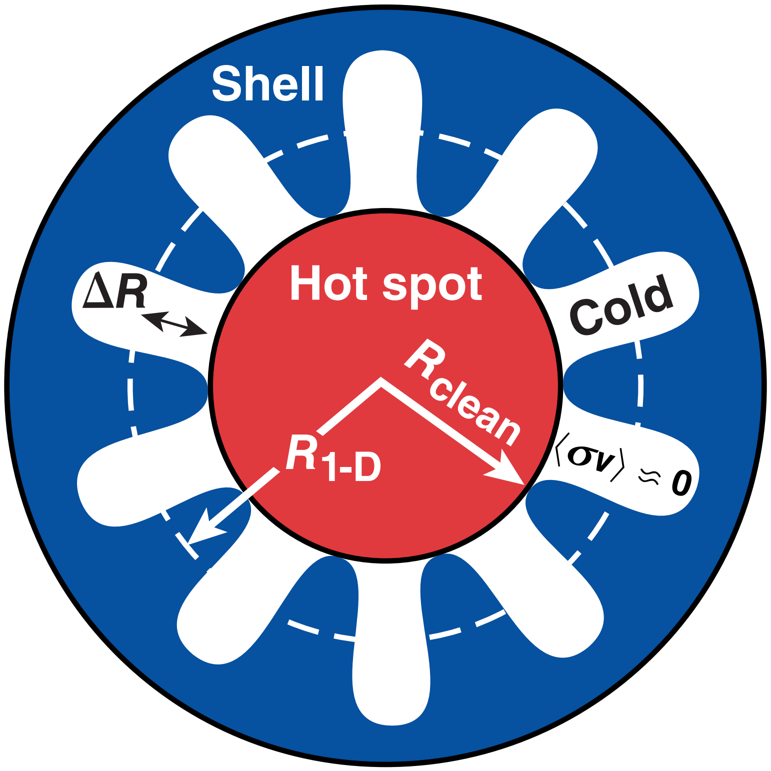
A Measurable Three-Dimensional Ignition Criterion for Inertial Confinement Fusion



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Summary

An analytic model of the hot-spot evolution provides a measurable 3-D ignition criterion for ICF

- The one-dimensional (1-D) measurable Lawson criterion of Zhou and Betti* is extended to 3-D using the Yield-Over-Clean (YOC) as a measure of the implosion uniformity
- The ignition parameter from the analytic theory depends on areal density, ion temperature, and yield-over-clean
- The analytic model is in reasonable agreement with a simulation database yielding
- Cryogenic implosions on OMEGA have achieved an ignition parameter $\chi^{\mathrm{fit}} \approx 0.008$. Hydro-equivalent ignition on OMEGA requires $\chi \approx 0.04$.

*C. D. Zhou and R. Betti, Phys. Plasmas 15, 102707 (2008).

