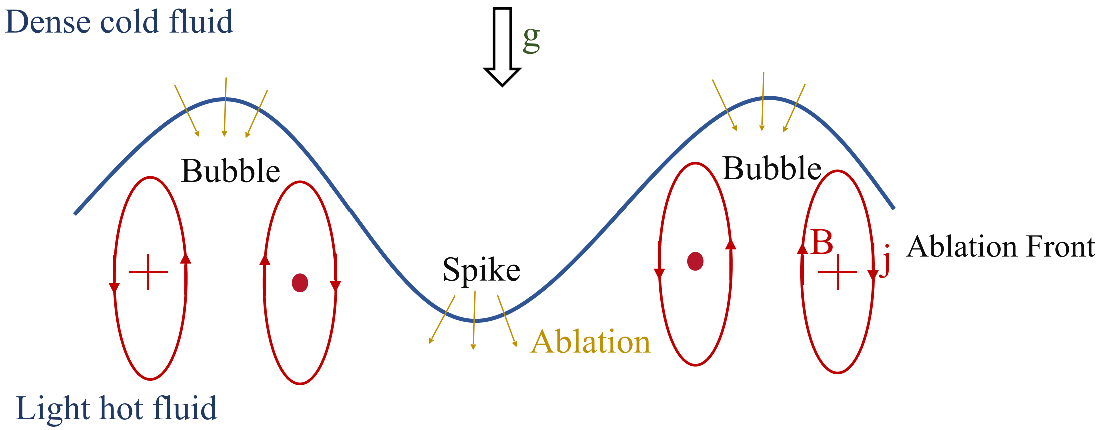
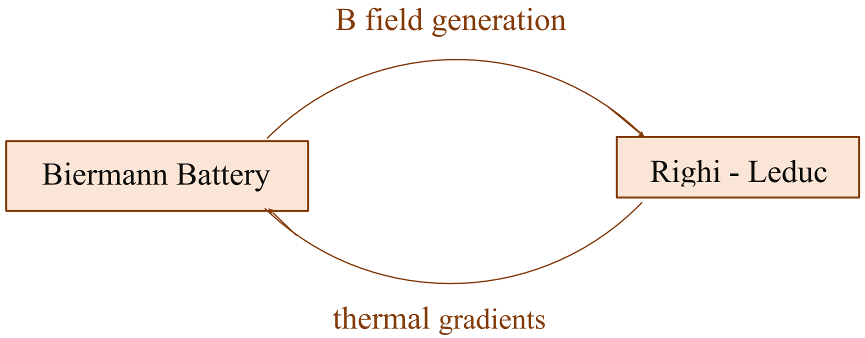
# The Effect of Self-Generated Magnetic Fields on the Ablative Rayleigh-Taylor Instability Dynamics





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- The Righi-Leduc term becomes important for perturbation wavelengths comparable to the distance between the ablation front and the critical surface, but computations show that the hydro - B field coupling is relatively weak
- The Darrieus-Landau instability dominates over the magneto-thermal instability in the subdense region

**Self-generated B field stabilizes the RTI up to a 20% in growth rate**

# Collaborators


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School of Aerospace Engineering

# Ablation fronts in Inertial Confinement Fusion are Rayleigh-Taylor unstable

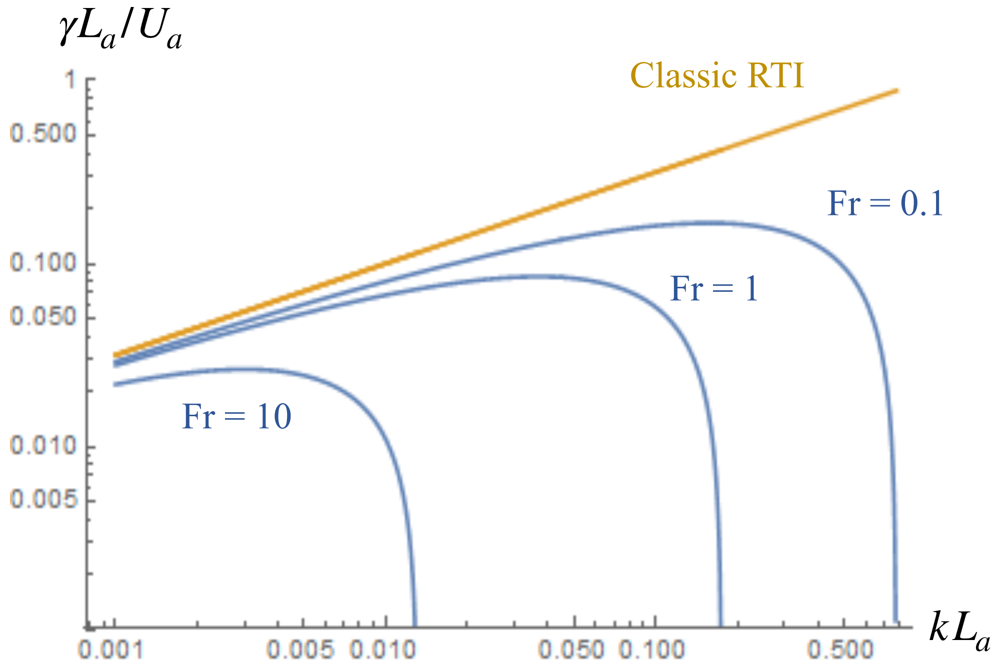
- **Dispersion relation**: Takabe formula

$$\gamma = 0.9\sqrt{kg} - 3kU_a$$

- Froude Number

$$\mathrm{Fr} = \frac{\text{Convection}}{\text{Gravity}} = \frac{U_a^2}{gL_a}$$

**Ablation stabilizes the RTI. The Froude number becomes the governing parameter**



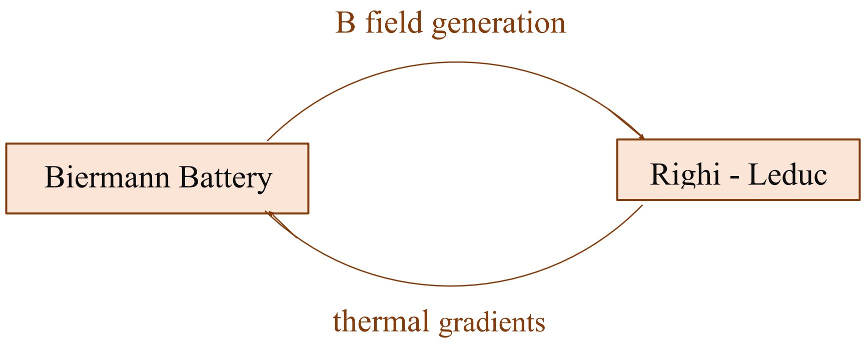
# During the Rayleigh-Taylor instability, the B-Field is self-generated due to the Biermann-Battery term

