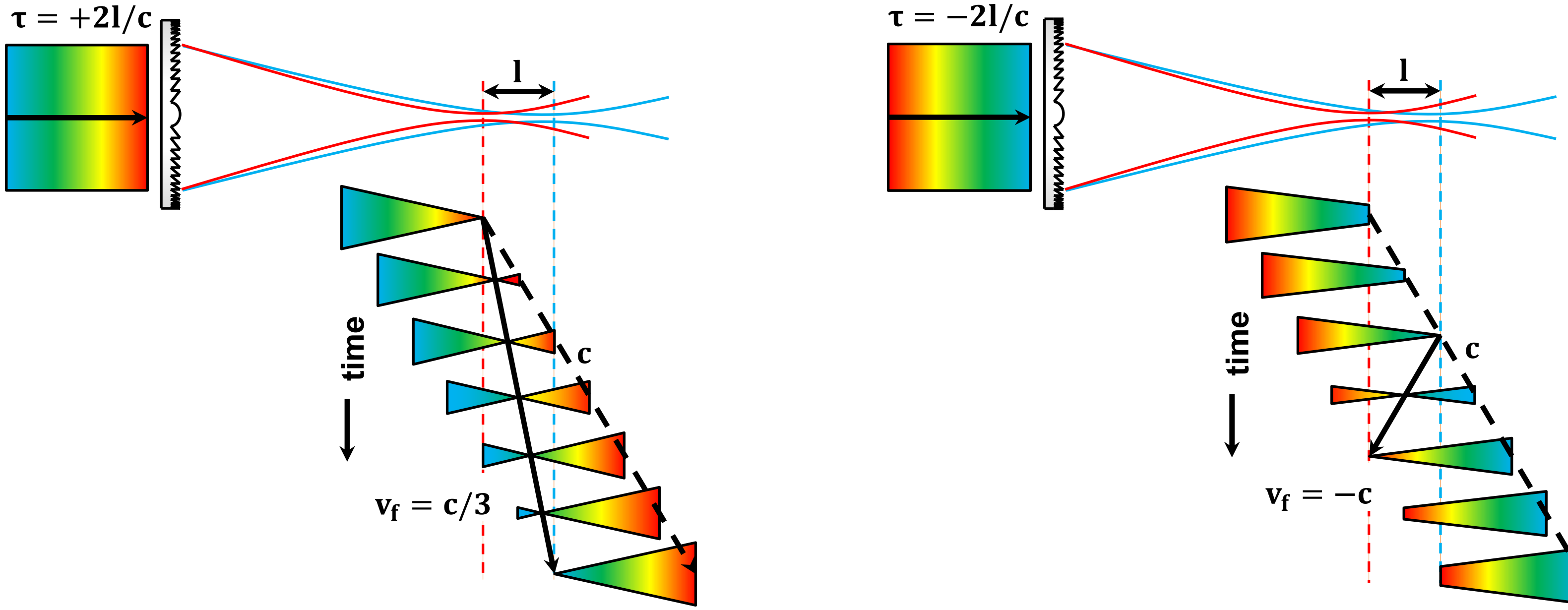
where $\beta_I = \mathrm{v}_I / c$

From the constant of motion:

$$\frac{p_z}{m_e c} = 2\beta_I \gamma_I^2$$

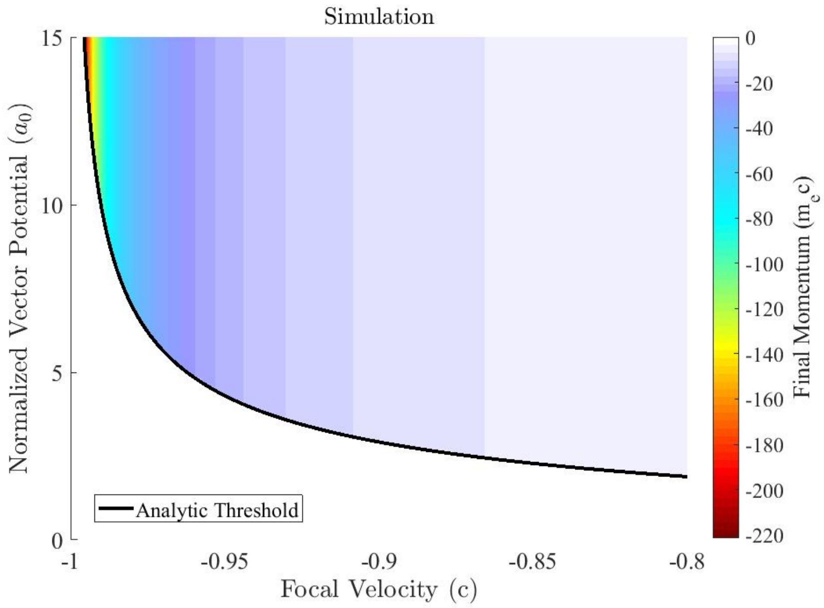
Once the electron has outrun the pulse, the electron retains energy gained in the pulse

