Relativistic Electron Beam Microinstabilities in the Fast-Ignition Regime

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Summary

A general dispersion relation for relativistic electron-beam microinstabilities is useful in addressing several problems relevant to fast ignition

- Previous work on Weibel, two-stream, and related instabilities in relativistic electron beams has employed assumptions and approximations that limit applicability.
- More reliable determination of plasma and beam temperature effects is obtained using Maxwell–Boltzman–Jüttner distribution functions (or suitable approximations) rather than delta-function or waterbag distributions.
- Inclusion of off-diagonal elements in the dielectric tensor incorporates the electrostatic component of beam filamentation, an important diagnostic signature which can affect growth rates.
- The generalization to complex wave vectors in arbitrary directions allows calculations of spatial growth rates and investigations of absolute versus convective instability.

Most fast-ignition scenarios require propagation of a relativistic electron beam through a plasma

- Large-scale beam instabilities (kinking, pinching) develop slowly on the FI timescale.
- Microinstabilities grow faster and include beam–plasma (electrostatic) and filamentation (electromagnetic or mixed) instabilities.
- These instabilities require impedance.
  - Reactive (electron inertia, Weibel and beam–plasma instability): dominant at low densities (few × critical).
  - Resisitive (collisional, resistive filamentation): dominant at high densities (compressed core).
  - A FI beam will transit both regions (reactive first.)
- A fully relativistic treatment of the collisionless case can be carried out analytically; the collisional case is more difficult.

Instabilities can be treated as a perturbed equilibrium provided the growth times are shorter than the beam slowing time

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- Assume that, in equilibrium, the charge densities, currents, and fields vanish and that all perturbed quantities have the space and time dependence $e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$

- Maxwell's equations relate the current to the perturbed electric field.

$$\mathbf{j} = \mathbf{j}_b + \mathbf{j}_p = -\frac{ic^2}{4\pi\omega}\left(k^2\mathrm{I} \quad \underbrace{-\,\mathbf{kk}}_{\substack{\text{Longitudinal}\\\text{(electrostatic)}\\\text{term}}} \quad -\frac{\omega^2}{\underbrace{c^2}_{\substack{\text{From}\\\text{displacement}\\\text{current}}}}\mathrm{I}\right)\cdot\mathbf{E}$$

- The rest of the problem consists of using the plasma properties to derive the perturbed current as a response to E (the dielectric tensor).

The simplest model treats the return current as purely resistive

- The beam current J_b is assumed collisionless and, in equilibrium, is balanced by a return current $J_p = -J_b$.
- In the resistive model (Gremillet *et al.*, 2002), the perturbed current is related to the field by $J_p = 1/\eta\, E$, where η is the resistivity.
- When the frequencies (real or growth rate) become comparable to η, inertial effects can be included using the result from a fluid treatment
$$\frac{1}{\eta}\mathbf{I} \rightarrow \frac{\omega_{p0}^2\left[\left(1-\frac{\mathbf{k}\cdot\mathbf{v}_0}{\omega}\right)\mathbf{I}+\frac{\mathbf{k}\mathbf{v}_0}{\omega}\right]+3\upsilon_T^2\mathbf{k}\mathbf{k}}{\omega_{p0}^2\eta-4\pi i(\omega-\mathbf{k}\cdot\upsilon_0)}+\frac{i}{4\pi}\mathbf{v}_0\mathbf{k}$$
, where v_0 is the equilibrium beam velocity.
- At low densities, inertial effects dominate the perturbed return current, and a collisionless kinetic treatment is appropriate.

The relativistic electron beam can be represented as a Maxwell–Boltzmann–Jüttner distribution

- The MBJ distribution is a relativistic generalization of the Maxwellian

$$f_0(\mathbf{w}) = \frac{\xi n_0}{4\pi\Gamma K_2(\xi)} e^{-\xi\Gamma(\gamma-\beta\cdot\mathbf{w})},\ \text{where}\ \xi \equiv \frac{mc^2}{k_B T_R} = \frac{c^2}{\upsilon_T^2},\ \mathbf{w} \equiv \frac{\mathbf{p}}{mc} = \gamma\frac{\mathbf{v}}{c}, \beta$$

is the average β, and $\Gamma \equiv \left(1-\beta^2\right)^{-1/2} = \sqrt{1+w^2}$.