Collaborators

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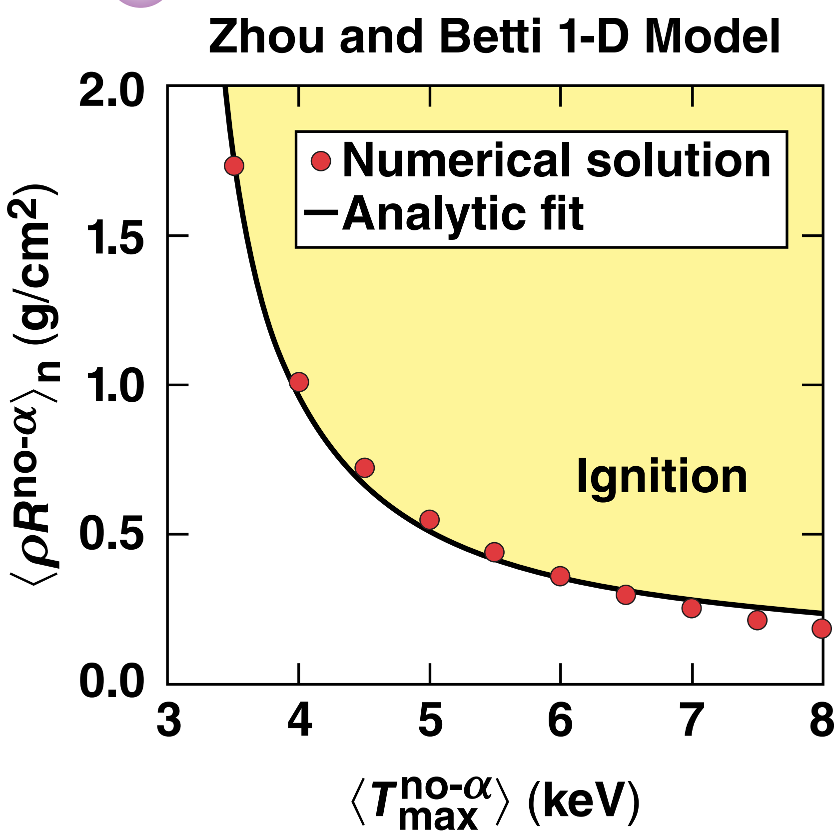
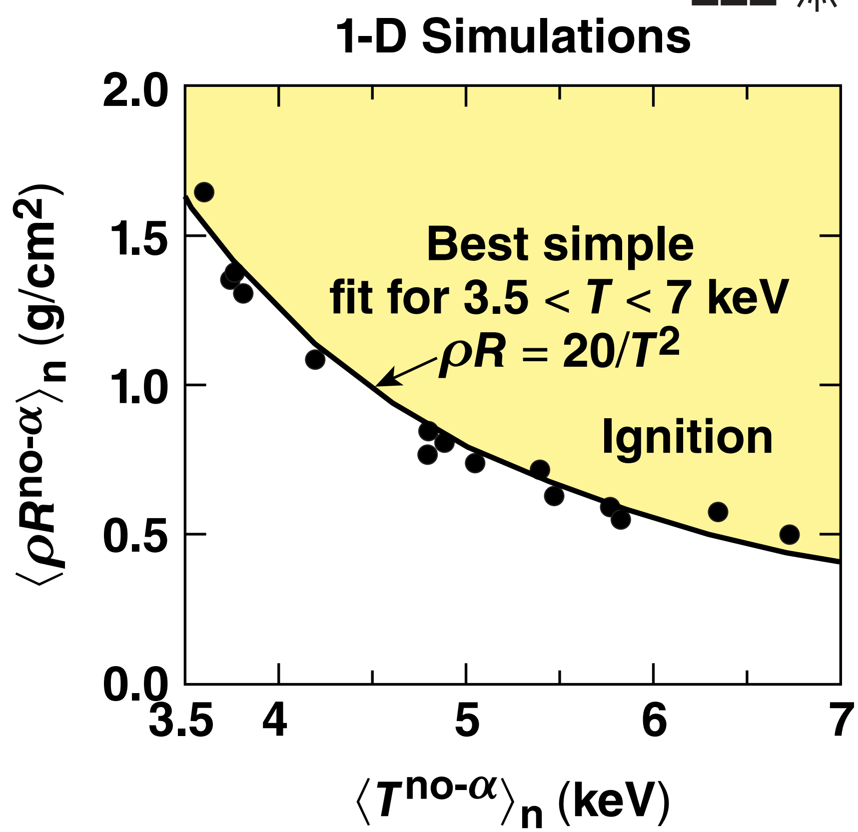
The one-dimensional ignition criterion depends on the total areal density and ion temperature (without calculated α-particle deposition)





$$\left\langle \rho R^{no\text{-}\alpha}_{g/cm^2} \right\rangle_n \left(\frac{\left\langle T^{no\text{-}\alpha}_{keV} \right\rangle}{4.5} \right)^2 > 1 \quad \text{for } 3.5 < T^{no\text{-}\alpha}_{keV} < 7$$

TC8649

C. D. Zhou and R. Betti, Phys. Plasmas 15, 102707 (2008).

The effects of nonuniformities in the deceleration phase are added to the ignition model through a clean volume analysis

