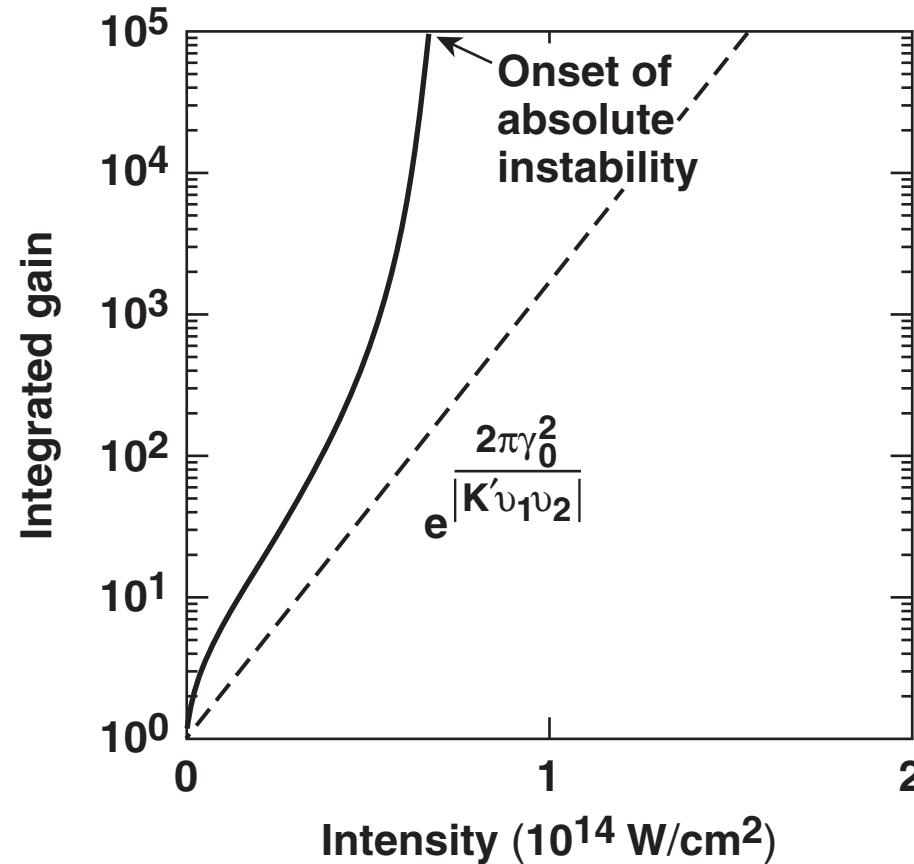


# Convective Versus Absolute Two-Plasmon Decay in Inhomogeneous Plasmas



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## Summary

# Accurate calculation of convective gain and absolute thresholds for TPD is greatly simplified in Fourier space

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- TPD is limited to a narrow range of densities. For such profiles, the eighth-order differential equation describing TPD becomes second-order in  $k$ -space.
- For small  $k_{\perp}/k_0$ , the convective spatial gain is roughly an order of magnitude larger than the Rosenbluth formula, and the absolute instability threshold is low.
- For larger  $k_{\perp}/k_0$  the Rosenbluth formula is a better fit to the gain, and the absolute threshold is larger.

# Outline

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- **Convective and absolute TPD in theory and experiment**
- **Advantages of the Fourier space approach**
- **Illustrative results**
- **Summary and conclusions**

