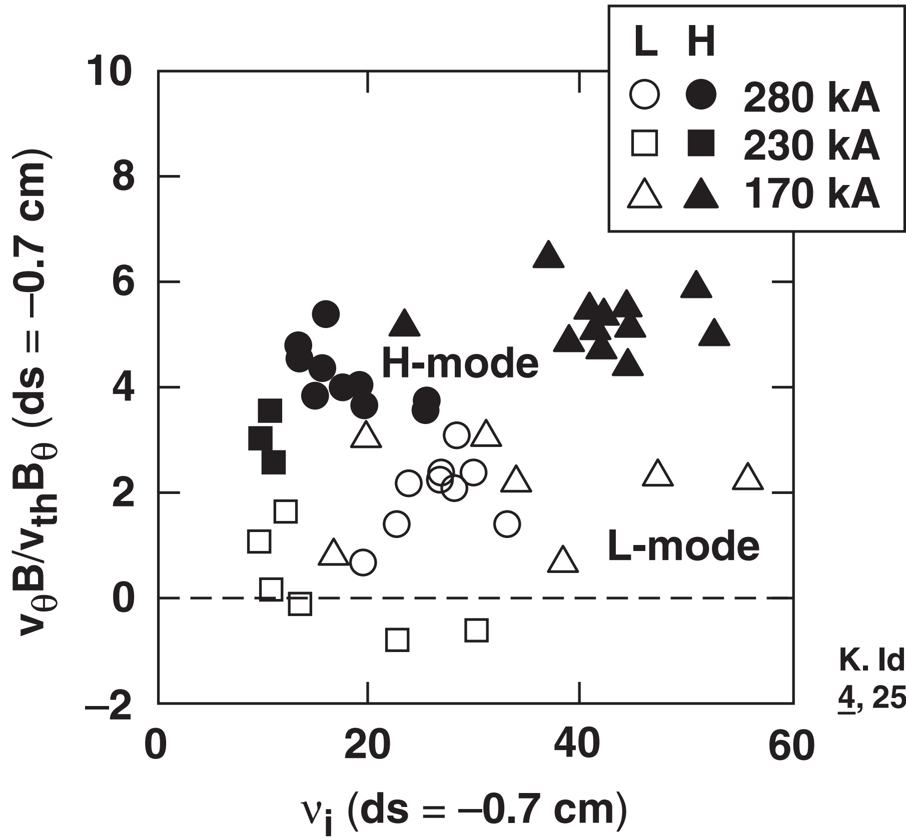
MHD Equilibria with Poloidal and Toroidal Flow



K. Ida *et al.*, Phys. Fluids B 4, 2552 (1992).

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Acknowledgements

- R. Betti, University of Rochester
- J. Manickam and S. Kaye, PPPL
- J. P. Freidberg, MIT
- J.-L. Gauvreau, UCLA

Outline

- Review of theory and numerics (with the code FLOW[1]) of tokamak equilibria with flow
- Equilibria with toroidal flow, effects of flow and anisotropy (application to NSTX)
- Equilibria with poloidal flow:
  - Equilibria with transonic poloidal flow
  - Equilibria with super-Alfvénic poloidal flow
    - inward-shifted equilibria
    - quasi-omnigenous equilibria

[1] L. Guazzotto, R. Betti, J. Manickam and S. Kaye, Phys. Plasmas 11, 604 (2004).
FLOW web site: http://www.me.rochester.edu/~guazzott/FLOW_manual.htm

TC6684

MHD equilibrium equations with flow

- **Continuity:**

$$\nabla \bullet (\rho \vec{v}) = 0$$

- **Momentum:**

$$\rho \vec{v} \bullet \nabla \vec{v} = \vec{J} \times \vec{B} - \nabla \bullet \overleftrightarrow{P}$$

$$\overleftrightarrow{P} \equiv p_{\perp} \overleftrightarrow{I} + \Delta \vec{B}\vec{B} \qquad \Delta \equiv \left(p_{\|} - p_{\perp}\right) / B^2$$

- **Maxwell equations (and Ohm's law):**

$$\nabla \times \left(\vec{v} \times \vec{B}\right) = 0 \qquad \nabla \bullet \vec{B} = 0$$

The MHD equations are reduced to a "Bernoulli" and a "Grad–Shafranov" equation

- **Plasma flow**

$$\vec{v} = \mathbf{M}_{\mathbf{A}\theta}\vec{v}_{\mathbf{A}} + \mathbf{R}\,\Omega(\Psi)\,\hat{\mathbf{e}}_{\varphi} \qquad \mathbf{M}_{\mathbf{A}\theta} = \mathbf{V}_{\theta}/\mathbf{V}_{\mathbf{A}\theta} = \Phi(\Psi)/\sqrt{\rho}$$

- **"Bernoulli" equation**

$$\frac{1}{2}\frac{(\mathbf{M}_{\mathbf{A}\theta}\mathbf{B})^2}{\rho} - \frac{1}{2}[\mathbf{R}\,\Omega(\Psi)]^2 + \mathbf{W} = \mathbf{H}(\Psi)$$

Enthalpy

- **"GS" equation**

$$\nabla \bullet \left[\left(1 - \mathbf{M}_{\mathbf{A}\theta}^2 - \Delta\right)\left(\frac{\nabla\Psi}{\mathbf{R}^2}\right)\right] =$$

$$-\frac{\partial \mathbf{p}_{||}}{\partial\Psi} - \frac{\mathbf{B}_{\varphi}}{\mathbf{R}}\frac{\mathbf{d}\,\mathbf{F}(\Psi)}{\mathbf{d}\Psi} - \vec{v}\bullet\vec{\mathbf{B}}\frac{\mathbf{d}\,\Phi(\Psi)}{\mathbf{d}\Psi} - \mathbf{R}\rho v_{\varphi}\frac{\mathbf{d}\,\Omega(\Psi)}{\mathbf{d}\Psi} - \rho\frac{\mathbf{d}\,\mathbf{H}(\Psi)}{\mathbf{d}\Psi} + \rho\frac{\partial \mathbf{W}}{\partial\Psi}$$

E. Hameiri, Phys. Fluids 26, 230 (1983).
R. Iacono *et al.*, Phys. Fluids B 2, 1794 (1990).

The input of the code FLOW uses quasi-physical free functions

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- Each physical quantity reduces to the corresponding free function in the cylindrical limit.
- The input functions can be supplied as analytical expressions or numerical tables.

