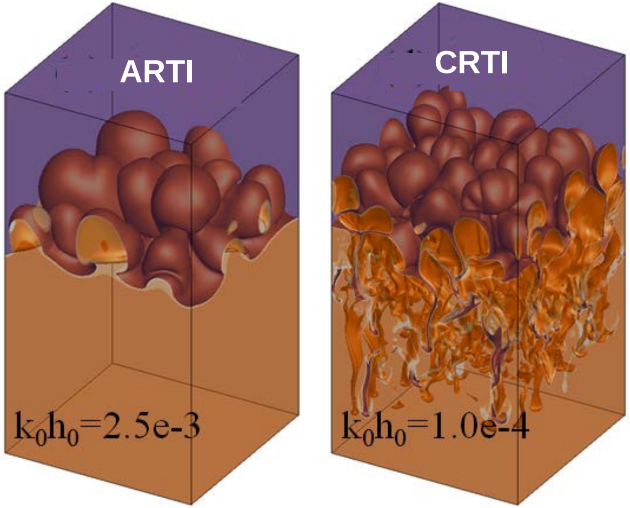
Self-Similar Multimode Bubble-Front Evolution of the Ablative Rayleigh–Taylor Instability in Two and Three Dimensions



H. Zhang
University of Rochester
Laboratory for Laser Energetics

60th Annual Meeting of the
American Physcial Society
Division of Plasma Physics
Portland, OR
5–9 November, 2018

Summary

The self-similar nonlinear evolution of the multimode ablative Rayleigh–Taylor instability (ARTI) is studied numerically in both two and three dimensions

- **The nonlinear multimode bubble-front penetration follows the $\alpha_b g t^2$ scaling law with α_b dependent on the initial conditions and ablation velocity**
- **Nonlinear ARTI is dominated by bubble competition, indicating that mass ablation reduces α_b with respect to the classical value for the same initial perturbation amplitude**
- **Ablation-driven vorticity accelerates the bubble velocity and prevents the transition from the bubble competition to the bubble merger regime at large initial amplitudes, leading to higher α_b than in the classical case**

Collaborators

R. Betti, D. Zhao, and H. Aluie

University of Rochester, Laboratory of Laser Energetics

R. Yan

University of Science and Technology of China

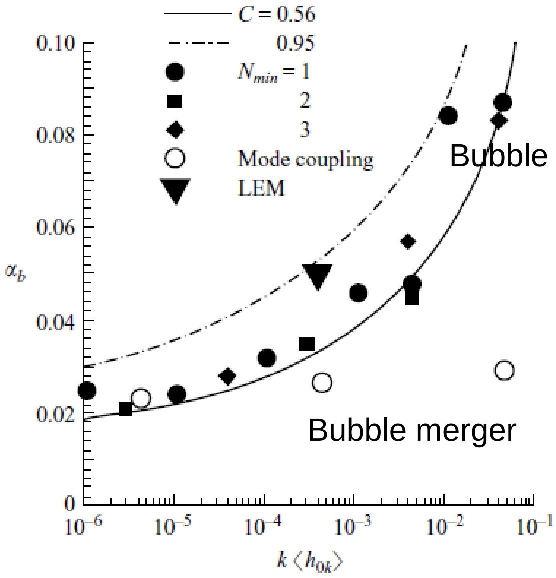
D. Shvarts

Department of Physics, NRCN, Israel

The nonlinear multimode bubble-front penetration of the classical RTI follows the $\alpha_b A_T g t^2$ scaling law

Self-similarity of nonlinear multimode RTI can be achieved in two ways:*

1. **Bubble merger: saturation of shorter wavelength modes leading to a universal α_b**
2. **Bubble competition: exponential growth and saturation of long wavelength modes, α_b increases logarithmically with initial perturbation**



3-D simulations:

Bubble competition:

$$\alpha_b = \frac{C\sqrt{\pi}}{4}\left[\ln\left(\frac{2C\sqrt{\pi}}{k\langle h_{0k}\rangle}\right)-1\right]^{-1} \quad (C \sim 0.56)$$

Bubble merger: $\alpha_b \sim 0.02$ to 0.04

3-D experiments:

$\alpha_b \sim 0.04$ to 0.08, $C = 0.95$

---

*G. Dimonte, Phys. Rev. E 69, 056305 (2004);
P. Ramaprabhu, G. Dimonte, and M. J. Andrews, J. Fluid Mech. 536, 285 (2005).

