Instabilities of Relativistic Electron Beams in Plasmas: Spatial Growth and Absolute Instability

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48th Annual Meeting of the American Physical Society
Division of Plasma Physics

Philadelphia, PA
30 October–3 November 2006

Summary

A general kinetic dispersion relation has been developed for relativistic electron beams in plasmas

- Previous kinetic instability analysis has been restricted to real transverse wave vectors.
- Generalizing the dispersion relation to complex wave vectors allows investigation of spatial growth and absolute instability.
- Generalization to arbitrary wave-vector angles allows investigation of the relative importance of filamentation, two-stream, and mixed modes.
- Some preliminary results: absolute instability is restricted to larger transverse wavelengths; filamentation generally grows faster than two-stream or mixed instabilities.

Most fast-ignition scenarios require propagation of a relativistic electron beam through a plasma

- Large-scale beam instabilities (kinking, pinching) develop slowly on the FI timescale.
- Microinstabilities grow faster and include beam–plasma (electrostatic) and filamentation (electromagnetic or mixed) instabilities.
- These instabilities require impedance.
  - reactive (electron inertia, Weibel and beam–plasma instability): dominant at low densities (few $\times$ critical).
  - resisitive (collisional, resistive filamentation): dominant at high densities (compressed core).
  - a FI beam will transit both regions (reactive first).
- A fully relativistic treatment of the collisionless case has been carried out analytically; the collisional case is more difficult.

Instabilities can be treated as a perturbed equilibrium provided the growth times are shorter than the beam slowing time

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- Assume that, in equilibrium, the charge densities, currents, and fields vanish and that all perturbed quantities have the space and time dependence $e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$.

- Maxwell's equations relate the current to the perturbed electric field.

$$j = j_b + j_p = -\frac{ic^2}{4\pi\omega}\left(k^2 I \quad \underbrace{-\,\mathbf{kk}}_{\substack{\text{Longitudinal}\\\text{(electrostatic)}\\\text{term}}} \quad \underbrace{-\,\frac{\omega^2}{c^2} I}_{\substack{\text{From}\\\text{displacement}\\\text{current}}}\right)\cdot E$$

- The rest of the problem consists of using the plasma properties to derive the perturbed current as a response to *E* (the dielectric tensor).

The simplest model treats the return current as purely resistive

- The beam current J_b is assumed collisionless and, in equilibrium, is balanced by a return current $J_p = -J_b$.
- In the resistive model (Gremillet *et al.*,*), the perturbed current is related to the field by $E = \eta J_p$, where η is the resistivity.
- When the frequencies (real or growth rate) become comparable to η, inertial effects can be included using the result from a fluid treatment

  $\eta \rightarrow \dfrac{\omega}{\omega + \mathbf{k} \cdot \upsilon_{0b}} \eta - \dfrac{4\pi i \omega}{\omega_p^2}$, where υ_{0b} is the equilibrium beam velocity.
- At low densities inertial effects dominate the perturbed return current, and a collisionless kinetic treatment is appropriate.

TC7004a

*L. Gremillet, G. Bonnaud, and F. Amiranoff, Phys. Plasmas 9, 941 (2002).

The relativistic electron beam can be represented as a Maxwell–Boltzmann–Jüttner (MBJ) distribution

- The MBJ distribution is a relativistic generalization of the Maxwellian

  $$f_0(w) = \frac{\xi n_0}{4\pi\Gamma K_2(\xi)} e^{-\xi\Gamma(\gamma - \beta\cdot w)}, \text{ where } \xi \equiv \frac{mc^2}{k_B T_R} = \frac{c^2}{\upsilon_T^2},\ w \equiv \frac{p}{mc} = \gamma\frac{v}{c}, \beta$$

  is the average beam β, and $\Gamma \equiv \left(1-\beta^2\right)^{-1/2} = \sqrt{1+w^2}$.