- When the thermal spread is small compared to the beam velocity, this can be approximated as a drifting Maxwellian

$$f_0(\mathbf{p}) = \frac{1}{(2\pi)^{3/2}} \frac{1}{(mv_T)^3\Gamma} n_b e^{-\frac{\left(\frac{p_z}{\Gamma} - mc\beta\right)^2 + p_\perp^2}{2(m\upsilon_T)^2}}.$$

- These forms are also used to represent the return current in the collisionless case.

In the collisionless case, the perturbed currents are calculated from the relativistic Vlasov equation

- **The linearized relativistic Vlasov equation can be written as**

$$\frac{\partial \mathbf{f}_{1\alpha}}{\partial \mathbf{t}} + \frac{\mathbf{cw}}{\gamma} \cdot \frac{\partial \mathbf{f}_{1\alpha}}{\partial \mathbf{x}} = -\frac{\mathbf{q}_\alpha}{\mathbf{m}_\alpha \mathbf{c}}\left(\mathbf{E} + \frac{\mathbf{w}}{\gamma} \times \mathbf{B}\right) \cdot \frac{\partial \mathbf{f}_{0\alpha}}{\partial \mathbf{w}}.$$

- **Solving for the perturbed current gives**

$$\mathbf{j}_\mathbf{b} = \frac{-\mathbf{ie}^2}{\mathbf{m}\omega}\left[\int \frac{\mathbf{p}}{\gamma}\frac{\partial \mathbf{f}_0}{\partial \mathbf{p}} \mathbf{d}^3\mathbf{p} + \int \frac{\left(\mathbf{k} \cdot \frac{\partial \mathbf{f}_0}{\partial \mathbf{p}}\right)\mathbf{pp}}{\gamma(\gamma \mathbf{m}\omega - \mathbf{k} \cdot \mathbf{p})} \mathbf{d}^3\mathbf{p}\right] \cdot \mathbf{E}.$$

- **In the drifting Maxwellian approximation, the integrals can be expressed in terms of the usual plasma *Z*-function.**

- **The exact relativistic integrals can be expressed in terms of integrals of the form** $\int_{-1}^{1} \mathbf{ds} \frac{\gamma}{\omega - \mathbf{cks}} \frac{\mathbf{K}_{\mathbf{n}/2}(\mathbf{z})}{\mathbf{z}^{\mathbf{n}/2}}$**, where** $\gamma = (1 - \mathbf{s}^2)^{-1/2}$ **and**

$$\mathbf{z} \equiv \xi \Gamma \sqrt{(1 - \beta_3 \mathbf{s})^2 \gamma^2 - \left(\beta_1^2 + \beta_2^2\right)}.$$

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The resulting dispersion relations are complicated algebraically but readily evaluated numerically

