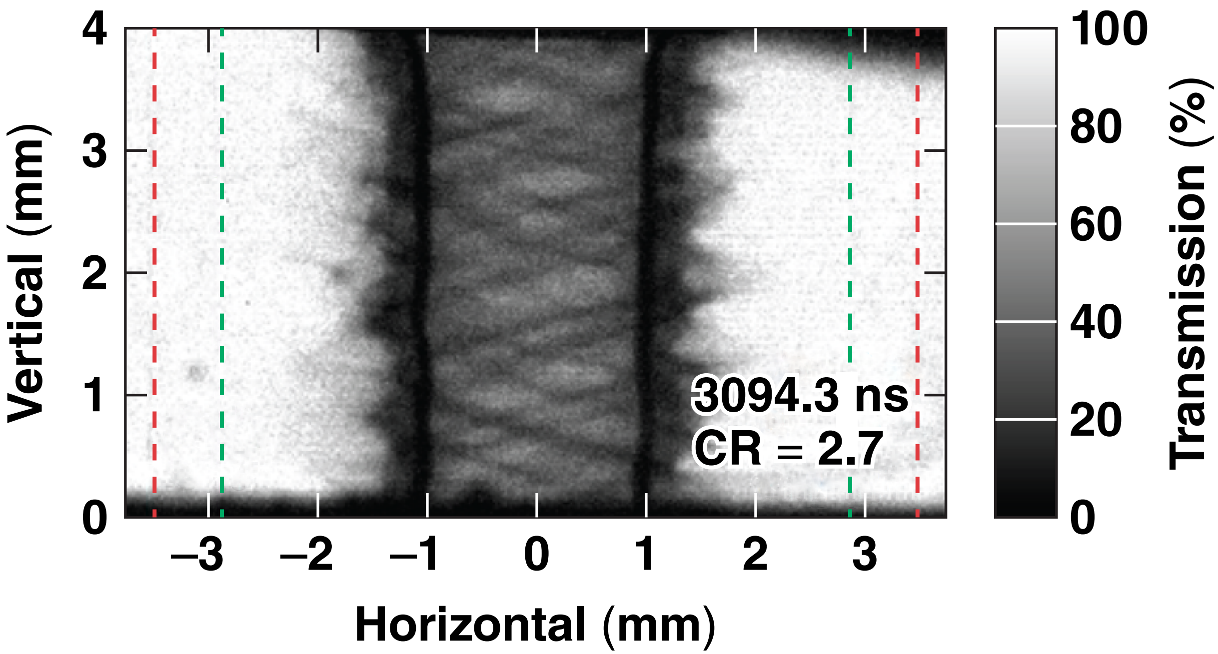
Instability Driven by Self-Generated Magnetic Fields: Relevance to Helical Structures in MagLIF Experiments



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Summary

A self-generated magnetic field can seed helices in liners with an axial magnetic field

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- A linear model shows that a self-generated magnetic field leads to the growth of density perturbations perpendicular to an existing magnetic field and temperature gradient
- Growth terminates before magnetic pressure becomes comparable to thermal pressure
- In a magnetically driven liner with an initial axial magnetic field, the self-generated magnetic field will drive helical perturbations at the heated surface before compression
- Resistivity *and* viscosity are required to obtain physical growth rates at large wave numbers