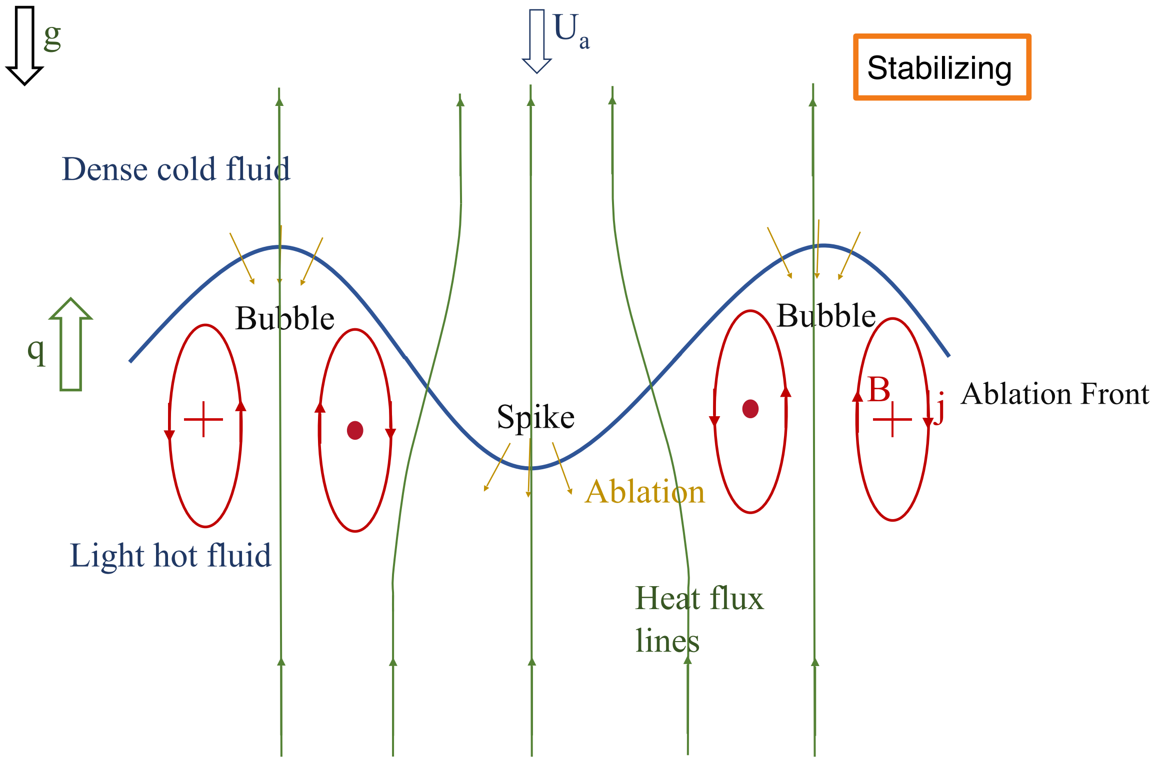
- **Biermann battery** term:

$$\frac{\partial \vec{B}}{\partial t} \sim \frac{\nabla p \times \nabla n}{n^2} \propto \frac{\partial \vec{\omega}}{\partial t}$$

- The magnetic field modifies the hydrodynamics
  - Linear term: **Righi-Leduc** $\vec{q}_{\text{RL}} \sim -\frac{T^4}{n}\vec{B} \times \nabla T$

- **Magnetothermal Instability**




**The Righi-Leduc effect bends the heat flux lines**

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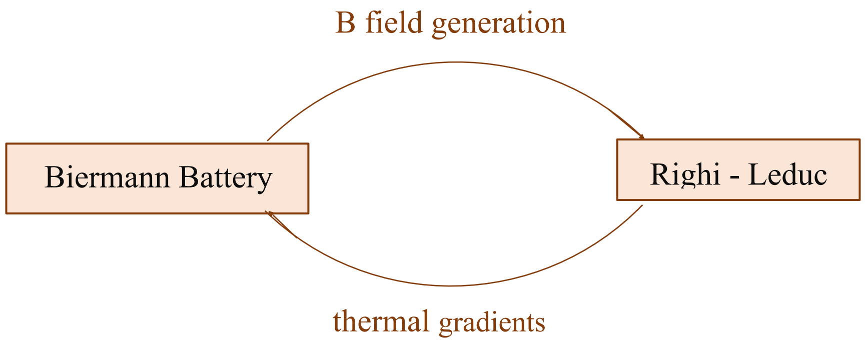