- Nonuniformities reduce the volume where fusion reactions occur:*

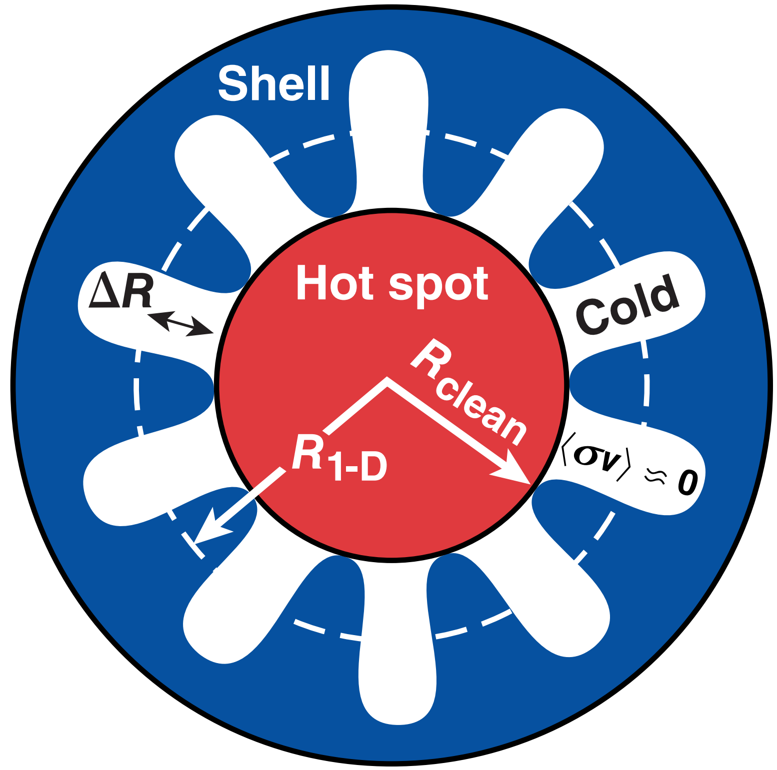
$$V_{\text{fusion}} \approx V_{\text{clean}} = \frac{4\pi}{3} R^3_{\text{clean}}$$

- The hot spot energy balance is affected by a reduced "clean" volume

$$\frac{d}{dt}\left(PR^3\right) = -2PR^2 \frac{dR}{dt} + \frac{\varepsilon_\alpha}{8\pi} \int_0^{V_{\text{clean}}} \langle \sigma \nu \rangle n^2 d\nu$$

$$\varepsilon_\alpha = 3.5 \text{ MeV}$$

- $\langle \sigma v \rangle \approx C_\alpha T^3 \leftarrow$ valid for $T \sim 4$ to 8 keV



Fusion reactions occur only within the clean volume.

*R. Kishony and D. Shvarts, Phys. Plasmas 8, 4925 (2001).

Zhou* and Betti's model is modified to include the clean radius in the hot-spot energy balance

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- The hot-spot formation and ignition model is governed by three ODE's

Hot-spot energy balance

$$\frac{d}{dt}\left(PR^3\right) = \underbrace{-2pR^2\,\frac{dR}{dt}}_{\text{Compression/ expansion}} + \underbrace{\boxed{\frac{\varepsilon_\alpha}{8} C_\alpha P^2 R^3_{clean} T}}_{\alpha\text{-particles deposition}}$$

Temperature equation (from hot-spot mass conservation)

$$\frac{d}{dt}\left(\frac{PR^3}{T}\right) = \underbrace{0.87\,\kappa_0 R T^{5/2}}_{\text{Heat conduction}} + \underbrace{\cancel{C_{rad}\frac{P^2 R^3}{T^{5/2}}}}_{\text{Radiation}}$$

$\kappa_{sp} = \kappa_0 T^{5/2}$

Neglected in this talk

Shell Newton's law

$$\underbrace{M_{sh}}_{\text{Shell mass}} \frac{d^2R}{dt^2} = 4\pi P R^2$$

TC8651

*C. D. Zhou and R. Betti, Phys. Plasmas 15, 102707 (2008).