Collaborators

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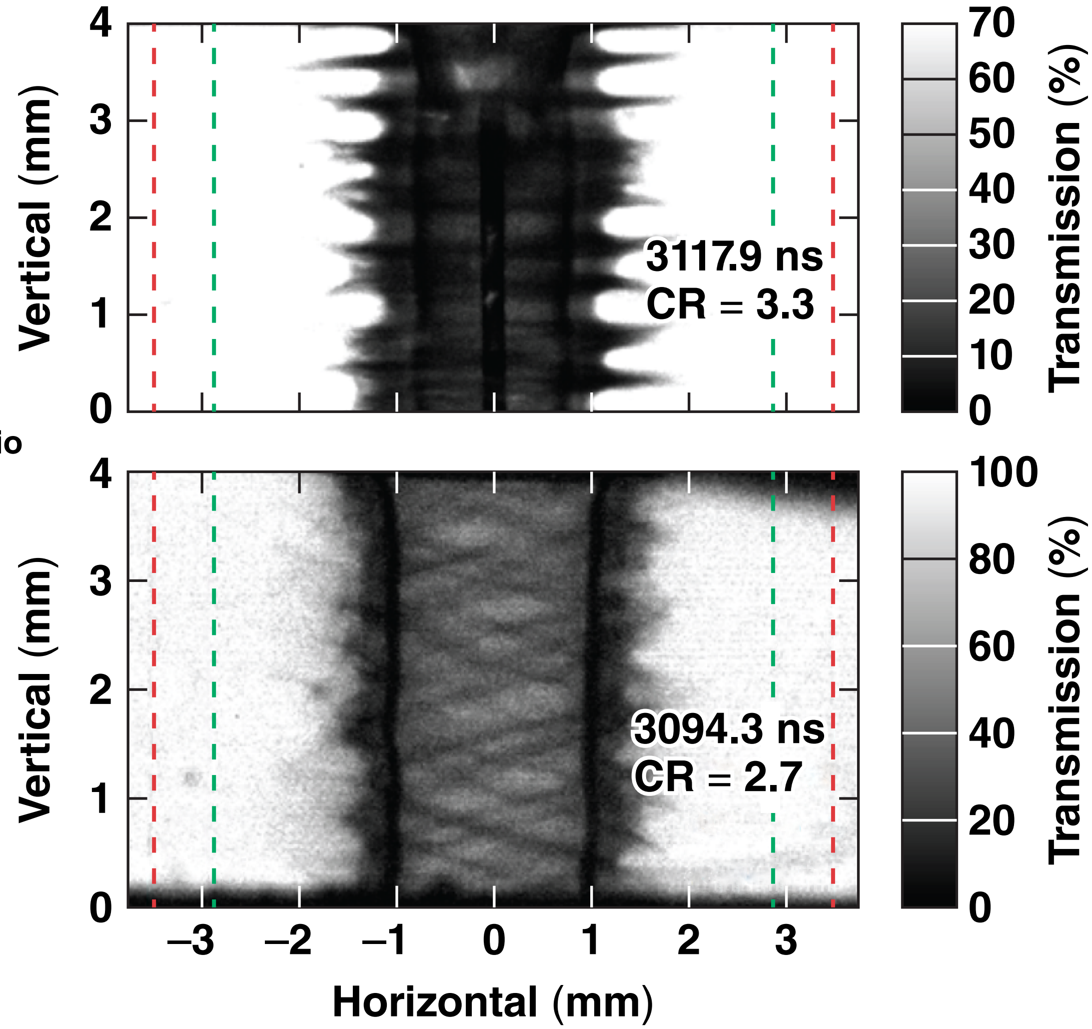
Experiments on Z show helices at pitch angles to the horizontal axis of 16° to 33° with an axial magnetic field more than 100× smaller than the azimuthal field*

$B_{z0} = 0$
$B_{\phi} \sim 4$ kT

CR: convergence ratio

$B_{z0} = 7$ T
$B_{\phi} \sim 4$ kT



*T. J. Awe *et al.*, Phys. Rev. Lett. 111, 235005 (2013); T. J. Awe *et al.*, Phys. Plasmas 21, 056303 (2014).

The 3-D magnetohydrodynamic (MHD) code *GORGON* accurately reproduced the results by adding helices with a pitch angle of 7.2° to the outer surface of the liner*

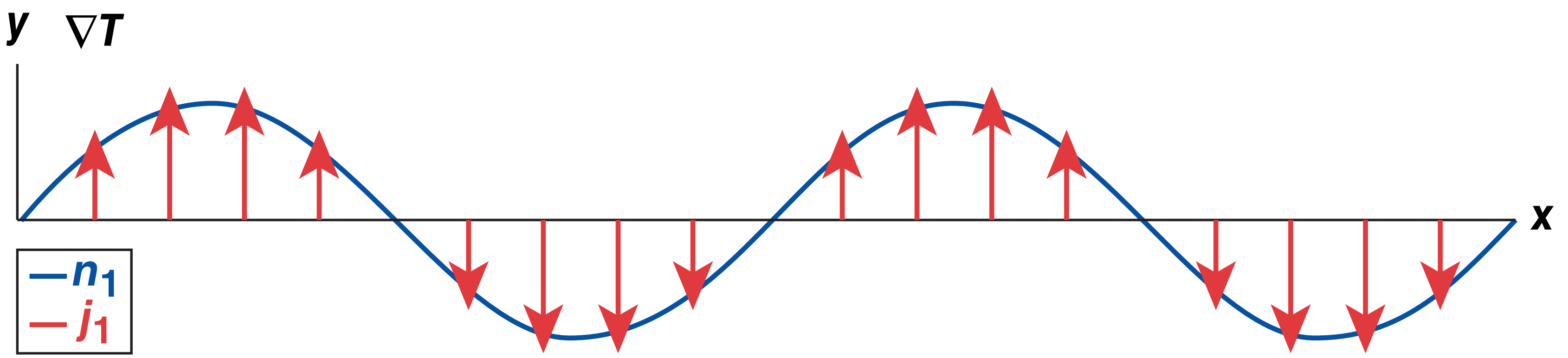
- **A mechanism is needed whereby the axial magnetic field seeds these helices before compression for this to be a consistent explanation**
- **A self-generated magnetic field—not included in *GORGON*—provides such a mechanism****
- **A self-generated magnetic field arises from the electron pressure gradient term in Ohm's law, which in Faraday's law gives**