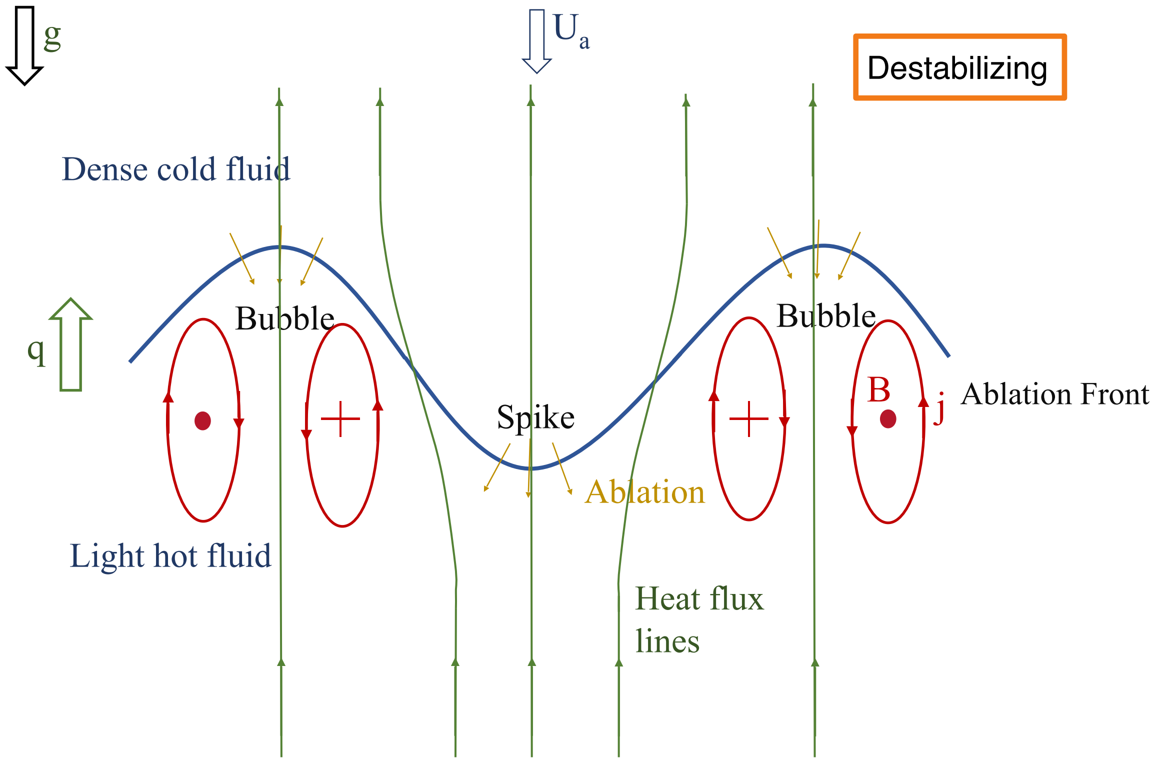


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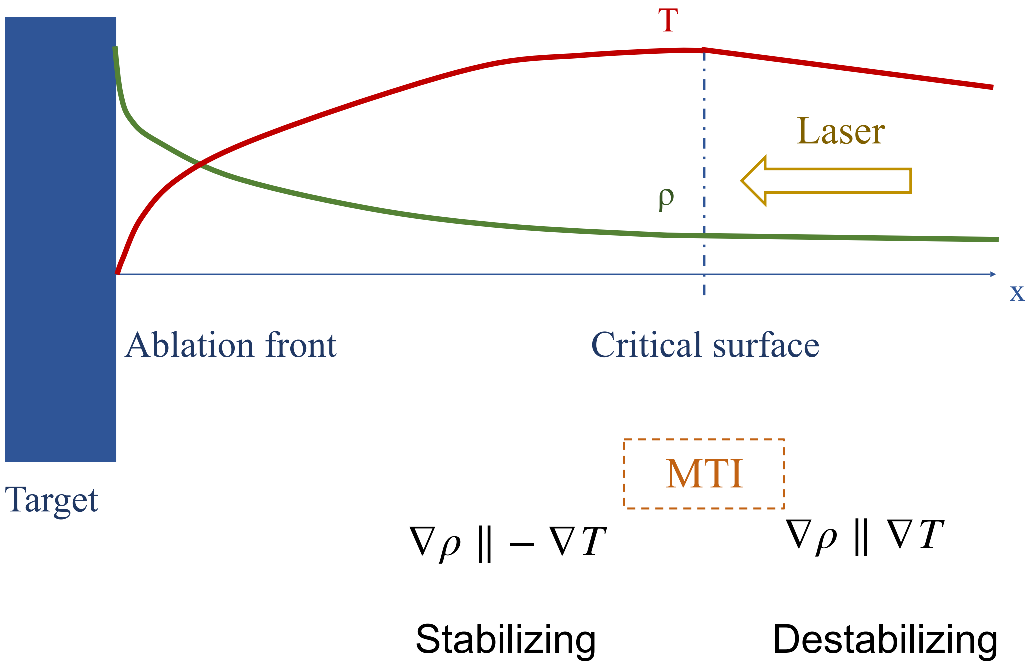
# The magneto-thermal instability takes place together with hydrodynamic instabilities

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  - Sanz, Phys. Rev. Let. 73, 20 (1994)
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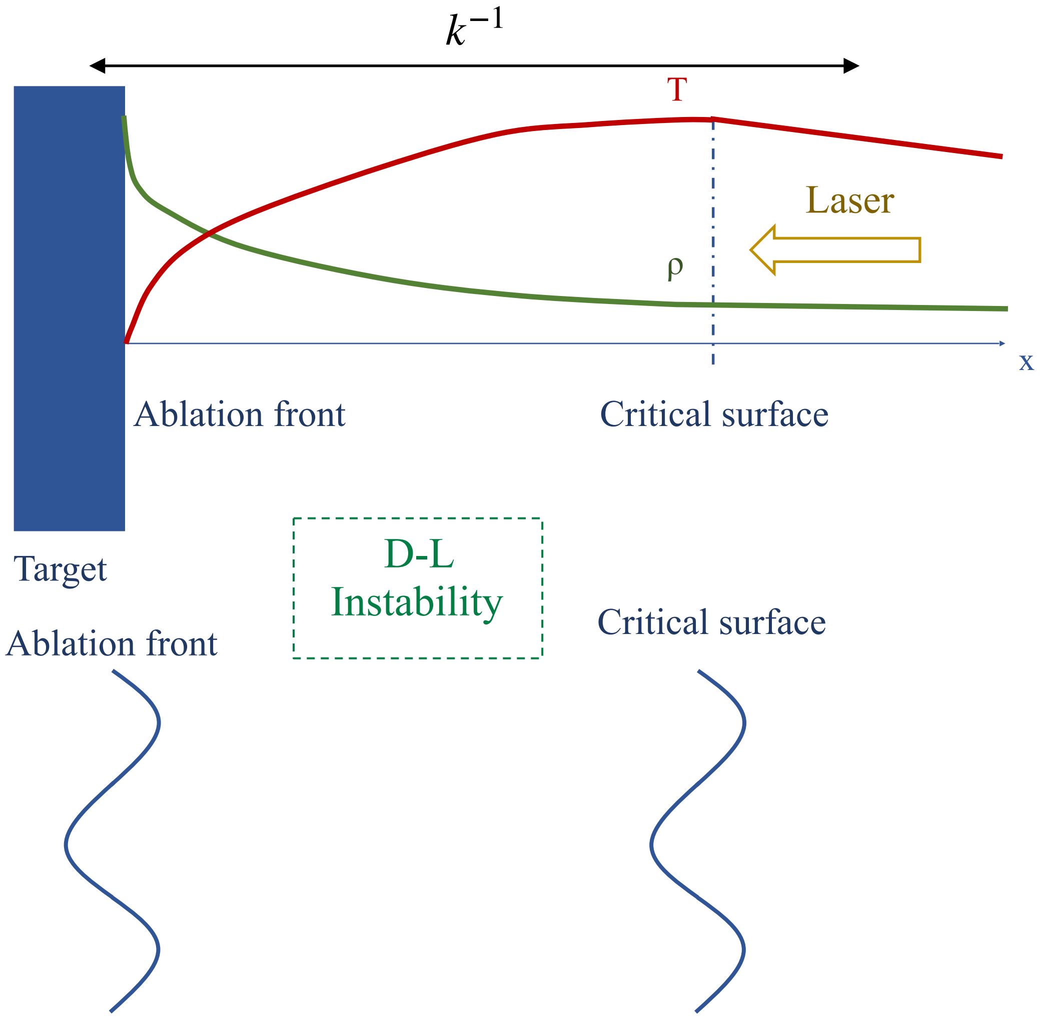


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# We use a magnetohydrodynamic model with Braginskii's expressions for transport terms

