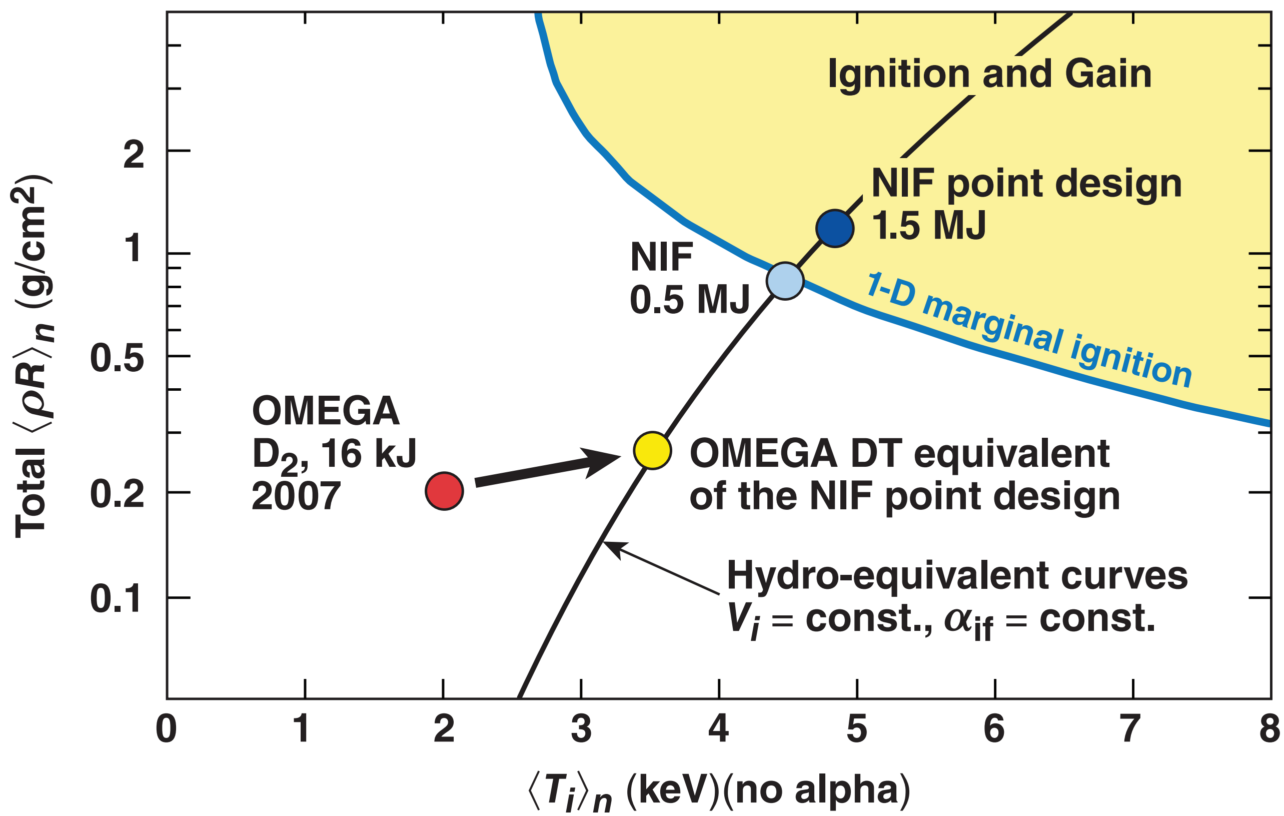
Measurable Lawson Criterion and Hydro-Equivalent Curves for Inertial Confinement Fusion



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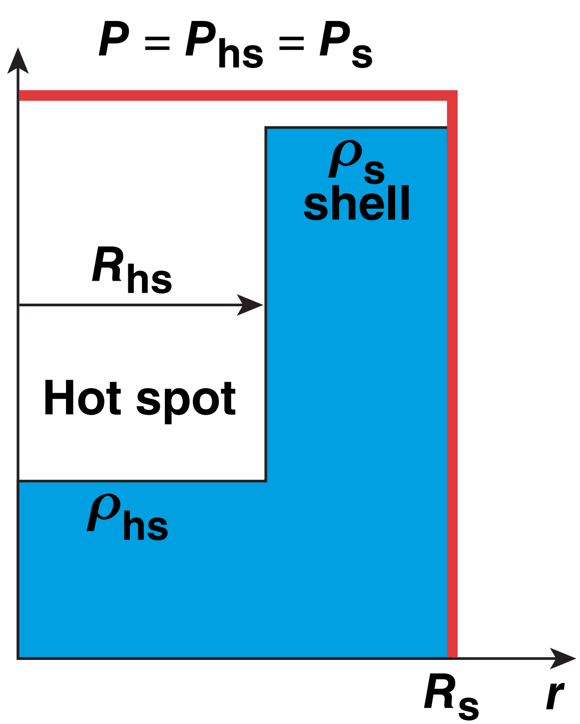
Summary