- When the thermal spread is small compared to the beam velocity, this can be approximated as a drifting Maxwellian

  $$f_0(p) = \frac{1}{(2\pi)^{3/2}} \frac{1}{(m\nu_T)^3\Gamma} n_b e^{-\frac{\left(\frac{p_z}{\Gamma} - mc\beta\right)^2 + p_\perp^2}{2(m\upsilon_T)^2}}.$$

- These forms are also used to represent the return current in the collisionless case.

In the collisionless case, the perturbed currents are calculated from the relativistic Vlasov equation

- The linearized relativistic Vlasov equation can be written as

$$\frac{\partial f_{1\alpha}}{\partial t}+\frac{cw}{\gamma}\cdot\frac{\partial f_{1\alpha}}{\partial x}=-\frac{q_\alpha}{m_\alpha c}\left(E+\frac{w}{\gamma}\times B\right)\cdot\frac{\partial f_{0\alpha}}{\partial w}.$$

- The dispersion relation is obtained by using Maxwell's equations to relate the fields to the perturbed current:

$$j_b=\frac{-ie^2}{m\omega}\left[\int\frac{p}{\gamma}\frac{\partial f_0}{\partial p}d^3p+\int\frac{\left(k\cdot\frac{\partial f_0}{\partial p}\right)pp}{\gamma(\gamma m\omega-k\cdot p)}d^3p\right]\cdot E.$$

- In the drifting Maxwellian approximation, the integrals can be expressed in terms of the usual plasma Z-function.

- The exact relativistic integrals can be expressed in terms of integrals of the form $\int_{-1}^{1}ds\frac{\gamma}{\omega-cks}\frac{K_{n/2}(z)}{z^{n/2}}$, where $\gamma=\left(1-s^2\right)^{-1/2}$ and $z\equiv\xi\Gamma\sqrt{(1-\beta_3 s)^2\gamma^2-\left(\beta_1^2+\beta_2^2\right)}$.

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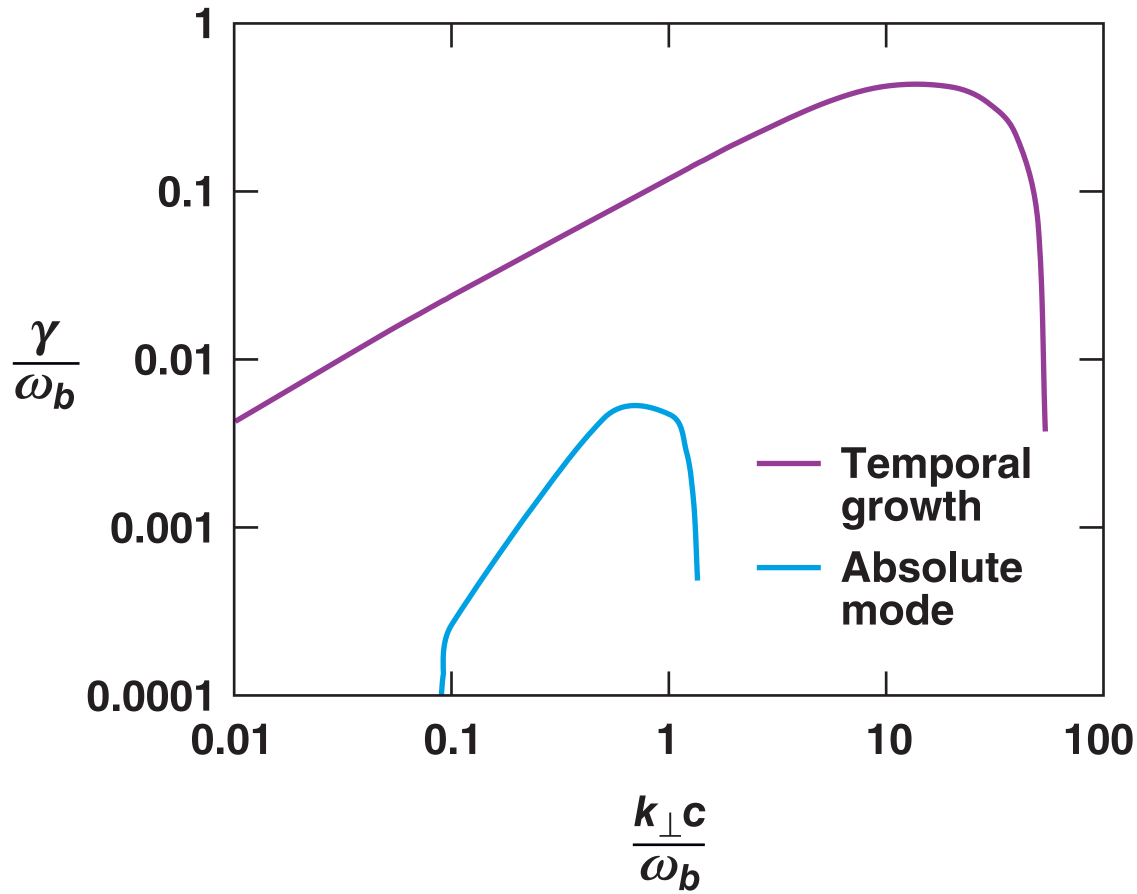
The filamentary absolute mode is limited to a smaller range of wave numbers than the temporal instability