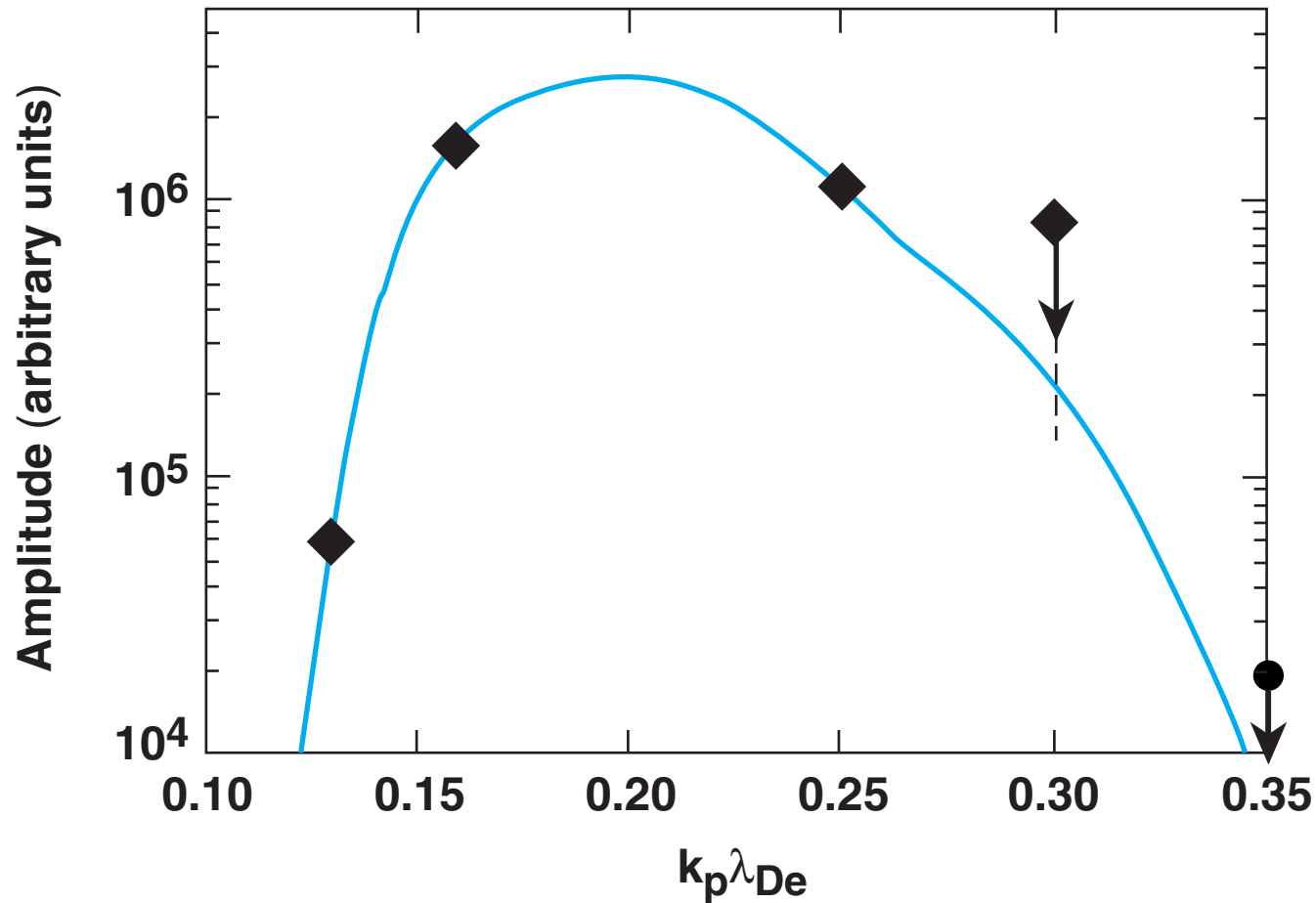
# Both convective and absolute forms of the two-plasmon-decay (TPD) instability are expected to play a role in laser-fusion experiments

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- **Convective instability:** Plasma waves arising from noise enter the interaction region, are amplified, and propagate out at an enhanced level.
  - **Spatial growth  $\rightarrow \infty$ : absolute instability**
- **Absolute instability:** Waves in the interaction region are amplified faster than they can propagate out; temporal growth continues until limited by nonlinear effects.
- **Absolute instability predominates at small plasmon wave vectors; small group velocity, large phase velocity.**
- **Convective instability predominates at large wave vectors; large group velocity, smaller phase velocity (traps electrons more effectively).**

# The current TPD experiments allow for a rough estimate of the plasma-wave spectrum



The absolute instability would be just above threshold for  $k_p \lambda_{De} < 0.13$ .

# The equations describing TPD are difficult to treat in configuration space

- Using the velocity potential defined by  $\mathbf{v} \equiv \nabla\psi$ , the equations governing TPD can be written

$$\frac{\partial\psi}{\partial t} = \frac{e\phi}{m} - \frac{3v_e^2 n_1}{n_0} - \mathbf{v}_0 \cdot \nabla\psi; \quad \frac{\partial n_1}{\partial t} + \nabla \cdot (n_0 \nabla\psi) + \mathbf{v}_0 \cdot \nabla n_1 = 0; \quad \nabla^2 \phi = 4\pi e n_1.$$

- These lead to an eighth-order ODE. Simplifications are of questionable validity near the plasma wave turning points.
- Simple generic three-wave convective instability theory gives the spatial gain formula  $G = \exp\left(\frac{2\pi\gamma_0^2}{|\mathbf{K}'v_1v_2|}\right)$ . Constant parameters, and the exponential function of intensity must break down at the absolute threshold ( $G \rightarrow \infty$  for finite intensity).