$D(\Psi)$ ⟶ Quasi-density

$P_{||}(\Psi)$ ⟶ Quasi-parallel pressure

$P_{\perp}(\Psi)$ ⟶ Quasi-perpendicular pressure

$B_0(\Psi)$ ⟶ Quasi-toroidal magnetic field

$M_{\theta}(\Psi)$ ⟶ Quasi-poloidal sonic Mach number

$M_{\varphi}(\Psi)$ ⟶ Quasi-toroidal sonic Mach number

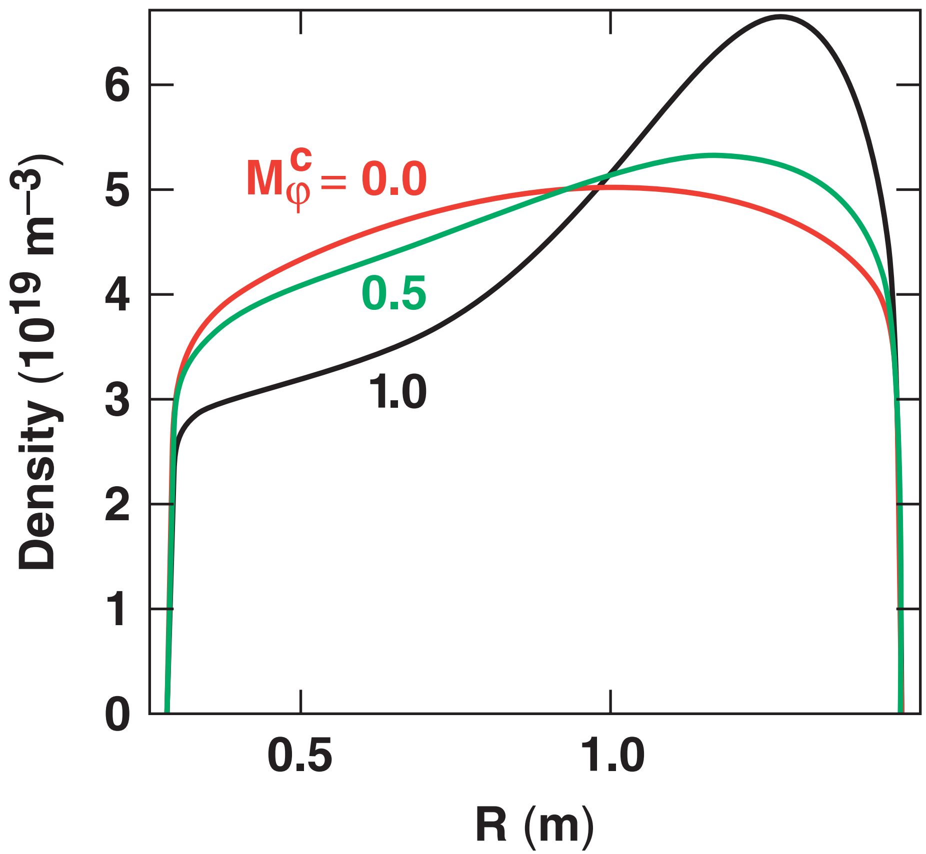
R. Betti and J. P. Freidberg, Phys. Plasmas 7, 2439 (2000).
L. Guazzotto, R. Betti, J. Manickam and S. Kaye, Phys. Plasmas 11, 604 (2004).

Equilibria with Purely Toroidal Flow. Applications to NSTX

$$\mathbf{v}_\varphi = \Omega(\Psi)\mathbf{R}$$
$$\mathbf{v}_\theta = \mathbf{0}$$

J. Menard, Bull. Am. Phys. Soc. 48, 18 (2003).

The centrifugal force causes an outward shift of the plasma



Effect of increasing rotation with constant plasma total mass for NSTX-like equilibria

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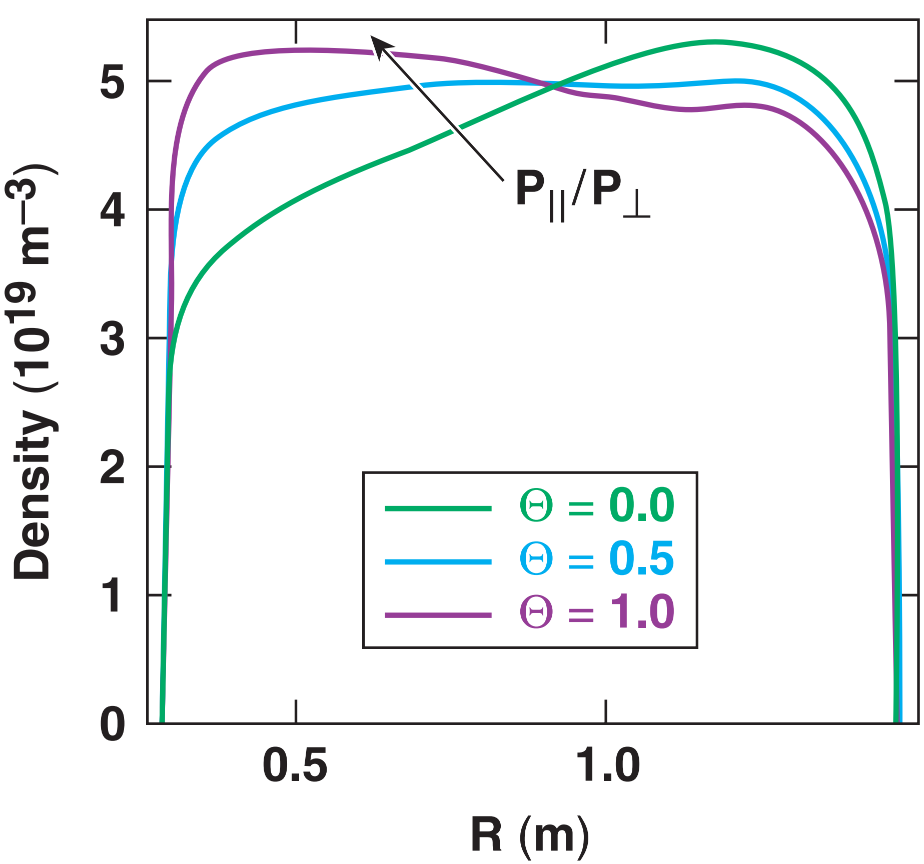
The parallel anisotropy ($p_{||} > p_{\perp}$) causes an inward shift



