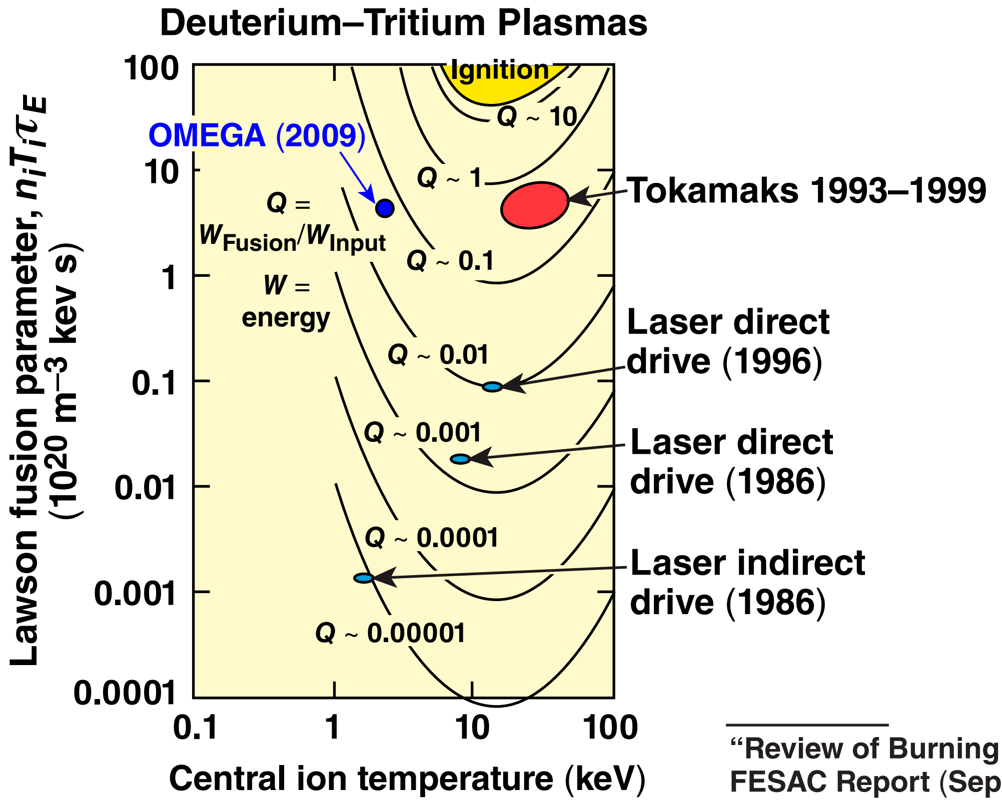
ICF Ignition, the Lawson Criterion and Comparison with MCF



"Review of Burning Plasma Physics," FESAC Report (September 2001).

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51st Annual Meeting of the
American Physical Society
Division of Plasma Physics
Atlanta, GA
2–6 November 2009

Collaborators

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Summary

The measurable Lawson criterion and hydro-equivalent curves determine the requirements for an early hydro-equivalent demonstration of ignition on OMEGA and on the NIF (THD)

- Cryogenic implosions on OMEGA have achieved a Lawson parameter $P\tau_E \approx 1$ atm-s comparable to large tokamaks
- Performance requirements for hydroequivalent ignition on OMEGA and NIF (THD)*

Hydro-equivalent ignition	$\langle \rho R \rangle_n$ g/cm^2	$\langle T_i \rangle_n$ keV	YOC	$P\tau_E$ (atm s)
OMEGA (25 kJ)	~0.30	~3.4	15% (~3 × 10^{13} neutrons)	2.6
NIF (THD)	~1.8	~4.7	~40%	20

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*NIF will begin the cryogenic implosion campaign using a surrogate Tritium–Hydrogen–Deuterium (THD) target. Only a very small fraction of Deuterium (<5%) is used to prevent fusion reactions from affecting the hydrodynamics.

See B. Spears (UO5.00013).

Outline

- A 1-D measurable Lawson criterion for ICF
- Hydro-equivalent curves and hydro-equivalent ignition
- The 3-D extension of the Lawson criterion
- Comparison with magnetic confinement

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"Classic" work on ICF ignition has focused on static models of the hot spot and has neglected the dense shell

- Lawson criterion applied to the hot spot

$nT_i\tau_E > 3 \times 10^{15}\ \text{cm}^{-3}\ \text{keV s} \Longrightarrow n, \tau$ cannot be measured

- Static models of the ignition condition use the hot-spot areal density and ion temperature

$\rho R_{\text{hot spot}} \approx 0.3\ \text{g/cm}^2 \qquad T_i \approx 5\ \text{to}\ 10\ \text{keV}$

$\rho R_{\text{hot spot}}$ cannot be measured

- J. D. Lawson, Proc. Phys. Soc. London B70, 6 (1957).
- S. Yu. Gus'kov *et al*., Nucl. Fusion 16, 957 (1976).
- S. Atzeni and A. Caruso, Phys. Lett. A 85, 345 (1981).
- S. Atzeni and A. Caruso, Nuovo Cimento B 80, 71 (1984).
- R. Kishony *et al*., Phys. Plasmas 4, 1385 (1997).
- J. D. Lindl, *Inertial Confinement Fusion: The Quest for Ignition and Energy Gain Using Indirect Drive* (Springer-Verlag, New York, 1998).
- S. Atzeni and J. Meyer-ter-Vehn, *The Physics of Inertial Fusion: Beam Plasma Interaction, Hydrodynamics, Hot Dense Matter*, International Series of Monographs on Physics (Clarendon Press, Oxford, 2004).

Ion temperatures, areal densities, and neutron yields are parameters of the fuel assembly that can be measured with existing diagnostics

- Neutron yields and neutron rates are measured with scintillators

$N_{\text{neutron}} \qquad \frac{dN_{\text{neutron}}}{dt}$

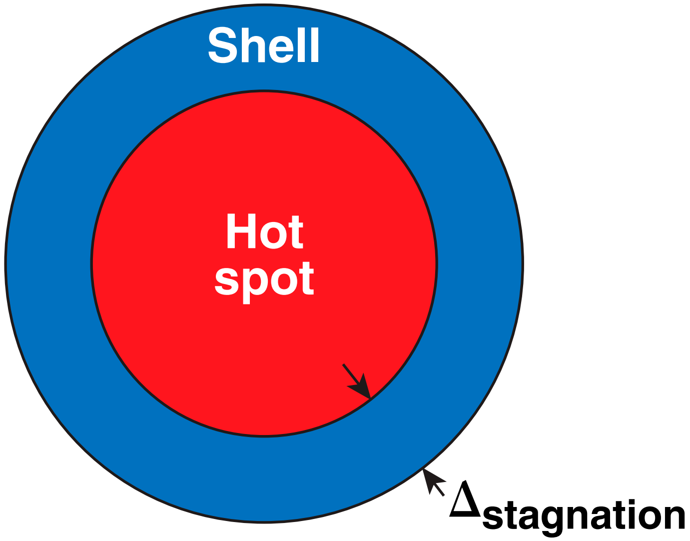
- Ion temperature (neutron averaged) is measured with the neutron time-of-flight detectors

$\langle T_i \rangle_{\text{neutron}}$

- Total areal density (neutron averaged) is measured with magnetic recoil spectrometer measuring the downscattered neutron fraction.*

$$\langle \rho R \rangle_{\text{neutron}} \equiv \left\langle \int_0^{\infty} \rho dr \right\rangle_{\text{neutron}}$$

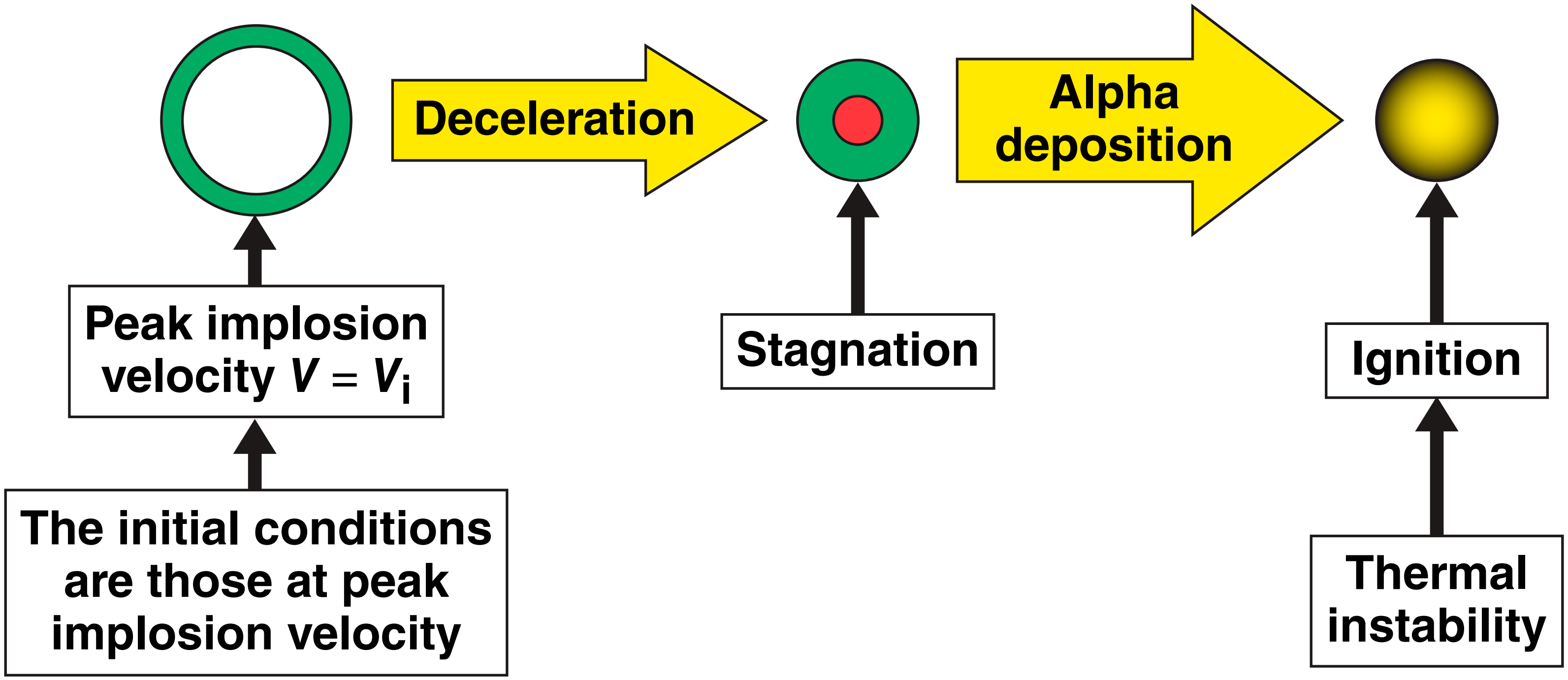
Total $\rho R \approx$ shell $(\rho\Delta)_{\text{stagnation}}$



See J. Frenje (NI2.00004).

