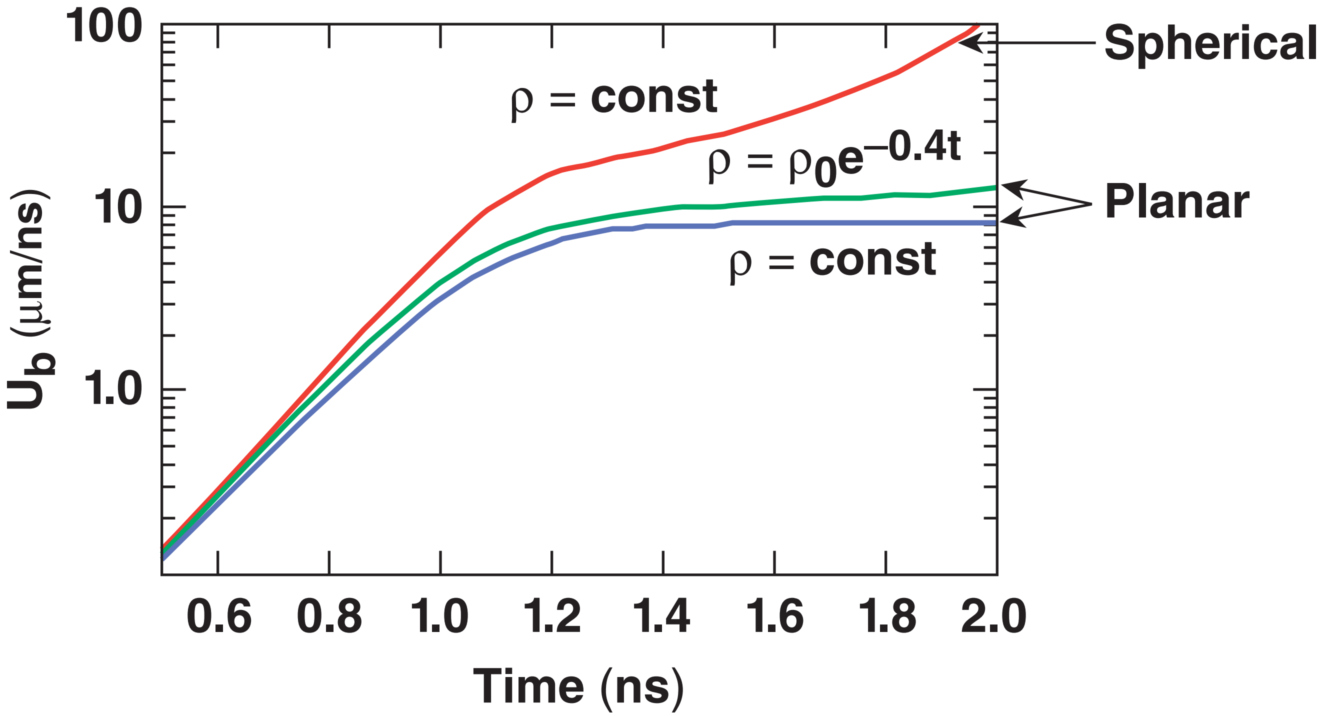
Effect of Temporal Density Variation and Convergent Geometry on Nonlinear Bubble Evolution in the Classical Rayleigh–Taylor Instability



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46th Annual Meeting of the
American Physical Society
Division of Plasma Physics
Savannah, GA
15–19 November 2004

Summary

Layzer's model has been extended to include temporal variation in density and convergence effect

UR LLE

- Layzer's model describes the nonlinear bubble evolution in planar geometry with constant density.
- Temporal density variation and convergent effect are important in ICF implosions.
- Density variation modifies the asymptotic bubble growth to

$$\eta_0 = U_L \int \rho(t')\, dt' / \rho(t), \qquad U_L = \sqrt{\frac{g}{3k}}\ .$$