## For a linear density profile, a more sophisticated treatment is feasible using Fourier transforms

- TPD is confined to a narrow range of densities below quarter-critical, so a linear density profile should be a good approximation.
- For a linear density profile, Fourier transforming in space leads to two coupled first-order ODEs in k-space:

$$\frac{dW_+}{d\kappa} = h(\kappa) W_-, \quad \frac{dW_-}{d\kappa} = -h^*(\kappa) W_+ \quad \text{for density profile } \frac{n_1}{n_0} = 1 + \frac{x}{L};$$

$$\text{coupling coefficient } h(\kappa) = \frac{\alpha \left( \frac{k_y}{k_0} \right) \kappa e^{i\alpha\sqrt{\beta}\kappa(\kappa-2\Omega)}}{\sqrt{\left[ \kappa^2 + \frac{1}{4} + \left( \frac{k_y}{k_0} \right)^2 \right]^2 - \kappa^2}} .$$

- Previous studies have employed this k-space formulation to treat the absolute instability (Liu and Rosenbluth, 1976; Simon *et al.*, 1983).

# Both absolute and convective forms of TPD can be studied using the k-space approach

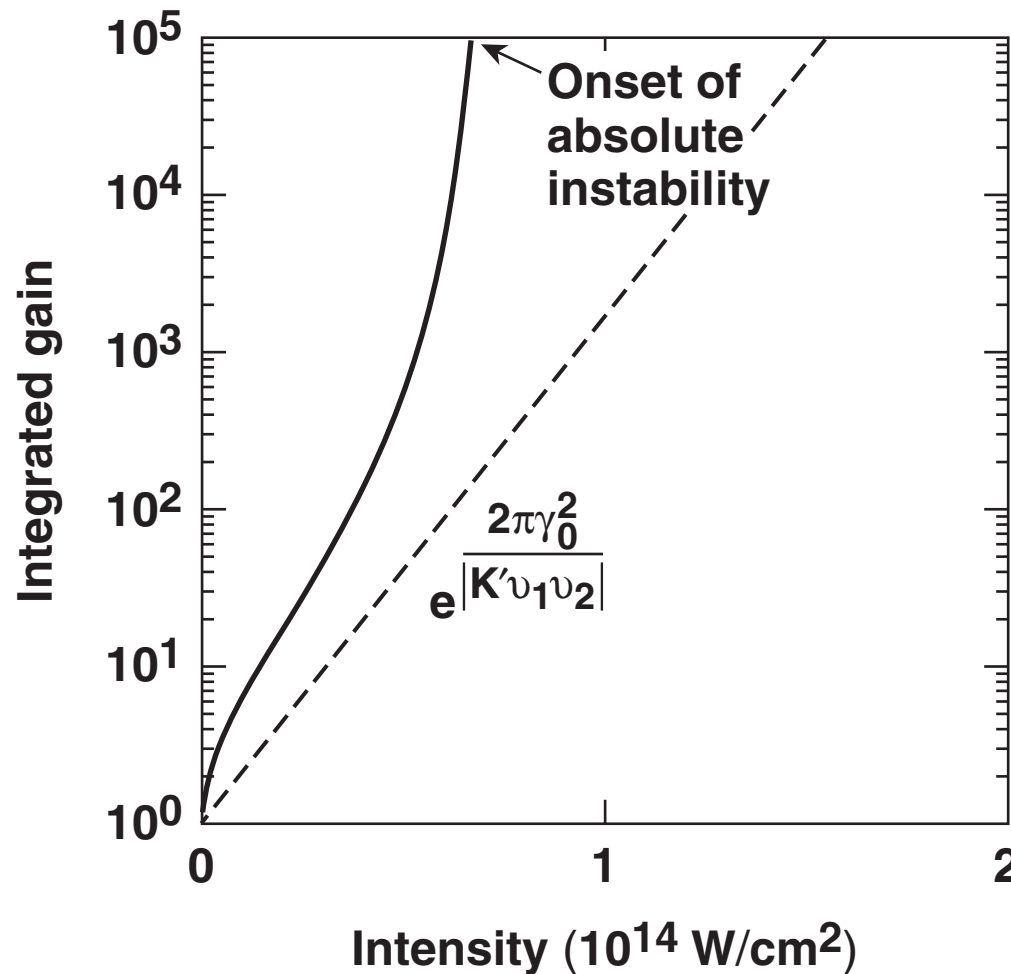
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- **Absolute modes are found by searching for temporally growing modes localized in k-space (Simon *et al.*, 1983). It can be difficult to obtain accurate results near the threshold.**
- **The convective instability can be studied using real  $k$  and  $\omega$ ; the absolute threshold can be identified with divergent spatial gain.**
- **$\begin{pmatrix} W_+ \\ W_- \end{pmatrix}$  represents the plasma wave amplitudes at  $\begin{pmatrix} k + k_0, \omega + \omega_0 \\ k - k_0, \omega - \omega_0 \end{pmatrix}$ .**
- **Incoming waves at a large negative  $x$  are represented by  $W_{\pm}(\kappa \rightarrow \pm\infty)$  and outgoing waves by  $W_{\pm}(\kappa \rightarrow \mp\infty)$ ; numerical integration in k-space gives the gain factor.**