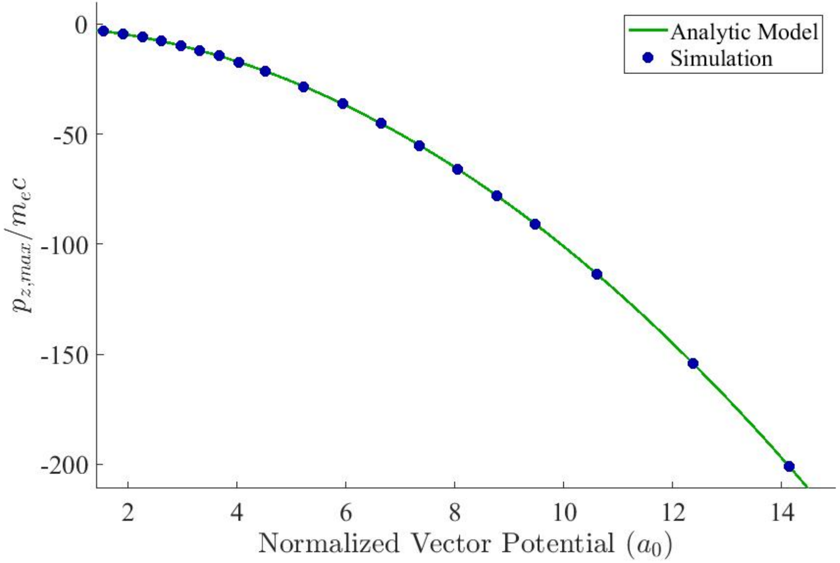
The flying focus combines a chromatic optic with a chirped laser pulse, decoupling the velocity of the intensity peak from the phase velocity



The chirp of the pulse determines the time at which color reaches focus, resulting in a peak intensity with a dynamic trajectory

The analytic model for the final momentum agrees with simulations of electron motion in flying focus pulses





Net energy gain requires the flying focus pulse to surpass a threshold a_0 value for a given focal velocity

Working in the Lorentz frame of the flying focus provides an intuitive explanation for the energy gain

In the flying focus frame, the ponderomotive potential is time-independent, implying the electron energy is conserved: $\gamma_i = \gamma_f$

There are two ways this can occur

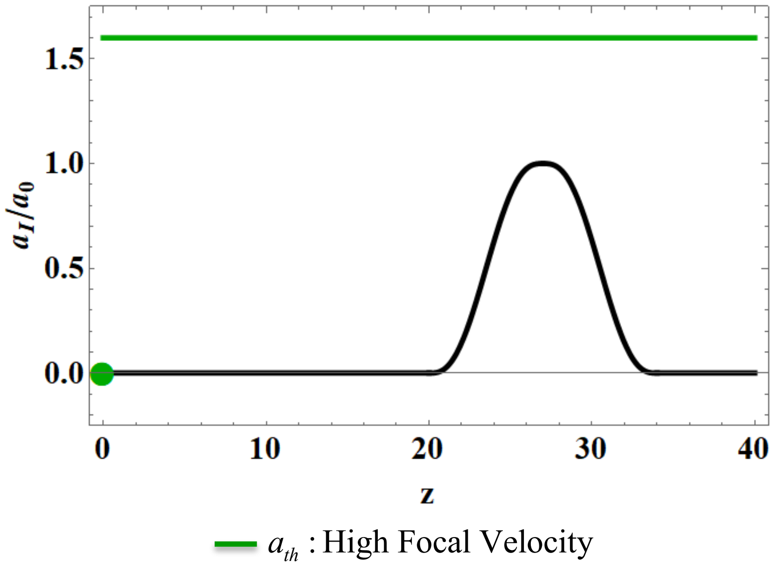
Working in the Lorentz frame of the flying focus provides an intuitive explanation for the energy gain