$$\frac{\partial \vec{B}}{\partial t} = \frac{\nabla P_e \times \nabla n_e}{n_e^2 e} = \frac{\nabla k_B T_e \times \nabla n_e}{n_e e}$$

*T. J. Awe *et al.*, Phys. Plasmas 21, 056303 (2014).
**J. R. Davies *et al.*, Plasma Phys. Control. Fusion 51, 035013 (2009).

Density perturbations perpendicular to a temperature gradient generate a magnetic field that can lead to instability

$$n_1 \propto \sin(kx) \quad T = T_0(y) \quad \frac{\partial B_{z1}}{\partial t} \propto \cos(kx) \quad \frac{\partial j_{y1}}{\partial t} \propto \sin(kx)$$



- **Similar currents attract and opposite currents repel, leading to the growth of density perturbations**
- **An initial magnetic field perpendicular to the density perturbation and temperature gradient (B_{z0}) is required to give a force in a linear model**
  - $F_{x1} = j_{y1} \times B_{z0}$
- **Carrying out the usual analysis of the linearized equations gives…**

The dispersion relation for magnetoacoustic modes perpendicular to a temperature gradient

$$\omega^3 - i\omega^2 k^2 (D_B + D_v) - \omega k^2 \left(v_A^2 + c_s^2 + D_B D_v k^2\right) + i c_s^2 D_B k^4 + \frac{B_0 k_B T}{\rho_0 \mu_0 e L} k^3 = 0$$

$$D_B = \frac{\eta}{\mu_0}, D_v = \frac{4}{3}\nu, v_A = \sqrt{\frac{B_0^2}{\rho_0 \mu_0}},\ c_s = \sqrt{\frac{P_0}{\rho_0}},\ L = \frac{T}{|dT/dr|}$$

TC11520

The dispersion relation for magnetoacoustic modes perpendicular to a temperature gradient

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$$\omega^3 - i\omega^2 k^2 (D_B + D_v) - \omega k^2 \left(v_A^2 + c_s^2 + D_B D_v k^2\right) + i c_s^2 D_B k^4 + \boxed{\frac{B_0 k_B T}{\rho_0 \mu_0 e L} k^3} = 0$$

$$D_B = \frac{\eta}{\mu_0}, D_v = \frac{4}{3}\nu, v_A = \sqrt{\frac{B_0^2}{\rho_0 \mu_0}}, \ c_s = \sqrt{\frac{P_0}{\rho_0}}, \ L = \frac{T}{|dT/dr|}$$

Driving term caused by a self-generated magnetic field

The driving frequency is determined by Alfvén velocity, sound speed, and the ratio of the ion collisionless skin depth δ_i to the temperature scale length L

$$\Omega = Vk = \left(\frac{B_0 k_B T}{\rho_0 \mu_0 e L}\right)^{1/3} k$$

$$V = \left(v_A c_s^2 \frac{Z}{Z+1} \frac{\delta_i}{L}\right)^{1/3}, \quad \delta_i = \frac{c}{\omega_{pi}}$$

$$\hat{L} = \frac{L}{\delta_i} \frac{Z+1}{Z} \qquad \delta_i \approx 0.3 \frac{A}{Z} \rho_{g/cm^3}^{-1/2} \, \mu m$$