1-MeV beam

$n_b = 10^{20}\ \mathrm{cm}^{-3}$

$\frac{n_b}{n_0} = 0.01$

$\frac{\upsilon_T}{c} = 0.03$

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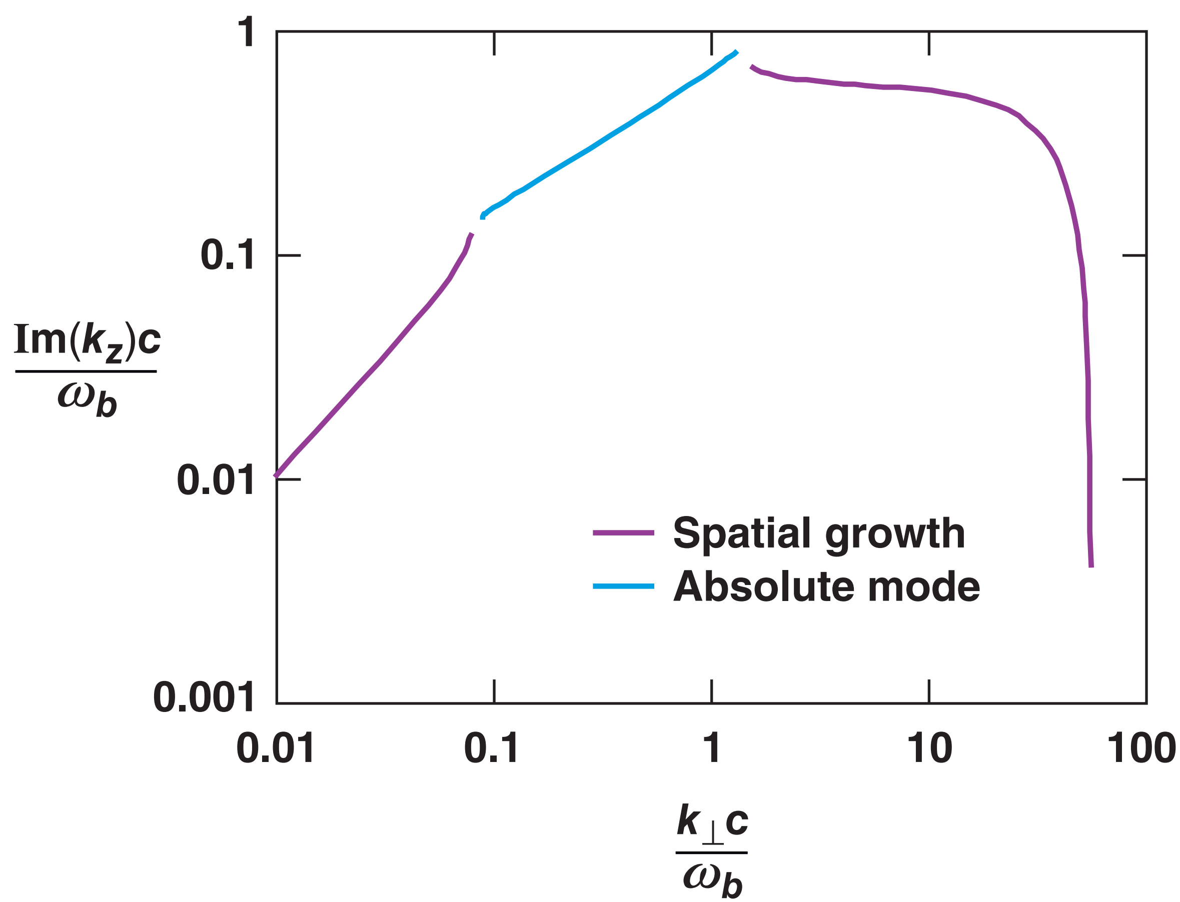
The spatial growth rates also peak at smaller wave numbers



