Kinetic and Fluid Models of the Filamentation Instability of Relativistic Electron Beams for Fast-Ignition Conditions

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Summary

Fluid approximations can differ significantly from kinetic results for relativistic beam filamentation

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- Fluid models represent an algebraic approximation to the kinetic plasma-dispersion function.
- This approximation breaks down near threshold (low frequencies) or for calculations of spatial growth or absolute instability.
- Kinetic calculations show that in general the instability has somewhat larger growth rates and extends over a wider range of transverse wave numbers than indicated by fluid approximations.
- Inclusion of mobile ions also tends to extend the range of the instability.

Most fast-ignition scenarios require propagation of a relativistic electron beam through a plasma

- Large-scale beam instabilities (kinking, pinching) develop slowly on the FI timescale.
- Microinstabilities grow faster and include beam–plasma (electrostatic) and filamentation (electromagnetic or mixed) instabilities.
- These instabilities require impedance.
  - reactive (electron inertia, Weibel and beam–plasma instability): dominant at low densities (few $\times$ critical).
  - resistive (collisional, resistive filamentation): dominant at high densities (compressed core).
  - a FI beam will transit both regions (reactive first).
- A fully relativistic treatment of the collisionless case can be carried out analytically; the collisional case is more difficult.

Instabilities can be treated as a perturbed equilibrium provided the growth times are shorter than the beam slowing time

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- Assume that, in equilibrium, the charge densities, currents, and fields vanish and that all perturbed quantities have the space and time dependence $e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$.

- Maxwell's equations relate the current to the perturbed electric field.

$$\mathbf{j}=\mathbf{j}_b+\mathbf{j}_p=-\frac{ic^2}{4\pi\omega}\Bigg(k^2\mathbf{I}\underbrace{-\mathbf{k}\mathbf{k}}_{\substack{\text{Longitudinal}\\\text{(electrostatic)}\\\text{term}}}\underbrace{-\frac{\omega^2}{c^2}\mathbf{I}}_{\substack{\text{From}\\\text{displacement}\\\text{current}}}\Bigg)\cdot\mathbf{E}$$

- The rest of the problem consists of using the plasma properties to derive the perturbed current as a response to *E* (the conductivity tensor).

The collisionless relativistic electron beam can be represented as a Maxwell–Boltzmann–Jüttner (MBJ) distribution

- The MBJ distribution is a relativistic generalization of the Maxwellian

$$f_0(w) = \frac{\xi n_0}{4\pi\Gamma K_2(\xi)} e^{-\xi\Gamma(\gamma - \beta \cdot w)}, \text{ where } \xi \equiv \frac{mc^2}{k_B T_R} = \frac{c^2}{\upsilon_T^2},\ w \equiv \frac{p}{mc} = \gamma \frac{v}{c},$$

β is the average beam β, and $\Gamma \equiv \left(1 - \beta^2\right)^{-1/2}$.

- When the thermal spread is small compared to the beam velocity, this can be approximated as a drifting Maxwellian

$$f_0(p) = \frac{1}{(2\pi)^{3/2}} \frac{1}{(m\nu_T)^3 \Gamma} n_b e^{-\frac{\left(\frac{p_z}{\Gamma} - mc\beta\right)^2 + p_\perp^2}{2(m\upsilon_T)^2}}.$$

- These forms are also used to represent the return current in the collisionless case.

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Generalizing the dispersion relation to complex *k* allows the study of spatial growth and absolute instability

- Roots of the dispersion relation with real *k* and complex ω indicate instability (pure temporal growth), usually convective.

- Start with a temporally growing mode and decrease $\mathrm{Im}(\omega)$; if one of the complex k_z roots crosses the real axis and acquires $\mathrm{Im}(k_z) < 0$ for $\mathrm{Im}(\omega) = 0$, it represents spatial growth.

- Perturbations introduced where the beam originates grow as it propagates into the plasma; spatial growth is most appropriate to the FI problem.

- If two k_z roots merge across the real axis to a double root with $\mathrm{Im}(k_z) < 0$ as $\mathrm{Im}(\omega)$ decreases to 0, absolute instability is indicated. Absolute modes grow at a fixed point independently of the original perturbation amplitudes and so eventually dominate.