A dynamic model of ignition relates the hot-spot stagnation properties to those of the shell

Dynamic model of hot-spot formation and ignition



Deceleration

Alpha deposition

Peak implosion velocity $V = V_\mathrm{i}$

The initial conditions are those at peak implosion velocity

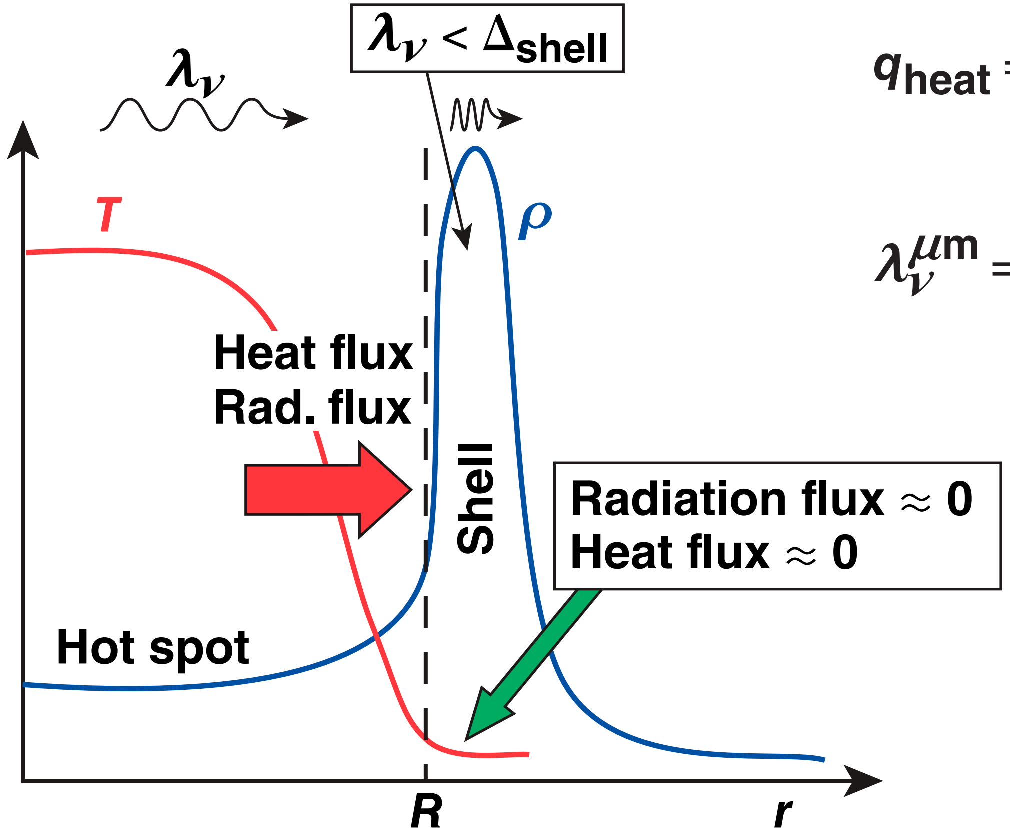
Stagnation

Ignition

Thermal instability

- R. Betti *et al*., Phys. Plasmas 8, 5257 (2001).
- R. Betti *et al*., Phys. Plasmas 9, 2277 (2002).
- J. Sanz *et al*., Phys. Plasmas 12, 112702 (2005).
- Y. Saillard, Nucl. Fusion 46, 1017 (2006).
- J. Garnier and C. Cherfils-Clérouin, Phys. Plasmas 15, 102702 (2008).
- C. D. Zhou and R. Betti, Phys. Plasmas 15, 102707 (2008).

The heat and radiation energy lost by the hot spot does not propagate through the dense cold shell; no heat or radiation flux at the hot-spot boundary



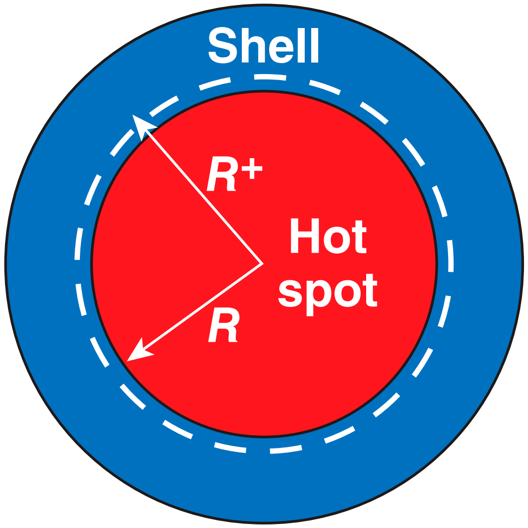
$\lambda_\nu < \Delta_{shell}$

Heat flux
Rad. flux

Radiation flux ≈ 0
Heat flux ≈ 0

$$q_{heat} = -\kappa(T)\nabla T \quad \kappa_{Sp}(T) \approx \kappa_0 T^{5/2}$$

$$\lambda_\nu^{\mu m} = \sqrt{\frac{T_{eV}^{shell}}{200}} \left(\frac{h\nu_{keV}}{5}\right)^3 \left(\frac{1000}{\rho_{g/cc}}\right)^2$$



$$q_{heat}\left(r = R^+\right) \approx 0 \quad q_{rad}\left(r = R^+\right) \approx 0$$

The hot-spot global energy balance determines the ignition condition

$$\frac{d}{dt}\left[\frac{3}{2}P(t)\,\mathrm{Vol}(t)\right] = \text{compression/decompression work}$$

$$+\ \alpha\_\text{heating} — (\text{conduction} + \text{radiation losses})$$

- ➡ Compression/decompression work $= -S(t)P(t)\dfrac{dR}{dt}$
- ➡ Alpha heating $= \dfrac{\varepsilon_\alpha}{16}P^2 \int_V \dfrac{\langle \sigma v \rangle}{T^2}\,dV$
- ➡ Conduction losses at $R^+ = \int_S \vec{q}_{\text{heat}} \cdot \vec{n}\,dS = 0$
- ➡ Radiation losses at $R^+ = \int_S \vec{q}_{\text{rad}} \cdot \vec{n}\,dS = 0$

ε_α = 3.5 MeV

$S = 4\pi R^2$

$\mathrm{Vol} = \dfrac{4\pi}{3}R^3$

TC8537

The dynamic ignition model requires an equation for the temperature evolution

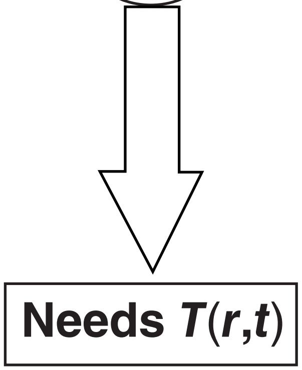
➡ Use a T^3 approximation of $\langle \sigma v \rangle \Rightarrow \langle \sigma v \rangle \approx C_\alpha T^3$

- Hot-spot energy equation

$$\boxed{\frac{d}{dt}\left[P(t)R(t)^3\right] = -2P(t)R(t)^2\frac{dR}{dt} + C_\alpha P(t)^2 \int_0^R Tr^2\,dr}$$

- Shell’s momentum equation (simple “thin-shell” approximation, $M_{sh} \equiv$ shell mass)

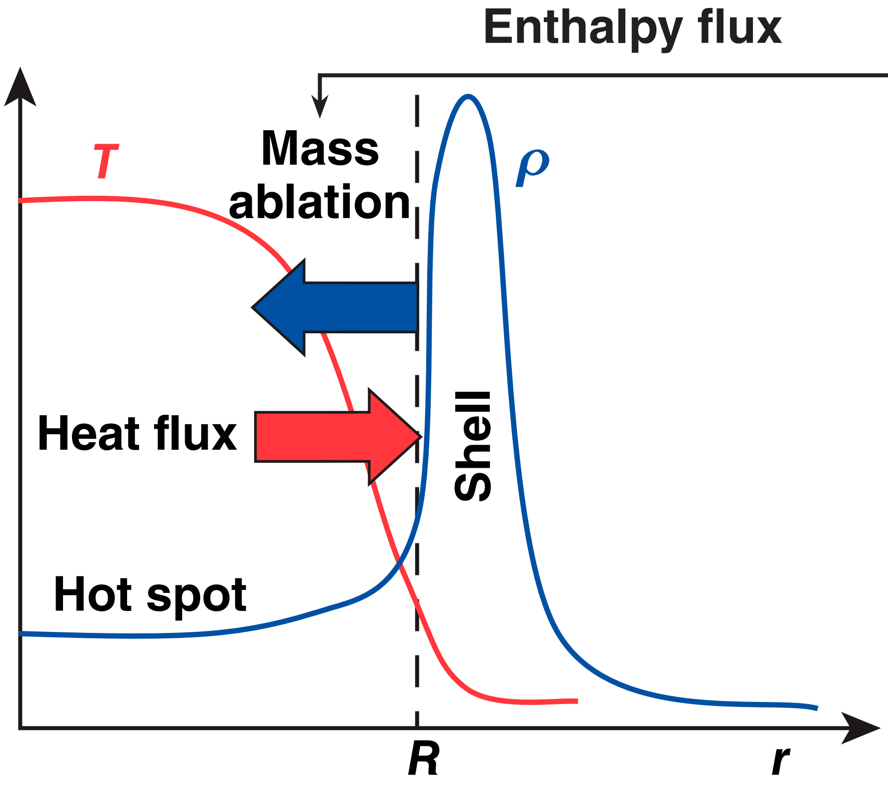
$$\boxed{M_{sh}\frac{d^2R}{dt^2} = 4\pi P(t)R(t)^2}$$



Needs $T(r,t)$

More realistic compressible “thick shell model” see: R. Betti *et al*., Phys. Plasmas 9, 2277 (2002).

The heat lost by the hot spot is deposited on the shell inner surface driving an ablative mass flow into the hot spot (ignore radiation losses*)



$$\boxed{\kappa_{sp}(T) = \kappa_0 T^{5/2}}$$

$$\frac{5}{2}Pv_a = -\kappa_{sp}(T)\nabla T$$

$$Pv_a = \frac{2\rho T}{m_i}v_a = 2\frac{T}{m_i}\dot{m}_a$$

$$\dot{m}_a = -\frac{m_i\kappa_0}{5}T^{3/2}\frac{dT}{dr}$$

use $T = T_0(t)\left(1-\frac{r^2}{R^2}\right)^{2/5}$

$$\dot{m}_a = \frac{4m_i\kappa_0}{25}\frac{T_0^{5/2}}{R(t)}$$

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*Radiation losses are ignored in this talk for simplicity.
They are included in C. D. Zhou and R. Betti, Phys. Plasmas 15, 102707 (2008).