The yield-over-clean (YOC) is used as a measure of the implosion uniformity

$$\mathrm{YOC} = \left[\frac{\mathrm{Yield}_{3\text{-D}}}{\mathrm{Yield}_{1\text{-D}}}\right] \sim \frac{R^3_{\mathrm{clean}}}{R^3_{1\text{-D}}}$$

Develop 3-D model

Compute yield-over-clean without α-particle energy deposition

YOC

Compute ignition criterion (with α-particle deposition)
ρR, T_i

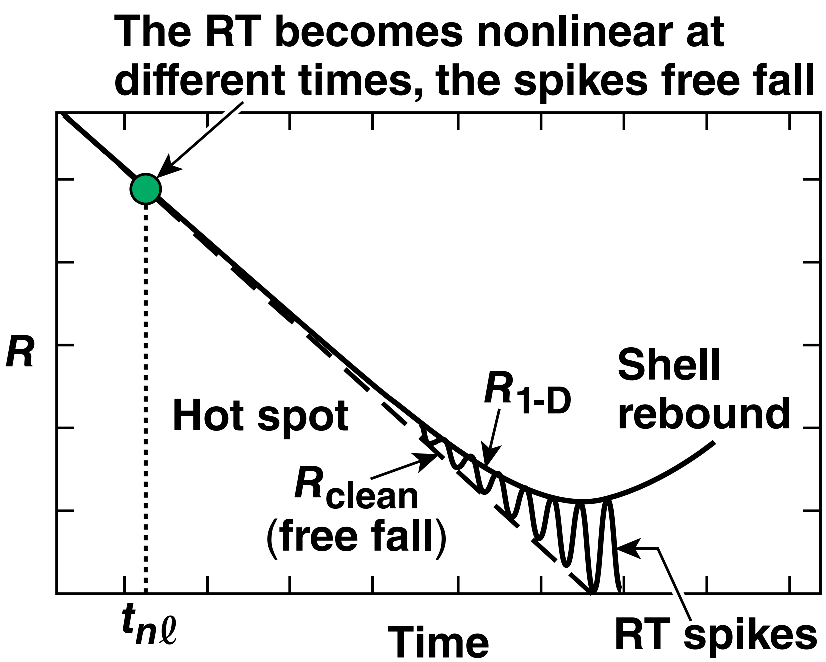
Relate ignition criterion to YOC

Ignition criterion $f\left(\rho R, T_i\ \mathrm{YOC}\right) = 0$

TC8652

Multiple models are used to assess the sensitivity of the ignition conditions to the hot-spot model

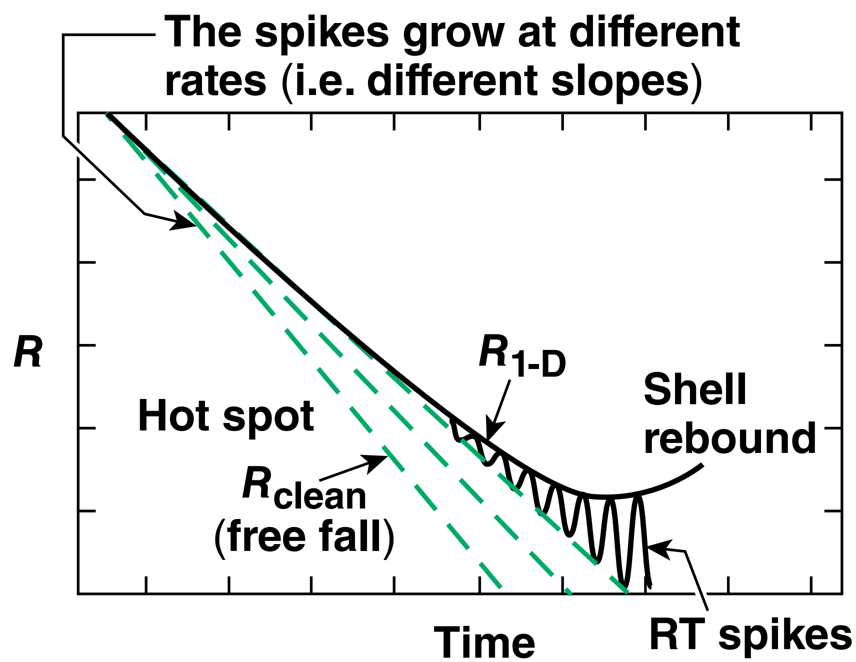
Model I



$$\mathrm{YOC}^{\text{no-}\alpha} = \mathrm{YOC}^{\text{no-}\alpha}\,(t_{n\ell})$$