Fluid models represent an algebraic approximation to the kinetic plasma-dispersion function $Z(\zeta)$

- For small arguments $Z(\zeta) \cong i\sqrt{\pi}\, e^{-\zeta^2} - 2\zeta\left(1 - 2\frac{\zeta^2}{3} + 4\frac{\zeta^4}{15} - 8\frac{\zeta^6}{105} + \dots\right).$

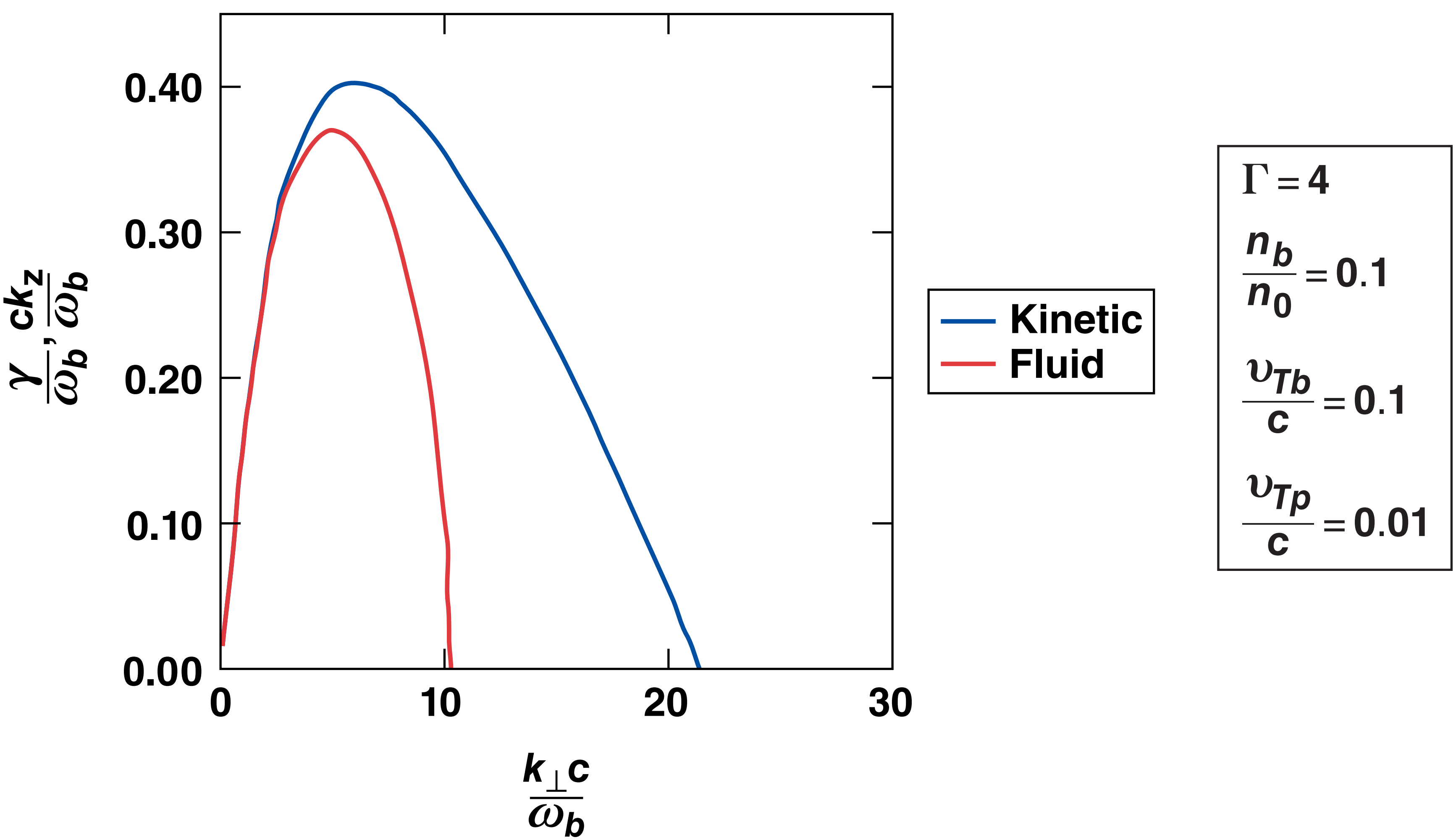
- For large arguments $Z(\zeta) \sim i\sqrt{\pi}\,\sigma e^{-\zeta^2} - \frac{1}{\zeta}\left(1 + \frac{1}{2\zeta^2} + \frac{3}{4\zeta^4} + \frac{15\zeta^6}{8\zeta^6} + \dots\right)$

  where $\sigma = \begin{cases} 0, \mathrm{Im}(\zeta) > 1/\left|\mathrm{Re}\,(\zeta)\right| \\ 1, \mathrm{Im}(\zeta) < 1/\left|\mathrm{Re}\,(\zeta)\right| \\ 2, -\mathrm{Im}(\zeta) > 1/\left|\mathrm{Re}\,(\zeta)\right| \end{cases}$