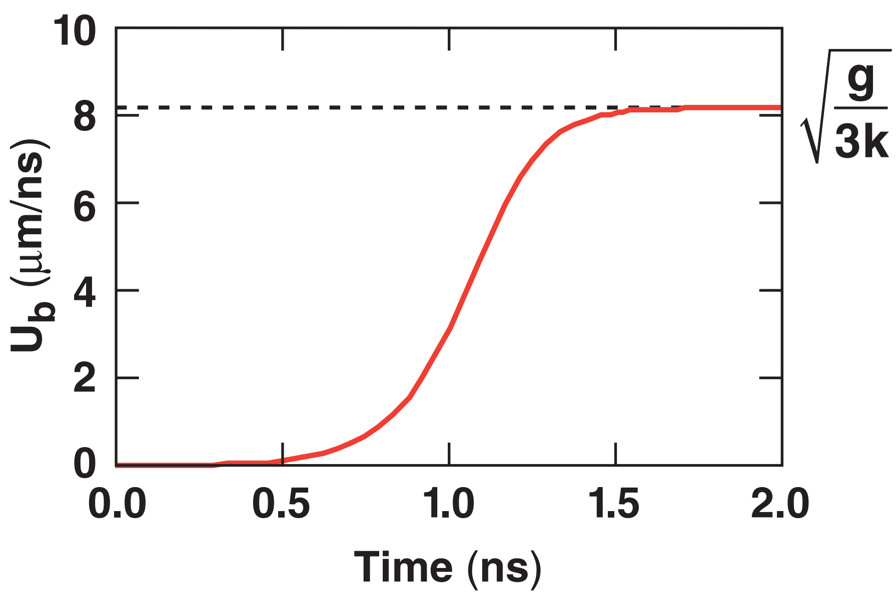
- Extension of Layzer's model to spherical geometry leads to

$$\eta_0 \rightarrow r_0(t) \int \frac{\bar{U}_L(t')}{r_0(t')}\, dt', \quad \bar{U}_L(t) = \sqrt{\frac{g r_0(t)}{\ell}}$$ for a solid sphere.

TC6712

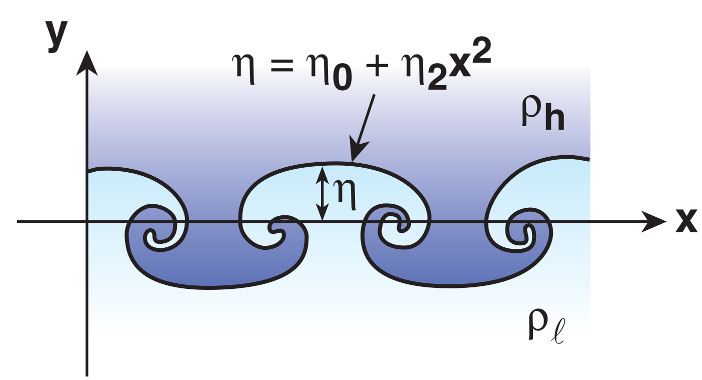
Layzer's nonlinear RT model is only valid for planar geometry

- $\mathbf{U} = \nabla\Phi$
- $\nabla^2\Phi = 0$ $\quad\quad \Phi = a(t)\cos(kx)e^{-k(y-\eta_0)}$



g = 100 (μm/ns^2)
k = 0.5 (μm^{-1})