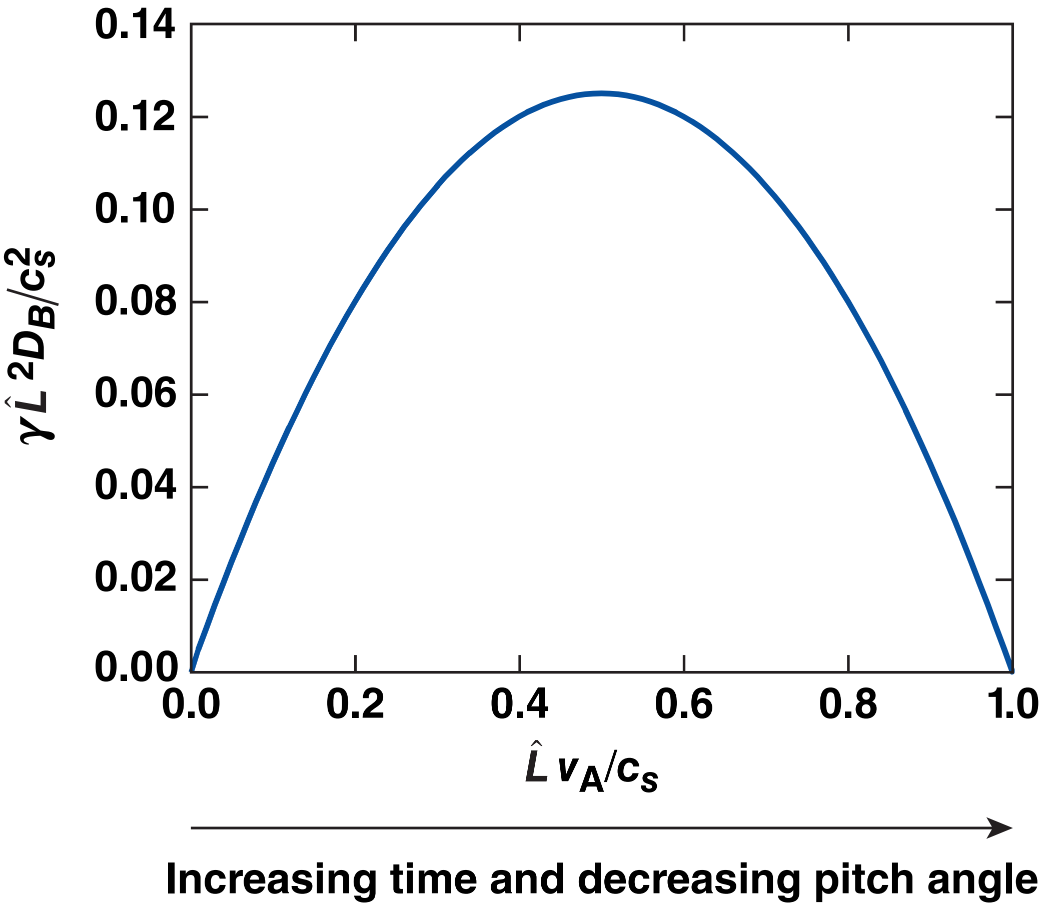
- Initially heat a resistive skin depth giving $\hat{L} \sim 1$
- $\hat{L}$ will tend to increase because of thermal conduction

$$\hat{L} \geq 1$$

TC11522

Resistivity but no viscosity: the maximum growth rate occurs for a wave number tending to infinity

$$\gamma \to \frac{v_A c_s}{2\hat{L} D_B}\left(1 - \hat{L}\frac{v_A}{c_s}\right),\ D_B > 0, D_v = 0, k \to \infty$$



TC11523

Stability is achieved before magnetic pressure exceeds thermal pressure

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$$\frac{v_A}{c_s} \geq \frac{1}{\hat{L}}, \; \frac{P_B}{P} \geq \frac{1}{2\hat{L}^2}$$