- $\zeta \equiv \dfrac{\Gamma(\omega - \beta c k_z)}{\sqrt{2}\sqrt{(k_y \upsilon_{T\perp})^2 + (\Gamma k_z \upsilon_{Tz})^2}}$,

  so the asymptotic approximation fails in several instances of interest for the filamentation instability.

Fluid model underpredicts instability at large transverse wave numbers



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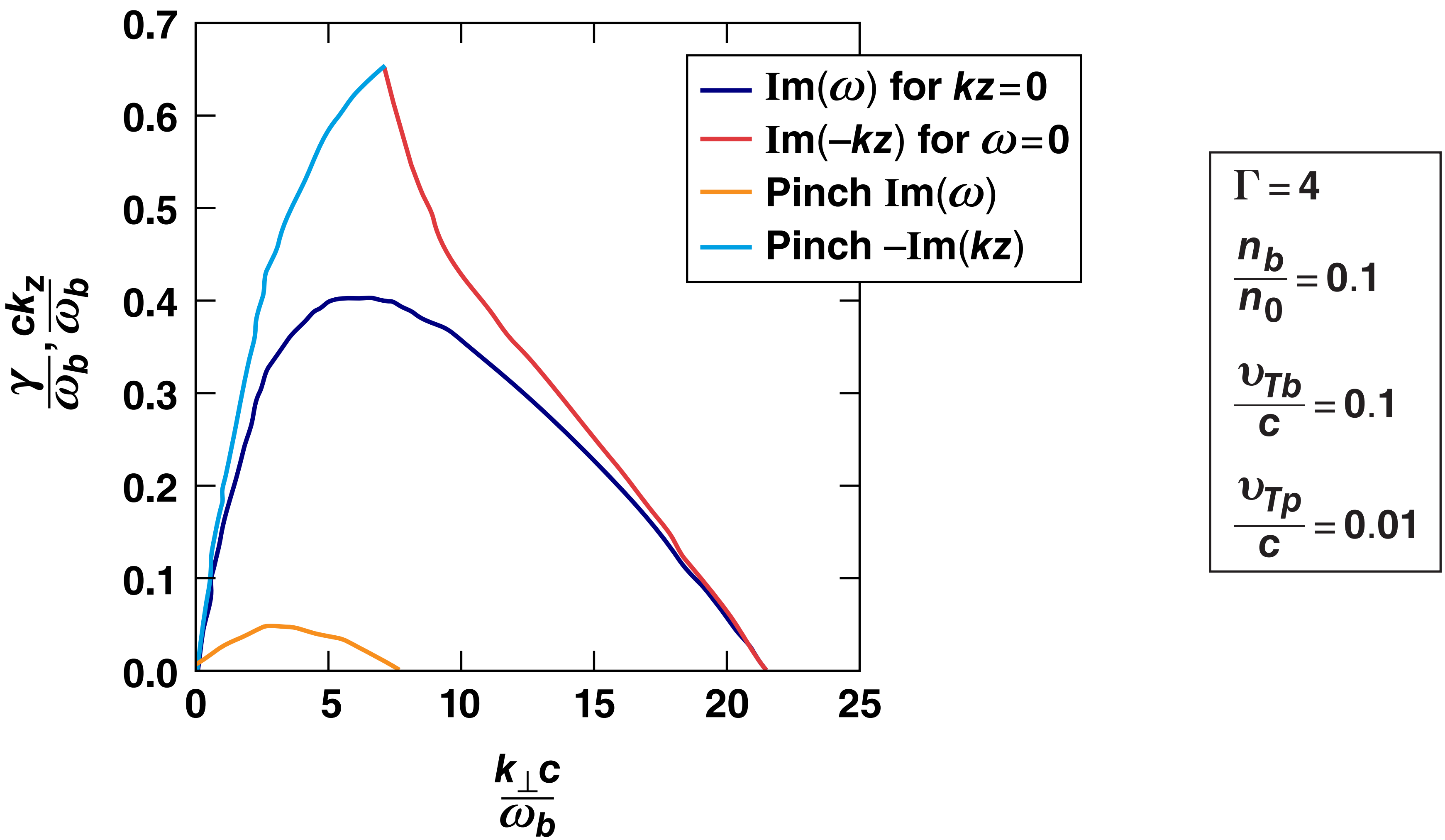
Absolute mode appears only in kinetic model