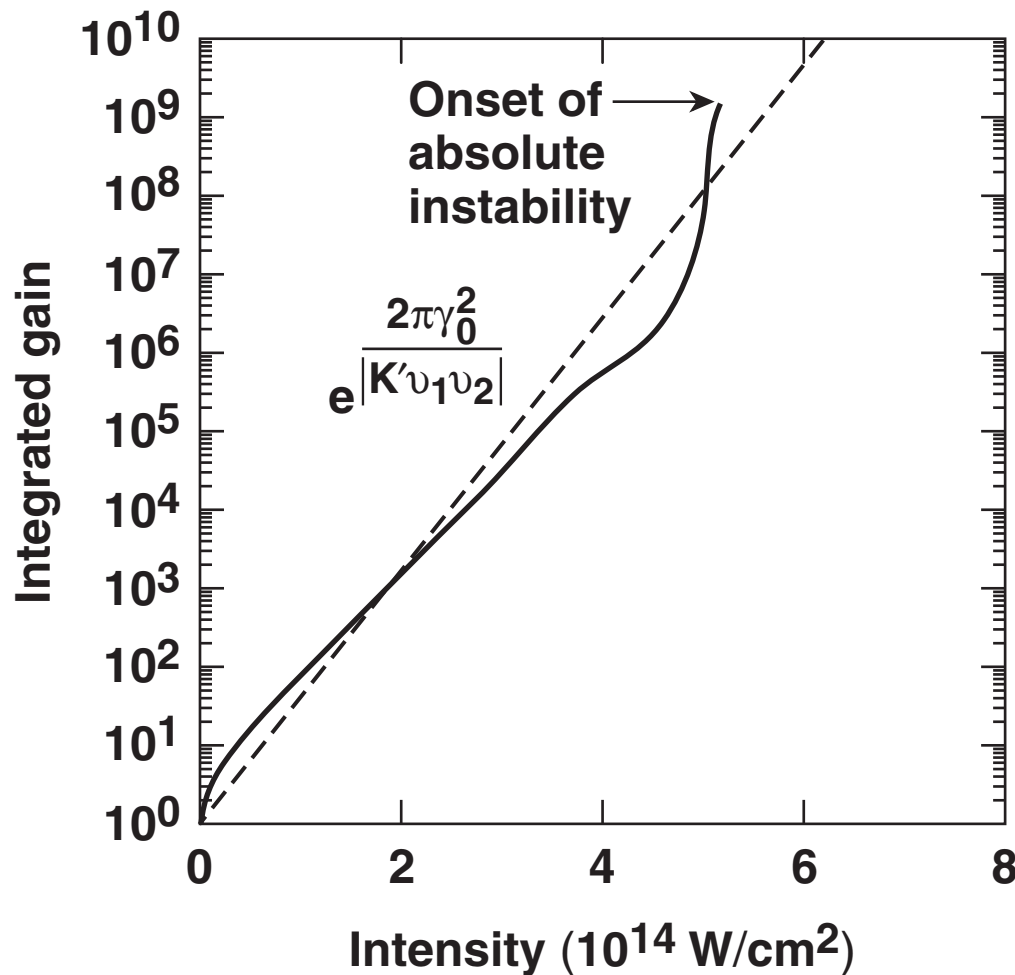


At small values of  $k_y/k_0$ , spatial amplification is larger than predicted by the simple model and shows transition to absolute mode