Expansion near bubble tip



Bubble curvature $R = \frac{1}{2\eta_2}$

Bubble amplitude η_0

Bubble velocity $U_b = \dot{\eta}_0$

Bubble tip velocity saturates at $\sqrt{g/3k}$

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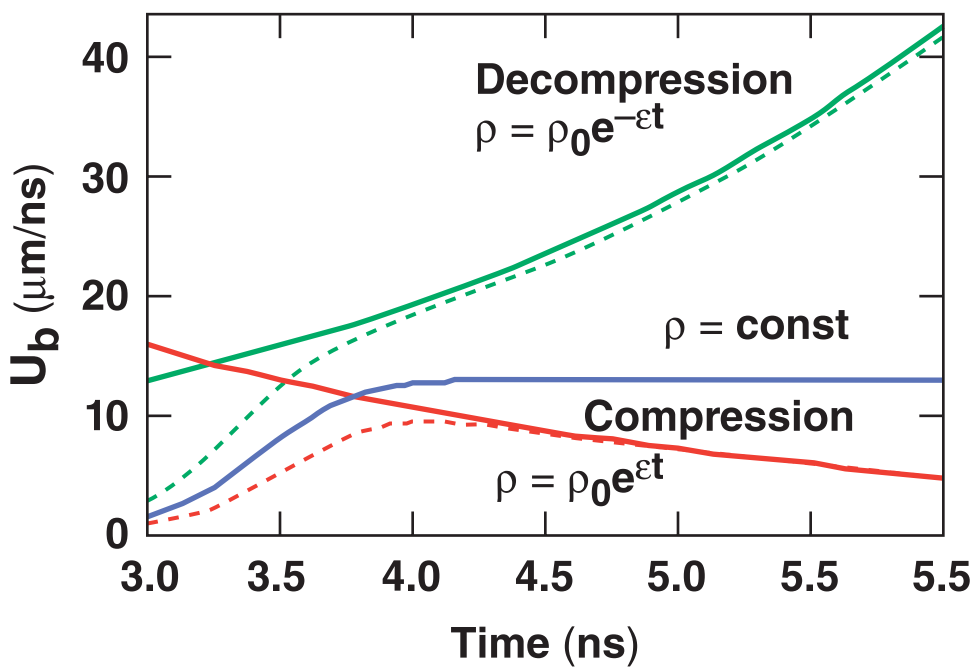
Density variation can be easily included in the model

$$\nabla^2\Phi = -\frac{\dot{\rho}}{\rho} \qquad \Phi = a(t)\cos(kx)e^{-k(y-\eta_0)} - \frac{\dot{\rho}}{\rho}\frac{y^2}{2}$$



Asymptotic solution

$$\eta_0 \xrightarrow{t\to\infty} \sqrt{\frac{g}{3k}}\,\frac{\int \rho(t')\,dt'}{\rho(t)}$$

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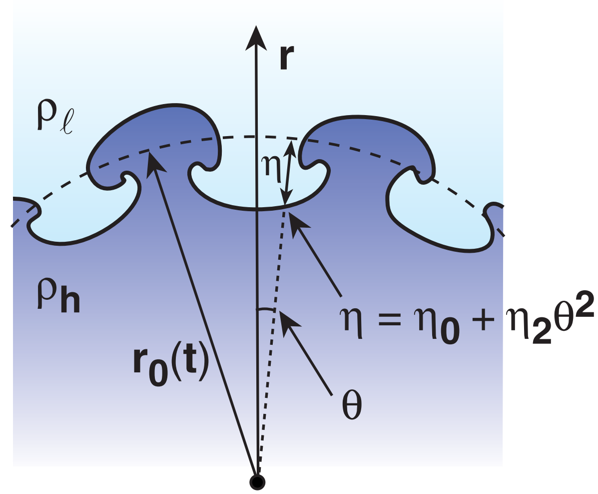
The Layzer's model is extended to include the spherical convergence effect