The ablation effect on the nonlinear multimode evolution is not well understood

- Bubble merger theory shows mass ablation reduces α_b:* $\alpha_\text{b} = (1-b\hat{V}_\text{a})\alpha_C$
- ARTI experiments on OMEGA show that $\alpha_\text{b} = 0.04$ is slightly lower than CRTI experiments** and spectrum shifts to longer wavelengths****
- Recent experiments on the NIF show that nonlinear ARTI can grow faster than Haan's model***

Ablation effect on single RTI mode:

- Suppress linear growth rate[†]

$$\gamma = \sqrt{A_T kg} - bkV_\text{a}$$

- Enhances nonlinear bubble velocity[‡]

$$U_\text{b}^\text{rot} = \sqrt{g(1-r_d)/C_g k + r_d\omega_0^2/4k^2}$$

- Nonlineraly destabilize small scale RTI[‡‡]

---

*D. Oron, U. Alon, and D. Shvarts, Phys. Plasmas 5, 1467 (1998); **O. Sadot *et al*., Phys. Rev. Lett. 95, 265001 (2005); ***A. Casner *et al*., Phys. Plasmas 22, 100702 (2015);
[†]H. Takabe *et al*., Phys. Fluids 28, 3676 (1985); [‡]R. Betti and J. Sanz, Phys. Rev. Lett. 97, 205002 (2006); R. Yan *et al*., Phys. Plasmas 23, 022701 (2016); [‡‡]H. Zhang *et al*., Phys. Rev. E 97, 011203(R) (2018;);
V. Smalyuek et al. Physical Review Letters 103 150001 (2009)

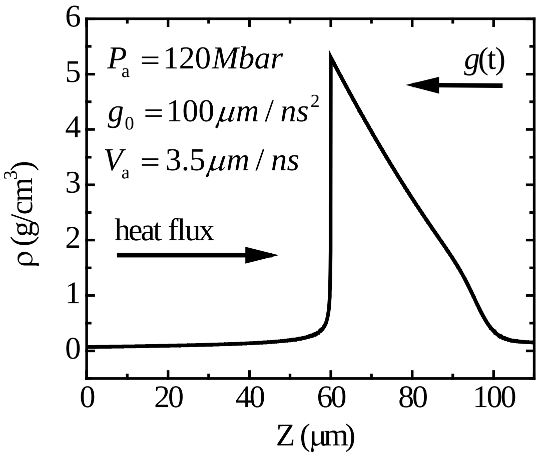
Both 2-D and 3-D planar simulations are used to investigate the multimode ARTI

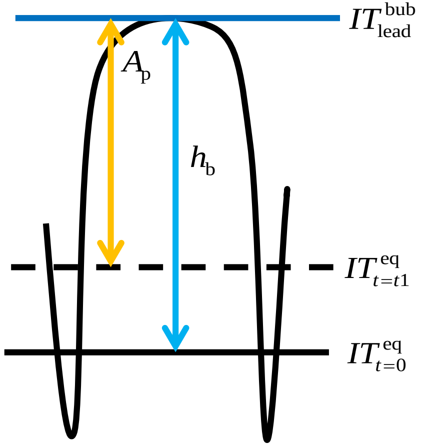
- **Simulation setup corresponds to a typical acceleration phase of a direct-drive target**
- **2-D simulations: $L_x = 100\ \mu\text{m}$, 3-D simulation: $L_x = L_y = 50\ \mu\text{m}$, Grid size: $0.1\ \mu\text{m}$, Linear cutoff: $k_{\text{cl}} = 1\ \mu\text{m}^{-1}$**

Bubble-front penetration: $h_{\text{b}} = IT_{\text{lead}}^{\text{bub}} - IT_{t=0}^{\text{eq}}$