$$\Theta = \frac{P_{||}(\Psi) - P_{\perp}(\Psi)}{P_{\perp}(\Psi)}$$

(Toroidal Mach number)
$M_{\varphi}^{c} = 0.5$

Total energy is conserved.

NSTX-like parameters

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Equilibria with Poloidal Flow

Viscosity is reduced in supersonic flows and omnigenous B-field

- Poloidal flows in tokamaks are damped.
- Poloidal viscosity $\nu_\theta \sim 1/M_p^2$

$$M_p = \frac{v_\theta}{C_{s\theta}}$$

$$C_{s\theta} = C_s \frac{B_\theta}{B}$$

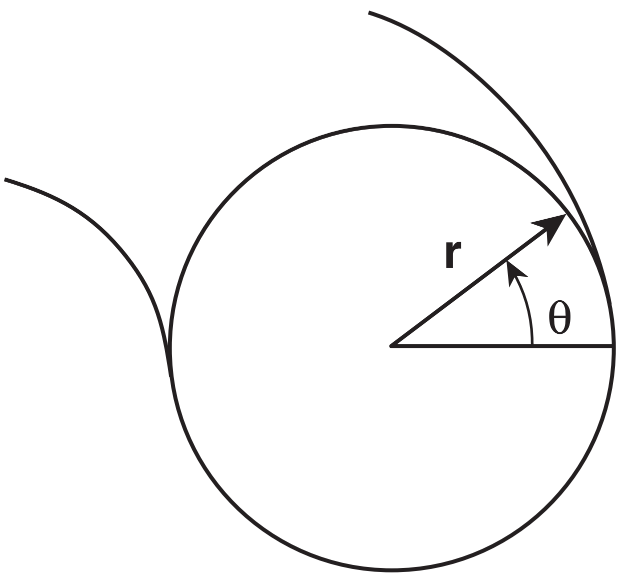
Poloidal sound speed

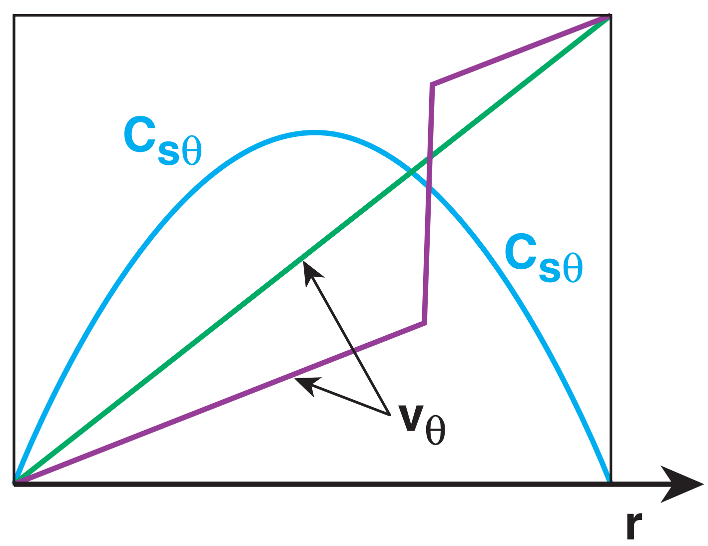
- Equilibria with supersonic poloidal flow have reduced viscosity.
- Omnigenous equilibria have low neoclassical viscosity.
- Specific applications shown for the UCLA Electric Tokamak;
  Results have general applicability.

K. C. Shaing *et al*., Phys. Plasmas (1996).
A. B. Hassam, Nucl. Fusion 36, 707 (1996).

Equilibria with transonic poloidal flow: flow profile ranging from subsonic to supersonic

$$v_\theta \sim C_{s\theta} = C_s \in / q$$





Transonic solution of Bernoulli equation is discontinuous and imposes constraints on the free functions