UR LLE

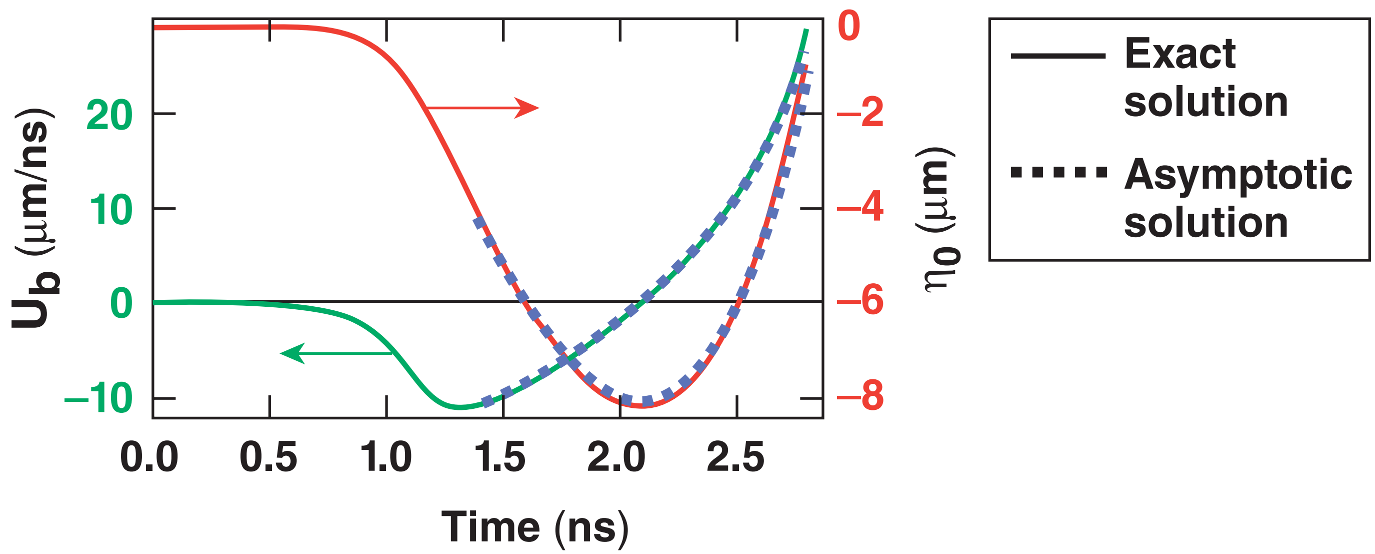


- **Solid sphere:** $\rho r_0^3 = \text{const}$
- $\Phi = a(t)\, r^\ell P_\ell(\cos\theta) - \left(\dot{r}_0 + \dfrac{r_0}{3}\dfrac{\dot{\rho}}{\rho}\right)\dfrac{r_0^2}{r} - \dfrac{r_0^2}{r}\dfrac{\dot{\rho}}{\rho}$
- $\dfrac{\partial\Phi}{\partial r} = \dot{r}_0$ **at equilibrium**
- $\dfrac{d}{dt}\left(\dfrac{\eta_2}{r_0}\right) = \dfrac{d}{dt}\left(\dfrac{\eta_0}{r_0}\right)\left(2\ell\dfrac{\eta_2}{r_0} - \dfrac{\ell(\ell+1)}{4}\right)$ $\qquad \dfrac{\eta_2}{r_0} \xrightarrow{t\to\infty} \dfrac{\ell+1}{8}$
- $-\dfrac{1}{r_0^2}\dfrac{d}{dt}\left(r_0^2\dot{\xi}\right) + \ell\dot{\xi}^2 = -\dfrac{\ddot{r}_0}{r_0}, \quad \xi = \dfrac{\eta_0}{r_0}$
- $\dot{\xi} = -\sqrt{-\dfrac{\ddot{r}_0}{\ell r_0}} \qquad \ell >> 1$

