MHD Effects in Laser-Produced Plasmas

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Abstract

The implementation of the magneto-hydrodynamic (MHD) module in the arbitrary Lagrange-Eulerian (ALE) hydrocode for laser-plasma simulation *DRACO*[1] is described. The MHD block accounts for convection, diffusion, and generation of the magnetic field by the thermoelectric/magnetic effects caused by the non-parallel temperature and density gradients and the Nernst term. The effect of the magnetic field on the transport coefficients for MHD equations is explicitly taken into account and the influence of the strong magnetic field on hydrodynamics and heating of the laser-imploded plasma pellets are studied.

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[1]P. B. Radha *et al*., Phys. Plasmas 12, 056307 (2005).

Summary

Megagauss magnetic fields are generated in spherical implosions

- Isotropic and anisotropic MHD equations are added to the ALE hydrocode *DRACO*.
- A generation of the megagauss magnetic field for spherical implosions is numerically demonstrated.
- The influence of the magnetic field on transport coefficients is analyzed and the important role of the Nerst term is demonstrated.

Governing equations for isotropic MHD
(no dependence of transport coefficients on the magnetic field)

$$-\frac{1}{c}\frac{\partial \vec{A}}{\partial t} = \vec{E}$$

Ohm's law:

$$\vec{E} = \eta \vec{j} - \frac{\vec{v} \times \vec{B}}{c} - \frac{\vec{\nabla} p_e}{e n_e} + \frac{(\vec{v} - \vec{v}_e) \times \vec{B}}{c}$$

$$\vec{j} = \frac{c}{4\pi} \vec{\nabla} \times \vec{B} = -e n_e (\vec{v} - \vec{v}_e), \quad Z n_i = n_e, \quad (\vec{v} - \vec{v}_e) << \vec{v}$$

$$\frac{\partial \vec{A}}{\partial t} = \vec{v} \times \vec{\nabla} \times \vec{A} - \frac{c^2 \eta}{4\pi} \vec{\nabla} \times \vec{\nabla} \times \vec{A} + \frac{c}{e n_e} \vec{\nabla} p_e$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}) - \vec{\nabla} \times \left(\frac{c^2 \eta}{4\pi} \vec{\nabla} \times \vec{B} \right) + \frac{c}{e} \vec{\nabla} \times \left(\frac{\vec{\nabla} p_e}{n_e} \right)$$

Equation of motion $+\frac{e}{c}[\vec{j} \times \vec{B}]$, **Thermal transport** $+\vec{E} \cdot \vec{j}$

TC8334

A_ϕ/B_ϕ representation of the magnetic field in the cylindrical geometry $\{r,\phi,z\}$ with rotational symmetry

$$\vec{B} = \left\{ -\frac{\partial A_\phi}{\partial z},\ B_\phi,\ \frac{1}{r}\frac{\partial}{\partial r}\left(rA_\phi\right) \right\};$$

Assumption: $\frac{\partial}{\partial \phi} = 0,\ v_\phi = 0$

$$\frac{\partial A_\phi}{\partial t} = -\left[v_r \frac{1}{r}\frac{\partial}{\partial r}\left(rA_\phi\right) + v_z \frac{1}{r}\frac{\partial}{\partial z}\left(rA_\phi\right) \right] + \frac{c^2\eta}{4\pi}\left\{ \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(rA_\phi\right)\right] + \frac{\partial}{\partial z}\left[\frac{1}{r}\frac{\partial}{\partial z}\left(rA_\phi\right)\right] \right\}$$

Advection: **Diffusion:** **Source:**

$$\frac{\partial B_\phi}{\partial t} = -\left[\frac{\partial}{\partial r}\left(v_r B_\phi\right) + \frac{\partial}{\partial z}\left(v_z B_\phi\right) \right] + \left\{ \frac{\partial}{\partial r}\left[\frac{c^2\eta}{4\pi}\frac{1}{r}\frac{\partial}{\partial r}\left(rB_\phi\right)\right] + \frac{\partial}{\partial z}\left[\frac{c^2\eta}{4\pi}\frac{1}{r}\frac{\partial}{\partial z}\left(rB_\phi\right)\right] \right\} + \frac{c}{e}\left[\frac{\partial}{\partial z}\left(\frac{1}{n}\frac{\partial p}{\partial r}\right) - \frac{\partial}{\partial r}\left(\frac{1}{n}\frac{\partial p}{\partial z}\right) \right]$$