Mass conservation in the hot spot determines the temperature evolution

- Ablation rate from shell to hot spot

$$\dot{m}_a = \frac{4m_i\kappa_0}{25}\frac{T_0(t)^{5/2}}{R(t)}$$

- Hot-spot-mass conservation

$$\frac{dM}{dt} = 4\pi R^2\dot{m}_a$$

- Mass–Temp relation

$$M = \int_V \rho dV = \frac{m_i}{2}\int_V \frac{P(t)}{T(r,t)}dV \approx \frac{4m_i P(t)R(t)^3}{T_0(t)}$$

- Temperature equation

$$\boxed{\frac{d}{dt}\left(\frac{PR^3}{T_0}\right) \approx \frac{4\pi}{25}\kappa_0 RT_0^{5/2}}$$

Hot-spot-ignition model: three ODE's for pressure, radius, and central temperature

- Hot-spot energy or pressure equation

$$\frac{d}{dt}\left[P(t)\,R(t)^3\right] = -2P(t)\,R(t)^2\,\frac{dR}{dt} + C_\alpha P(t)^2\,T_0(t)\,R(t)^3$$

- Shell momentum or hot-spot-radius equation

$$M_{sh}\frac{d^2R}{dt^2} = 4\pi P(t)\,R(t)^2$$

- Hot-spot-mass or temperature equation*

$$\frac{d}{dt}\left[\frac{P(t)\,R(t)^3}{T_0(t)}\right] \approx \frac{4\pi}{25}\kappa_0 R(t)\,T_0(t)^{5/2}$$

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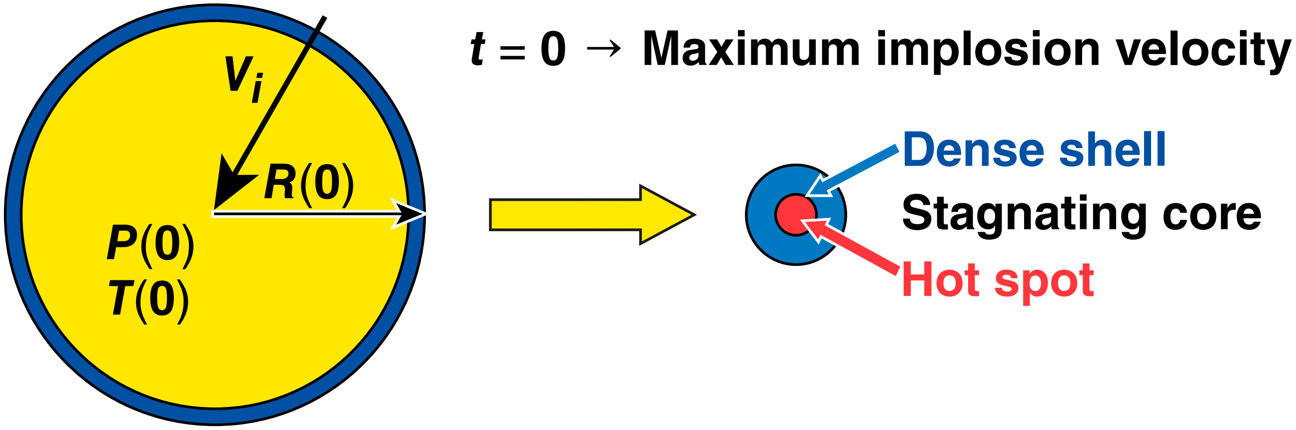
*Radiation losses are neglected in this talk for simplicity

The initial conditions are related to the time when the imploding shell reaches the maximum implosion velocity

$t = 0 \rightarrow$ **Maximum implosion velocity V_i**



$$P(0) = P_0 \quad T_0(0) = T_0^0 \quad R(0) = R_0 \quad \dot{R}(0) = -V_i$$

- Expansion parameter $\in \ll 1$

$$\boxed{\in \equiv \frac{\text{Hot-spot thermal energy}\,(t=0)}{\text{Shell kinetic energy}\,(t=0)} = \frac{2\pi P(0)\,R(0)^3}{(1/2)\,M_{sh} V_i^2} \ll 1}$$

The model has an analytic solution in the absence of α-particle energy deposition

- Stagnation pressure (without alphas)

$$P_{st}^{no\text{-}\alpha} \approx P_0 \in^{-5/2}$$

- Stagnation radius (without alphas)

$$R_{st}^{no\text{-}\alpha} \approx \in^{1/2} R_0$$

- Stagnation temperature (without alphas)

$$T_{st}^{no\text{-}\alpha} = \left[\frac{0.1}{\kappa_0} \frac{M_{sh} V_i^3}{\left(R_{st}^{no\text{-}\alpha}\right)^2} \right]^{2/7}$$

Expansion parameter

$$\in \equiv \frac{2\pi P(0) R(0)^3}{(1/2) M_{sh} V_i^2} \ll 1$$

The no-α stagnation values are used in the derivation of the dimensionless ignition model (with α's); *ignition depends on the one dimensionless parameter γ_α*

Dimensionless variables

$$\boxed{\hat{P} = P \Big/ P_{st}^{no\text{-}\alpha}} \quad \boxed{\hat{R} = R \Big/ R_{st}^{no\text{-}\alpha}} \quad \boxed{\hat{T} = T \Big/ T_{st}^{no\text{-}\alpha}} \quad \boxed{\tau = tV_i \Big/ R_{st}^{no\text{-}\alpha}}$$

Energy

$$\boxed{\frac{d}{d\tau}\left(\hat{P}\hat{R}^5\right) = \boxed{\gamma_\alpha}\, \hat{P}\hat{R}^5\hat{T}}$$

Critical parameter—determines ignition

$$\boxed{\gamma_\alpha = C_\alpha \frac{P_{st}^{no\text{-}\alpha} R_{st}^{no\text{-}\alpha} T_{st}^{no\text{-}\alpha}}{8V_i}}$$

Mass

$$\boxed{\frac{d}{d\tau}\left(\frac{\hat{P}\hat{R}^3}{\hat{T}}\right) = RT^{5/2}}$$

Momentum

$$\boxed{\frac{d^2\hat{R}}{d\tau^2} = \hat{P}\hat{R}^2}$$

Initial conditions

$$\hat{P}(0) = \in^{5/2}$$

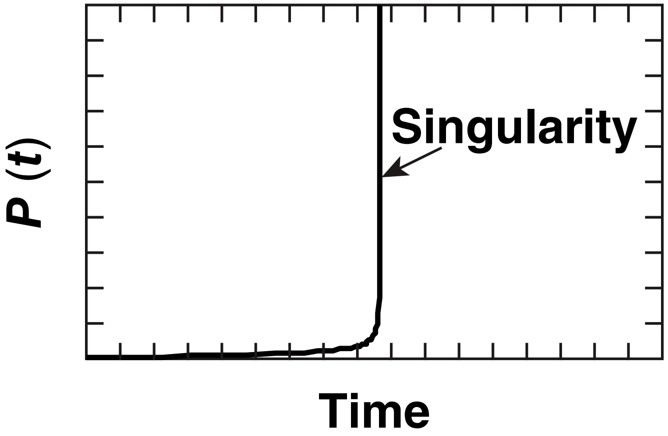
$$\hat{R}(0) = \in^{-1/2}$$

$$\dot{\hat{R}}(0) = -1$$

$$\hat{T}(0) = \in^{1/2}$$

$$\boxed{\in \rightarrow 0}$$

Ignition condition for the model equations: find the critical value of γ_α leading to an explosive solution



Condition for singular explosive solution → $\boxed{\gamma_\alpha > 1.2}$ ⇐ Singularity caused by $\langle \sigma v \rangle \sim C_\alpha T^3$

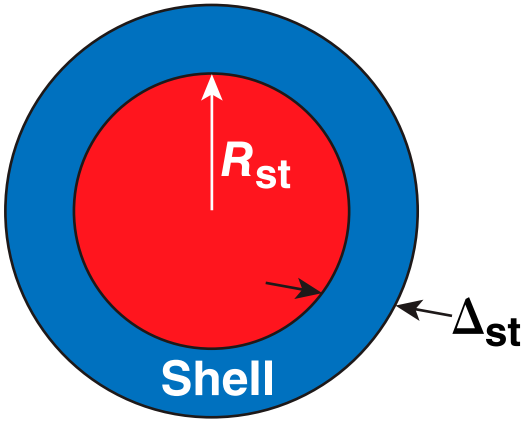
$$\gamma_\alpha \sim C_\alpha \frac{P_{st}^{no\text{-}\alpha} R_{st}^{no\text{-}\alpha} T_{st}^{no\text{-}\alpha}}{8\, V_i}$$

Cannot be measured

The ignition parameter γ_α can be written in terms of the measurable parameters ρR_{total} and T_i

Ignition condition: $\gamma_\alpha > 1.2$

$$\gamma_\alpha \sim C_\alpha \frac{P_{\text{st}}^{\text{no-}\alpha} \; R_{\text{st}}^{\text{no-}\alpha} \; T_{\text{st}}^{\text{no-}\alpha}}{8V_i}$$



Energy conservation

$$2\pi P_{\text{st}} R_{\text{st}}^3 = \frac{1}{2} M_{\text{sh}} V_i^2$$

Shell mass at stagnation

$$M_{\text{sh}} \approx 4\pi \rho_{\text{st}} \Delta_{\text{st}} R_{\text{st}}^2$$

Temperature-velocity relation

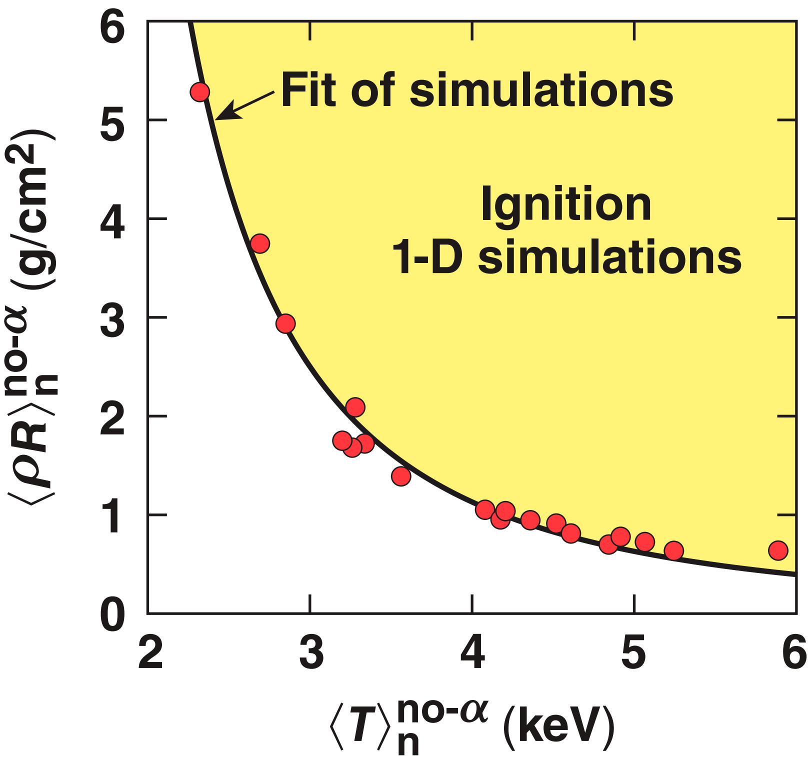
$$T_{\text{st}}^{\text{no-}\alpha} = \left[\frac{0.4\pi}{\kappa_0} (\rho\Delta)_{\text{st}}^{\text{no-}\alpha} V_i^3\right]^{2/7}$$