Time-varied acceleration:* $S = [\int \sqrt{g}\,\text{d}t]^2$

Assuming $A_T \sim 1$: $\alpha_{\text{b}} = \partial h_{\text{b}} / \partial S$



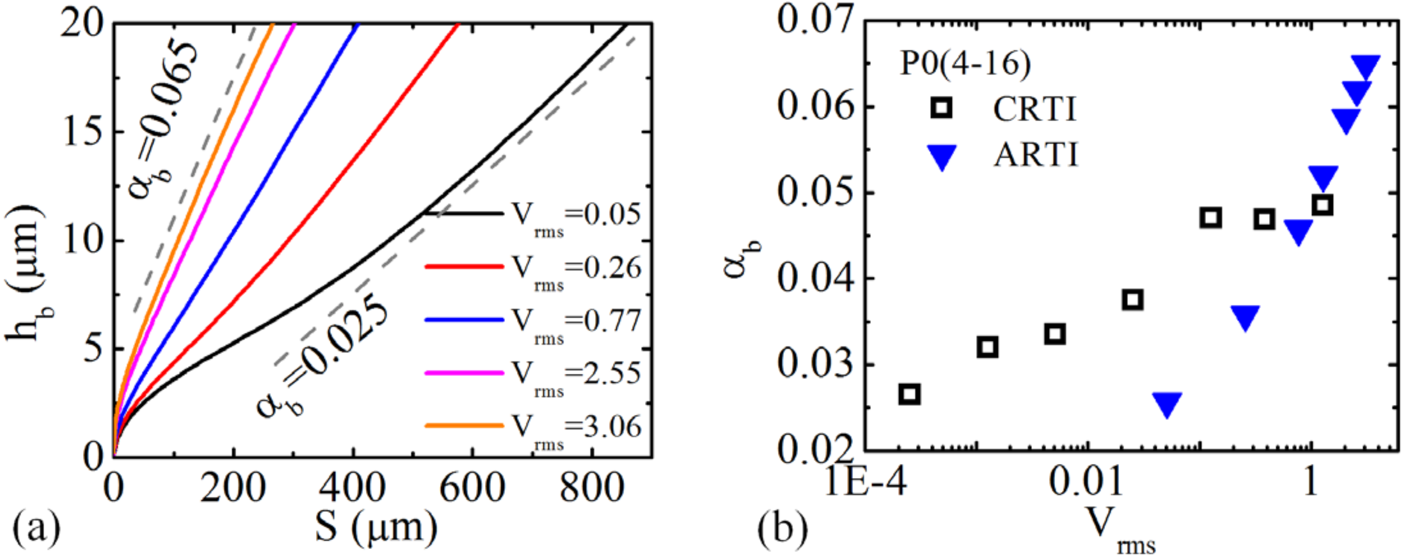


---

*G. Dimonte *et al*., Phys. Fluids 16, 1668 (2004).

The nonlinear multimode bubble-front penetration follows the $\alpha_b g t^2$ scaling law

- Nonlinear ARTI is dominated by bubble competition and α_b depends on initial perturbation
- Mass ablation reduces α_b with respect to the classical value for the same initial perturbation amplitude
- α_b in ARTI can be higher than CRTI when initial perturbation is large



The dependence of α_b on initial perturbation and ablation is derived from the bubble competition model* modified by ablation

CRTI bubble competition:

$$\alpha_C = \frac{C\sqrt{\pi}}{4}\left[\ln\left(\frac{2C\sqrt{\pi}}{k\left\langle h_{0k}\right\rangle}\right)-1\right]^{-1} * \qquad \textbf{Eq. (1)}$$

ARTI linear growth:

$$\gamma \approx \sqrt{gk} - bkV_\mathrm{a} = \gamma_\mathrm{cl}(1-b\hat{V}_\mathrm{a}) \qquad \hat{V}_\mathrm{a} = \sqrt{k/g}\,V_\mathrm{a}$$

Linear phase:

$$h_\mathrm{b} = A_\mathrm{b} + V_\mathrm{a}t = h_0\exp(\gamma t) + V_\mathrm{a}t$$