1-MeV beam

$n_b = 10^{20}\ \mathrm{cm}^{-3}$

$\frac{n_b}{n_0} = 0.01$

$\frac{\upsilon_T}{c} = 0.03$

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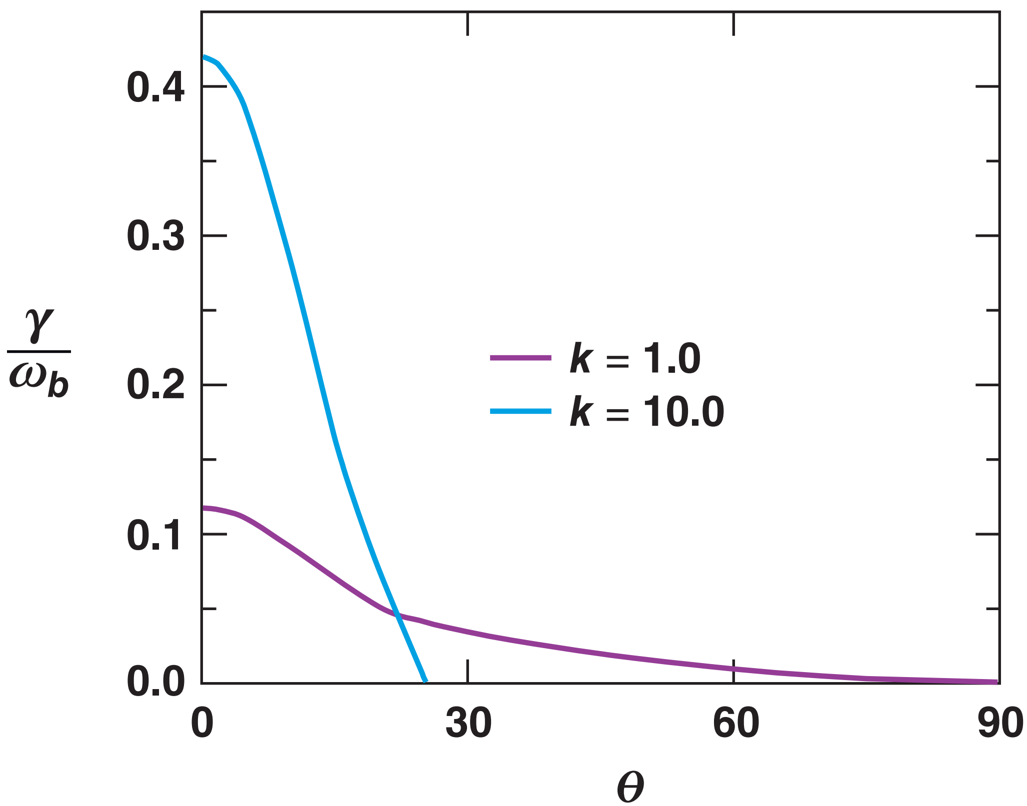
In general, the filamentary mode predominates over the two-stream and mixed modes



