Non-inertial Eulerian Hydrodynamic Code for ICF Implosion Simulations

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Objective and Motivation

Objective

- Develop a Eulerian hydrodynamic code in spherical geometry that solves the equations of motion of a fluid in the frame of reference of the target center of mass, then study the effects of Rayleigh–Taylor instability on the deceleration phase

Motivation

- A non-inertial frame of reference can be defined so that the imploding shell boundaries are static; a fine-mesh region can be constructed in non-inertial coordinates that covers the dense shell and accurately resolves hydrodynamic instabilities

TC10329

Coordinate transformation to a non-inertial frame is used

Coordinate transformation:

$$x \equiv \frac{r}{R(t)}, \quad t \equiv t, \quad \theta \equiv \theta$$

- r is the radial coordinate in spherical geometry
- $R(t)$ is a time dependent reference (e.g., the shell's center of mass)

This coordinate transformation leads to:

$$\partial_r \equiv \frac{1}{R(t)}\partial_x, \quad \partial_t \equiv \frac{-x}{R}\dot{R}\partial_x + \partial_t, \quad \partial_\theta \equiv \partial_\theta$$

TC10332

The polar and azimuthal components of the momentum equation have no inertial forces

B. **Polar component:** a term proportional to $\dot{R}$ is introduced as a result of the transformation

$$\partial_t(\rho v_\theta) + \frac{1}{r^2}\partial_r(r^2 \rho v_r v_\theta) + \frac{1}{r\sin\theta}\partial_\theta(\rho v_\theta^2 \sin\theta) + \frac{\rho v_\theta v_r}{r} - \frac{\rho v_\phi^2 \cot\theta}{r} = -\frac{1}{r}\partial_\theta p$$

$$\partial_t(Qw_\theta) + \frac{1}{R}\partial_x(Qw_\theta w_x) + \frac{1}{xR\sin\theta}\partial_\theta(Qw_\theta^2\sin\theta) + \frac{Qw_\theta w_x}{xR} - \frac{Qw_\phi^2\cot\theta}{xR} = -xR^2\partial_\theta p - \frac{Q\dot{R}w_\theta}{R}$$

C. **Azimuthal component:** if $v_\phi = w_\phi = 0$, then this equation does not contribute and the system is reduced to 2-D geometry

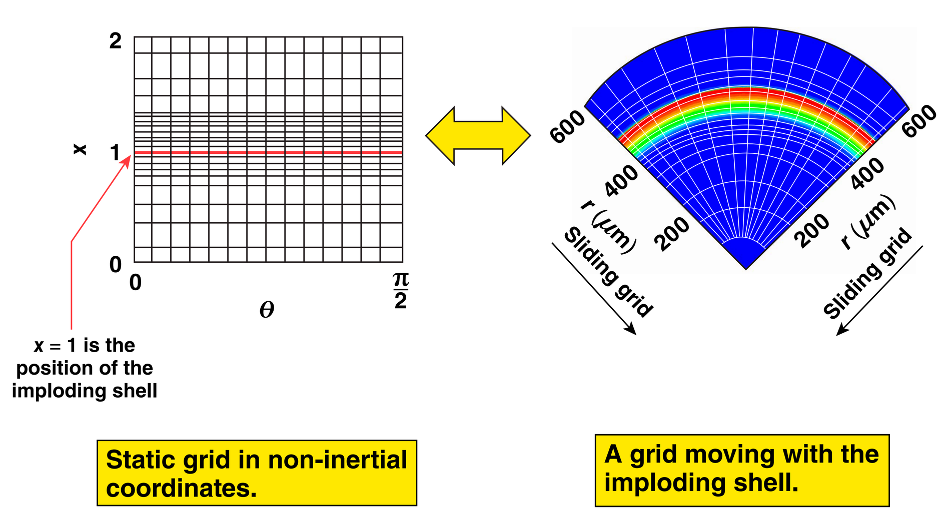
TC10335

The benefits from the new center-of-mass frame of reference include a static grid, allowing high resolution when needed