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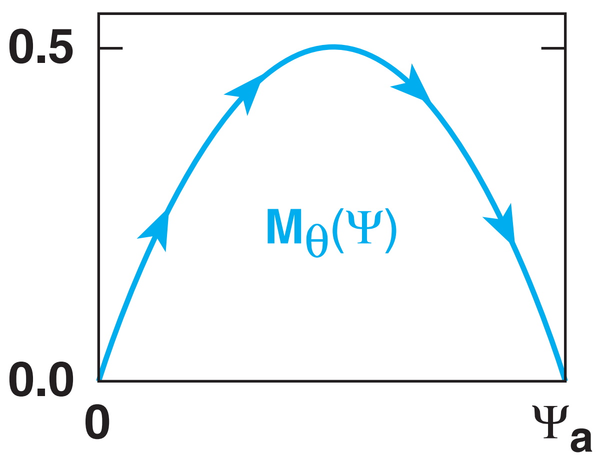
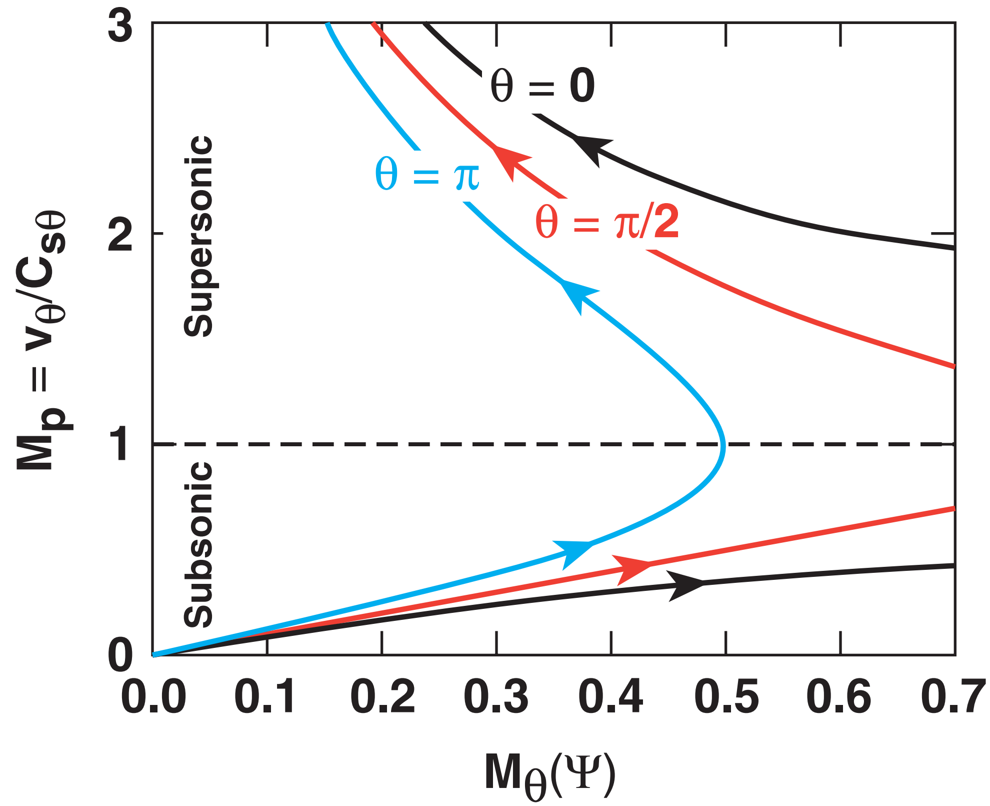
- $M_\theta(\Psi)$ cannot be chosen arbitrarily.





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Transonic solution of Bernoulli equation is discontinuous and imposes constraints on the free functions