The ignition condition is derived in terms of two parameters ($\langle T_i \rangle_n$ and total $\langle \rho R \rangle_n$) that can be directly measured in ICF implosions

- This new form of the Lawson's criterion is used to determine how close to ignition OMEGA's cryogenic implosions are currently performing.
- Hydro-equivalent curves in the (ρR, T_i) plane show how current OMEGA implosions perform when scaled up to the NIF.
- The conventional relation hot-spot $\rho R \sim 0.3$ g/cm^2, $T_i \sim 10$ keV does not correctly reproduce the ignition conditions.

The hot-spot ignition condition is given by the balance of alpha heating with the energy losses, including the expansion losses

Isobaric fuel assembly



$$\boxed{\frac{\dot{E}_{hs}}{E_{hs}} = \frac{1}{\tau_\alpha} - \frac{1}{\tau_{rad}} - \frac{1}{\tau_{exp}} > 0}$$

$1/\tau_\alpha \sim n_{hs}^2 \langle \sigma v \rangle / P_{hs}$ — **Alpha heating**

$1/\tau_{rad} \sim n_{hs}^2 \sqrt{T_{hs}} / P_{hs}$ — **Radiation cooling**

$1/\tau_{exp} \sim \sqrt{\ddot{R}_{hs}/R_{hs}}$ — **Expansion**

$$\boxed{M_s \ddot{R}_{hs} = 4\pi P_{hs} R_{hs}^2 \quad \text{shell Newton's law}}$$

TC7595

The expansion losses represent the internal energy lost by the hot spot and transferred to the surrounding dense shell as kinetic energy

$1/\tau_{exp} \sim \sqrt{\ddot{R}_{hs}/R_{hs}}$ Expansion

$M_s \ddot{R}_{hs} = 4\pi P_{hs} R_{hs}^2$ Shell Newton's law

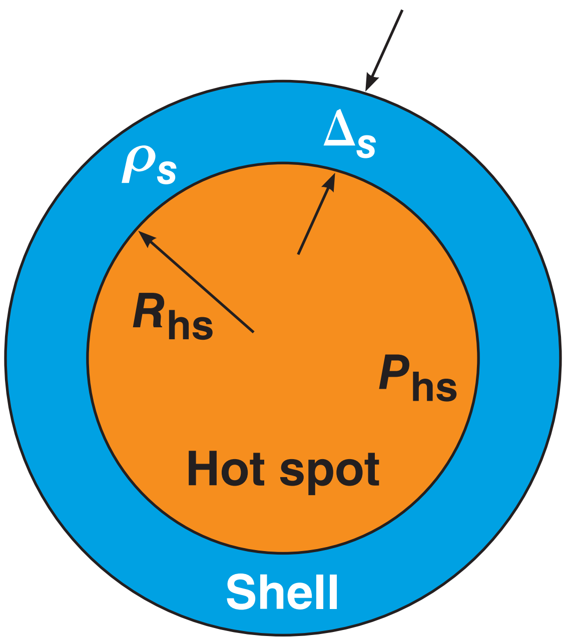
$M_s \sim \rho_s \Delta_s R_s^2$

Δ_s = shell thickness

Shell areal density

$$\frac{1}{\tau_{exp}} \sim \sqrt{\frac{P_{hs} R_{hs}}{M_s}}$$

Shell mass



TC7914

The ignition condition depends on shell areal density, implosion velocity, and hot-spot ion temperature

FSC

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Hot-spot pressure and temperature

$$\frac{P_{hs}}{R_{hs}} M_s f(T_i)^2 > \text{const}$$

Hot-spot radius Shell mass

$$f(T) \sim T_i \left[1 - \left(\frac{T_{brem}}{T_i}\right)^{2.5}\right]$$

for $4 < T_i < 13$ $T_{brem} \leq 4.4$ keV

- $P_{hs} \sim \left(P_{hs} R_s^3\right) / R_s^3 \sim \left(M_s V^2\right) / R_s^3$
- $M_s \sim \rho_s \Delta_s R_s^2$
- $R_s \sim R_{hs}$

$$(\rho_s \Delta_s) V T_i \left[1 - \left(\frac{T_{brem}}{T_i}\right)^{2.5}\right] > \text{const}$$