- Defining $R(t)$ to be the position of the center of mass of the shell at time t. In non-inertial coordinates, $x = 1$ is always the location of the center of mass
- The sharp density gradients occur at the shell boundaries that would be around $x = 1$. Therefore, the finer mesh would be static around this region
- The center of the system is $r = 0$, which is mapped to $x = 0$ and is always fixed
- As $R(t)$ changes with time the coordinates are rescaled. Since $R(t)$ decreases, the new coordinate allows automatic zooming to the region between the origin and the center of mass
- $\dot{R}$ and $\ddot{R}$ are the velocity and acceleration of center of mass, respectively

TC10340

A non-inertial frame of reference is equivalent to a moving grid with high resolution in the region including the imploding shell



TC10330

The continuity equation requires redefining the density, in transformed coordinates

$$\partial_t \rho + \vec{\nabla}\cdot(\rho\vec{v}) = 0$$

In spherical polar coordinates, assuming azimuthal symmetry

$$\partial_t \rho + \frac{1}{r}\partial_r(r^2\rho v_r) + \frac{1}{r\sin\theta}\partial_\theta(\rho v_\theta \sin\theta) = 0$$

In new coordinates:

$$\partial_t Q + \frac{1}{R}\partial_x(Qw_x) + \frac{1}{xR\sin\theta}\partial_\theta(Qw_\theta\sin\theta) = 0$$

With new variables in non-inertial coordinates

$$Q = x^2R^3\rho$$ → Redefined density parameter

$$w_x = v_r - \dot{R}x, \quad w_\theta = v_\theta, \quad w_\phi = v_\phi$$

TC10333

The energy equation is written in a conserved-Eularian form with a redefined total energy ξ

Assuming ideal gas EOS,

$$\partial_t\left(\frac{p}{\gamma-1} + \frac{\rho v^2}{2}\right) + \vec{\nabla}\cdot\left[\vec{v}\left(\frac{\gamma p}{\gamma-1} + \frac{\rho v^2}{2}\right)\right] = 0$$

In new coordinates,

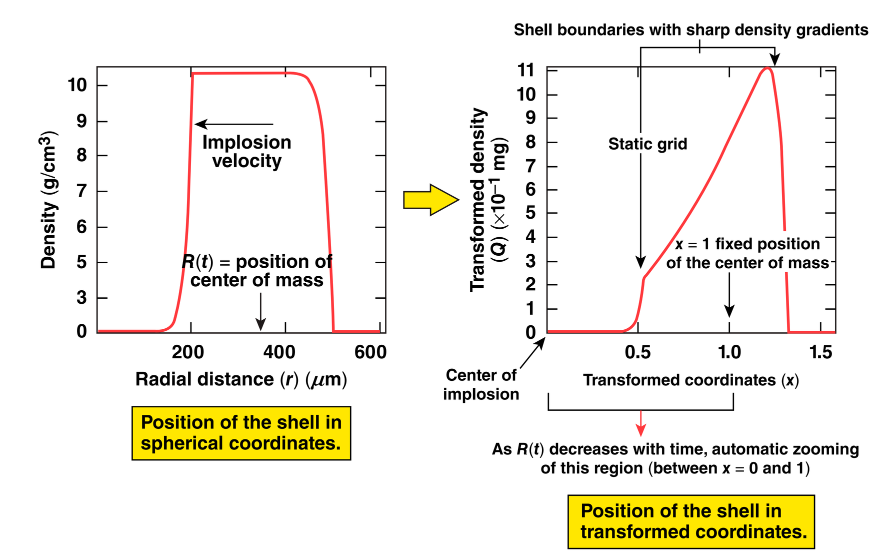
$$\partial_t\xi + \frac{1}{R}\partial_x(\xi w_x) + \frac{1}{xR\sin\theta}\partial_\theta(w_\theta\xi\sin\theta) = -\frac{1}{R}\partial_x\left(\frac{Qpw_x}{\rho}\right) - \frac{1}{xR\sin\theta}\partial_\theta\left(\frac{Qpw_\theta}{\rho}\sin\theta\right) - \frac{\dot{R}}{R}\partial_x\left(\frac{Qpx}{\rho}\right)$$

with total energy $\xi = \left[\frac{Qp}{\rho(\gamma-1)} + \frac{Qw^2}{2}\right]$