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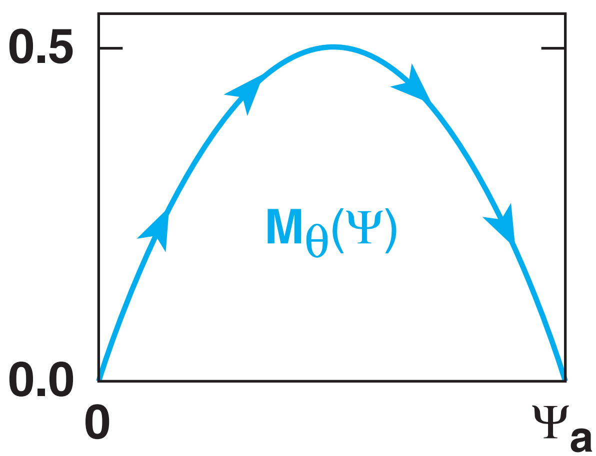
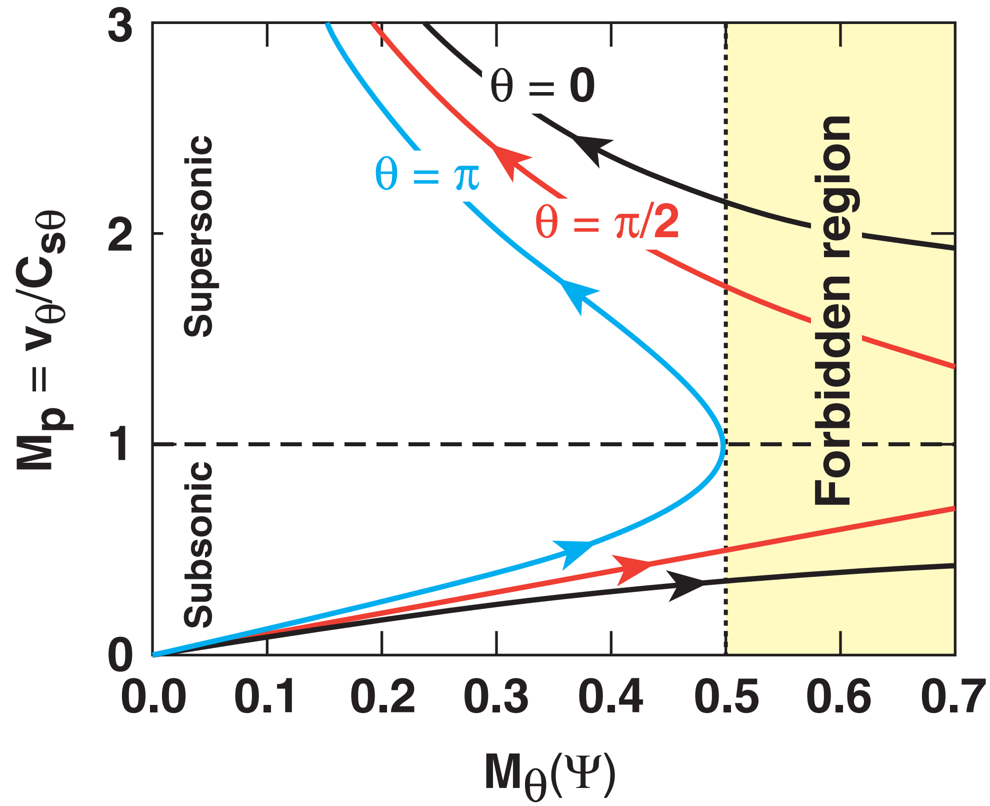
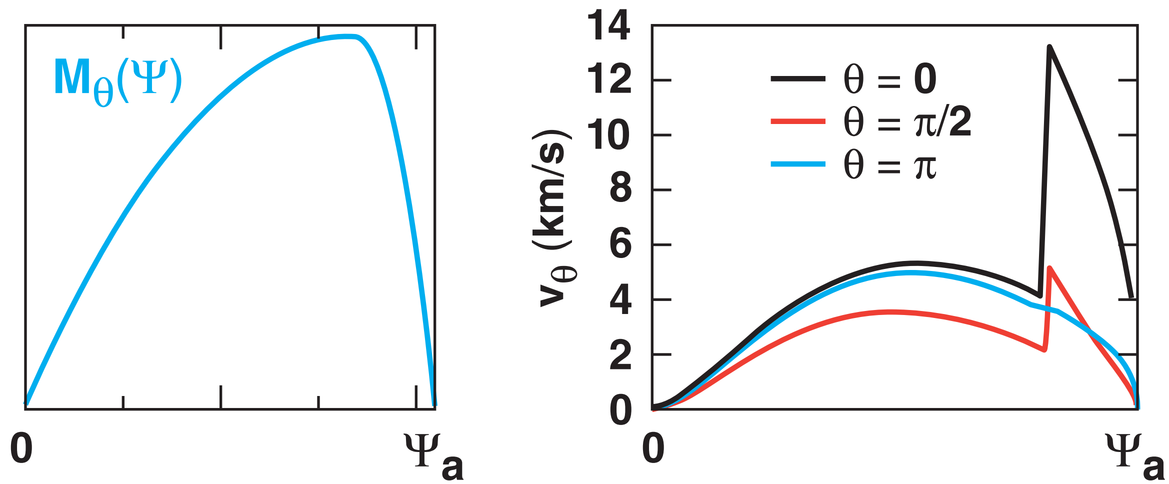
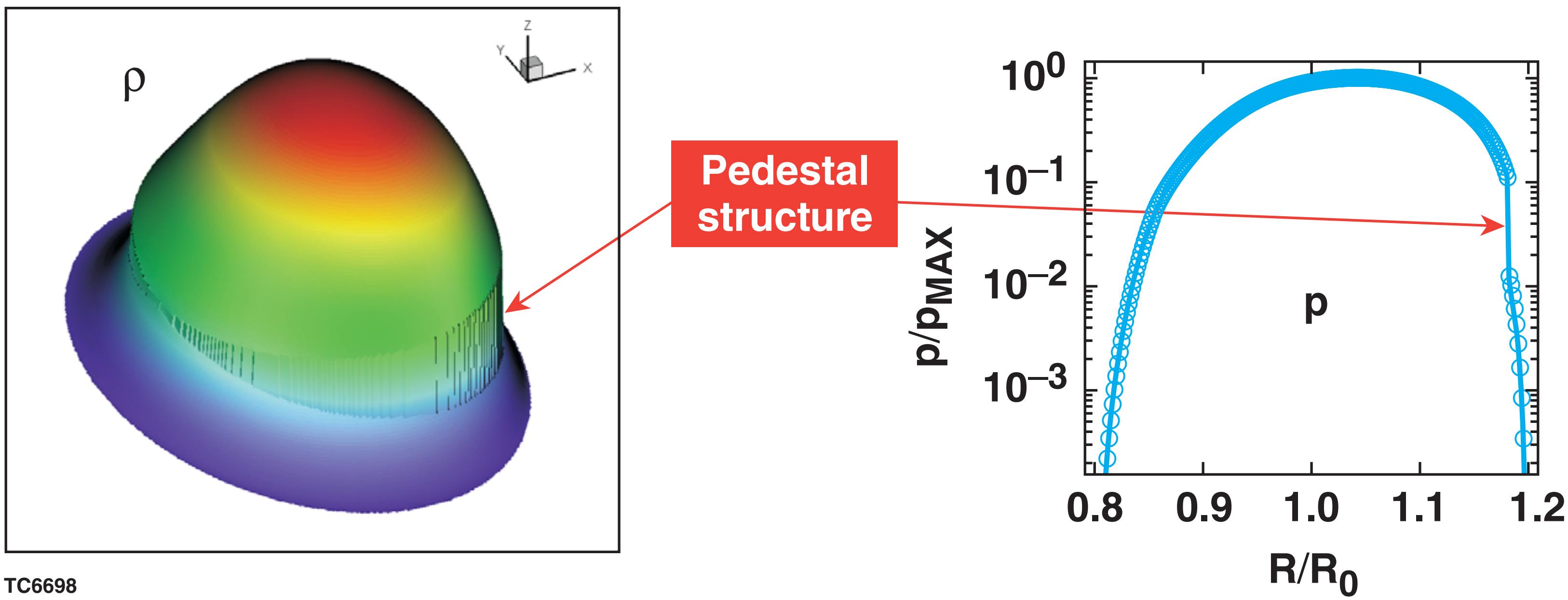




Numerical transonic equilibria exhibit an edge pedestal structure in the pressure profile

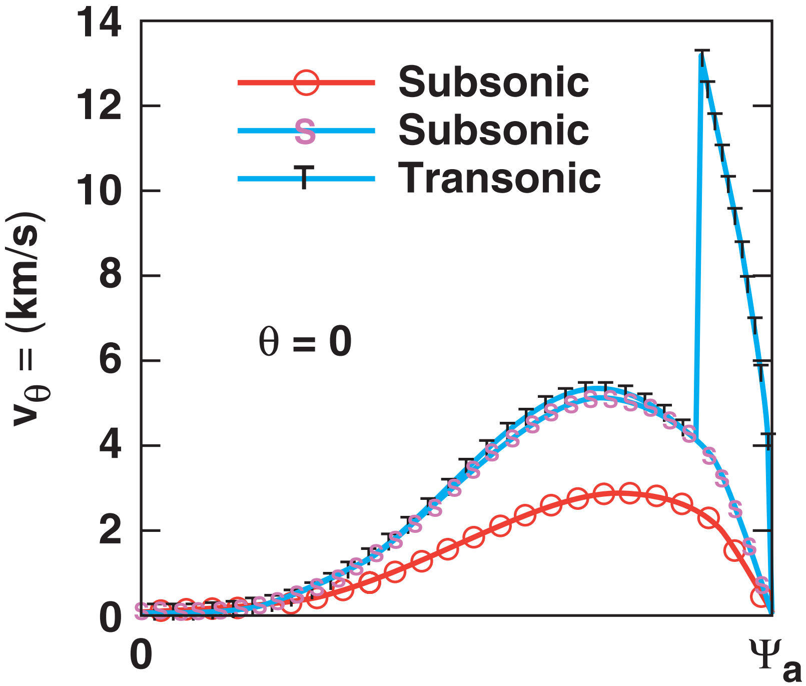
- **Low-β ET equilibrium**
- **Results are general**