$$t_{n\ell} = t_{n\ell}\,(\mathrm{YOC}^{\text{no-}\alpha})$$

Model II



$$\mathrm{YOC}^{\text{no-}\alpha} = \mathrm{YOC}^{\text{no-}\alpha}\,(\text{slope})$$

$$\text{slope} = \text{slope}\,(\mathrm{YOC}^{\text{no-}\alpha})$$

$$\boxed{R_{\text{clean}} = R_{\text{clean}}\,(\mathrm{YOC}^{\text{no-}\alpha})}$$

TC8653

The ignition model is cast in a dimensionless form using stagnation properties calculated without α-particle deposition

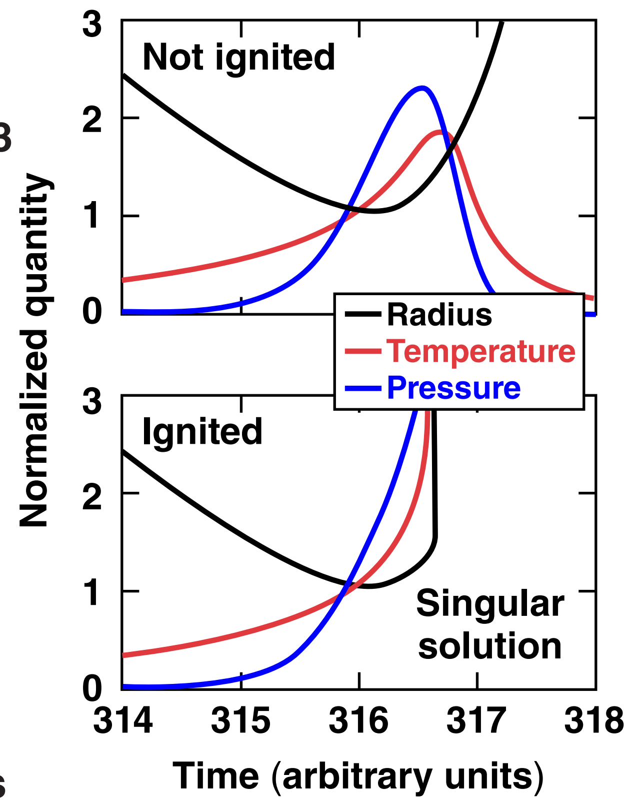
- Ignition depends on two parameters: γ_α and $YOC^{\text{no-}\alpha}$

Dimensionless equations

$$\begin{cases} \dfrac{d}{d\tau}\left(\hat{P}\hat{R}^3\right) = -2\hat{P}\hat{R}^2\dfrac{d\hat{R}}{d\tau} + \gamma_\alpha \hat{T}\hat{P}^2\left[\hat{R}_{\text{clean}}\left(YOC^{\text{no-}\alpha}\right)\right]^3 \\ \dfrac{d}{d\tau}\left(\dfrac{\hat{P}\hat{R}^3}{\hat{T}}\right) = \hat{R}\hat{T}^{5/2} \\ \dfrac{d^2\hat{R}}{d\tau^2} = \hat{P}\hat{R}^2 \\ \tau = \dfrac{tV_i}{R_{\text{stag}}^{\text{no-}\alpha}}, \quad \hat{R} = \dfrac{R}{R_{\text{stag}}^{\text{no-}\alpha}}, \quad \hat{P} = \dfrac{P}{P_{\text{stag}}^{\text{no-}\alpha}}, \quad \hat{T} = \dfrac{T}{T_{\text{stag}}^{\text{no-}\alpha}} \end{cases}$$

$$\boxed{\gamma_\alpha = 0.07\left(\rho R\right)^{3/4}\left(T_{\text{stag}}^{\text{no-}\alpha}\right)^{15/18}}\ ^*$$