Shell areal density Implosion velocity

TC7916

Eliminating the velocity and energy leads to an ignition condition depending on shell areal density and hot-spot ion temperature

$$V \sim T_i^{0.8}\, \alpha_{\mathrm{if}}^{0.15}\, E_L^{0.056}$$

$$E_L \sim (\rho_s \Delta_s)^3\, \alpha_{\mathrm{if}}^{1.75}\, V^{-0.18}$$

Hydro-variables scaling from Ref. 1

$$\boxed{(\rho\Delta) T_i^{2.1}\, \cancel{\alpha_{\mathrm{if}}^{0.03}} \left[1 - \left(T_{\mathrm{brem}}/T_i\right)^{2.5}\right]^{1.2} > \mathrm{const}}$$

[1]C. Zhou and R. Betti, Phys. Plasmas 14, 072703 (2007).

The simple scaling relation compares favorably with a set of simulations of marginally ignited capsules

Scaling $\rightarrow$ $$(\rho_s \Delta_s) T_i^{2.1} \left[1 - \left(\frac{T_{\text{brem}}}{T_i} \right)^{2.5} \right]^{1.2} > \text{const}$$

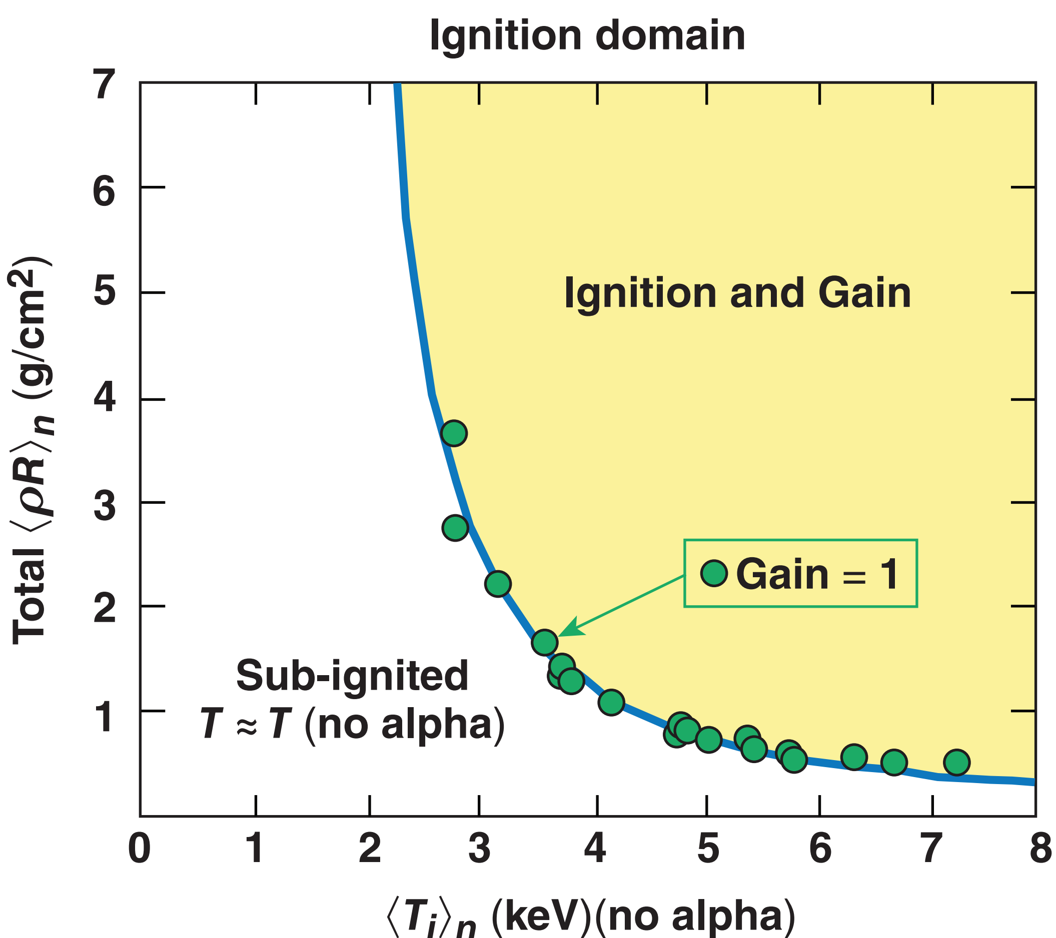
1-D *LILAC* $\rightarrow$ simulations
Gain = 1

$$\langle \rho R \rangle_n \left(\frac{\langle T_i \rangle_n}{4} \right)^{1.4} \left[1 - \left(\frac{2.6}{\langle T_i \rangle_n} \right)^{2.5} \right]^{0.37} > 1 \text{ g/cm}^2$$