TC6715

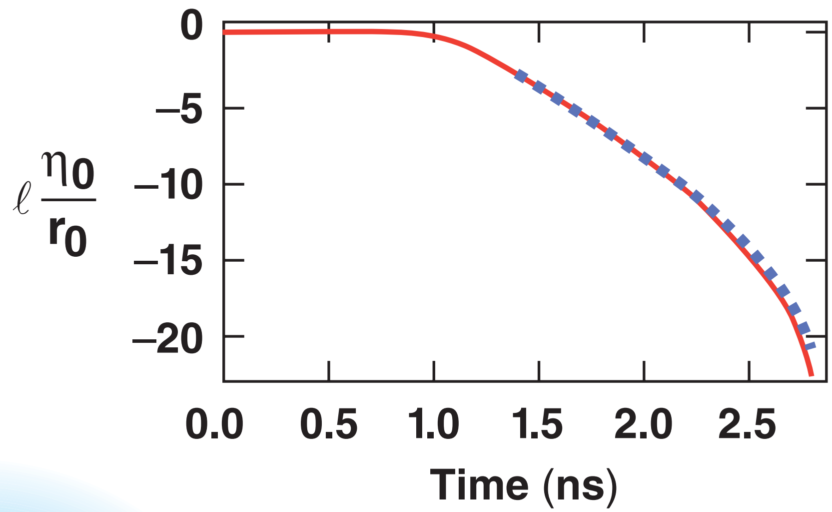
Asymptotic analysis agrees with an exact solution of the model

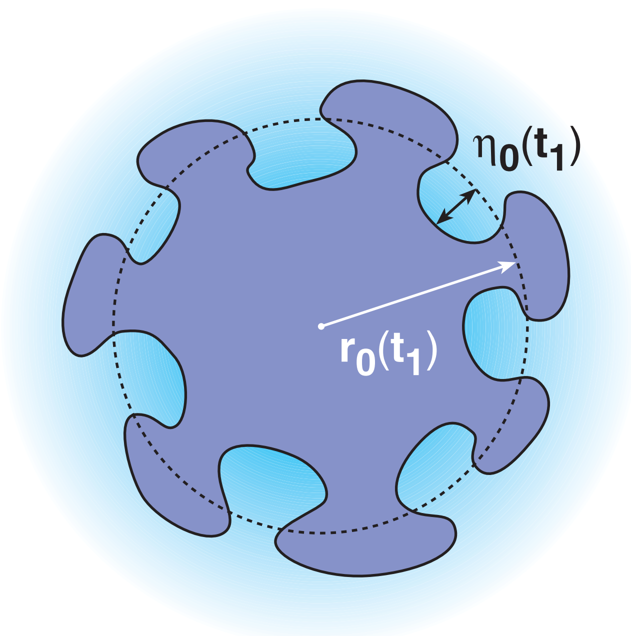
- $\ell = 200$, $g = 100\left(\mu m / ns^2\right)$, $r_0(0) = 400\,(\mu m)$
- $r_0(t) = r_0(0) - \dfrac{gt^2}{2}$, $\eta_0(0) = -10^{-3}\,(\mu m)$



$$\eta_0 \to r_0(t) \int \frac{\bar{U}_L(t')}{r_0(t')}\,dt', \quad \bar{U}_L(t) = \sqrt{\frac{g r_0(t)}{\ell}}$$

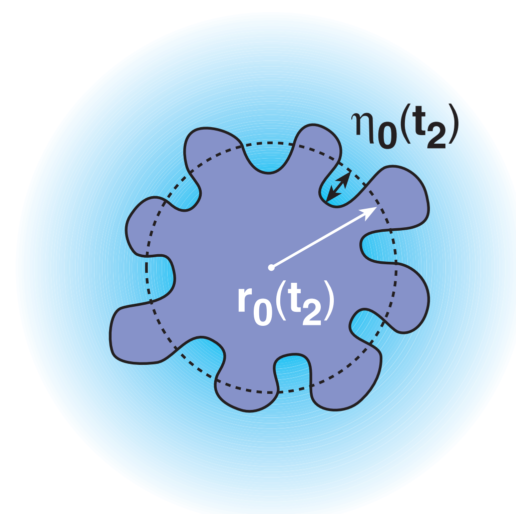
Even though bubble amplitude decreases in solid sphere, η_0/r_0 always increases