Stagnation properties calculated without α's

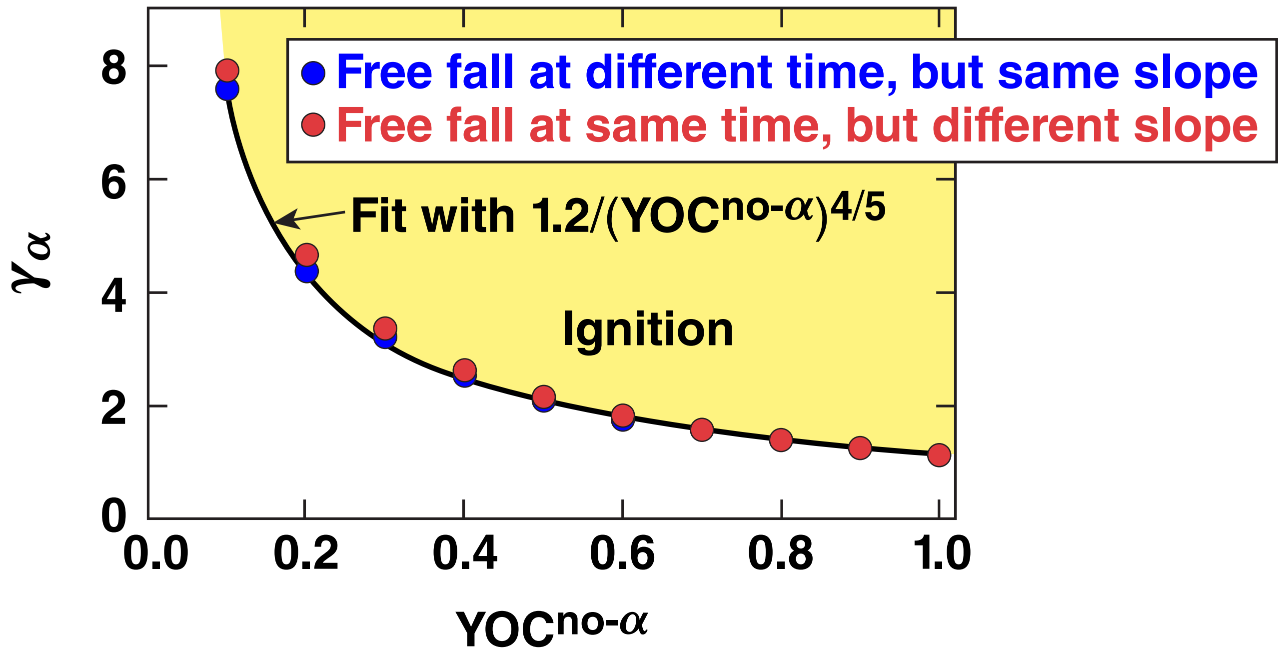


Ignition condition: for a fixed $YOC^{\text{no-}\alpha}$, find critical γ_α leading to singular solutions.

TC8654

*C. D. Zhou and R. Betti, Phys. Plasmas 15, 102707 (2008).