using $w^2 = (w_x + x\dot{R})^2 + w_\theta^2 + w_\phi^2$

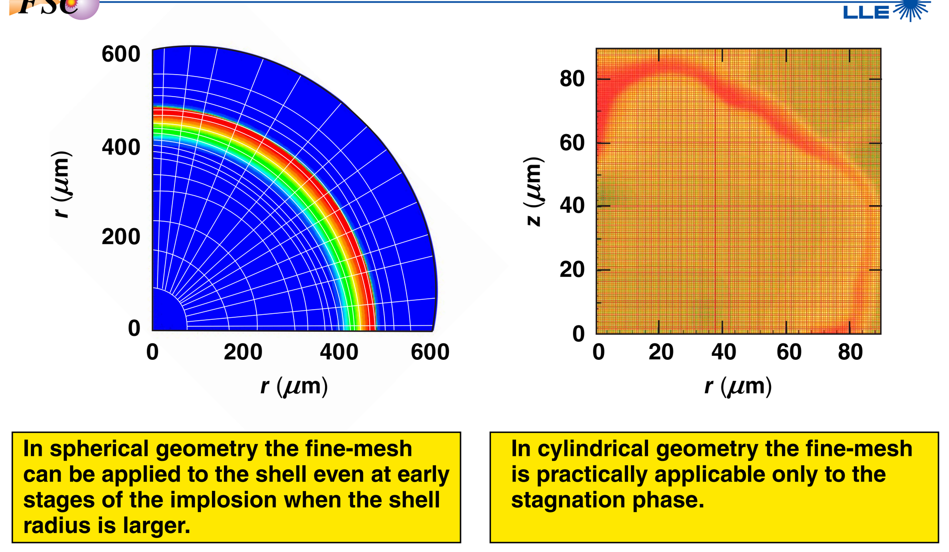
TC10336

The transformed density shown in non-inertial coordinates using a static grid



TC10338

The code makes use of spherical geometry which allows accurate positioning of the fine mesh region on the imploding shell unlike in cylindrical coordinates



TC10331

*K. Anderson et al., Bull. Am. Phys. Soc. 46, 280 (2001).

The momentum equation has pseudo-force terms in non-inertial coordinates

$$\partial_t(\rho\vec{v}) + \vec{\nabla}\cdot(\rho\vec{v}\vec{v}) = -\vec{\nabla}p$$

A. **Radial equation:**
In spherical coordinates

$$\partial_t(\rho v_r) + \frac{1}{r^2}\partial_r(r^2\rho v_r v_r) + \frac{1}{r\sin\theta}\partial_\theta(\rho v_\theta v_r \sin\theta) - \frac{\rho(v_\theta^2 + v_\phi^2)}{r} = -\partial_r p$$

Transformed equation:

$$\partial_t(Qw_x) + \frac{1}{R}\partial_x(Qw_x^2) + \frac{1}{xR\sin\theta}\partial_\theta(Qw_xw_\theta\sin\theta) - \frac{Q}{xR}(w_\theta^2 + w_\phi^2) = -x^2R^2\partial_x p - \underbrace{Q\left(x\ddot{R} + \frac{w_x\dot{R}}{R}\right)}_{\text{Inertial forces}}$$

TC10334

The hydrodynamic equations are summarized for a 1-D non-inertial Eulerian frame

Transformed parameters in 1-D

$$Q = x^2R^3\rho$$
$$\xi = \left[\frac{Qp}{\rho(\gamma-1)} + \frac{Qw^2}{2}\right]$$
$$w_x = v_r - x\dot{R}$$
$$w_\theta = w_\phi = 0$$
$$w^2 = (w_x + x\dot{R})^2$$