Measurable ignition condition

$$\gamma_\alpha \sim C_\alpha \left[(\rho\Delta)_{\text{st}}^{\text{no-}\alpha}\right]^{2/3} \left(T_{\text{st}}^{\text{no-}\alpha}\right)^{13/6} > 1.2$$

$$(\rho\Delta)_{\text{st}} \approx \rho R_{\text{total}}$$

Verify that the theory is right. Simulate many marginally igniting targets and plot the total areal density versus the ion temperature



- Simulation database
  - marginal ignited targets (Gain = 1)
  - laser energy 35 kJ to 10 MJ
  - implosion velocity 170 to 530 km/s

$\langle \rho R \rangle_n^{no\text{-}\alpha}$ = neutron-averaged total areal density

$\langle T_i \rangle_n^{no\text{-}\alpha}$ = neutron-averaged ion temperature

1-D simulations show that all marginally ignited targets lie on a single curve in the $\rho R^{no\text{-}\alpha}, T_i^{no\text{-}\alpha}$ plane.

TC8234a

C. D. Zhou and R. Betti, Phys. Plasmas 15, 102707 (2008).

The 1-D Lawson criterion can be written in terms of the measurable parameters ρR_{total}, T_i

- Simple model (this talk)

$$\boxed{(\rho R)_{\text{st}}^{\text{no-}\alpha} \left(T_{\text{st}}^{\text{no-}\alpha}\right)^{39/12} > \text{const} \big/ C_{\alpha}^{3/2}}$$

- Include more physics: finite shell thickness

$$\boxed{(\rho R)_{\text{st}}^{\text{no-}\alpha} \left(T_{\text{st}}^{\text{no-}\alpha}\right)^{5/2} > 40\,\text{g/cm}^2\ \text{keV}^{5/2}}$$

- Include radiation transport (analytic model)*

$$(\rho R)_{\text{st}}^{\text{no-}\alpha} \left(T_{\text{st}}^{\text{no-}\alpha}\right)^{5/4} \approx \frac{2.5}{\underbrace{1-\left(3 \big/ T_{\text{st}}^{\text{no-}\alpha}\right)^{5/2}}_{\text{Bremmsstrahlung pole}}}$$

T in keV
ρR in g/cm^2

TC8549a

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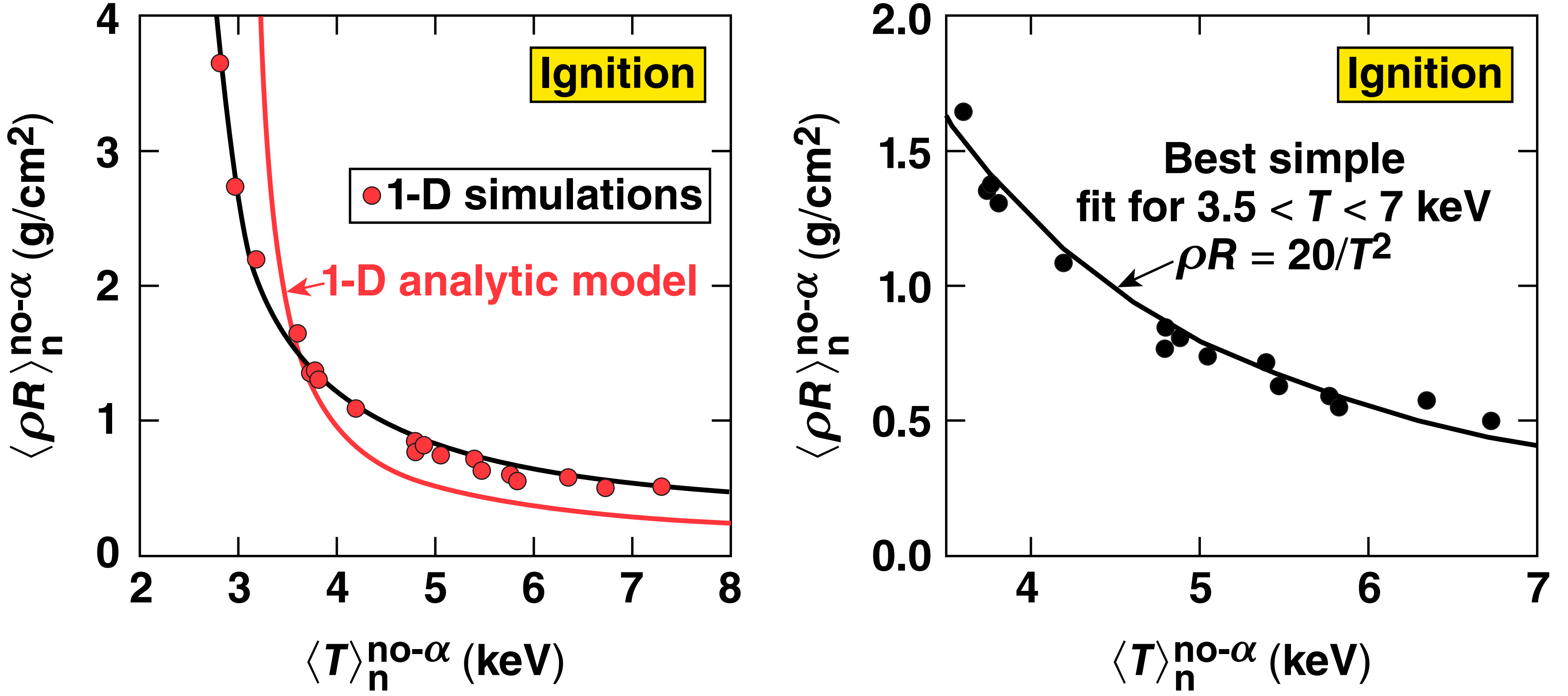
*C. D. Zhou and R. Betti, Phys. Plasmas 15, 102707 (2008).

The analytic model agrees reasonably well with the simulations; the latter can be accurately fit by a simple power law $\rho R \times T^2 > 20$ for $3.5 < T < 7$ keV



Ignition condition

1-D ignition parameter ➡

$$\boxed{\chi_{\text{1-D}} \equiv \left\langle \rho R^{\text{no-}\alpha}_{\text{g/cm}^2} \right\rangle_n \left(\frac{\left\langle T^{\text{no-}\alpha}_{\text{keV}} \right\rangle_n}{4.6} \right)^2 > 1}$$

$$3.5 < T^{\text{no-}\alpha}_{\text{keV}} < 7$$

TC8634

When can $T^{\text{no-}\alpha}$, $\rho R^{\text{no-}\alpha}$ be measured and the Lawson criterion be assessed?

- Surrogate D_2 implosions[1]
- Surrogate THD implosions[2]
- DT implosions on OMEGA[3,4]
- DT implosions on the NIF with Gain < 0.1

For all of the above $T \approx T^{\text{no-}\alpha}$, $\rho R \approx \rho R^{\text{no-}\alpha}$.
Use the measurable Lawson criterion.

- DT implosions on the NIF with $0.1 <$ Gain < 1.0: $T > T^{\text{no-}\alpha}$
- DT implosions on the NIF with Gain > 1:
  $T > T^{\text{no-}\alpha}$ and $\rho R < \rho R^{\text{no-}\alpha}$

For those, no need for a Lawson criterion.
Just look at the neutron detector.

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[1] T. C. Sangster *et al.*, Phys. Rev. Lett. 100, 185006 (2008).
[2] B. Spears (UO5.00013).
[3] V. N. Goncharov (UO5.00002).
[4] T. C. Sangster (NI2.00002).

Outline

- A 1-D measurable Lawson criterion for ICF
- Hydro-equivalent curves and hydro-equivalent ignition
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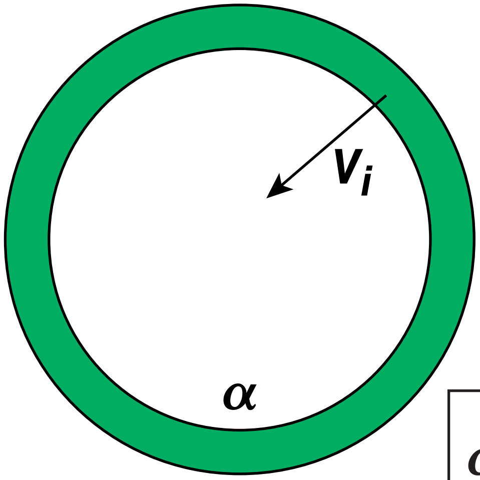
Hydro-equivalent implosions have the same implosion velocity, in-flight adiabat, and laser intensity leading to equal stagnation density and pressure



α = shell adiabat $\equiv \dfrac{P}{P_F} \Rightarrow$ measure the entropy

$$P_{\text{st}}\,(\text{Gbar}) \approx \frac{350}{\alpha^{0.9}}\left[\frac{V_i\,(\text{km/s})}{300}\right]^2 \qquad \rho_{\text{st}}^{\max}\,(\text{g/cc}) \approx \frac{780}{\alpha}\left[\frac{V_i\,(\text{km/s})}{300}\right] I_{15}^{0.13}$$

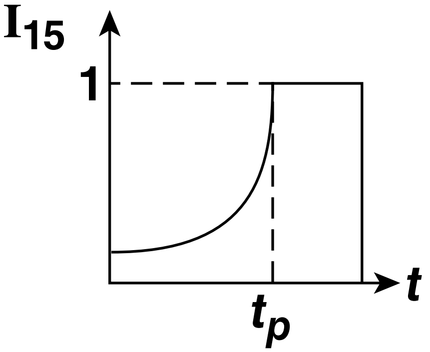
- In hydro-equivalent targets, mass and size depend on laser energy

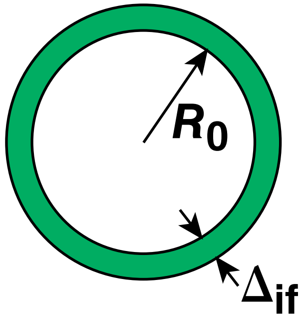
Mass ~ E_L Radius = $R \sim E_L^{1/3}$ Thickness = $\Delta \sim E_L^{1/3}$