- The dispersion relation is obtained from

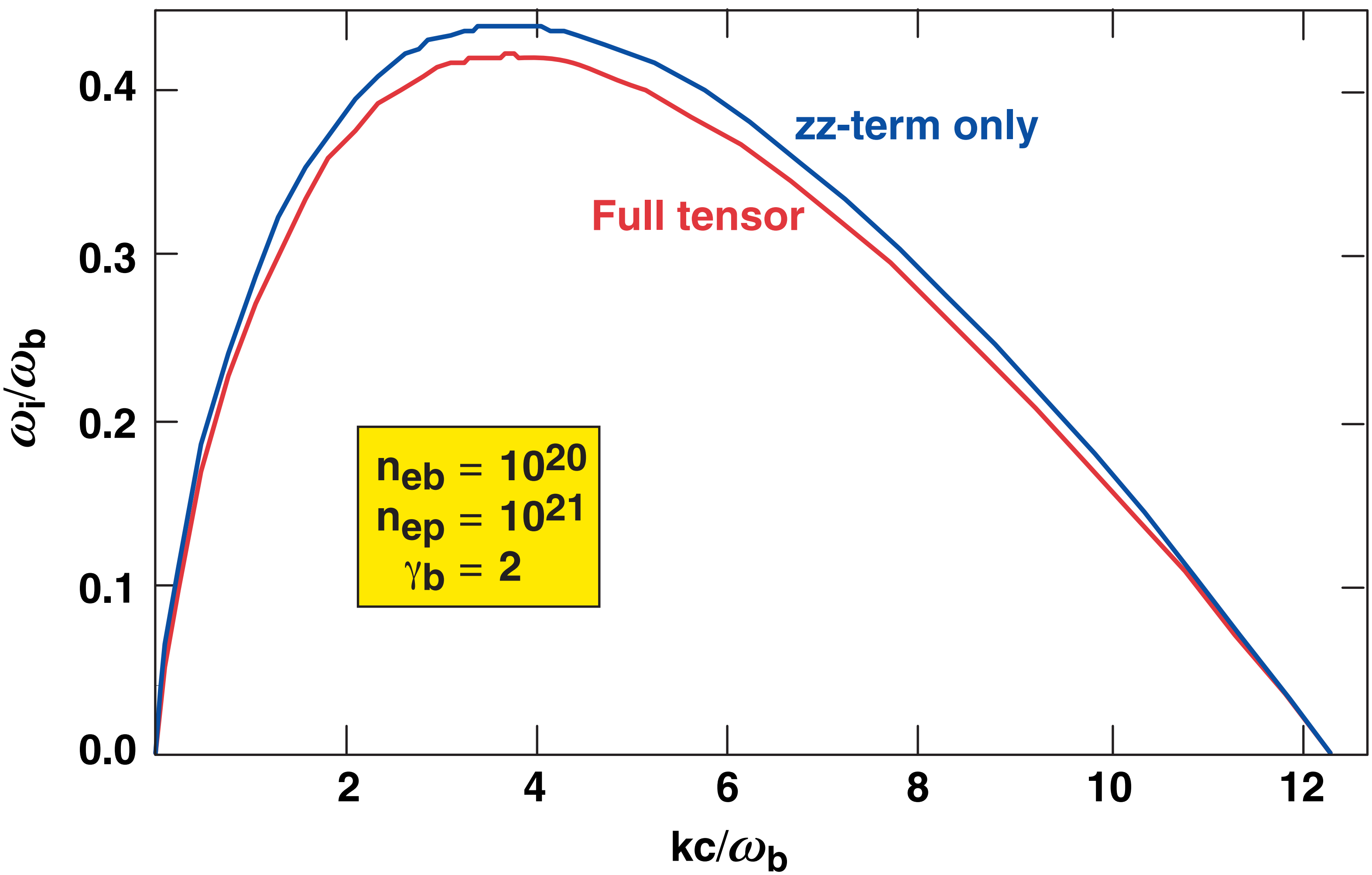
$$\left[\left(c^2k^2-\omega^2+\frac{\omega_p^2}{\Gamma}-\frac{4\pi i\omega}{\eta}\right)\underset{\sim}{I}-c^2kk-\frac{\omega_p^2}{\Gamma}\underset{\sim}{R}\right]\cdot E=0.$$

- A typical R-component in the drifting Maxwellian approximation with the beam propagating in the z-direction: $R_{zz}\equiv\left(k_y^2\upsilon_{T\perp}^2+\Gamma^2k_z^2\upsilon_{Tz}^2\right)^{-2}$

$$\times\left[\begin{array}{l}[1+\Omega Z(\Omega)]\left\{\begin{array}{l}\Gamma^2\left(k_y^2+k_z^2\right)\left[\left(\sqrt{2\Gamma k_z\upsilon_{Tz}^2\Omega+\beta c\sqrt{k_y^2\upsilon_{T\perp}^2+\Gamma^2k_z^2\upsilon_{Tz}^2}}\right)^2+k_y^2\upsilon_{Tz}^2\upsilon_{T\perp}^2\right]\\+2k_y^2k_z^2\upsilon_{Tz}^2\left(\upsilon_{T\perp}^2-\Gamma^2\upsilon_{Tz}^2\right)\end{array}\right\}\\+k_z\Gamma\left[\sqrt{2}\beta ck_y^2\sqrt{k_y^2\upsilon_{T\perp}^2+\Gamma^2k_z^2\upsilon_{Tz}^2}\left(\upsilon_{T\perp}^2-\Gamma^2\upsilon_{Tz}^2\right)Z(\Omega)+\Gamma^3\left(k_y^2+k_z^2\right)k_z\upsilon_{Tz}^4\right]\end{array}\right]$$

where $\Omega\equiv\dfrac{\gamma\omega-\Gamma\beta ck_z}{\sqrt{2}\sqrt{(k_y\upsilon_{T\perp})^2+(\Gamma k_z\upsilon_{Tz})^2}}$

The electrostatic component typically has little effect on filamentation growth rates, but is responsible for density perturbations seen in experiments