In the flying focus frame, the ponderomotive potential is time-independent, implying the electron energy is conserved: $\gamma_i = \gamma_f$

There are two ways this can occur

1. $v_f = v_i$

   The initial kinetic energy of the electron is sufficient to overcome the ponderomotive potential hill



Working in the Lorentz frame of the flying focus provides an intuitive explanation for the energy gain

In the flying focus frame, the ponderomotive potential is time-independent, implying the electron energy is conserved: $\gamma_i = \gamma_f$

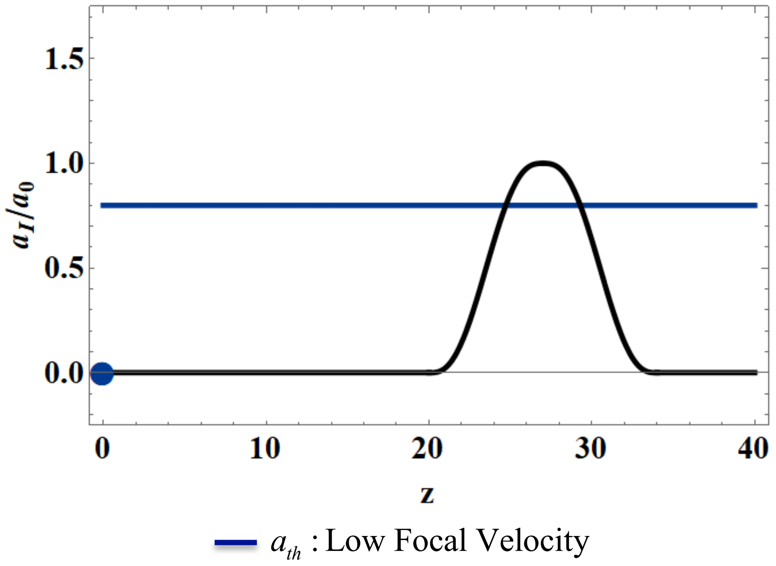
There are two ways this can occur

1. $\mathrm{v}_f = \mathrm{v}_i$

   The initial kinetic energy of the electron is sufficient to overcome the ponderomotive potential hill

2. $\mathrm{v}_f = -\mathrm{v}_i$

   The initial kinetic energy of the electron is insufficient to overcome the ponderomotive potential hill



The flying focus enables a novel mechanism for vacuum acceleration

- Typical planar pulses cannot impart net momentum to electrons

  The energy gained during ponderomotive acceleration in the leading edge of the pulse is lost during ponderomotive deacceleration in the trailing edge

- Planar-like flying focus pulses can accelerate electrons to relativistic momenta (either positive or *negative*)

  The ponderomotive force propagates at a subluminal velocity

  The electron gains enough energy during ponderomotive acceleration in the leading edge of the pulse that it outruns the pulse and never experiences the trailing edge