1-MeV beam

$n_b = 10^{20}$ cm^{-3}

$\frac{n_b}{n_0} = 0.01$

$\frac{\upsilon_T}{c} = 0.03$

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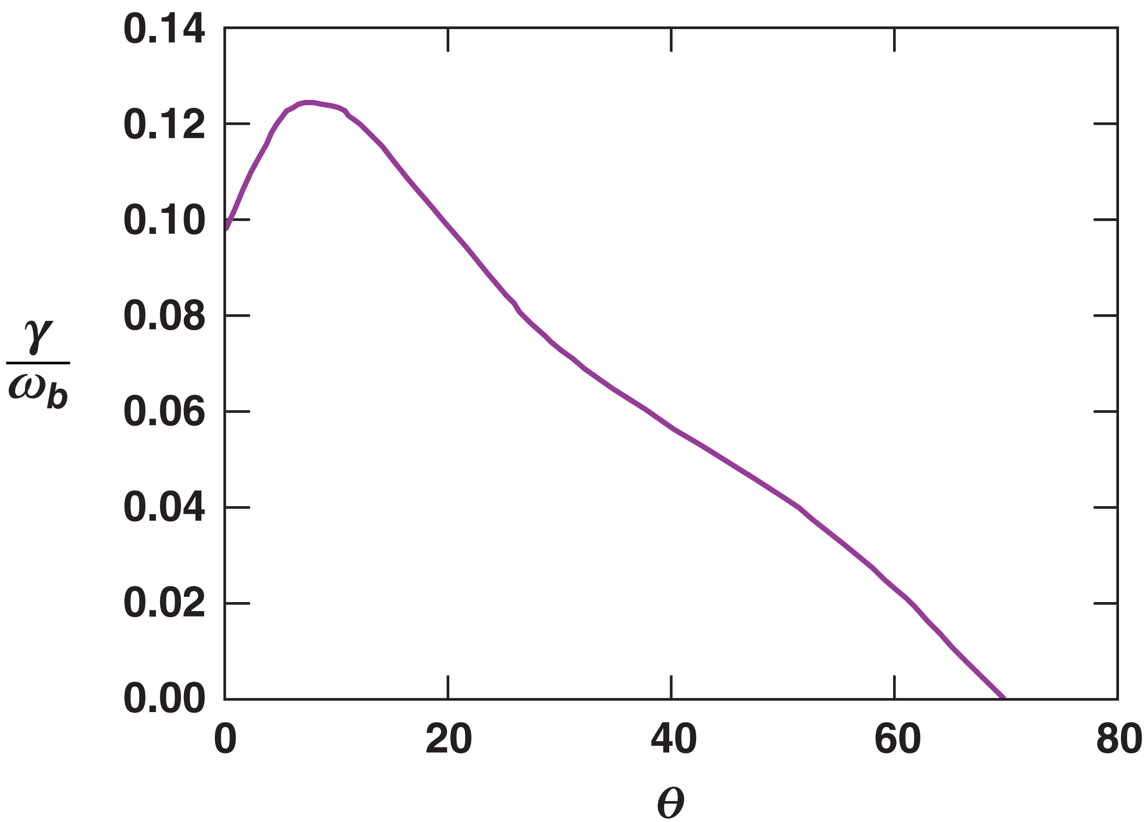
But for some values of the wave number the growth rate may peak for a mixed mode



1-MeV beam

$n_b = 10^{20}\ \mathrm{cm}^{-3}$

$\frac{n_b}{n_0} = 0.01$

$\frac{\upsilon_T}{c} = 0.03$

$\frac{k_\perp c}{\omega_b} = 15.0$

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