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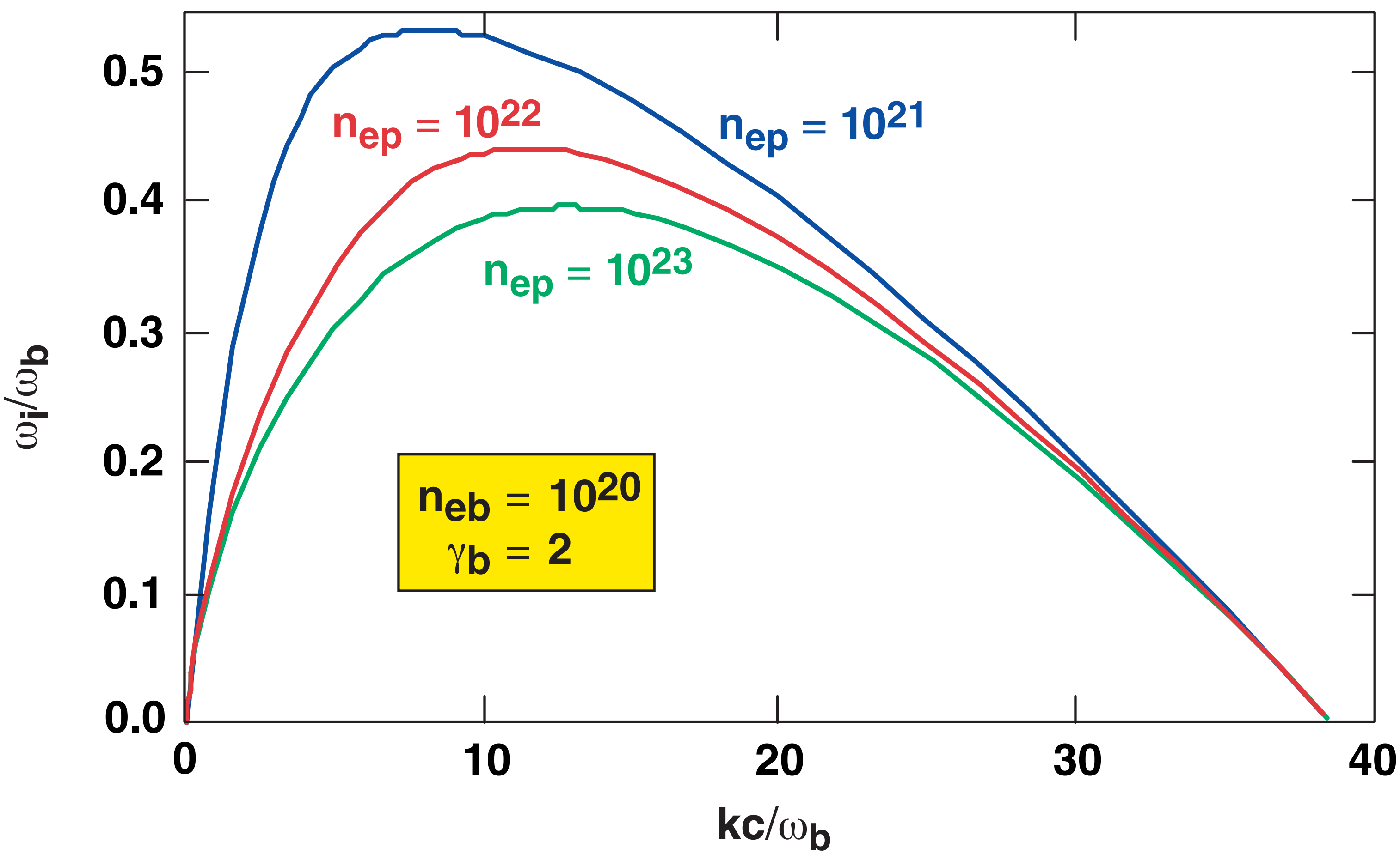
Inertial terms increase the growth rates at lower ratios of background to beam densities



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The general dispersion relation can be used to address several further problems of interest in FI experiments

- Using real ω and complex k spatial growth rates can be calculated, which are of greater relevance to FI than the temporal growth rates.

- The transition from convective to absolute intabilities can be studied; this requires both ω and k to be complex.

- Arbitrary wave-vector directions allow the comparison of two-stream and filamentation instabilities and the identification of the most unstable mode, which may lie between these instabilities [A. Brett *et al.*, PRL 94 (2005).]

- The analytic theory can be used to benchmark simulation codes such as LSP and optimize simulation parameters.

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