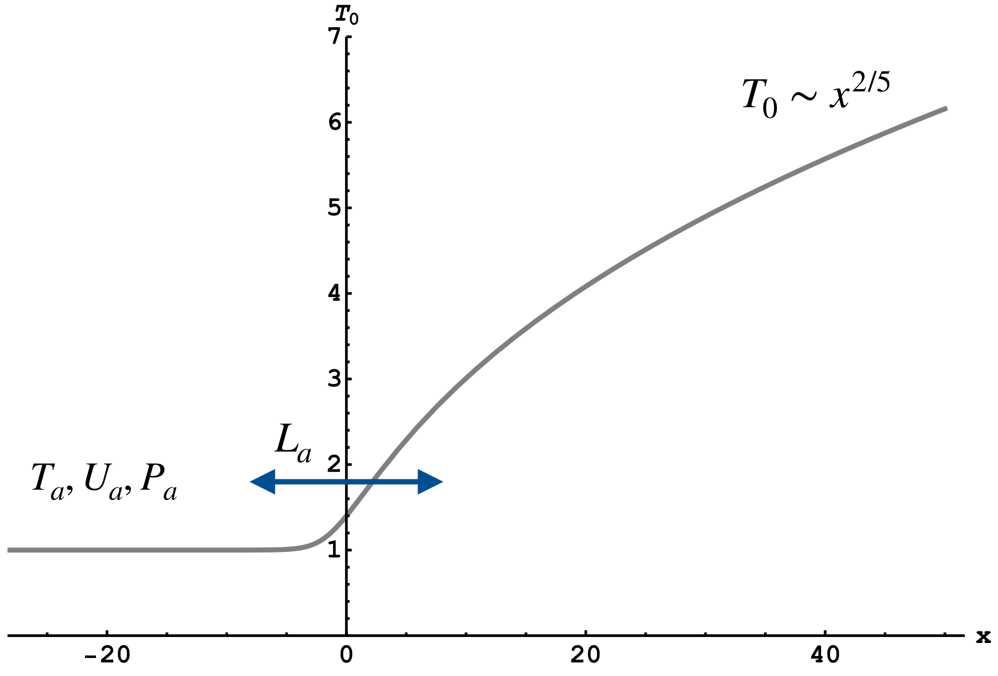
Induction equation

$$\frac{\partial \vec{B}}{\partial t} + \overbrace{\frac{c}{en^2}\nabla n \times \nabla p}^{\text{Biermann-Battery}} \overbrace{- \nabla \times \left(\vec{v} \times \vec{B}\right)}^{\text{B-Convection}} + \overbrace{\frac{c^2 \alpha_0 m_e}{4\pi e^2} \nabla \times \left(\frac{1}{n\tau} \nabla \times \vec{B}\right)}^{\text{B-Diffusion}} =$$

$$= \underbrace{\frac{c}{4\pi e}\left(1 + \frac{\alpha_0''}{\delta_0}\right) \nabla \times \left[\frac{1}{n}\vec{B} \times \left(\nabla \times \vec{B}\right)\right]}_{\text{Hall}} + \underbrace{\frac{c\beta_0''}{\delta_0 m_e} \nabla \times \left(\tau \vec{B} \times \nabla T\right)}_{\text{Nernst}}.$$

# A linear analysis of the equations has been performed to derive the stability spectrum

Ansatz: $q = q_0(x) + q_1(x)\exp(\gamma t + iky)$




Energy

$$\underbrace{\frac{\mathrm{d}u_1}{\mathrm{d}s} + k v_1}_{\text{Enth. Convection}} = \overbrace{\left(\frac{\mathrm{d}^2}{\mathrm{d}s^2} - k^2\right)\left(T_0^{5/2}T_1\right)}^{\text{Heat flux}} + \underbrace{\mathrm{Ma}^2\left(T_0^5 + \frac{1}{\mathrm{Re}}\frac{1}{T_0^2}\right)\frac{\mathrm{d}T_0}{\mathrm{d}s}k b_1}_{\text{Righi - Leduc + electrothermal terms}}$$

**The Righi-Leduc term is proportional to the square of the Mach number**

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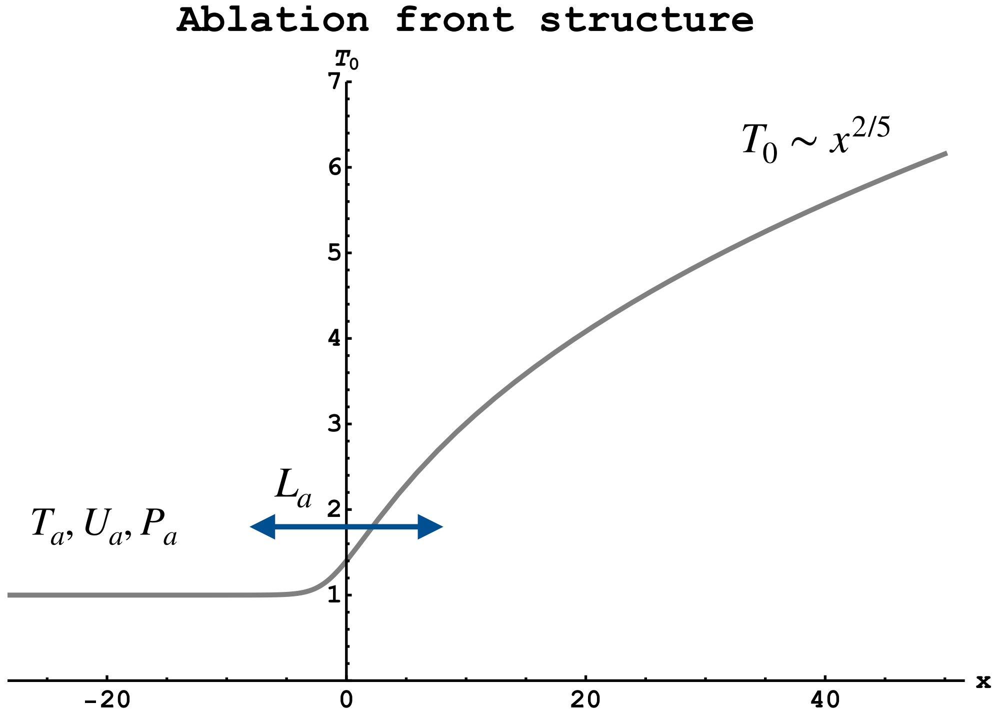