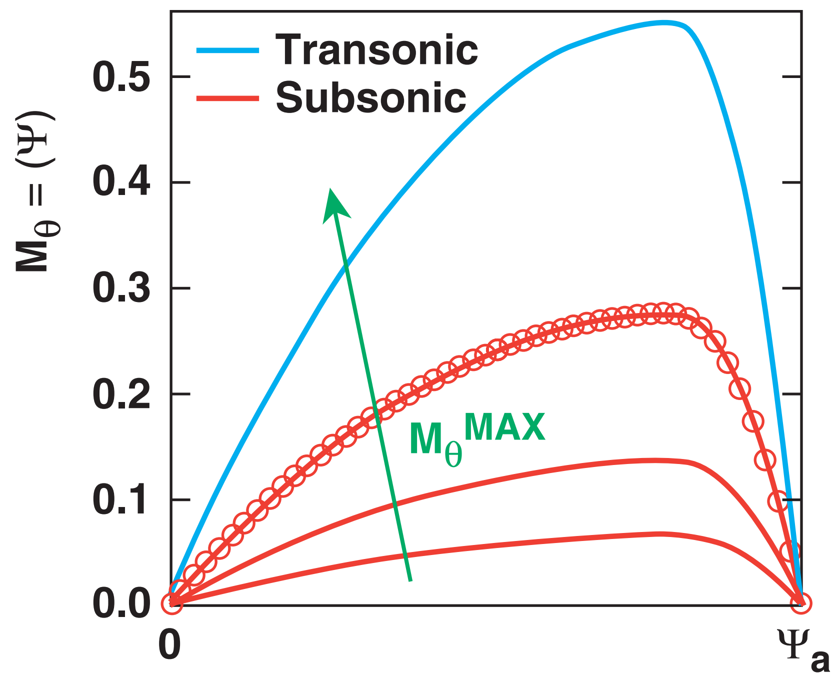




A bifucarted equilibrium exists for a critical poloidal velocity

A bifurcated equilibrium exists when the free function $M_\theta(\Psi)$ reaches the critical value.





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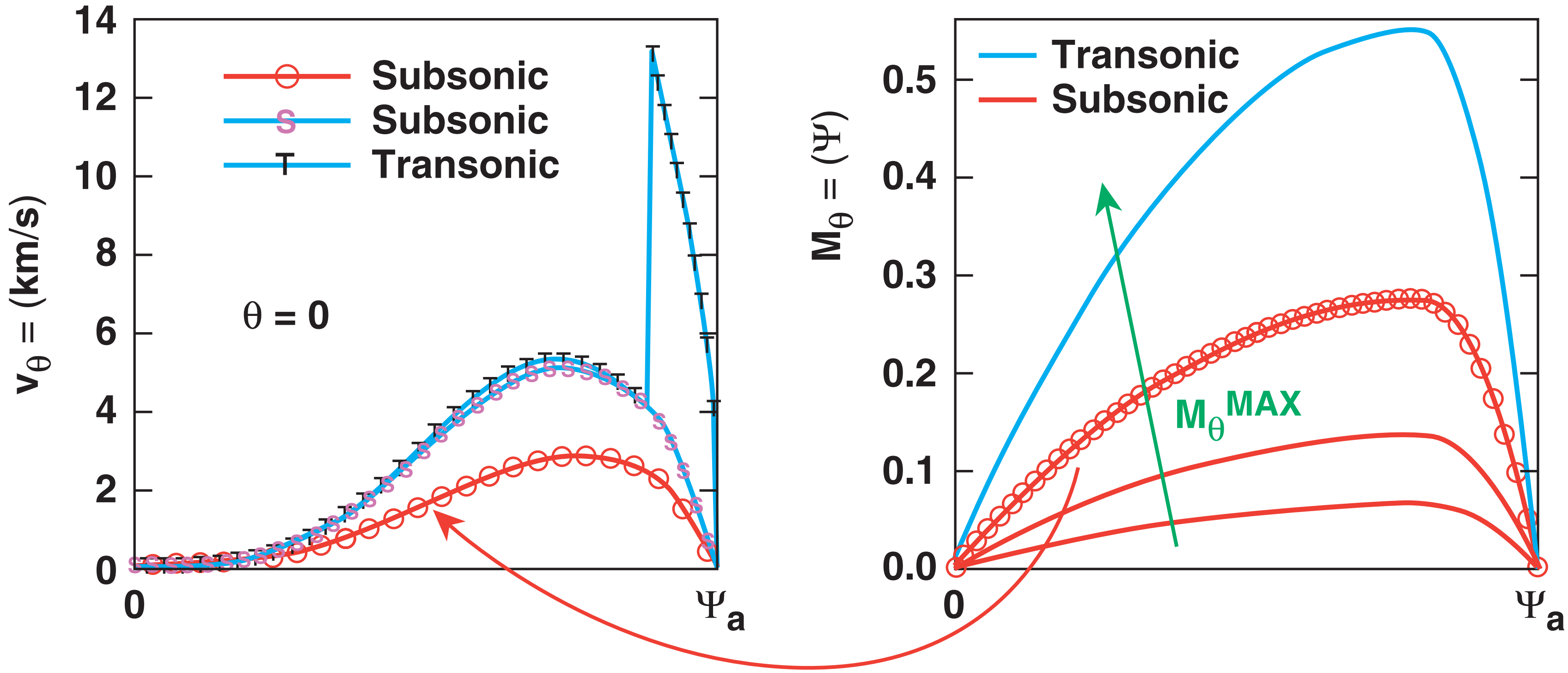