Hydro-equivalent implosions have similar Rayleigh–Taylor growth factors and nonlinear bubble-front penetration

- Hydro-equivalent implosions have the same in-flight aspect ratio (IFAR)*

$$\frac{R_0}{\Delta_{if}} \equiv \mathrm{IFAR} \approx \frac{40}{\alpha^{0.6}}\left[\frac{V_i\,(\mathrm{km/s})}{300}\right]^2 I_{15}^{-1/4}$$





- Hydro-equivalent implosions have a similar linear growth factor

$$\eta = \eta_0 e^{\gamma t} \qquad \gamma t \sim \sqrt{k\,\underset{\substack{\downarrow\\ \sim R}}{\boxed{gt^2}}} \sim \sqrt{\underset{\substack{\downarrow\\ \sim 1}}{k\Delta_{if}}\,\frac{R}{\Delta_{if}}} \sim \sqrt{\frac{R}{\Delta_{if}}} \sim \sqrt{\mathrm{IFAR}}$$

- Hydro-equivalent implosions have a similar nonlinear growth

$$\frac{h_{\mathrm{bubble}}}{\Delta_{if}} \sim \frac{\beta\,gt^2}{\Delta_{if}} \sim \frac{\beta R}{\Delta_{if}} \sim \underset{\substack{\downarrow\\ \boxed{0.05}}}{\beta} \cdot \mathrm{IFAR}$$

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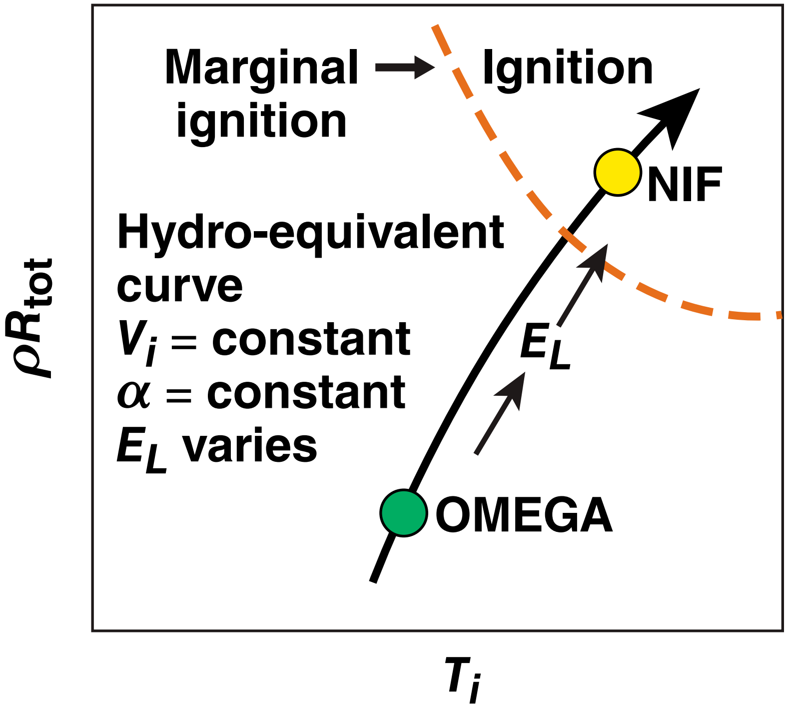
*J. D. Lindl, "Inertial Confinement Fusion: The Quest for Ignition and Energy Gain Using Indirect Drive (Springer-Verlag, New York, 1998), Chap 6, p. 61.

Total areal densities and ion temperatures increase as hydro-equivalent implosions are scaled up in energy

$$\rho R_{\text{tot}} \sim \frac{V_i^{0.06}}{\alpha^{0.6}} E_L^{1/3}$$

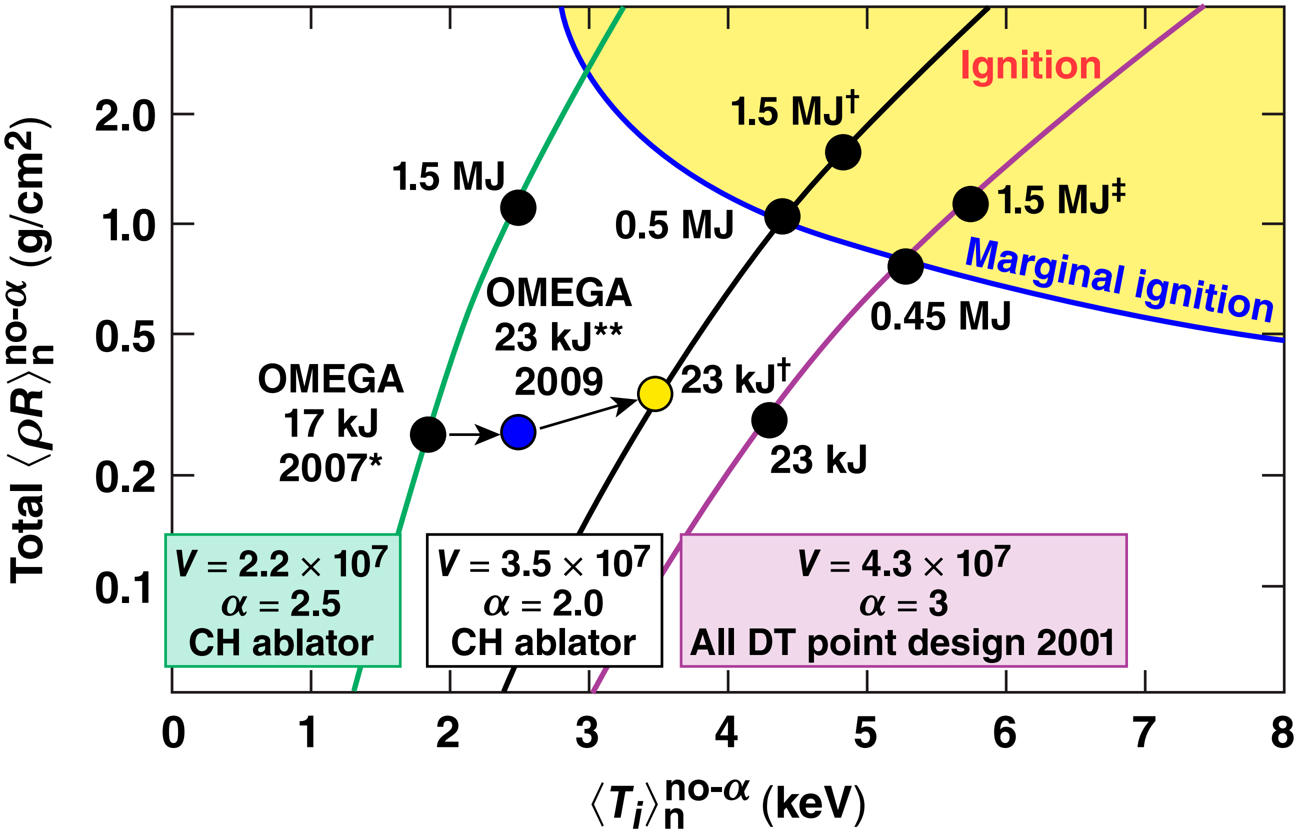
$$T_i \sim \frac{V_i^{1.25}}{\alpha^{0.15}} E_L^{0.07}$$

C. D. Zhou and R. Betti, Phys. Plasmas 14, 072703 (2007).



Hydro-equivalent curves show how OMEGA implosions would perform (in 1-D) when scaled up in energy.

The progress in direct-drive ICF can be assessed on the ρR, T_i plane through hydro-equivalent curves; OMEGA's goal is to achieve hydro-equivalent ignition



This diagram measures progress in 1-D compression; it does not include a good measure of implosion uniformity.

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* T. C. Sangster *et al.*, Phys. Rev. Lett. 100, 185006 (2008).

** T. C. Sangster (NI2.00002).

† V. N. Goncharov (UO5.00002).

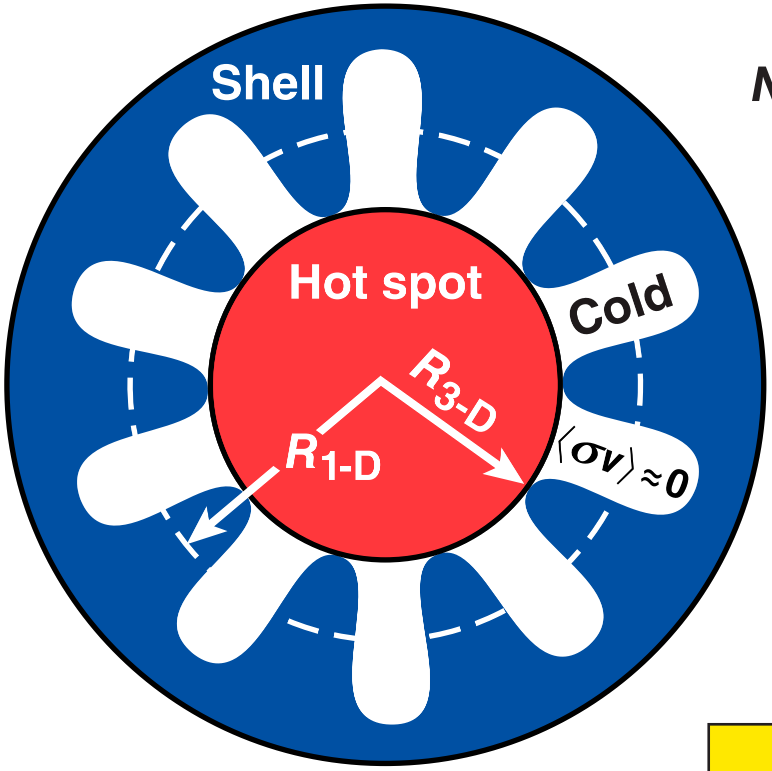
‡ P. W. McKenty *et al.*, Phys. Plasmas 8, 2315 (2001).

TC8638

Outline

- A 1-D measurable Lawson criterion for ICF
- Hydro-equivalent curves and hydro-equivalent ignition
- The 3-D extension of the Lawson criterion
- Comparison with magnetic confinement

In 3-D the fusion yield is reduced by the Rayleigh–Taylor instability that cools down parts of the hot spot



$$V_{3\text{-D}} \sim R^3_{3\text{-D}} < V_{1\text{-D}} \sim R^3_{1\text{-D}}$$

$$N^{3\text{-D}}_{\text{neutron}} \sim n_i^2 \langle \sigma v \rangle V_{3\text{-D}}\, \tau_{\text{burn}} \sim N^{1\text{-D}}_{\text{neutron}} \frac{V_{3\text{-D}}}{V_{1\text{-D}}}$$

- The yield-over-clean YOC = 3-D fusion yield; 1-D yield is approximately equal to the ratio unmixed volume/1-D volume

Can be measured

$$\text{YOC} \equiv \frac{N^{3\text{-D}}_{\text{neutron}}}{N^{1\text{-D}}_{\text{neutron}}} \approx \frac{V_{3\text{-D}}}{V_{1\text{-D}}}$$

YOC without α-deposition $\text{YOC}^{\text{no-}\alpha}$

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*R. Kishony and D. Shvarts, Phys. Plasmas 8, 4925 (2001).