- $\vec{\nabla}\cdot\vec{B} \equiv 0$ by construction
- A_ϕ and B_ϕ are evolved independently from each other
- The self-generated magnetic field is azimuthal as the source term goes only in the equation for B_ϕ

When implementing:

The advection, diffusion, and source terms are split from each other

The advection part of the equations for azimuthal magnetic field and vector potential represented as flow derivatives and solved on the moving *DRACO* mesh

Advection contribution:

$$\frac{\partial A_\phi}{\partial t} = -\left[v_r \frac{1}{r}\frac{\partial}{\partial r}\left(rA_\phi\right) + v_z \frac{1}{r}\frac{\partial}{\partial z}\left(rA_\phi\right)\right] \Rightarrow \boxed{\frac{d}{dt}\left(rA_\phi\right) = 0}$$

$$\frac{\partial B_\phi}{\partial t} = -\left[\frac{\partial}{\partial r}\left(v_r B_\phi\right) + \frac{\partial}{\partial z}\left(v_z B_\phi\right)\right] \Rightarrow \boxed{\frac{d}{dt}\left(\frac{B_\phi}{r\rho}\right) = 0}$$

DRACO implementation
A—for cell nodes; B—for cell centers

$$A^{n+1} = A^n \frac{yl^n}{yl^{n+1}} \qquad B^{n+1} = B^n \frac{ycent^{n+1}}{ycent^n}\frac{rho^{n+1}}{rho^n}$$

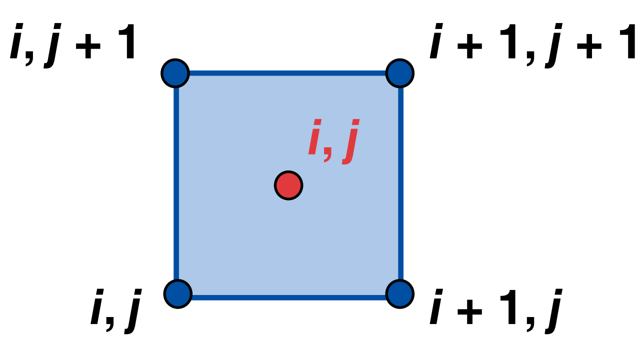
Differential operators are discretized on a non-orthogonal, non-even mesh

Symbolic representation:

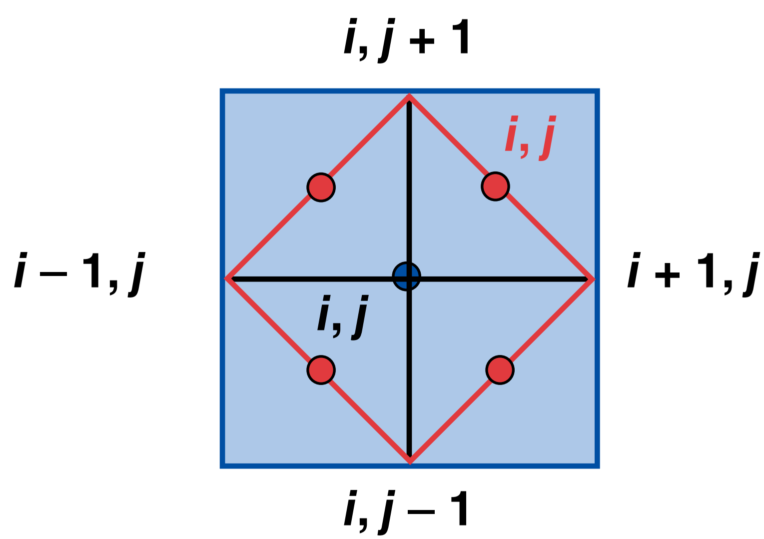
$$\vec{\nabla} = \lim_{s \to 0} \frac{1}{V} \oint_S \vec{n}\, d\vec{S}$$

- Operators CURL, GRAD, and DIV are discretized by the "control volume" approach. They are represented as fluxes through the boundaries of corresponding "control volumes:"