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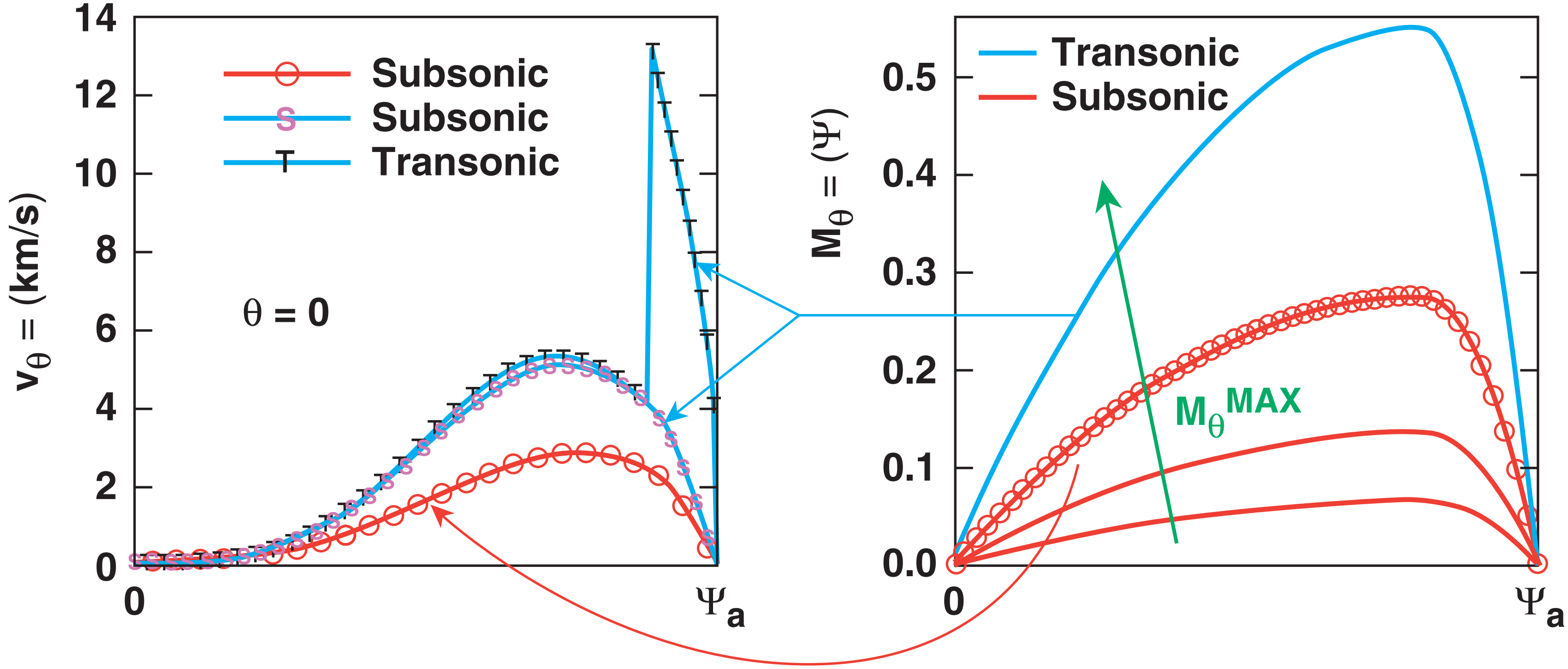

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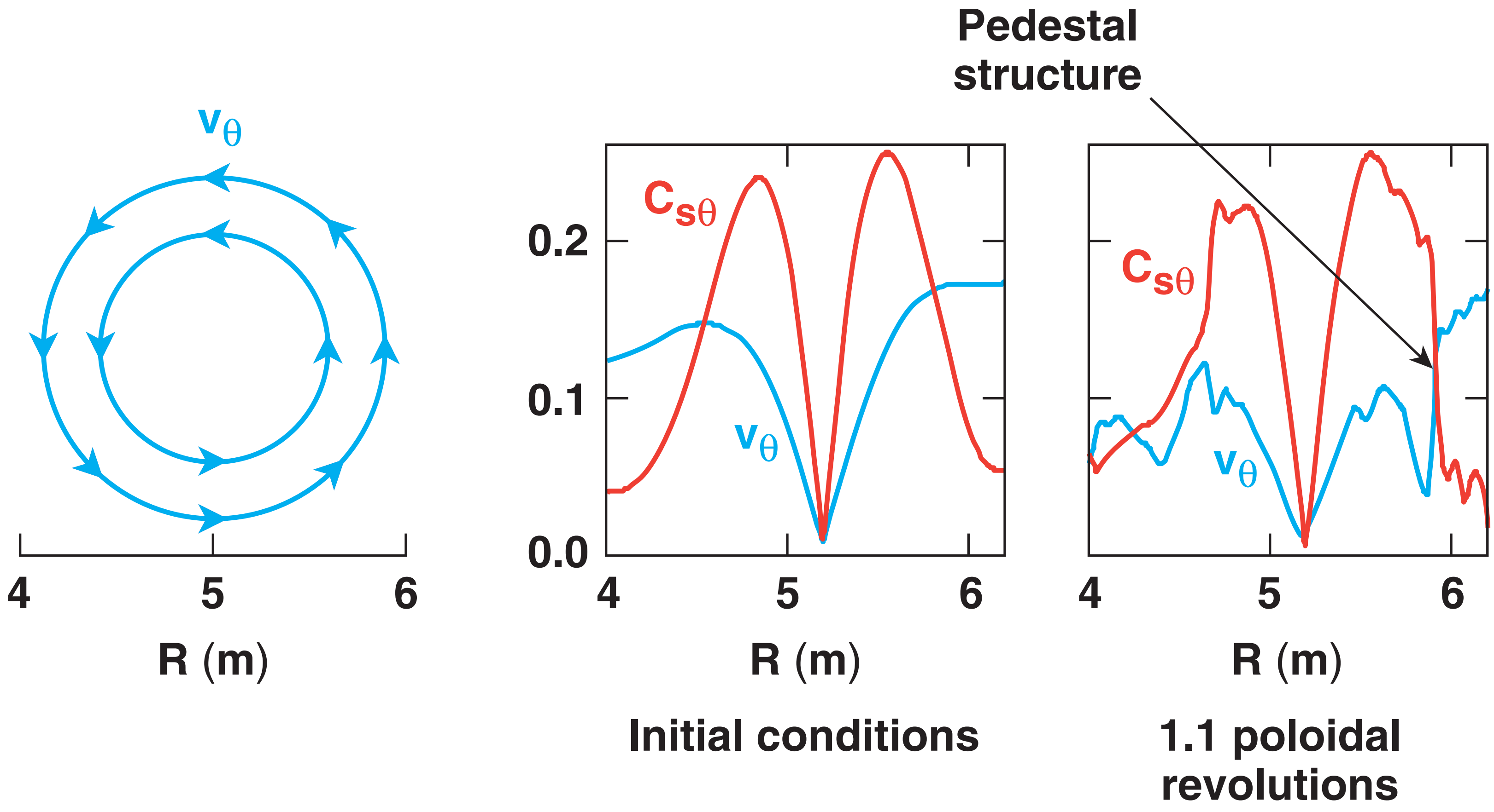
Initial value 2-D MHD simulations of transonic flow show the generation of discontinuities