Nonlinear bubble penetration:

$$h_\mathrm{b} = U_\mathrm{b}(t-t_\mathrm{NL}) + h_\mathrm{b}^\mathrm{NL} \qquad U_\mathrm{b} = C\sqrt{g\lambda/2}$$

Apply self-similar condition:

$$\frac{\partial h_\mathrm{b}}{\partial\lambda} = 0 \Rightarrow \alpha_\mathrm{b} \sim \frac{(1-b\hat{V}_a)C\sqrt{\pi}}{4}\left(\ln\frac{2C\sqrt{\pi}}{k_0h_0}-1\right)^{-1} = (1-b\hat{V}_\mathrm{a})\alpha_C \qquad \textbf{Eq. (2)}$$

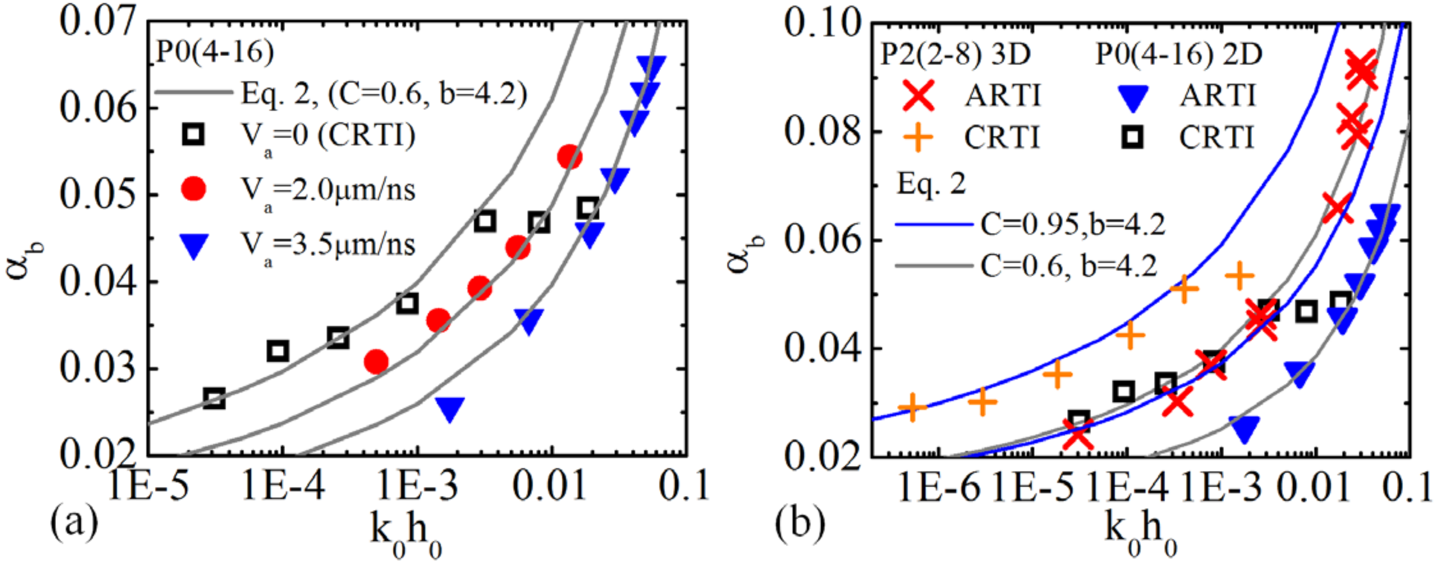
Mass ablation suppresses nonlinear bubble growth by reducing γ

---

*G. Dimonte, Phys. Rev. E 69, 056305 (2004);

Simulations are used to quantify the dependence of α_b on h_0 and V_a

- $C_{3\text{-D}}/C_{2\text{-D}}$ ~ 1.6: 3-D bubble velocity is 1.7× larger than 2-D
- b = 4.2 for both 2-D and 3-D: the same linear-dispersion relation
- Pj(m,n)= initial mode spectrum that decays ~k^{-j} with modes m through n with k=m×2π/L