Nodal (*A*) to centered
$\left(d\vec{A}/dr, d\vec{A}/dz\right)$



Centered (*B*) to nodal
$\left(d\vec{B}/dr, d\vec{B}/dz\right)$



TC8337

Implementation of the diffusion and source terms

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Diffusion contribution: $\frac{\partial B_\phi}{\partial t} = -\vec{\nabla} \times \left(D_B \vec{\nabla} \times \vec{B} \right)\Big|_\phi$

1. Subroutine *rotrotB*(*i,j,rrB*) discretizes the operator $\vec{\nabla} \times \left(D_B \vec{\nabla} \times \vec{B} \right)$ on the mesh by the "control volume" approach.
2. Subroutine *Coef*(*CoeB*) calculates the "diagonal" coefficients *CoeB*[*i,j*] at $B_{i,j}$.
3. *B* at the next step are found from an implicit scheme by the "hyperSOR" iterative approach:

$$B_{i,j} = (1-\omega) \cdot B_{i,j} + \omega \cdot \left[B_{i,j}^{\mathrm{old}} - dt \cdot \left(rrB - CoeB_{i,j} \right) B_{i,j} \right] \Big/ \left(1 + dt \cdot CoeB_{i,j} \right)$$

Source contribution: $\frac{\partial B_\phi}{\partial t} = \frac{c}{e} \vec{\nabla} \times \left(\frac{\vec{\nabla} p_e}{n_e} \right)\Bigg|_\phi$

1. Discretized by the modifed "control volume" technique along the contour corresponding to the cell's boundaries.
2. Discretization numerically satisfies $\vec{\nabla} \times \vec{\nabla} f(r,z)\Big|_\phi = 0$ to round off errors.

TC8338

Governing equations for anisotropic MHD
(transport coefficients depend on the magnetic field)

Ohm's law: $\vec{E} = -\frac{\vec{v} \times \vec{B}}{c} - \frac{\vec{\nabla} p_e}{e n_e} + \frac{\vec{R}_T + \vec{R}_j}{e n_e}$

Friction force/Diffusion:

$$\vec{R}_j = \frac{1}{e n_e}\left\{a_{\parallel} \vec{h}\left(\vec{j} \cdot \vec{h}\right) + a_{\perp}(B)\vec{h} \times \left[\vec{j} \times \vec{h}\right] - a_{\wedge}(B)\left[\vec{h} \times \vec{j}\right]\right\}$$

$$\vec{h} = \frac{\vec{B}}{B}$$

Thermal force/"quasi-sources":

Nernst term

$$\vec{R}_T = -\beta_{\parallel}^{uT} \vec{h}\left(\vec{\nabla} T_e \cdot \vec{h}\right) - \beta_{\perp}^{uT}(B)\vec{h} \times \left[\vec{\nabla} T_e \times \vec{h}\right] - \beta_{\wedge}^{uT}(B)\left[\vec{h} \times \vec{\nabla} T_e\right]$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}) + \frac{c}{e}\vec{\nabla} \times \left(\frac{\vec{\nabla} p_e}{n_e}\right) - \frac{c}{e}\vec{\nabla} \times \left(\frac{\vec{R}_T + \vec{R}_j}{n_e}\right)$$

TC8339

The thermal transport equation for anisotropic MHD

$$C\rho \frac{dT}{dt} = -P_e \frac{1}{V}\frac{dV}{dt} - C\rho \frac{2m_e}{m_i}\frac{1}{\tau_e}(T_e - T_{ion}) + Q_e^{L,R,F,CP} - \vec{\nabla}\cdot(\vec{q_T}+\vec{q_j}) + Q_e^{Magn}$$

Thermal heat flux:

$$\vec{q_T} = -\kappa_\parallel \vec{h}(\vec{\nabla}T_e\cdot\vec{h}) - \kappa_\perp(B)\vec{h}\times[\vec{\nabla}T_e\times\vec{h}] - \kappa_\wedge(B)[\vec{h}\times\vec{\nabla}T_e] =$$

$$-\kappa_\perp(B)\vec{\nabla}T_e - \kappa_\wedge(B)[\vec{h}\times\vec{\nabla}T_e] - (\kappa_\parallel-\kappa_\perp)\vec{h}(\vec{\nabla}T_e\cdot\vec{h})$$

Frictional heat flux:

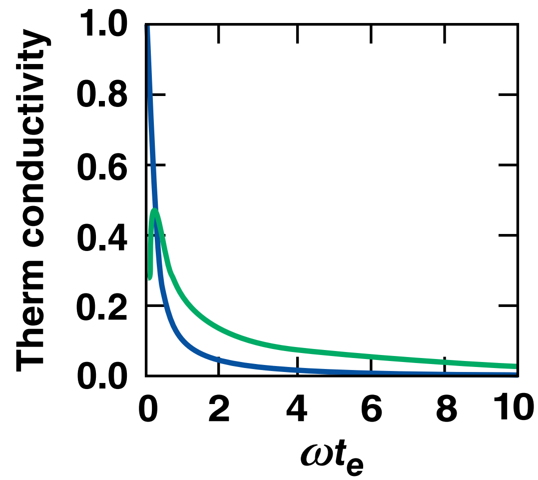
$$\vec{q}_j = -\frac{1}{en_e}\{\beta_\parallel^{Tu}\vec{h}(\vec{j}\cdot\vec{h}) + \beta_\perp^{Tu}(B)\vec{h}\times[\vec{j}\times\vec{h}] + \beta_\wedge^{Tu}(B)[\vec{h}\times\vec{j}]\} =$$

$$-\frac{1}{en_e}\{\beta_\perp^{Tu}(B)\vec{j} + \beta_\wedge^{Tu}(B)[\vec{h}\times\vec{j}] + (\beta_\parallel^{Tu}-\beta_\perp^{Tu})\vec{h}(\vec{j}\cdot\vec{h})\}$$

Joule heating:

$$Q_e^{Magn} = \vec{E}\cdot\vec{j} = -\frac{\vec{\nabla}p_e}{en_e}\cdot\vec{j} + \frac{\vec{R}_T+\vec{R}_j}{en_e}\cdot\vec{j}$$

For $B\to 0$,
$\kappa_\perp \to \kappa_\parallel \equiv \kappa$, and $\kappa_\wedge \to 0$
For $B\to\infty$,
$\kappa_\perp, \kappa_\wedge \to 0$, but $\kappa_\wedge > \kappa_\parallel$



$$\omega = \frac{eB}{mc},$$

τ_e – electron/ion collision time

TC8340

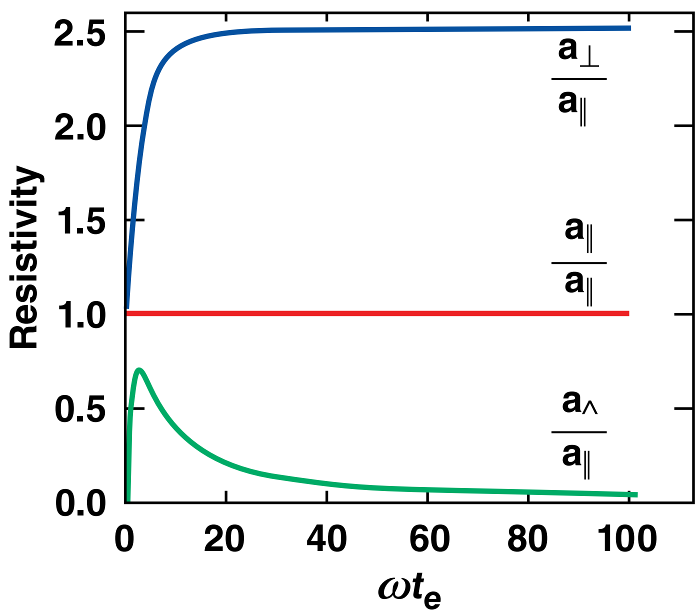
The equation for azimuthal magnetic field B_ϕ self-generated by *gradN* × *gradT* and *Nerst* terms for anisotropic MHD