The YOC is used to extend the measurable Lawson criterion to three dimensions

Back to the 1-D Lawson criterion

$$\boxed{(\rho R)_{\mathrm{st}}^{\mathrm{no}\text{-}\alpha}\left(T_{\mathrm{st}}^{\mathrm{no}\text{-}\alpha}\right)^{39/12} > \mathrm{const} \Big/ C_{\alpha}^{3/2}}$$

$$C_{\alpha} \sim \int_{V} \langle \sigma v \rangle \, dV$$

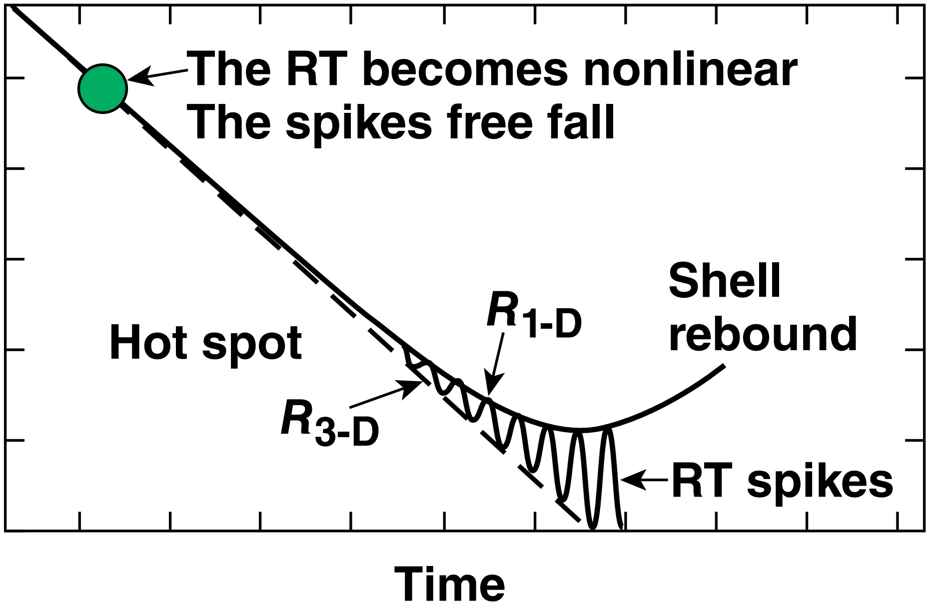
$$\boxed{C_{\alpha}^{\text{3-D}} \sim \int_{V_{\text{3-D}}} \langle \sigma v \rangle \, dV \approx C_{\alpha}^{\text{1-D}} \frac{V_{\text{3-D}}}{V_{\text{1-D}}} \approx C_{\alpha}^{\text{1-D}} \bullet \mathrm{YOC}^{\mathrm{no}\text{-}\alpha}}$$

3-D measurable Lawson criterion

$$\boxed{(\rho R)_{\mathrm{st}}^{\mathrm{no}\text{-}\alpha}\left(T_{\mathrm{st}}^{\mathrm{no}\text{-}\alpha}\right)^{39/12}\left(\mathrm{YOC}^{\mathrm{no}\text{-}\alpha}\right)^{3/2} > \mathrm{const}}$$

An analytic model based on the free-fall lines provides a quantitative estimate of the 3-D Lawson criterion*

R = shell trajectory



- **Analytic 3-D measurable Lawson criterion**

$$\chi_{3\text{-D}}^{\text{analytic}} \equiv (\rho R)_{\text{st}}^{\text{no-}\alpha}\left(\frac{T_{\text{st}}^{\text{no-}\alpha}}{4.6}\right)^{5/2} \quad \text{YOC}^{\text{no-}\alpha} > 1$$

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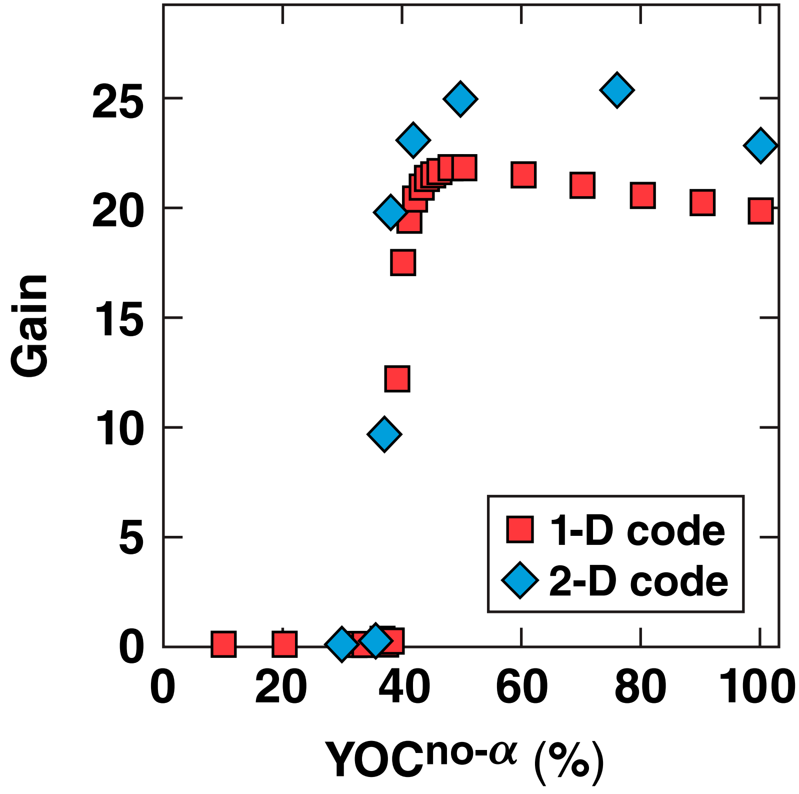
*P. Chang (TO5.00004).

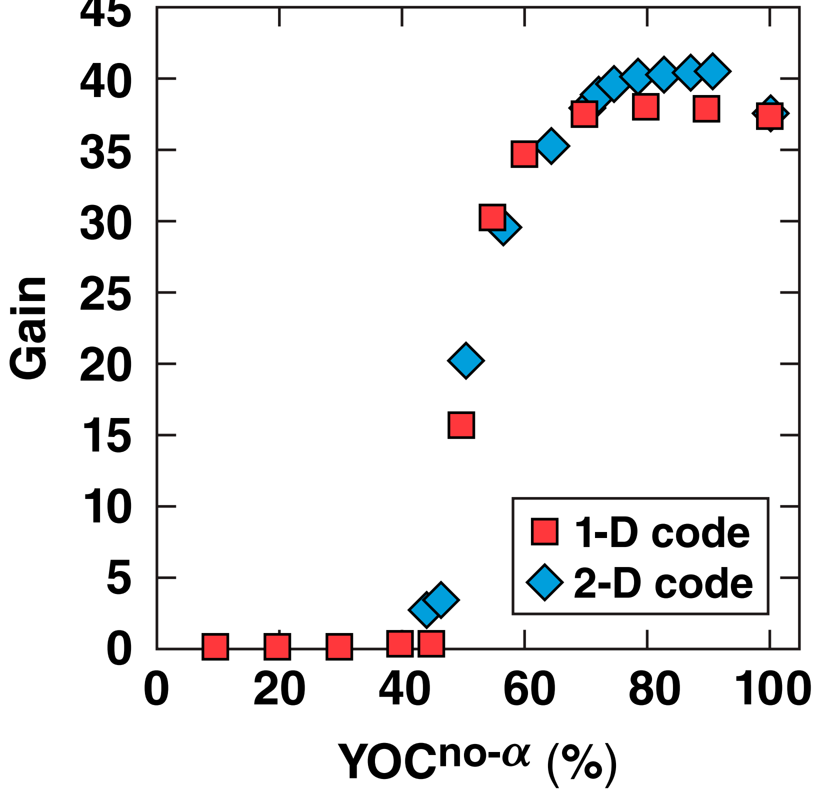
The clean volume analysis is validated by comparing 2-D simulations with inner-surface roughness and 1-D simulations having reduced $\langle\sigma v\rangle\rightarrow\langle\sigma v\rangle V_{3\text{-D}}/V_{1\text{-D}} \approx \langle\sigma v\rangle \mathrm{YOC}^{\text{no-}\alpha}$

- In the 1-D simulations, $\langle\sigma v\rangle$ is reduced by the YOC (or clean volume fraction) until the hot-spot temperature reaches 10 keV.

Direct-drive surrogate of NIF 1.0-MJ indirect drive

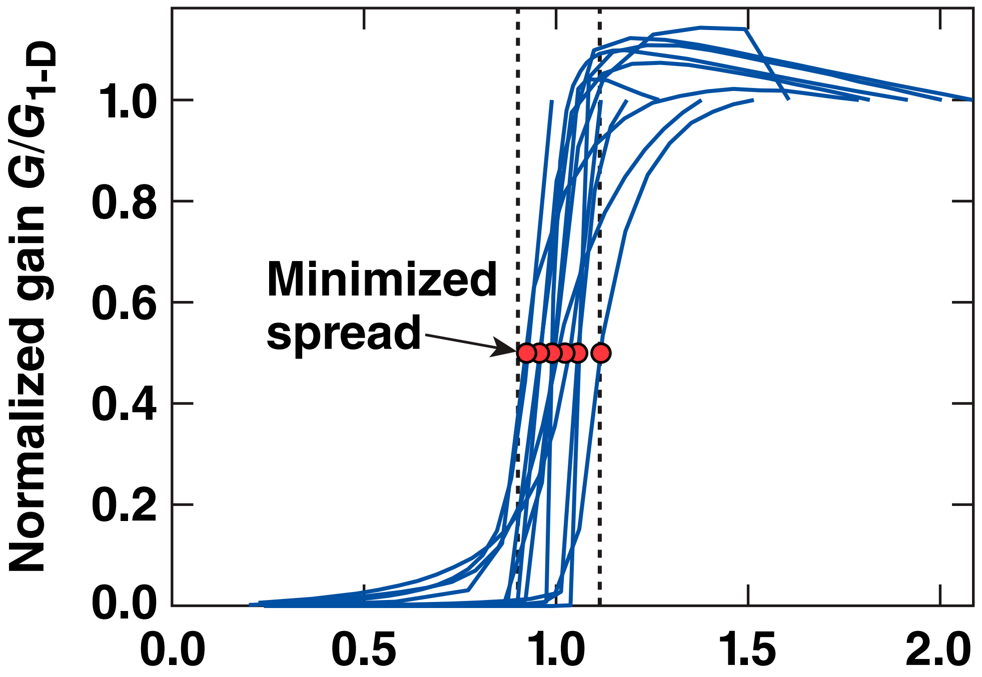


Direct-drive 1.5-MJ all-DT target design



K. S. Anderson (UO5.00004).

Results from a 2-D + pseudo 2-D simulation database are in reasonable agreement with the ignition model



$$\chi = (\rho R)_{\text{st}}^{\text{no-}\alpha} \left(\frac{T_{\text{st}}^{\text{no-}\alpha}}{4.7} \right)^2 \left(\text{YOC}^{\text{no-}\alpha} \right)^{0.7}$$