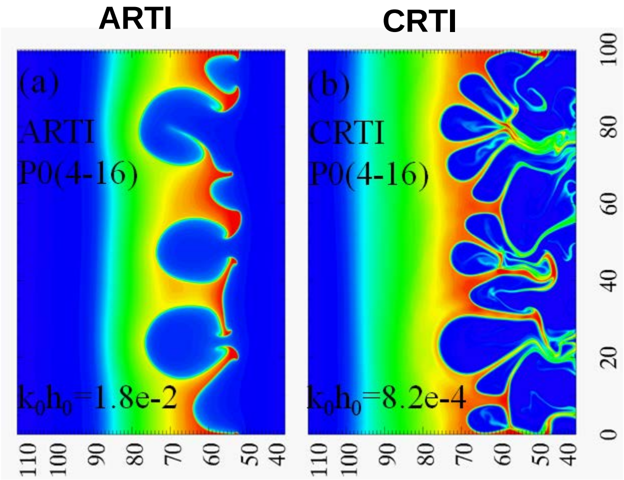
$$k_0 = 0.063\mu\text{m}^{-1}$$

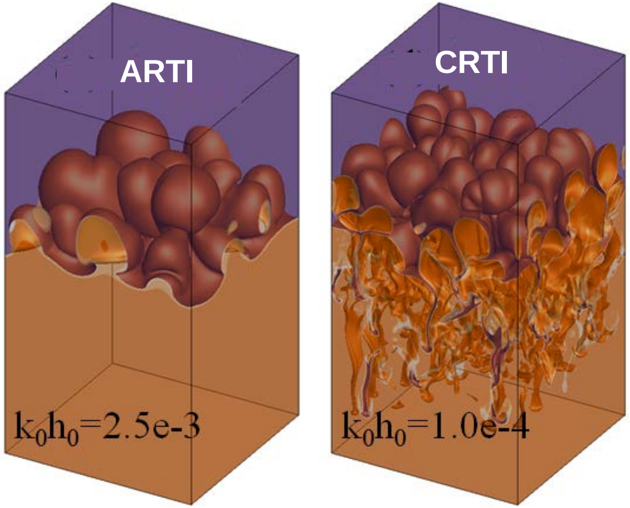
$$b = 4.2$$

$$C_{2\text{-D}} = 0.6$$

$$C_{3\text{-D}} = 0.95$$

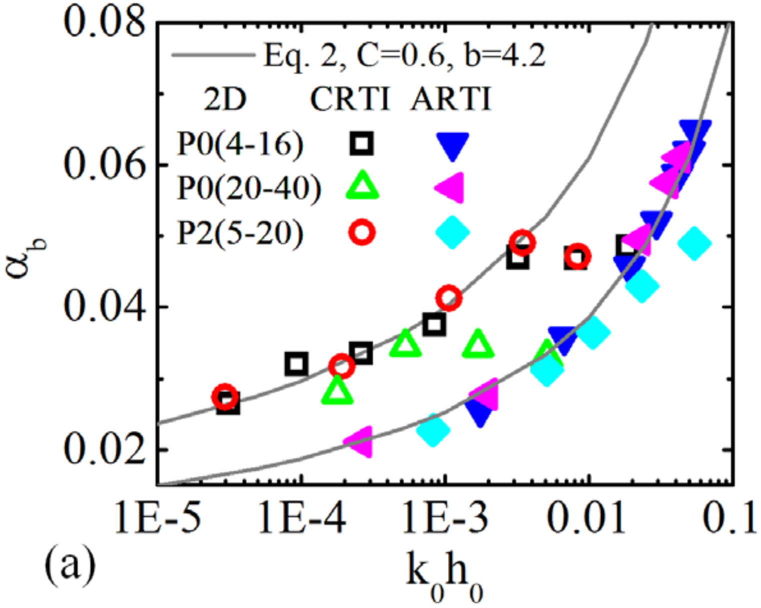
The mode-structure comparison between classical and ablative RTI shows larger bubbles dominate the asymptotic behavior