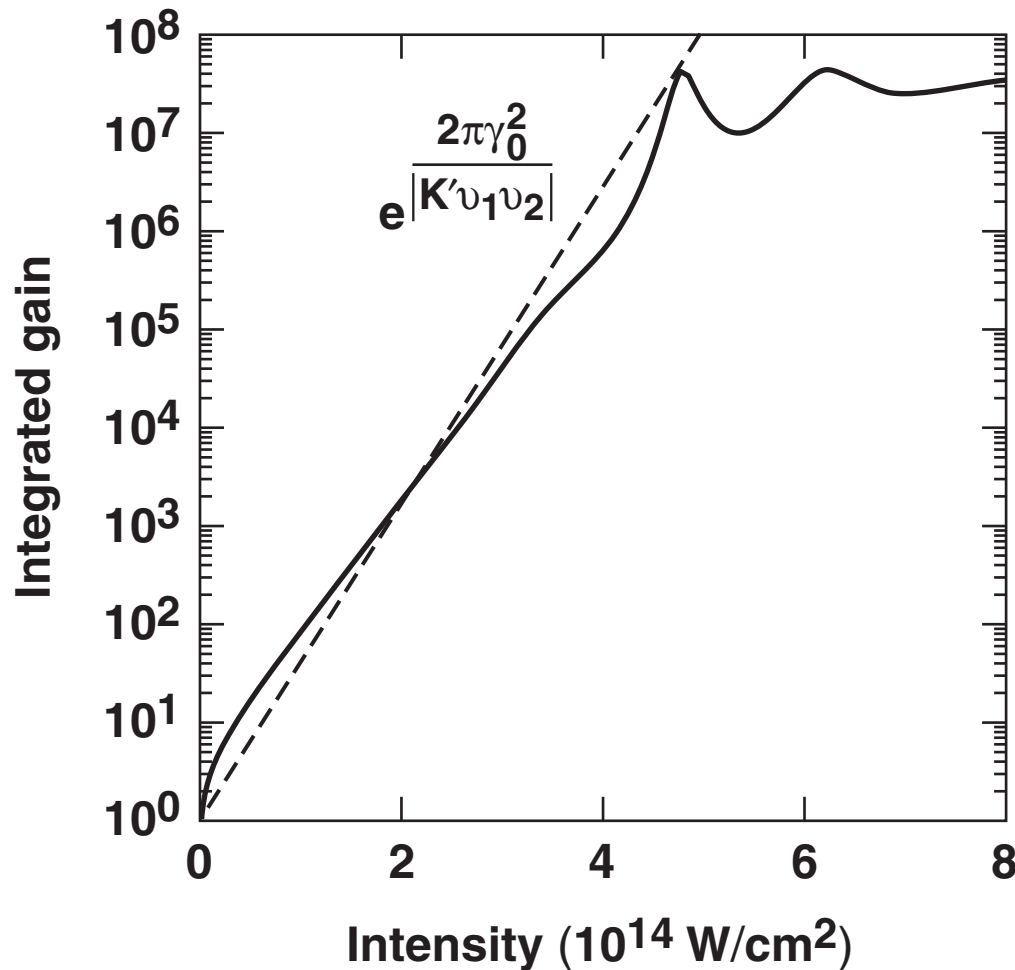
$L_\mu = 400$   
 $T_{\text{keV}} = 2.0$   
 $k_y/k_0 = 0.1$

At larger values of  $k_y/k_0$ , spatial amplification is closer to the simple model; absolute threshold is higher



$L_\mu = 400$   
 $T_{\text{keV}} = 2.0$   
 $k_y/k_0 = 0.1$

# Off-resonance, instability remains convective, but gain may be large enough to make it effectively absolute



$L_\mu = 400$

$T_{\text{keV}} = 2.0$

$k_y/k_0 = 0.1$

Off-resonant  $\omega$

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