- 3-D measurable Lawson criterion (fit from simulations)

$$\chi_{3\text{-D}}^{\text{fit}} = (\rho R)_{\text{st}}^{\text{no-}\alpha} \left(\frac{T_{\text{st}}^{\text{no-}\alpha}}{4.7} \right)^2 \left(\text{YOC}^{\text{no-}\alpha} \right)^{0.7} > 1$$

TC8640

Hydrodynamic scaling laws provide the requirements for an hydro-equivalent demonstration (1:60) of ignition on OMEGA

- Areal density: OMEGA needs to demonstrate $\langle \rho R \rangle \approx 0.3\ \text{g/cm}^2$
- Ion temperature: OMEGA needs to demonstrate $\langle T_i \rangle \approx 3.4\ \text{keV}$
- Yield-over-clean (YOC): the required YOC on OMEGA is difficult to estimate. Use simple clean volume analysis:

$$R_{3\text{-D}} = R_{1\text{-D}} - \underset{\boxed{\text{RT spike amplitude}}}{\underset{\uparrow}{\Delta R_{\text{RT}}}}$$

$$\Delta R_{\text{RT}} \sim \underset{\boxed{\text{Initial seed}}}{\underset{\nearrow}{\sigma_0}}\ \underset{\boxed{\text{Growth factor}}}{\underset{\uparrow}{G_{\text{RT}}}}$$

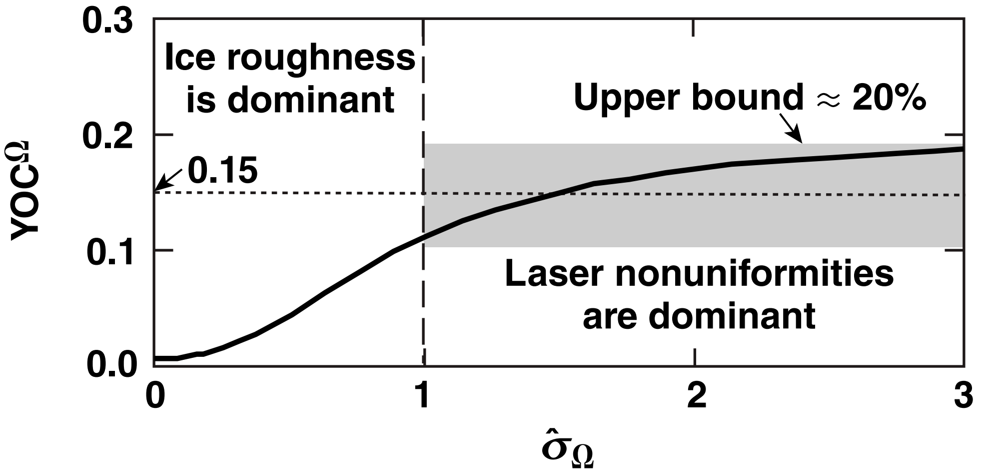
$$\underset{\boxed{\text{Hydro-equivalency}}}{G_{\text{RT}}^{\text{NIF}} \approx G_{\text{RT}}^{\Omega}}$$

$$\text{YOC}^{\Omega} \approx \left[1 - \frac{\sigma_0^{\Omega}}{\sigma_0^{\text{NIF}}} \left(\frac{E_L^{\text{NIF}}}{E_L^{\Omega}} \left(1 - \text{YOC}^{\text{NIF}}\right) \right)^{1/3} \right]^3$$

A simple estimate indicates that OMEGA must achieve YOC's around 15% (weakest point of the theory)

- Seeds for the RT come from the ice roughness and the laser nonuniformities: $\sigma_0 \approx \sqrt{\sigma_{\text{ice}}^2 + \sigma_{\text{laser}}^2}$
- Beta layering makes NIF targets as smooth as OMEGA's: $\sigma_{\text{ice}}^{\Omega} \approx \sigma_{\text{ice}}^{\text{NIF}}$
- Laser nonuniformities grow with size ($E_L{}^{1/3}$) and are reduced by a larger number of overlapping beams (N_b) $\sigma_{\text{laser}} \sim E_L^{1/3} N_b^{-1/2}$

$$\frac{\sigma_0^{\Omega}}{\sigma_0^{\text{NIF}}} \approx \sqrt{\frac{1+\hat{\sigma}_{\Omega}^2}{1+\left(\frac{E_L^{\text{NIF}}}{E_L^{\Omega}}\right)^{2/3}\left(\frac{N_b^{\Omega}}{N_b^{\text{NIF}}}\right)\hat{\sigma}_{\Omega}^2}} \qquad \hat{\sigma}_{\Omega} \equiv \frac{\sigma_L^{\Omega}}{\sigma_{\text{ice}}^{\Omega}}$$



YOC on OMEGA
that extrapolates
to a YOC ≈ 50%
on the NIF

(very approximate!)

OMEGA (DT) and NIF (THD) campaigns aim to achieve an early hydro-equivalent demonstration of ignition (on different scales)

- Where does OMEGA currently stand?

  $\langle \rho R \rangle_n \approx 0.2\ \mathrm{g/cm^2}$ $\langle T_i \rangle_n \approx 2.1\ \mathrm{keV}$ $\mathrm{YOC} \approx 10\%$

  Igniton parameter $\chi = 0.008$ ⇨ $$\boxed{\chi \equiv \rho R \left(\frac{T}{4.7}\right)^2 \mathrm{YOC}^{0.7}}$$

- A scale 1:60 (25 kJ:1.5 MJ) hydro-equivalent demonstration of ignition on OMEGA requires

  $\langle \rho R \rangle_n \approx 0.3\ \mathrm{g/cm^2}$ $\langle T_i \rangle_n \approx 3.4\ \mathrm{keV}$ $\mathrm{YOC} \approx 15\%$

  Ignition parameter $\chi = 0.04$

- An early scale 1:1 demonstration of ignition on the NIF-THD* requires

  $\langle \rho R \rangle_n \approx 1.8\ \mathrm{g/cm^2}$ $\langle T_i \rangle_n \approx 4.8\ \mathrm{keV}$ $\mathrm{YOC} \approx 40\%$

  Ignition parameter $\chi = 1$

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*THD = Tritium–Hydrogen–Deuterium targets, see B. Spears (UO5.00013).

Outline

- A 1-D measurable Lawson criterion for ICF
- Hydro-equivalent curves and hydro-equivalent ignition
- The 3-D extension of the Lawson criterion
- Comparison with magnetic confinement

A confinement time for ICF can be derived from the ignition condition

- Start from the analytic measurable Lawson criterion

$$\frac{\gamma_\alpha \text{YOC}^{4/5}}{1.2} = (\rho R)^{3/4}\left(\frac{T}{4.6}\right)^{15/8} \text{YOC}^{4/5} \equiv \chi^{3/4} > 1$$

- Rewrite γ_α and define a 1-D confinement time

$$\gamma_\alpha \equiv \frac{\varepsilon_\alpha \langle \sigma v \rangle}{24\, T_{st}^2} P_{st}\left(\frac{R_{st}}{V_i}\right) \qquad \tau_E^{\text{1-D}} \equiv \left(\frac{R_{st}}{V_i}\right)$$

Energy conservation

$$\frac{1}{2} M_{sh} V_i^2 = 2\pi\, P_{st}\, R_{st}^3$$

- The 1-D confinement time is the time required to displace the shell by a distance of the order of the hot-spot radius

$$\tau_E^{1\text{-}D} \equiv \frac{R_{st}}{V_i} = \sqrt{\frac{M_{sh}}{4\pi P_{st} R_{st}}}$$

- The 3-D confinement time is reduced by the RT instability. Its degradation is measured through the YOC

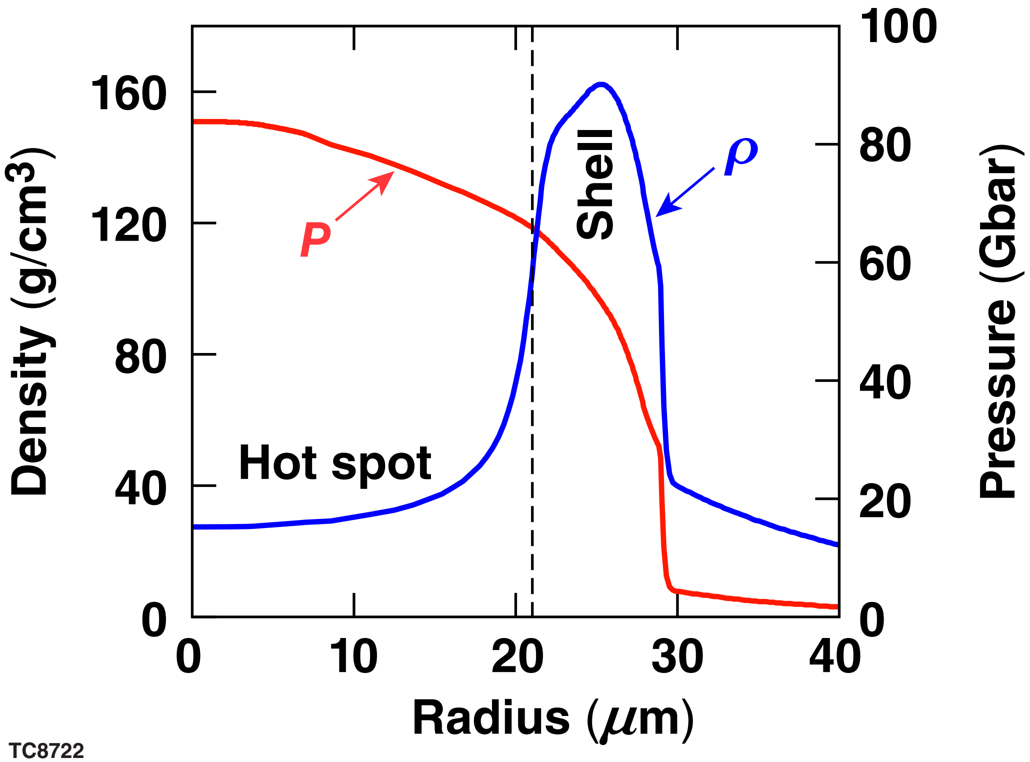
$$\tau_E^{3\text{-}D} = \tau_E^{1\text{-}D} \cdot \text{YOC}^{4/5}$$