$$\eta_0(t_1) > \eta_0(t_2)$$

$$\frac{\eta_0(t_1)}{r_0(t_1)} < \frac{\eta_0(t_2)}{r_0(t_2)}$$



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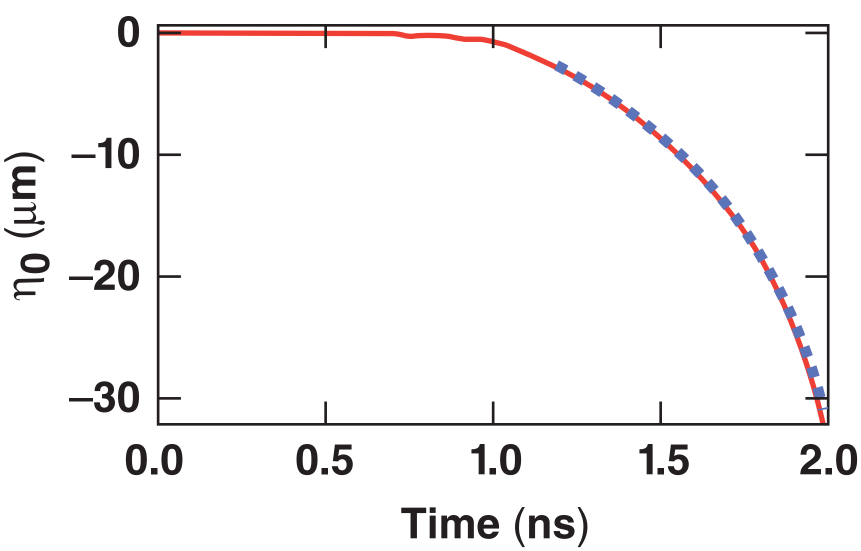
If $\rho r_0^3 \neq$ const (shell), asymptotic amplitude is determined by a first-order nonlinear differential equation

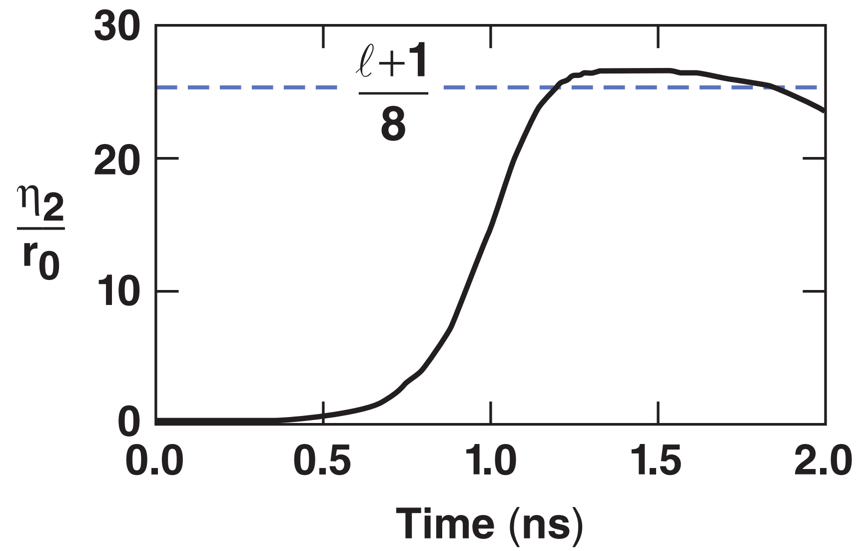
- $\ell \dot{\xi}^2 - \dot{m}\left\{\dot{\xi}\left[1 + 2(\ell+1)\left(\frac{\xi}{m}\right)^2\right] - 2\xi\frac{\dot{r}_0}{r_0}\right\} + \frac{\ddot{r}_0}{r_0}m^2 = 0$