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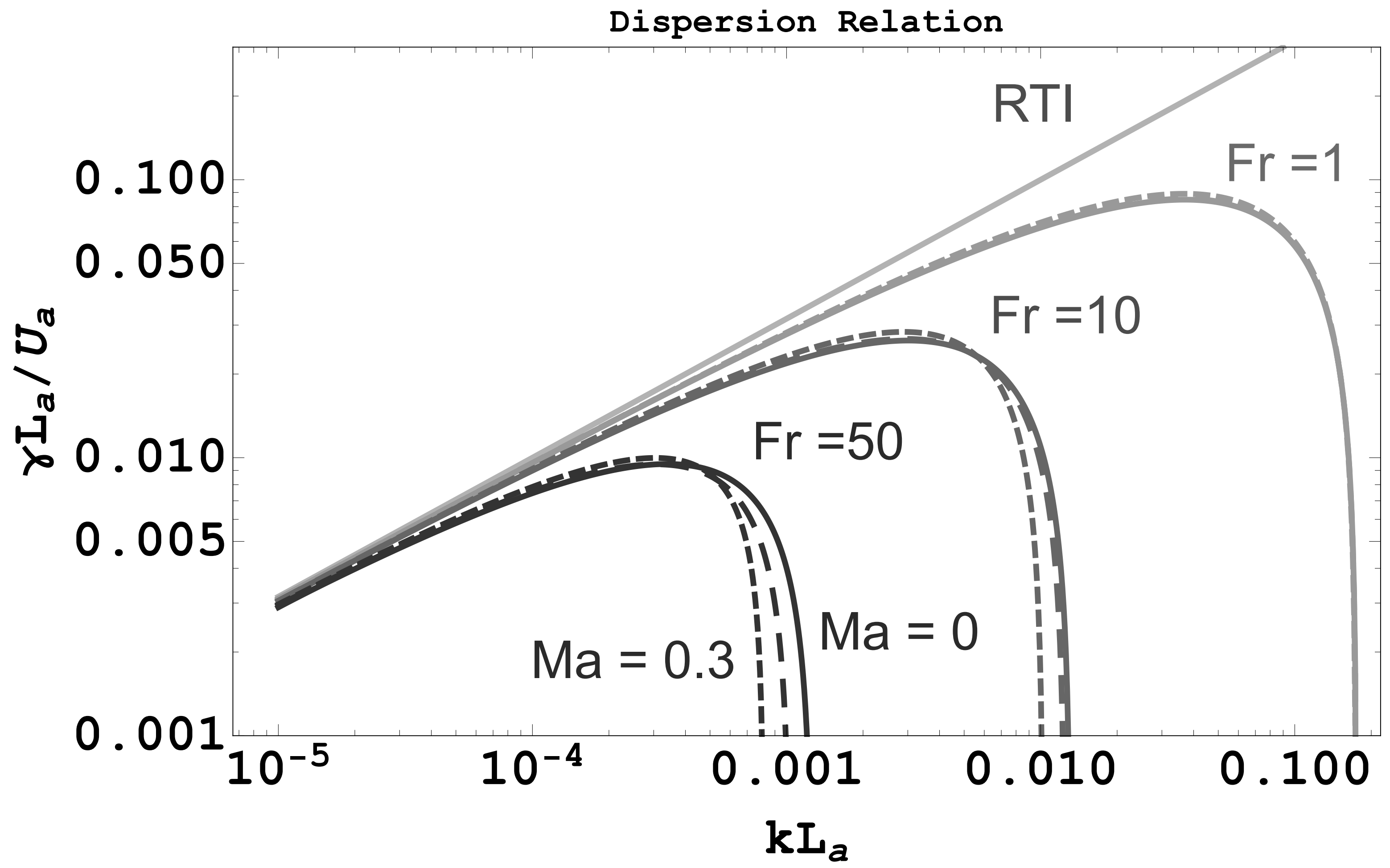
# Dispersion relation for isobaric ablation fronts with self-generated B-fields

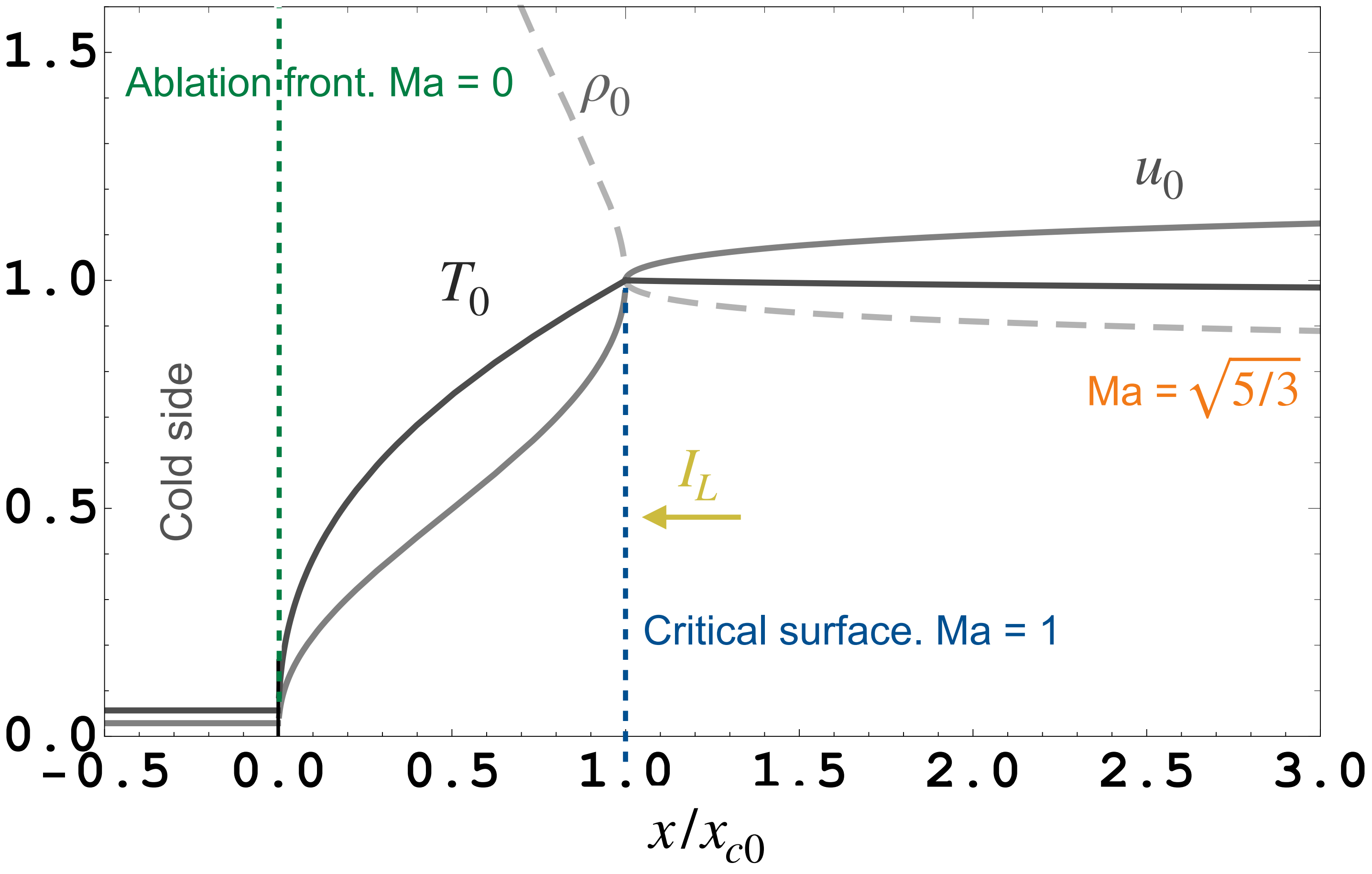



- The self-generated B-field has a **stabilizing** role close to the cutoff
- Its effect is more important for **large Froude numbers**, where the unstable wavelengths are large compared to the ablation from length