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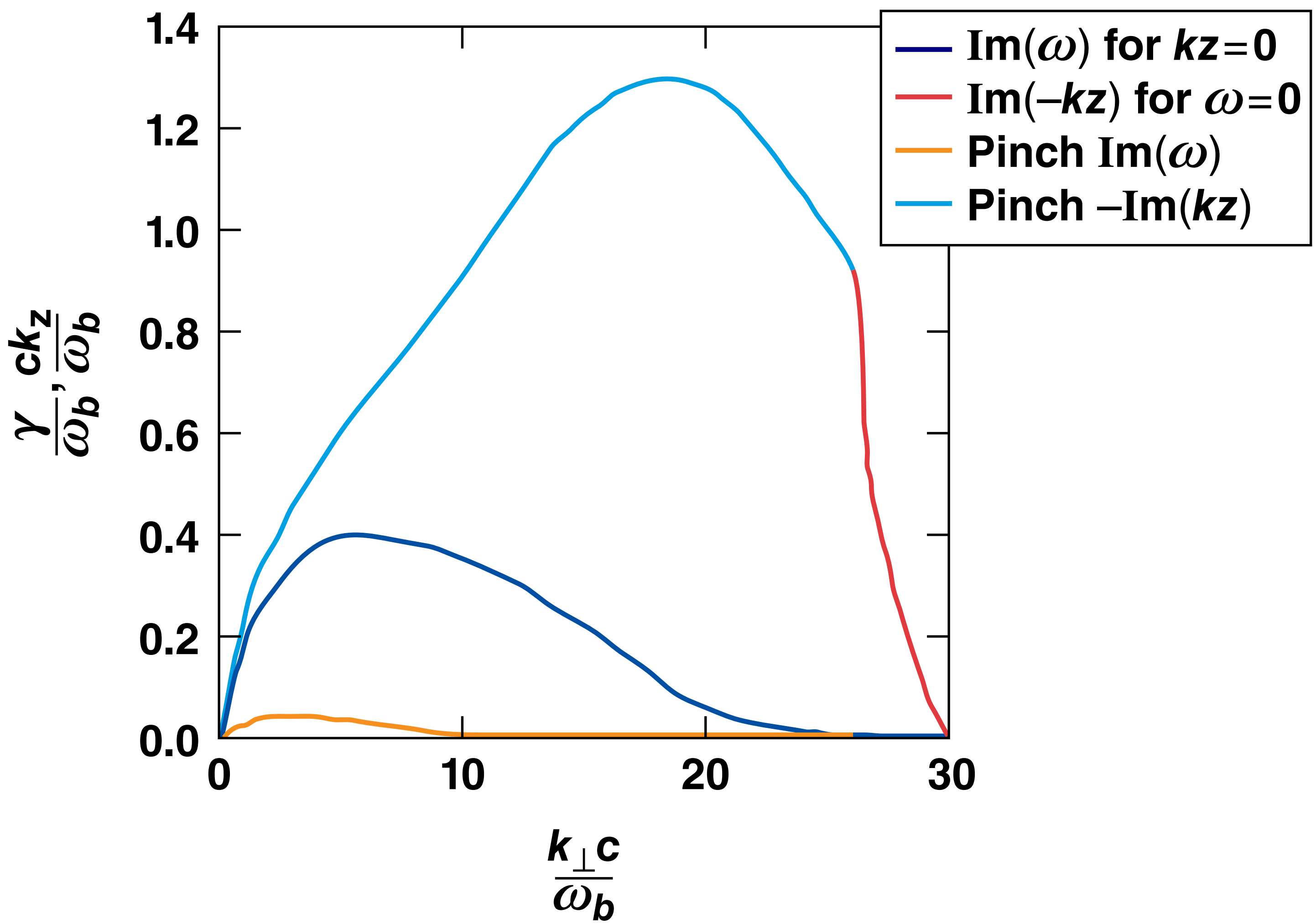
Mobile ions increase the range of instability