- **Simulations are carried out for $1 < \alpha_{\text{if}} < 5$ and $2 \times 10^7 < V_i < 5.5 \times 10^7$ cm/s.**

TC7919

The T_i and total ρR from simulations of marginally ignited capsules lay on a single curve; a measurable Lawson's criterion depends on burn average T_i and total ρR



- 1-D simulations

$\langle T_i \rangle_n^{\text{no alpha}}$ → Neutron-average ion temperature without alpha deposition

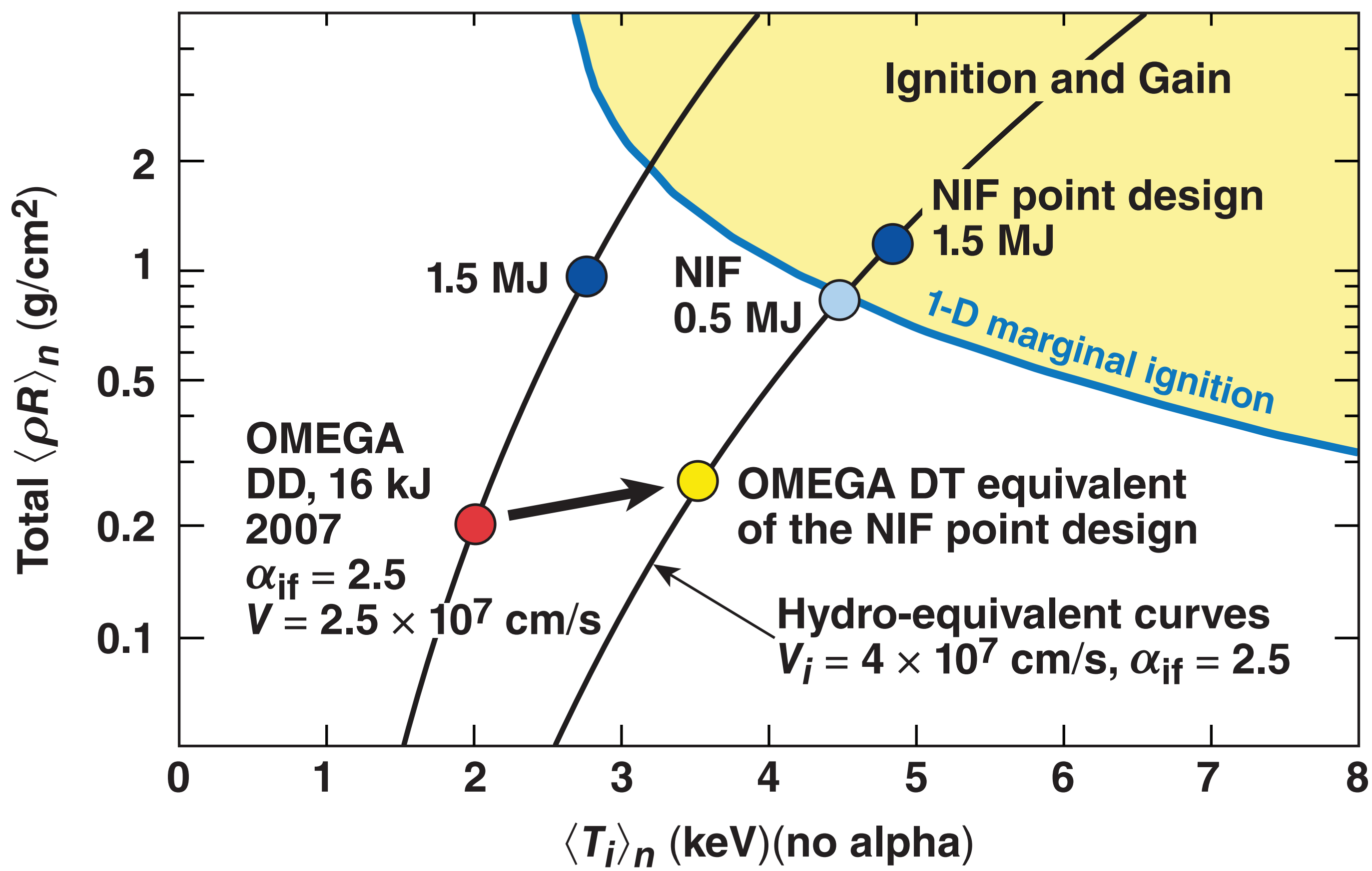
$\langle \rho R \rangle_n$ → Neutron-average areal density