**The effect of the B Field is only important for long wavelengths**

# A consistent model needs to consider non-isobaric effects and perturb the critical surface

Expansion structure




- Ablation front

$$x_a = \xi_a \exp(\gamma t + iky)$$

- Critical surface

$$x_c = x_{c0} + \xi_c \exp(\gamma t + iky)$$

- Boundary conditions:
  - Perturbed Mach number = 1
  - Unbounded thermal mode $\sim \exp(\lambda_T s)$
  - Unbounded magnetic mode $\sim \exp(\lambda_B s)$

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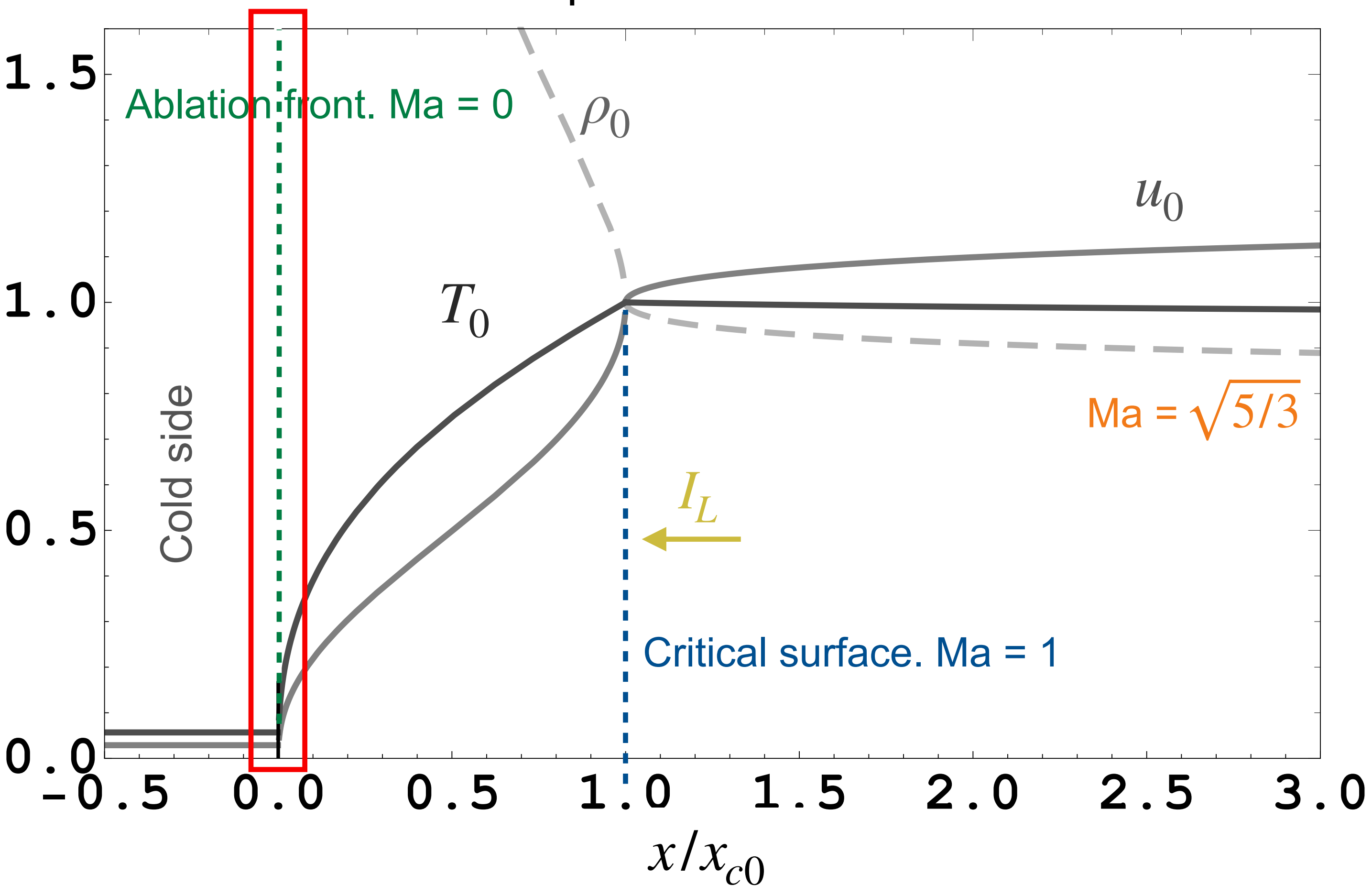


- Perturbed mass and momentum fluxes

$$\rho_0 u_1 + \rho_1 u_0 = fk \quad \text{Mass}$$

$$2u_1 + \rho_1 u_0^2 + p_1 = qk^{3/5} \quad \text{Momentum}$$

**Dispersion relation**

$$\gamma^2 + k\left(1+f\right)\gamma - k^2 f - \frac{k}{\mathrm{Fr}}\left(1 - q\mathrm{Fr}k^{3/5}\right) = 0$$