If $A_\phi\,(t=0)=0 \Rightarrow A_\phi\,(t)=0$, and only B_ϕ is generated

$$\frac{\partial B_\phi}{\partial t} - \vec{\nabla}\times(\vec{v}\times\vec{B})\Big|_\phi - \frac{c}{e}\vec{\nabla}\times\left(\frac{\vec{\nabla}p}{n_e}\right)\Big|_\phi = -\vec{\nabla}\times\left[D(B)_\perp \vec{\nabla}\times\vec{B}\right]\Big|_\phi + \left\{\frac{\partial}{\partial z}\left[D(B)_\wedge h_\phi \frac{1}{r}\frac{\partial}{\partial r}(rB_\phi)\right] - \frac{\partial}{\partial r}\left[D(B)_\wedge h_\phi \frac{1}{r}\frac{\partial}{\partial z}(rB_\phi)\right]\right\} +$$

$$+\vec{\nabla}\times\left[\frac{c}{en_e}\beta(B)_\perp^{uT}\vec{\nabla}T_e\right]\Big|_\phi + \vec{\nabla}\times\left[\frac{c}{en_e}\beta(B)_\wedge^{uT}\left(\vec{h}\times\vec{\nabla}T_e\right)\right]\Big|_\phi$$

Nernst term



Diffusion coefficient: $D_{\perp,\wedge}(B) = \dfrac{c^2 a_{\perp,\wedge}}{4\pi e^2 n_e^2}$

For B_ϕ self-generation, only $D_\perp(B)$ and $\beta(B)_\wedge^{uT}$ are essential

$\omega = \dfrac{eB}{mc}$, τ_e – electron/ion collision time

TC8341

The heat-flux limiters for transverse heat conductivities for the anisotropic case are a generalization of the limiter for the isotropic case

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Isotropic case:

$$q \sim \kappa \left|\vec{\nabla} T_e\right| \sim \frac{n_e T_e \tau_e}{m} \left|\vec{\nabla} T_e\right| \sim \frac{n_e T_e \tau_e}{m} \frac{T}{\lambda_{mfp}} \sim \frac{n_e T_e \tau_e}{m} \frac{T_e}{v_{T_e} \tau_e} \sim \frac{n_e T_e^{3/2}}{\sqrt{m}}$$

Anisotropic case:

$$q_{\perp,\wedge} \sim \kappa(B)_{\perp,\wedge} \left|\vec{\nabla} T_e\right|$$

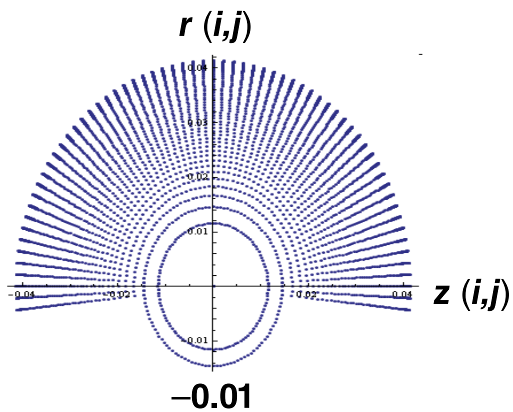
$$q_{\perp,\wedge}^{\lim} \sim f \times \kappa(B)_{\perp,\wedge} T_e \sqrt{\frac{1}{\lambda_{mfp}^2} + \frac{1}{r_B^2}} = f \times \kappa(B)_{\perp,\wedge} \frac{\sqrt{m T_e}}{\tau_e} \sqrt{1 + \tau_e^2 \omega_B^2}$$

$$q = \min\left[q, q^{\lim}\right], \qquad f \sim 0.06$$

TC8342

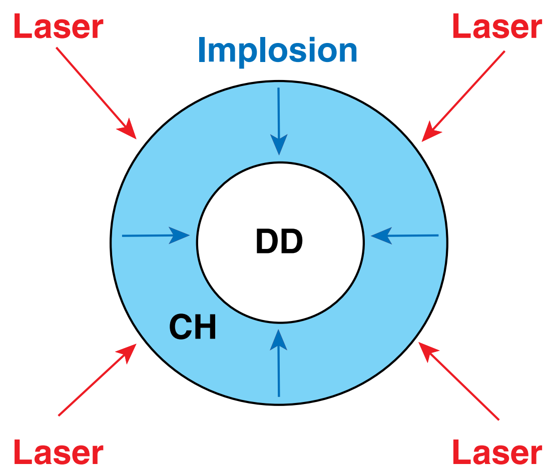
Initial input data for *DRACO*/MHD simulation of the spherical target implosion driven by the square laser pulse