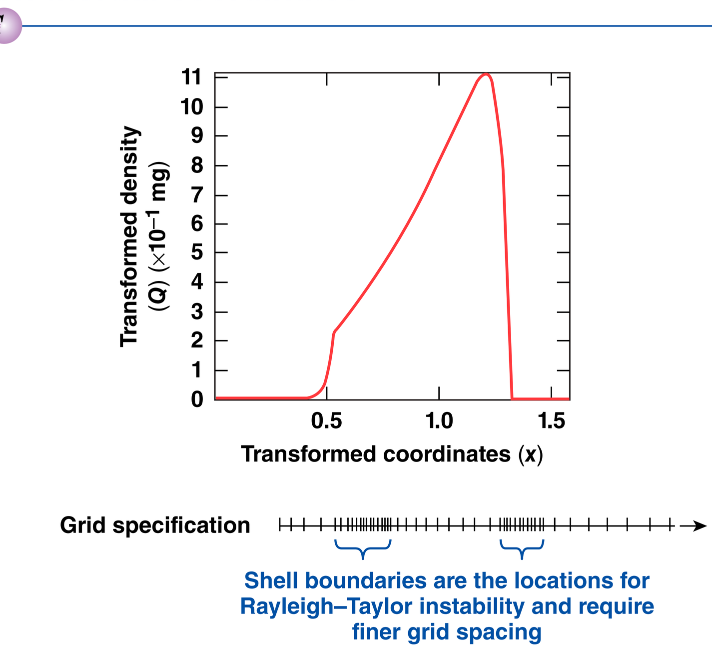
Transformed hydrodynamic equations

$$\partial_t Q + \frac{1}{R}\partial_x(Qw_x) = 0$$
$$\partial_t(Qw_x) + \frac{1}{R}\partial_x(Qw_x^2) = -x^2R^2\partial_x p - Q\left(x\ddot{R} + \frac{w_x\dot{R}}{R}\right)$$
$$\partial_t\xi + \frac{1}{R}\partial_x(\xi w_x) = -\frac{1}{R}\partial_x\left(\frac{Qpw_x}{\rho}\right) - \frac{\dot{R}}{R}\partial_x\left(\frac{Qpx}{\rho}\right)$$
$$p = \rho T/A \qquad A = m_i/(1+Z)$$

These are the closed set of equations used to develop the 1-D non-inertial Eulerian hydrodynamic code to facilitate comparison with existing Langrangian and Eulerian codes.

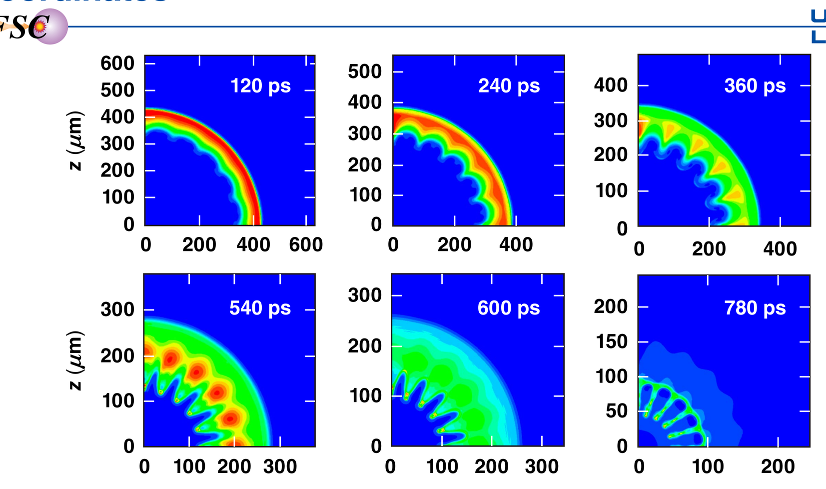
TC10337

Regions with sharp density gradients are static in non-inertial coordinates



TC10339

Simulations of the deceleration phase showing single-mode Rayleigh–Taylor growth in cylindrical non-inertial coordinates



Cylindrical non-inertial transformation $\alpha = \frac{r}{R(t)}$ $\beta = \frac{z}{R(t)}$ *

TC10341

*K. Anderson et al., Bull. Am. Phys. Soc. 46, 280 (2001).

Conclusion

- The accelerated coordinate system allows the use of a static Eulerian grid
- The new coordinates result in automatic zooming of the region between the origin and the center of mass
- In the new-coordinate system, the sharp density gradients at the shell boundaries are pinned around the center of mass, therefore specifying the location of fine mesh region. As a result the sharp density gradients at the shell boundaries are not missed

TC10342