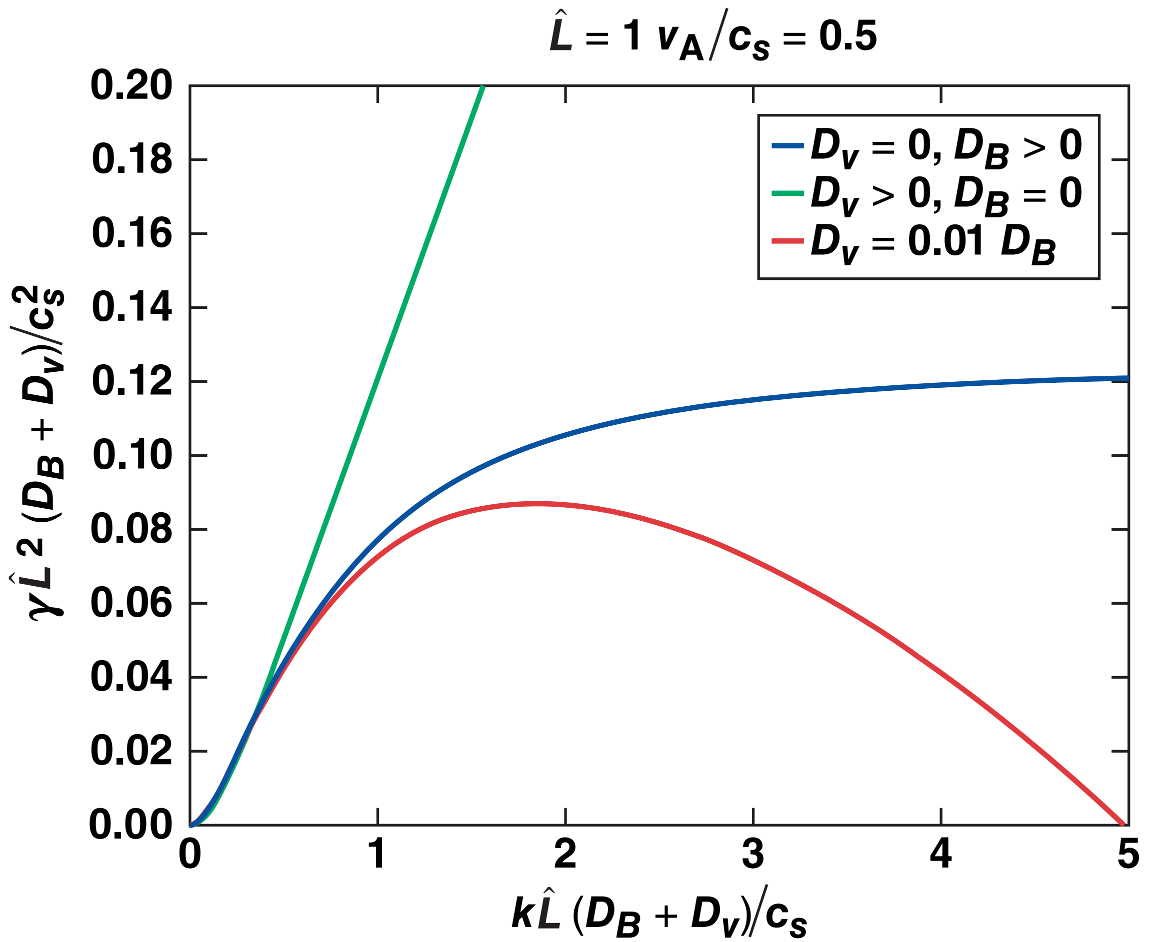
- Growth will stop before compression starts and before the pitch angle of the magnetic field becomes small
- The maximum growth rate is

$$\gamma \approx 41\left(\frac{A}{Z+1}\right)\left(\frac{Z\ln\Lambda}{10}\right)^{-1} \rho_{\mathrm{g/cm^3}}^{-1} T_{\mathrm{keV}}^{5/2} L_{\mu\mathrm{m}}^{-2} \ \mathrm{ns}^{-1}$$

$$\frac{v_A}{c_s} = \frac{1}{2\hat{L}}, \; \frac{P_B}{P} = \frac{1}{8\hat{L}^2}$$

This instability could seed helices because it acts perpendicular to the magnetic field; it can only occur prior to compression and has a growth time of the order of a nanosecond.

Resistivity *and* viscosity are required to give physical growth rates at large wave numbers



$$\lambda_{min} \geq \sqrt{\frac{\hat{L} D_B D_v}{v_A c_s}} \approx 20\left(\frac{Z+1}{5}\right)^{1/4}\left(\frac{B}{100\ \mathrm{T}}\right)^{-1/2}\left(\frac{kT/e}{100\ \mathrm{eV}}\right)^{1/4}\left(\frac{L}{1\ \mu\mathrm{m}}\right)^{1/2} \mu\mathrm{m}, D_v \ll D_B$$

TC11525

Self-generated magnetic field can seed helices in liners with an axial magnetic field

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