$\mathrm{Fr} \gg 1$

$$\gamma = \sqrt{\frac{k}{\mathrm{Fr}}\left(1 - q\mathrm{Fr}k^{3/5}\right)}$$

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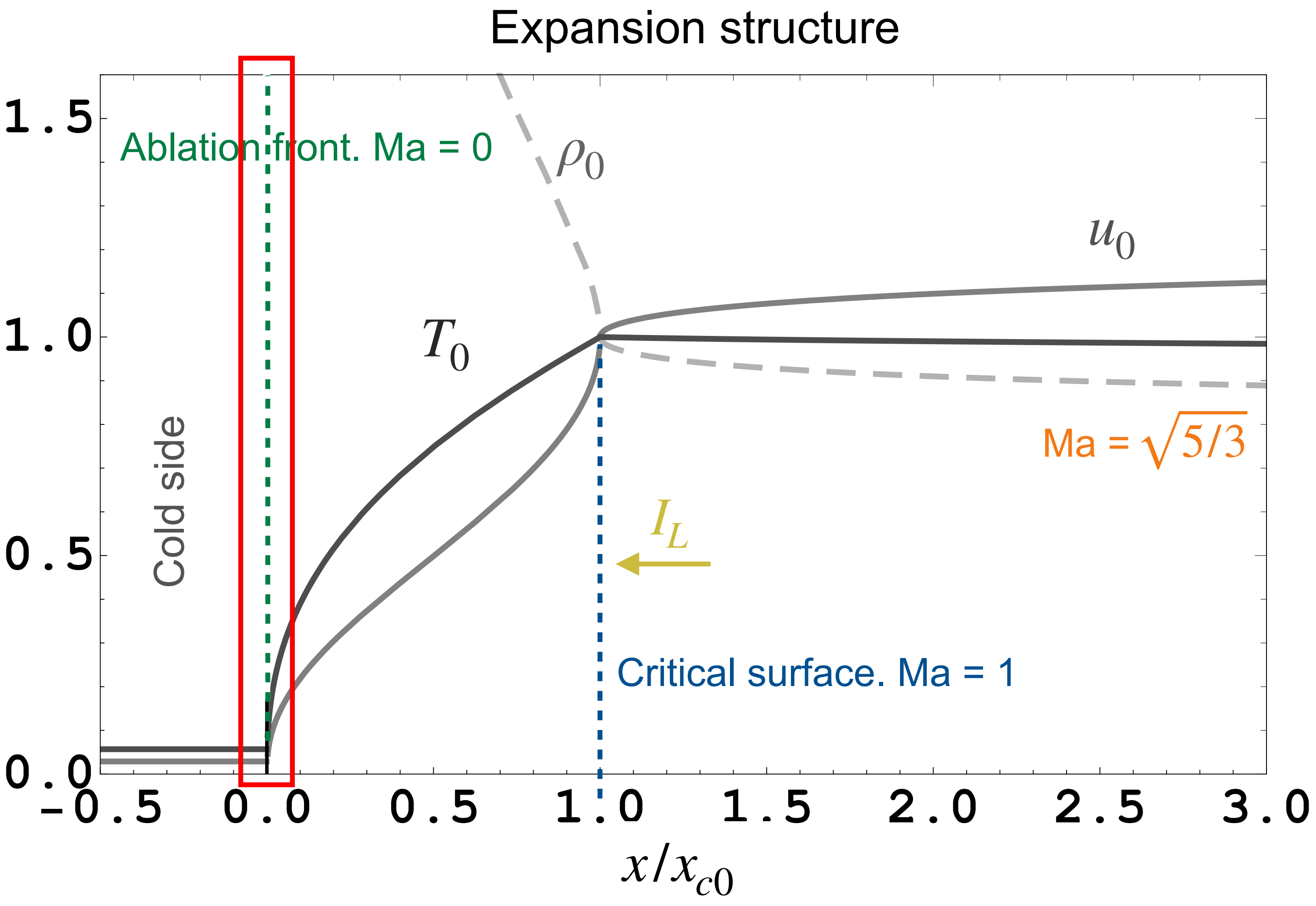


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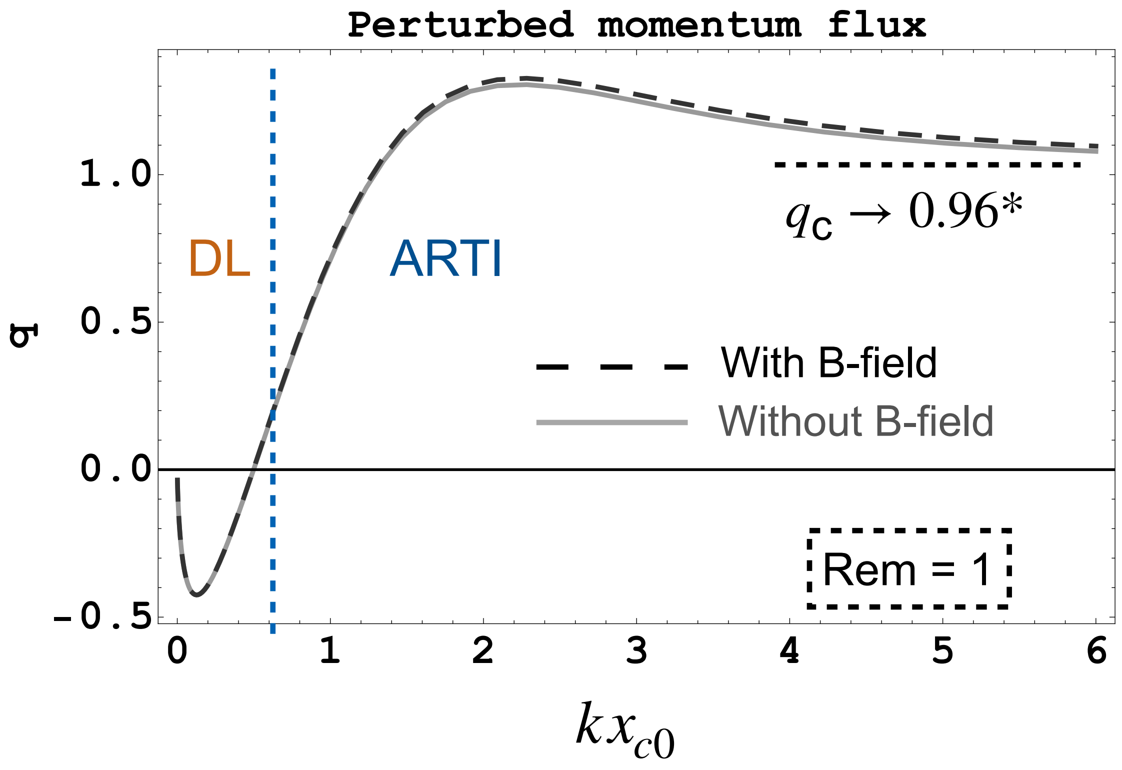
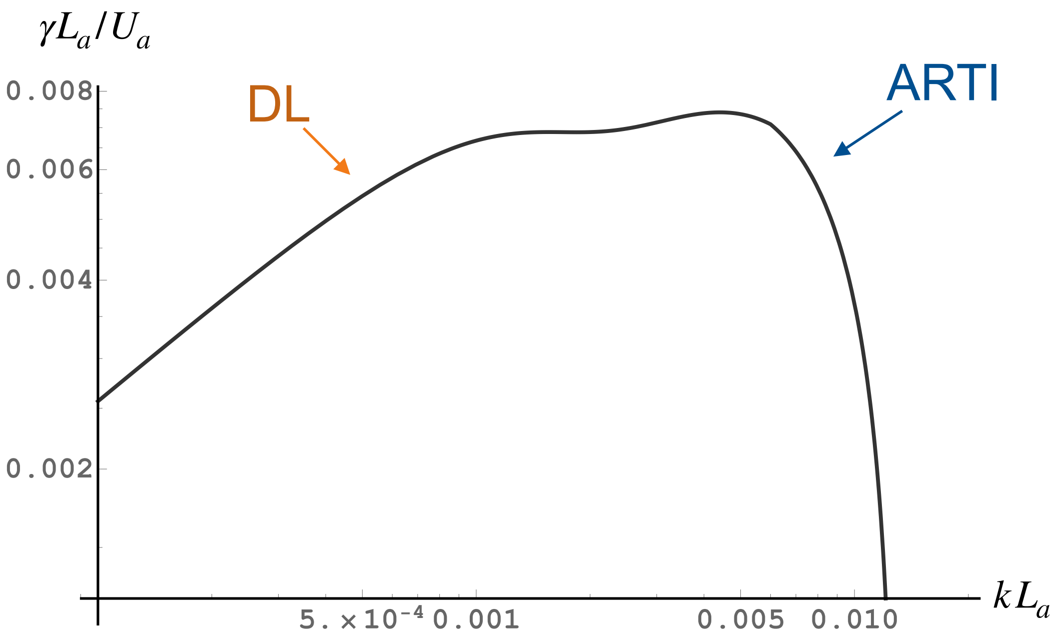
# The effect of the B field is small for Rem $\sim O(1)$