TC8025

Hydro-equivalent curves on the (ρR, T_i) show how close current OMEGA cryogenic implosions are to ignition and how they perform when scaled up to the NIF

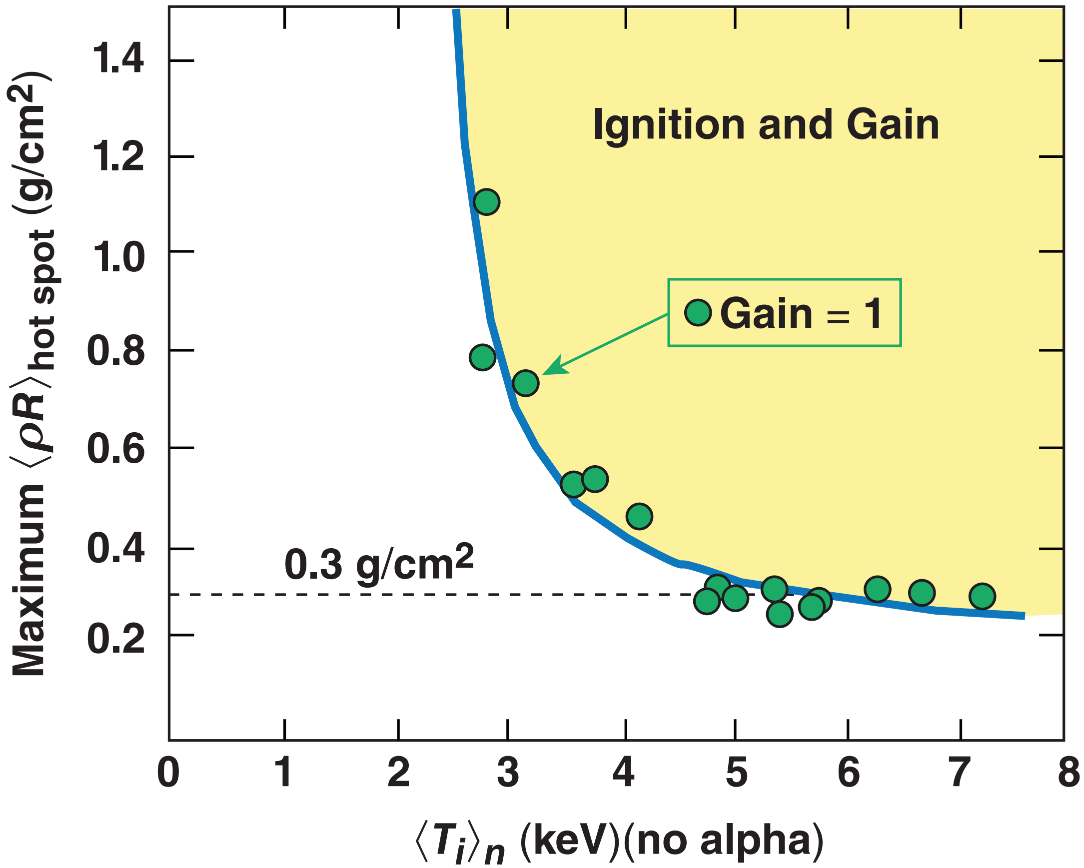
- Using the scaling relations for ρR and T_i, hydro-equivalent curves are plotted versus laser energy for fixed values of the implosion velocity and in-flight adiabat.

$$(\rho_s \Delta_s) \sim E_L^{0.33} \alpha_{if}^{-0.55} V^{0.06} \qquad T_i \sim V^{1.25} \alpha_{if}^{-0.19} E_L^{0.07}$$



TC8026

The condition for alpha particle confinement (hot-spot $\rho R \geq 0.3$ g/cm^2) is always satisfied



At low T_i, ignition requires hot-spot ρR well above 0.3 g/cm^2

TC8027

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