Simulations show a confinement time for OMEGA of about 10 ps and a pressure of about 100 Gbar, leading to $P\tau_E \sim 1$ atm × s

- Cryogenic DT implosions: simulation/experimental results

$R_{st}^{sim} \approx 22\ \mu m$ $\quad V_i^{sim} \approx 3 \times 10^7\ cm/s$ $\quad YOC^{exp} \approx 0.1$

Simulation of shot 52729



$$\tau_E \equiv \frac{R_{st}}{V_i} YOC^{4/5} \approx 12\ ps$$

$$P_{st} \approx 80\ Gbar$$

$$\boxed{P_{st}\,\tau_E \approx 1\ atm \times\ s}$$

TC8722

An effective $P\tau_E$ for ICF can be constructed using the ignition model and compared with $P\tau_E$ in MCF

- Define a τ_E from the ignition model

$$\gamma_\alpha \sim P_{\text{stag}}\tau_E \sim \chi^{3/4} \qquad P_{\text{stag}}\,\tau_E \approx \frac{24}{\varepsilon_\alpha}\frac{T^2}{\langle\sigma\nu\rangle}\chi^{3/4}$$

- Use $\langle\sigma v\rangle \sim C_\alpha T^3$ consistent with the analytic model

$$(P\tau_E)_{\text{ICF}} \approx 98\frac{\chi^{3/4}}{\langle T\rangle_n\,(\text{keV})}\text{atm}\times s$$

- OMEGA: $\chi \approx 0.008, T \approx 2.1\ \text{keV} \Rightarrow P\tau_E \approx 1.2\ \text{atm} \times s$

- JET: $P\tau_E \sim 1.2\ \text{atm} \times \text{s}$*

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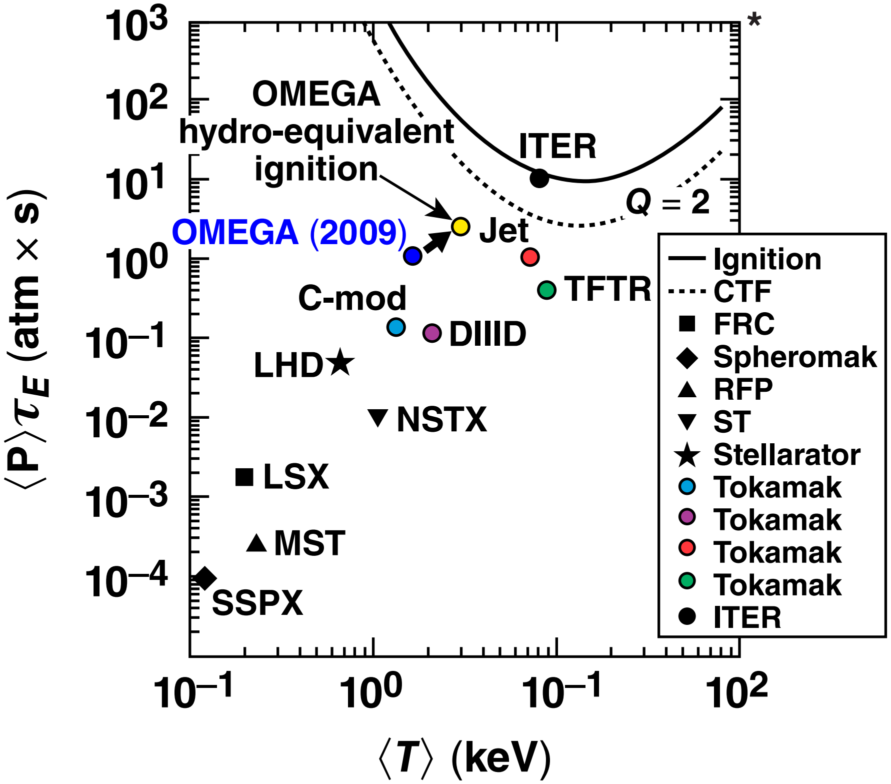
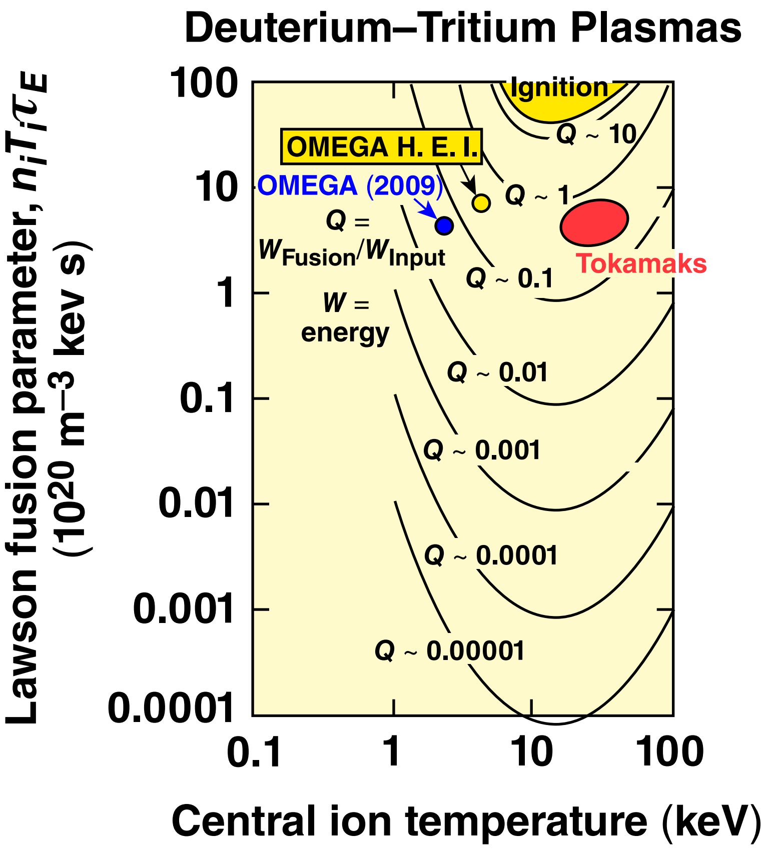
* "Review of Burning Plasma Physics," FESAC Report DOE/SC-0041 (September 2001).

The Lawson plots show the current performance and the future directions for OMEGA





OMEGA hydro-equivalent ignition:**
$\chi \approx 0.04$, $\langle T \rangle \approx 3.4$ keV, $p\tau_E \approx 2.6$ atm × s

How to resolve the discrepancy with the thermonuclear $Q = P_{fus}/P_{input}$?

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*Figure courtesy of J. P. Freidberg (MIT)
**Assuming the YOC prediction is correct

The discrepancy on *Q* is resolved by defining a physics-based thermonuclear *Q*

- "Conventional" hot-spot energy balance

$$W_\alpha + W_{\text{in}} = W_{\text{losses}} = \frac{3}{2}\frac{p}{\tau_E} \qquad W_\alpha\left(1+\frac{5W_{\text{in}}}{W_{\text{fus}}}\right) = W_\alpha\left(1+\frac{5}{Q}\right) = \frac{3}{2}\frac{p}{\tau_E}$$

- Fine $p\tau_E$ from the energy balance

$$p\tau_E = \frac{24}{\varepsilon_\alpha}\frac{T^2}{\langle\sigma\nu\rangle}\frac{Q}{5+Q}$$

- Define a physical thermonuclear *Q* for ICF

$$Q_{\text{ICF}}^{\text{physics}} \neq \frac{W_{\text{fusion}}}{W_{\text{laser}}} \qquad Q_{\text{ICF}}^{\text{physics}} = \frac{W_{\text{fusion}}}{W_{\text{hot spot}}}$$

$$W_{\text{laser}}^{\text{OMEGA}} = 24\ \text{kJ}$$

$$W_{\text{hot spot}}^{\text{OMEGA}} \approx 440\ \text{J}$$

From 1-D simulations

Measured neutron yield: 6×10^{12} ➡ $Q_{\text{OMEGA}}^{\text{physics}} \approx 3.8\%$

For a measured $T = 2.1$ keV, the previous result is recovered

$$p\tau_E = \frac{24}{\varepsilon_\alpha}\frac{T^2}{\langle\sigma\nu\rangle}\frac{Q}{5+Q} \approx 1\ \text{atm} \times s$$

Hydro-equivalent ignition on OMEGA requires $Q \sim 20\%$ to 25% and a neutron yield $\sim 3 \times 10^{13}$

- Hydro-equivalent ignition requires $p\tau_E \approx 2.5$ atm/s with $T \approx 3.4$ keV
- Determine Q from energy balance

$$p\tau_E = \frac{24}{\varepsilon_\alpha}\frac{T^2}{\langle\sigma v\rangle}\frac{Q}{5+Q} \qquad Q \approx 0.25$$

- Determine neutron yield from Q

$$W_{\text{fusion}} \approx 0.25\, W_{\text{hot spot}} \approx 110\text{ J} \Rightarrow 3.8 \times 10^{13}\text{ neutrons}$$

- Check with 1-D simulation of hydro-equivalent design

$$\text{1-D – neutron-yield}^{\text{sim}} \approx 2 \times 10^{14},\ \text{YOC} \approx 15\% \Rightarrow 3 \times 10^{13}\text{ neutrons}$$

- Summary for hydro-equivalent ignition requirements on OMEGA

$$\langle\rho R\rangle \approx 0.3\text{ g/cm}^2 \qquad \langle T_i\rangle_n \approx 3.4\text{ keV} \qquad \text{neutron-yield} \approx 3 \times 10^{13}$$

Summary/Conclusions

The measurable Lawson criterion and hydro-equivalent curves determine the requirements for an early hydro-equivalent demonstration of ignition on OMEGA and on the NIF (THD)

- Cryogenic implosions on OMEGA have achieved a Lawson parameter $P\tau_E \approx$ 1 atm-s comparable to large tokamaks
- Performance requirements for hydroequivalent ignition on OMEGA and NIF (THD)*

Hydro-equivalent ignition	$\langle \rho R \rangle_n$ g/cm^2	$\langle T_i \rangle_n$ keV	YOC	$P\tau_E$ (atm s)
OMEGA (25 kJ)	~0.30	~3.4	15% (~3 × 10^{13} neutrons)	2.6
NIF (THD)	~1.8	~4.7	~40%	20

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*NIF will begin the cryogenic implosion campaign using a surrogate Tritium–Hydrogen–Deuterium (THD) target. Only a very small fraction of Deuterium (<5%) is used to prevent fusion reactions from affecting the hydrodynamics.

See B. Spears (UO5.00013).