DRACO **mesh**
$t = 0$



i: along target radius
j: along target circumference

Imploding shell



Initialize_grid

layer 1 = 80 i_cells of DD
xlay(1) = 0.0395 cm

layer 2 = 50 i_cells of CH
xlay(2) = 0.0018 cm

j_cells = 100 cells

Simulation_input

Initial_laser_uniformity = "Legendre mode"
mode_num = 4
Laser_ampl_perturb = 1×10^{-2}

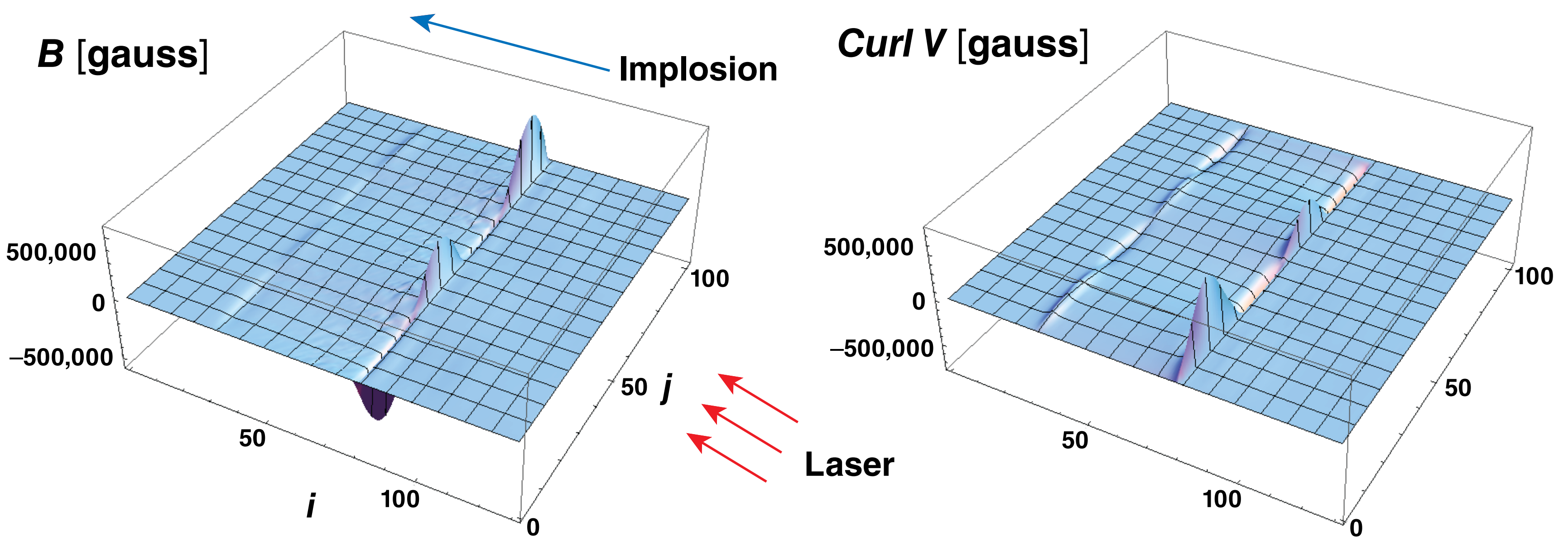
t1 = 0 s	power1 = 0
t2 = $1\,0 \times 10^{-10}$	power2 = 25 TW
t3 = $1\,0 \times 10^{-9}$	power3 = 25 TW
t4 = 1.1×10^{-9}	power4 = 0

TC8343

Vorticity in a hydrodynamic flow of a conducting fluid serves as a good indicator of the presence of a magnetic field