Focal velocity chirp expression

The expression

$$\frac{v_f}{c} = \left(1 \pm \frac{cT}{L_f}\right)^{-1}$$

With

$$\frac{cT}{L_f} = \frac{\eta c \tau^2 \omega_0}{2f}$$

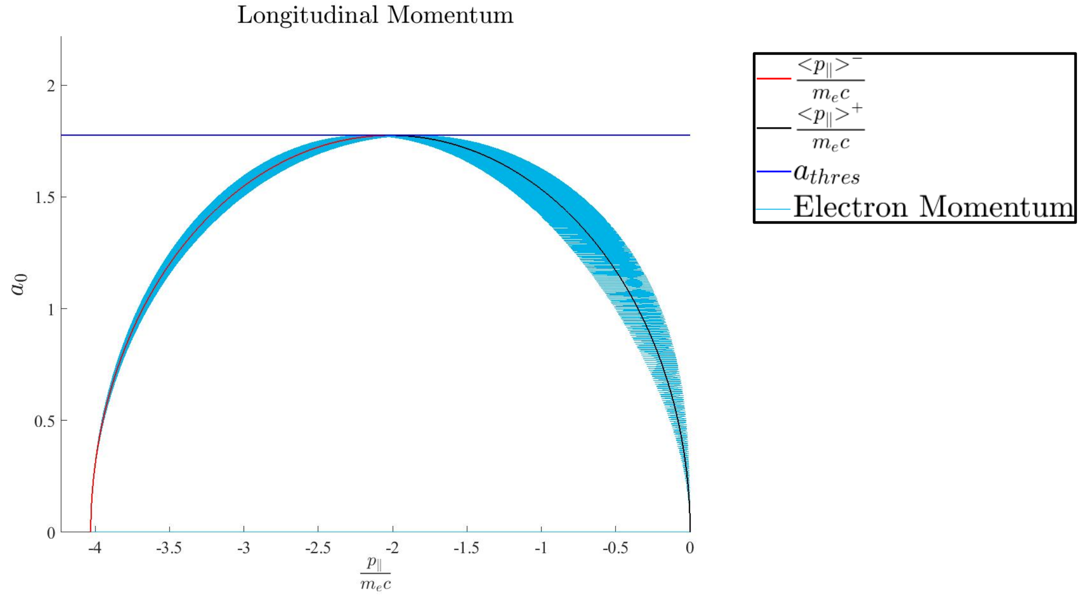
Can be written in terms of the chirp parameter η as

$$\frac{v_f}{c} = \frac{2f}{\eta c \tau^2 \omega_0}\left(1 + \frac{2f}{\eta c \tau^2 \omega_0}\right)^{-1}$$

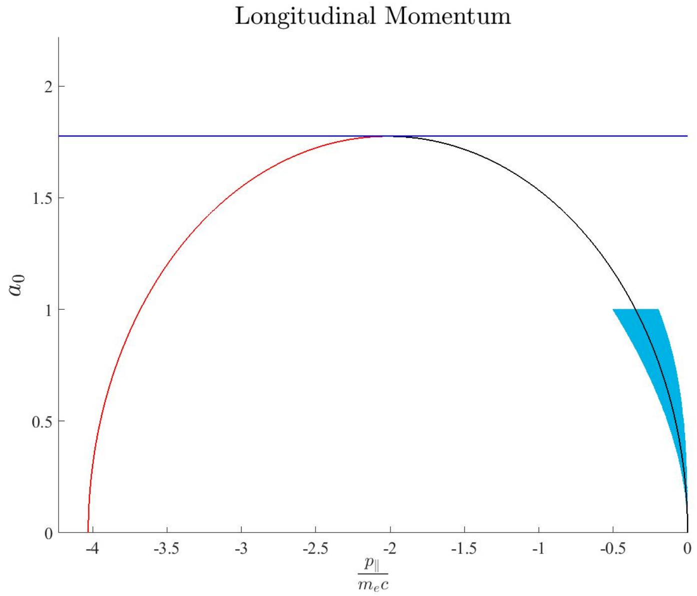
Electrons that overtake the pulse access the second root in phased average solution gaining net energy as $a_0 \rightarrow 0$

$$\frac{\langle p_z \rangle}{m_e c} = \beta_f \gamma_f^2 \pm \beta_f \gamma_f^2 \left[1 - \frac{1}{2}(\beta_f \gamma_f)^{-2} |a_0|^2\right]^{1/2}$$

Overtaken Electron



Overtaken Pulse



The flying focus decouples the velocity of the intensity peak from the group velocity

Model for flying focus

$$A(z,t) = a_0[\delta + (1-\delta)A_{FF}(z - v_f\, t)]A_P(z - c\, t)\, sin[k_0 z - \omega_0 t\ - \frac{\eta(ct-z)^2}{c\, T_P}]\,\hat{x}$$

$$T_{FF} = \frac{4\, c\, \tau^2 (1+\eta^2)^{\frac{1}{2}} f}{w_0^2}$$

$$T_P = 2\,(1+\eta^2)^{\frac{1}{2}}\tau$$

$$\delta \approx \frac{1}{4}$$

$0 < v_f < c$