Nonlinear ARTI is still in the bubble-competition regime even for large-amplitude small-scale initial perturbations

- Ablation-generated vorticity can keep the nonlinear ARTI in the bubble-competition regime
- α_b in ARTI can reach higher values than in CRTI for sufficiently large initial perturbations



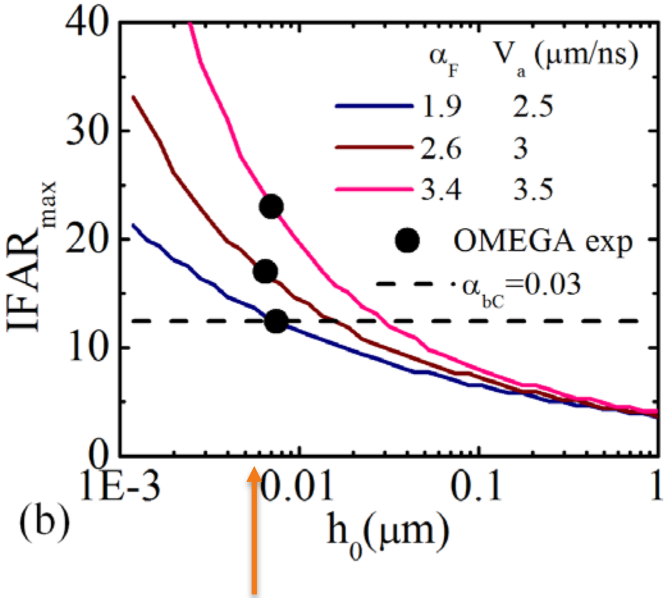
Linear cutoff: *m* ~ 16
P0(20-40): small-scale initial perturbation
P2(5-20): large-scale initial perturbation

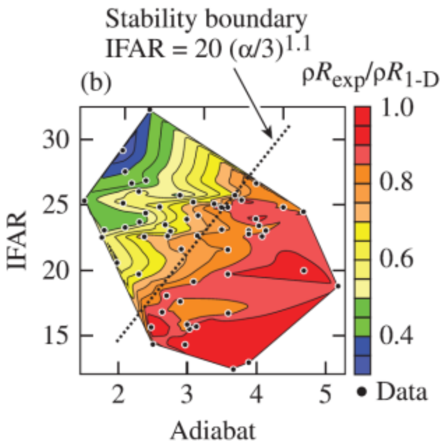
The self-similar nonlinear evolution of the multimode ablative Rayleigh–Taylor instability (ARTI) is studied numerically in both two and three dimensions

- **The nonlinear multimode bubble-front penetration follows the $\alpha_b A_T g t^2$ scaling law with α_b dependent on the initial conditions and ablation velocity**
- **Nonlinear ARTI is dominated by bubble competition, indicating that mass ablation reduces α_b with respect to the classical value for the same initial perturbation amplitude**
- **Ablation-driven vorticity accelerates the bubble velocity and prevents the transition from the bubble competition to the bubble merger regime at large initial amplitudes, leading to higher α_b than in the classical case**

The bubble-competition theory may be used to explain the hydrodynamic stability boundary observed in laser-fusion implosion experiments

- The allowed IFAR depends on the initial perturbation
- The Omega experiments indicate that $h_0 \sim 0.01\ \mu$m





In OMEGA experiments: low-adiabat ($\alpha_F < 3.5$) implosions are degraded mainly by small-scale RTI*

$\text{IFAR}_0 = R_0/\Delta_0$,
R_0: in-flight capsule radius
Δ_0: in-flight shell thickness

$$V_a = \dot{m}\alpha_F^{1/\gamma} / P_a^{1/\gamma} \Rightarrow \alpha_F = 3.4(V_a / 3.5)^{5/3}$$

$$\Rightarrow \text{IFAR} = 20(\alpha_F / 1.1)^{1.1} = 23(V_a / 3.5)^{1.83}$$ Stability cliff

Assume same initial perturbation for RT (does not account for RM)

---

* V. N. Goncharov *et al.*, Phys. Plasmas 21, 056315 (2014).