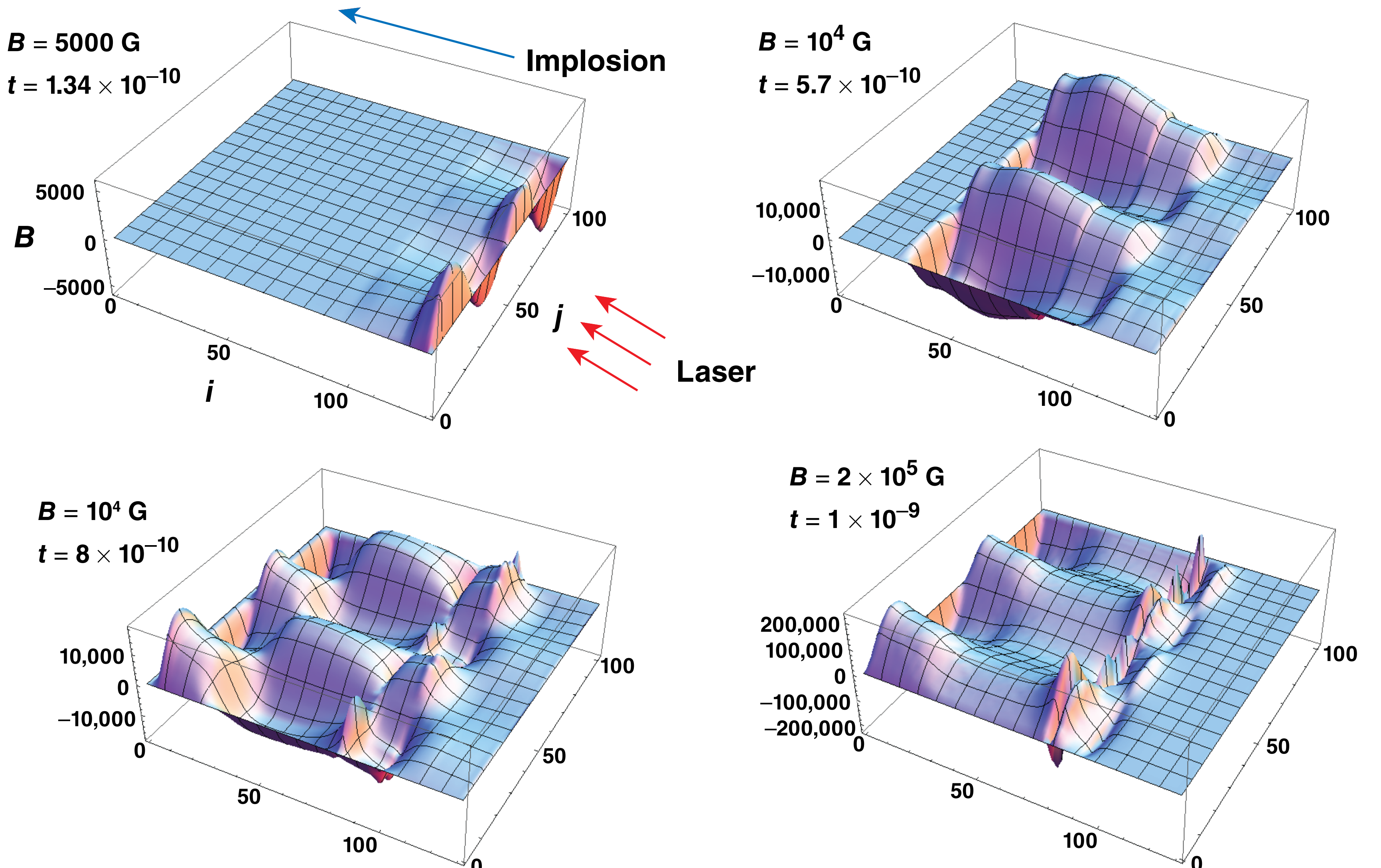
$$\frac{\partial \vec{\mathbf{B}}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{\mathbf{B}}) + \frac{c}{e}\vec{\nabla} \times \left(\frac{\vec{\nabla} p_e}{n_e}\right) \qquad \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{v}) = \vec{\nabla} \times [\vec{v} \times (\vec{\nabla} \times \vec{v})] - \vec{\nabla} \times \left(\frac{\vec{\nabla} p_{tot}}{\rho}\right)$$