$\xi = \rho r_0^2 \eta_0, \quad m(t) = \rho(t) r_0^3(t),$

$t \to \infty, \qquad \frac{\eta_2}{r_0} \to \frac{\ell+1}{8}$

- $r_0(t) = 400 - \frac{100t^2}{2}, \quad \rho(t) = \rho_0 e^{\varepsilon t}$





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Bubble amplitude in spherical geometry does not decrease with radius

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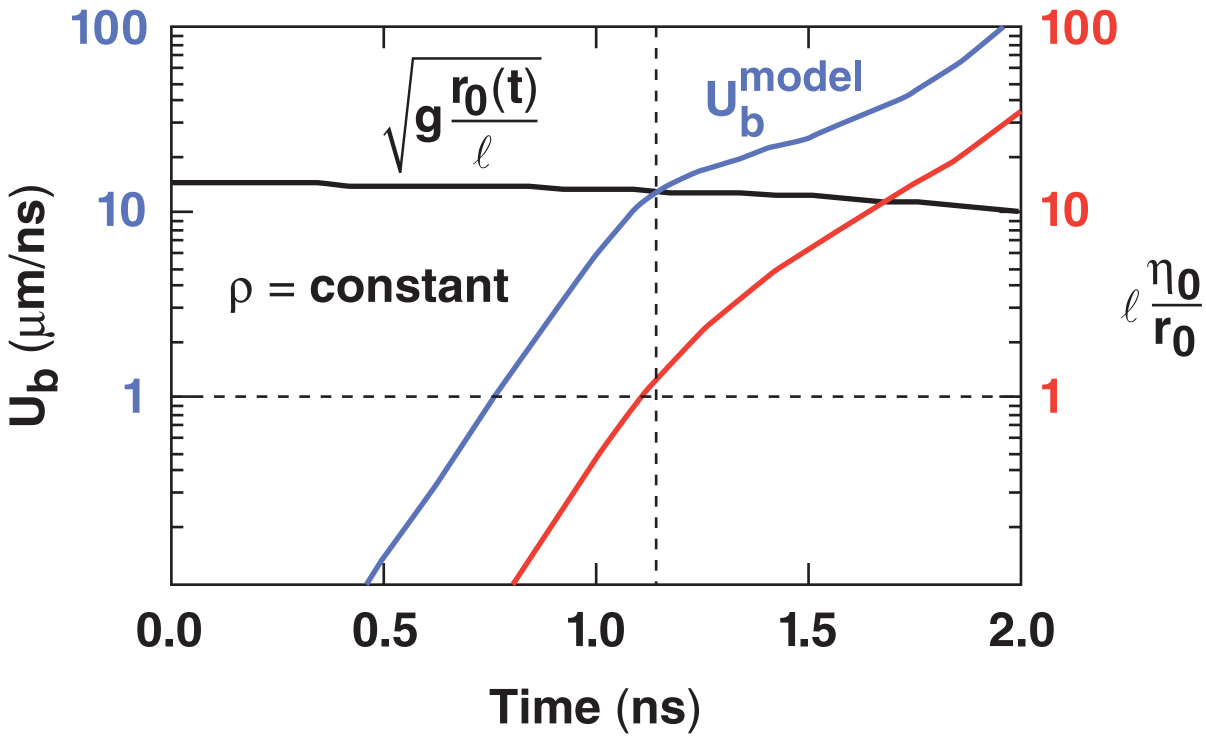
$$U_b^{planar} = \sqrt{\frac{g}{k}} \xrightarrow{?} U_b^{spherical} = \sqrt{g\frac{r_0(t)}{\ell}}$$



- Similar results are derived for cylindrical geometry in Reference 1.

[1] Y. Yedvab *et al*., presented at the 6th IWPCTM, Marseille, France, 18–21 June 1997, p. 528.

TC6718

Summary/Conclusions

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