The 3-D ignition condition is approximately independent of the hot-spot models



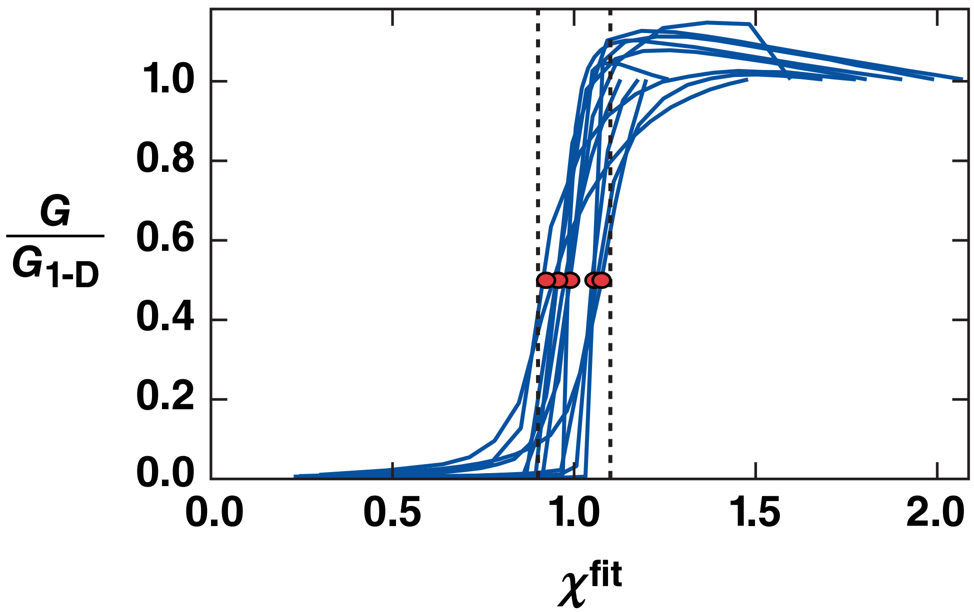
Model independent ignition criterion $\gamma_\alpha \left(\text{YOC}^{\text{no-}\alpha}\right)^{4/5} > 1.2$

Ignition parameter: $\chi = \rho R_{\text{tot}}^{\text{no-}\alpha} \left(\dfrac{T^{\text{no-}\alpha}}{4.6}\right)^{5/2} \left(\text{YOC}^{\text{no-}\alpha}\right)^{16/15} > 1$

TC8655

The clean volume model is implemented in *LILAC* and is used to generate a database of gain curves. The ignition condition is tuned with the simulation database

- In the 1-D code *LILAC*, $\langle \sigma v \rangle$ is replaced by $\langle \sigma v \rangle$ YOC $\approx \langle \sigma v \rangle \dfrac{V_{\text{clean}}}{V_{1\text{-D}}}$



Find exponents and constant (μ, δ, T_0) in ignition conditions to minimize the spread at 1/2 $G_{1\text{-D}}$

$$\chi^{\text{fit}} = (\rho R)^{\text{no-}\alpha} \left(\frac{T^{\text{no-}\alpha}}{T_0}\right)^{\mu} \left(\text{YOC}^{\text{no-}\alpha}\right)^{\delta}$$

Ignition parameter: $\boxed{\chi^{\text{fit}} = (\rho R)^{\text{no-}\alpha} \left(\frac{T^{\text{no-}\alpha}}{4.7}\right)^{2} \left(\text{YOC}^{\text{no-}\alpha}\right)^{0.7} > 1}$

K. Anderson (U05.00004).

Cryogenic implosions on OMEGA have achieved $\chi^{\text{fit}} \approx 0.008$; hydro-equivalent ignition on OMEGA requires $\chi \approx 0.04$

- For cryogenic implosions on OMEGA (have achieved)

$$(\rho R)_{\text{stag}}^{\text{no-}\alpha} \approx 0.2\ g/\text{cm}^2,\quad T_{\text{stag}}^{\text{no-}\alpha} \approx 2.1\,\text{keV},\quad \text{YOC} = 0.1^*$$

$$\boxed{\chi^{\text{fit}} = 0.008}$$

- Hydro-equivalent ignition on OMEGA requires

$$(\rho R)_{\text{stag}}^{\text{no-}\alpha} \approx 0.3\ g/\text{cm}^2,\quad T_{\text{stag}}^{\text{no-}\alpha} \approx 3.4\,\text{keV},\quad \text{YOC} = 0.15$$

$$\boxed{\chi^{\text{fit}} = 0.04}^{**}$$

*T. C. Sangster (NI2.00002).
**R. Betti (PT3.00001).

Summary/Conclusions

An analytic model of the hot-spot evolution provides a measurable 3-D ignition criterion for ICF and is in good agreement with simulation results

- The one-dimensional (1-D) measurable Lawson criterion of Zhou and Betti* is extended to 3-D using the Yield-Over-Clean (YOC) as a measure of the implosion uniformity
- The ignition parameter from the analytic theory depends on areal density, ion temperature, and yield-over-clean
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TC8648a

*C. D. Zhou and R. Betti, Phys. Plasmas 15, 102707 (2008).