$$B \sim -\frac{c}{e}\frac{m_{ion}}{2Z}\vec{\nabla} \times \vec{v}$$



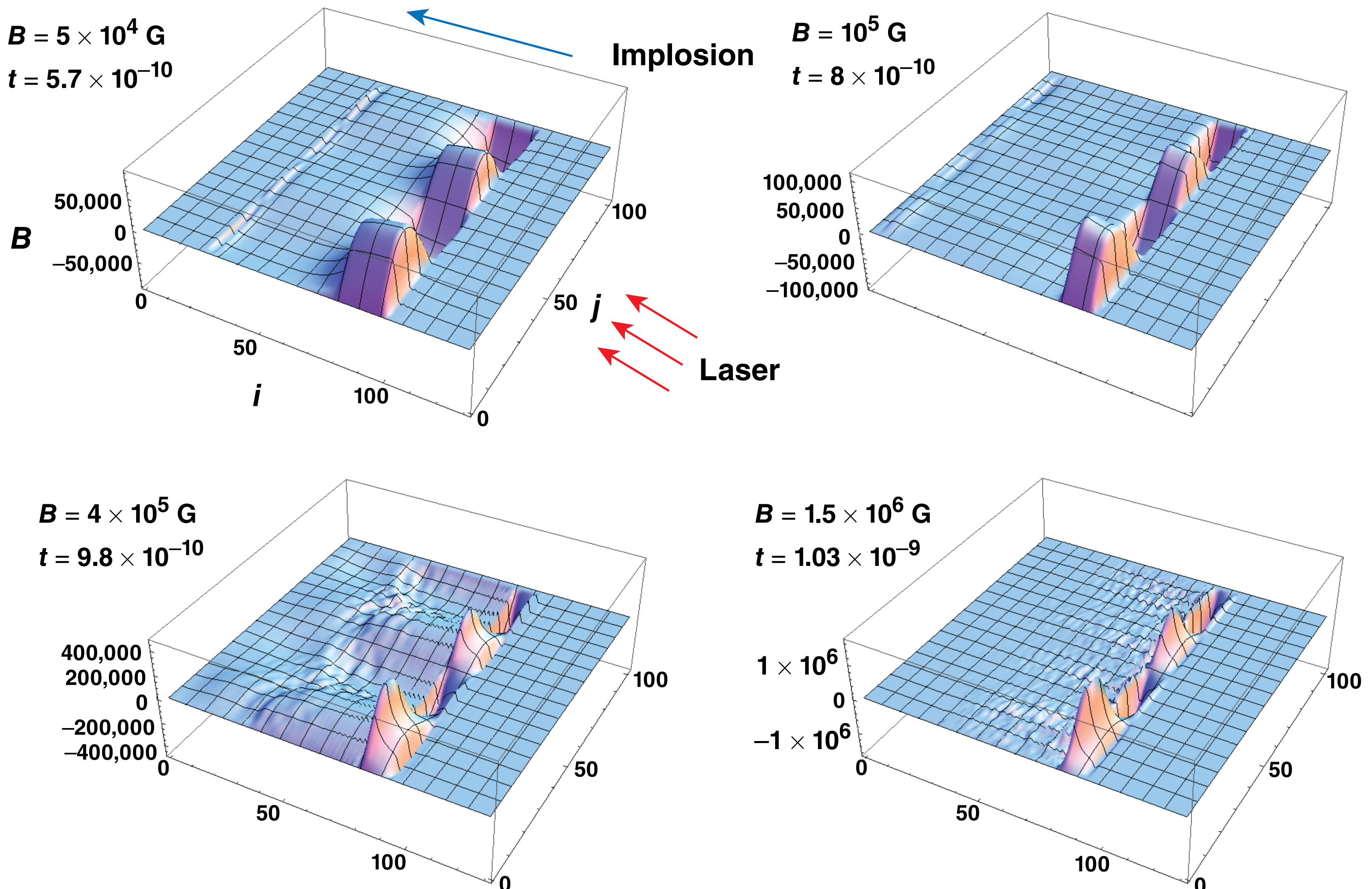
TC8344

Dynamics of self-generated by (*gradN* × *gradT*) azimuthal magnetic field B_ϕ for isotropic MHD



TC8345

Dynamics of self-generated (by *gradN* × *gradT* and *Nernst terms*) azimuthal magnetic field B_ϕ for anisotropic MHD



TC8346

Summary/Conclusions

Megagauss magnetic fields are generated in spherical implosions

- Isotropic and anisotropic MHD equations are added to the ALE hydrocode *DRACO*.
- A generation of the megagauss magnetic field for spherical implosions is numerically demonstrated.
- The influence of the magnetic field on transport coefficients is analyzed and the important role of the Nerst term is demonstrated.