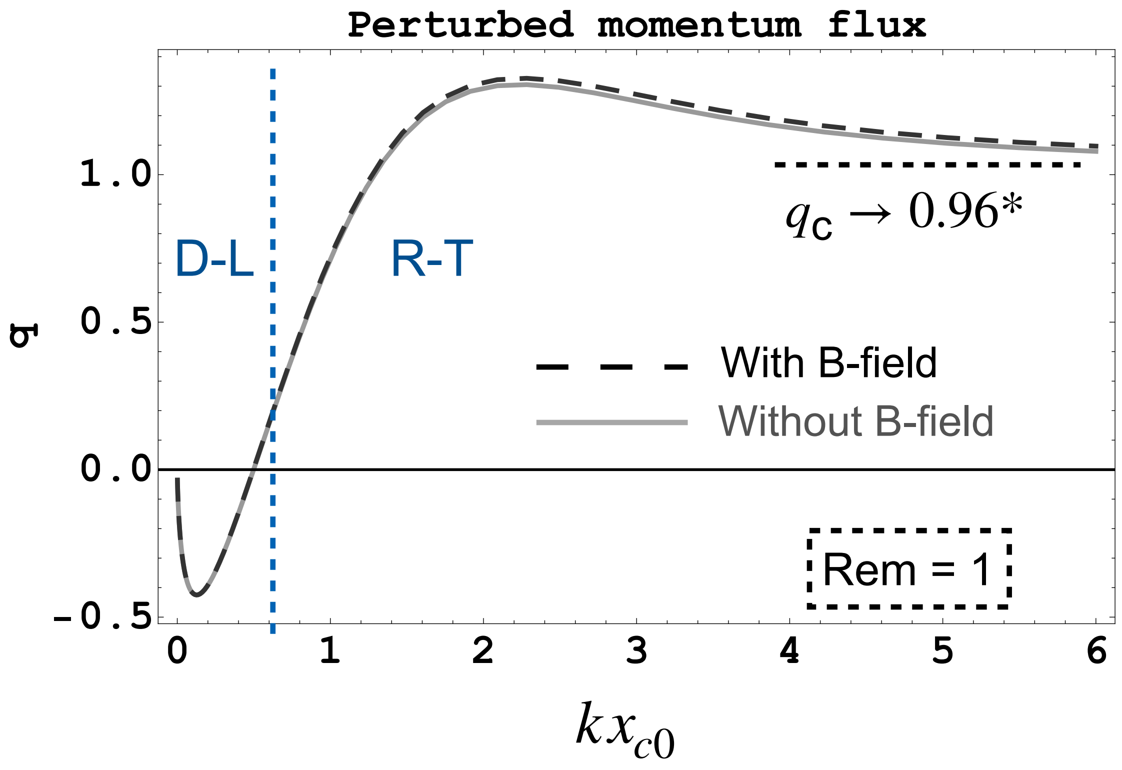


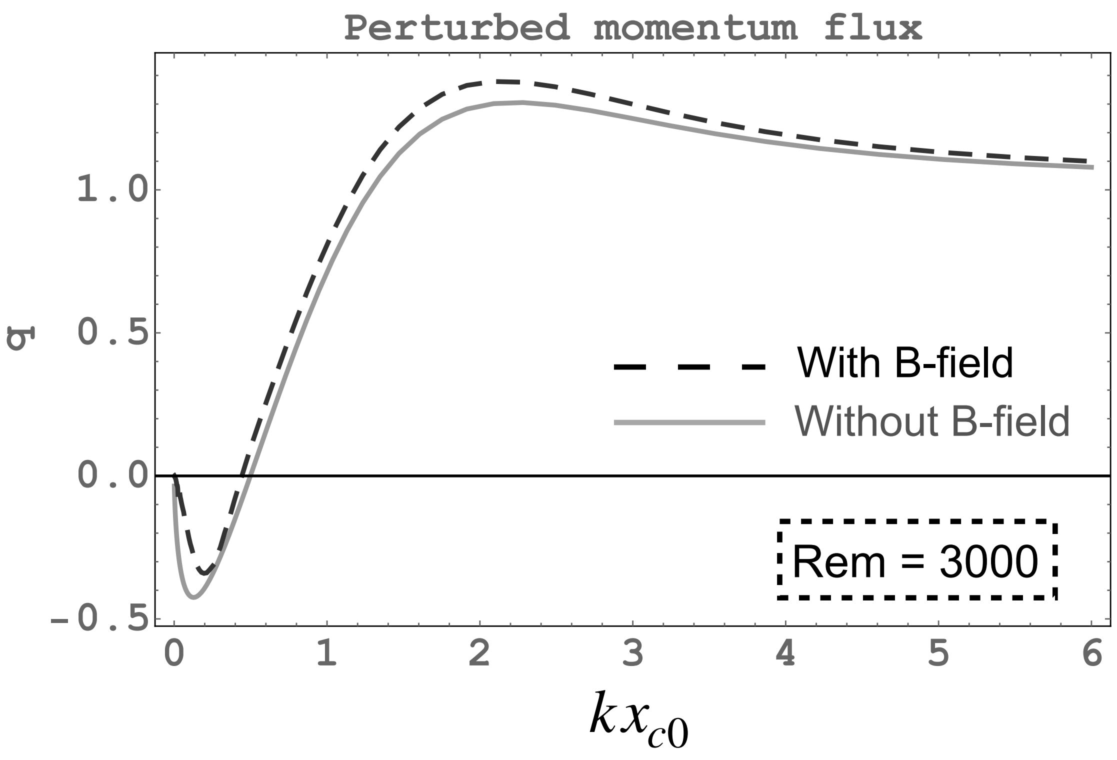




**The self-generated B-field stabilizes ~ 5% for low Rem**

* Sanz, Phys. Review Letters 73, 20 (1994), Sanz Phys. Review E 53, 4 (1996)

# Increasing Rem enhances the stabilizing effect of the B field, but it remains small for moderate Rem




**The self-generated B-field stabilizes up to ~ 20% for moderate Rem**

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