$\Gamma = 4$

$\frac{n_b}{n_0} = 0.1$

$\frac{\upsilon_{Tb}}{c} = 0.1$

$\frac{\upsilon_{Tp}}{c} = 0.01$

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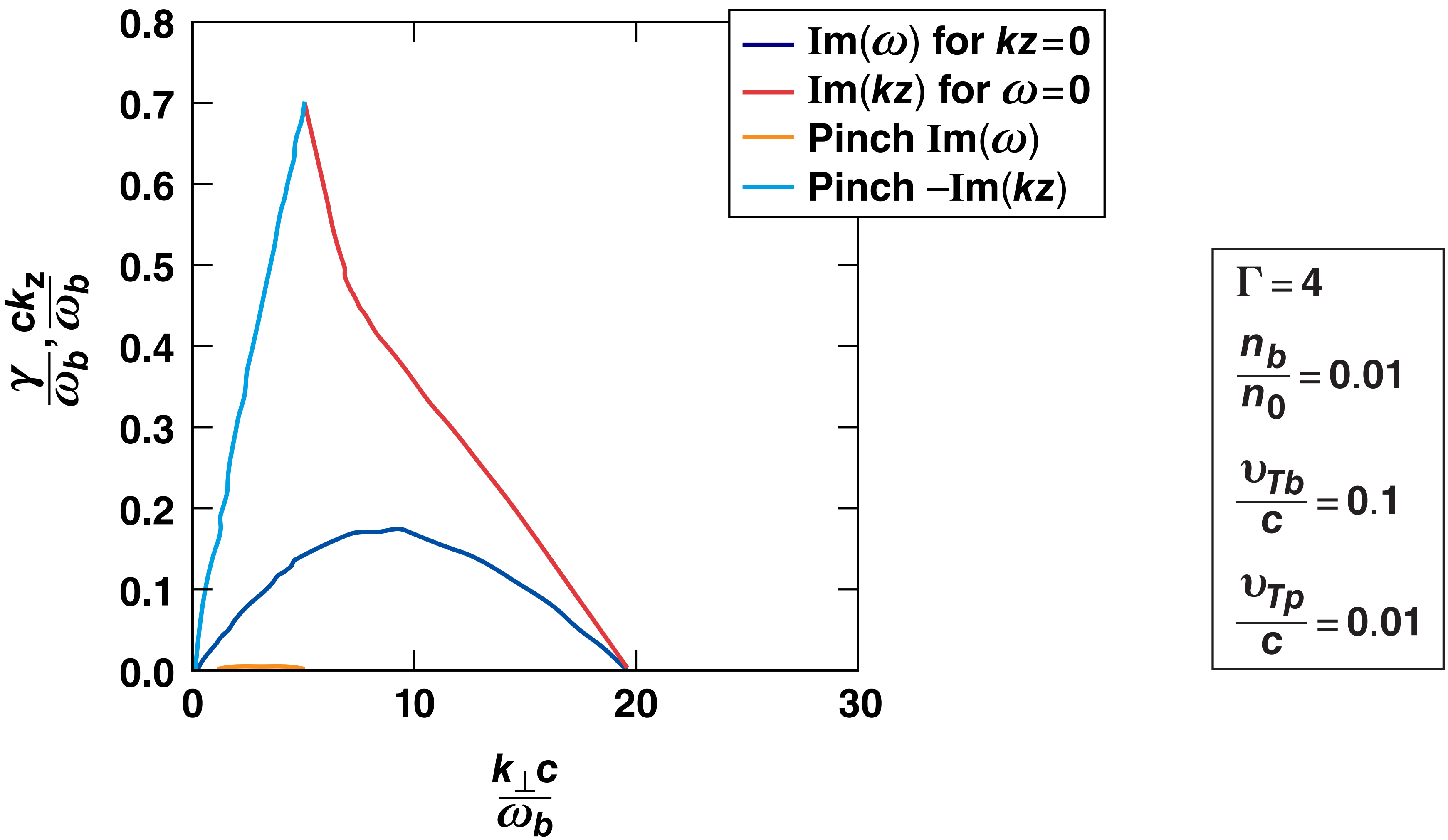
At smaller beam/plasma-density ratios only the kinetic model gives instability



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