- Poloidal sound speed (red) and poloidal velocity (blue) evolve to a discontinuous state.



TC6702

Equilibria with Super-Alfvénic Poloidal Flow

$$\mathbf{v}_{\theta} \gtrsim \mathbf{V}_{\mathbf{A}\theta}$$

Equilibria with super-Alfvénic poloidal flow (with respect to the poloidal Alfvén speed)

Simple model →

- Inverted Shafranov shift
- Poloidal flow >> toroidal flow

Quasi-omnigenous equilibria →

- $|\mathbf{B}| = |\mathbf{B}|(\Psi) + O(\epsilon^2)$
- Adjust flow to satisfy the above condition
- Small poloidal viscosity

Fully omnigenous equilibria[1] →

- $|\mathbf{B}| = |\mathbf{B}|(\Psi)$
- Negligible poloidal viscosity
- Not tokamak relevant

[1] H. Tasso and G. N. Throumoulopoulos, Phys. Plasmas 5, 2378 (1998).

An analytic model gives inverted Shafranov shift for Super-Alfvénic equilibria

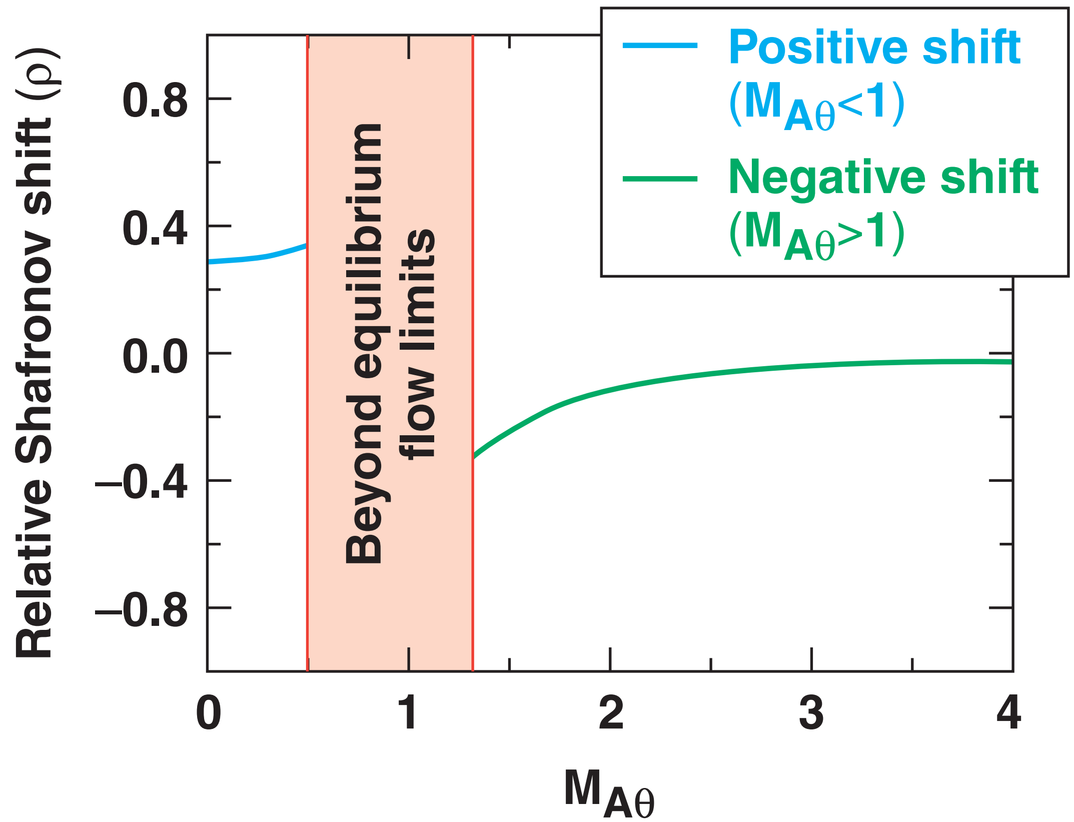
Assumptions:

$\in << 1$ $\quad \beta \sim \in$

$M_{A\theta} \approx$ const. $\quad \nu_\varphi << \nu_\theta$

For a fixed β the Shafranov shift is computed as a function of $M_{A\theta}$.

Shafronov shift depending on poloidal flow

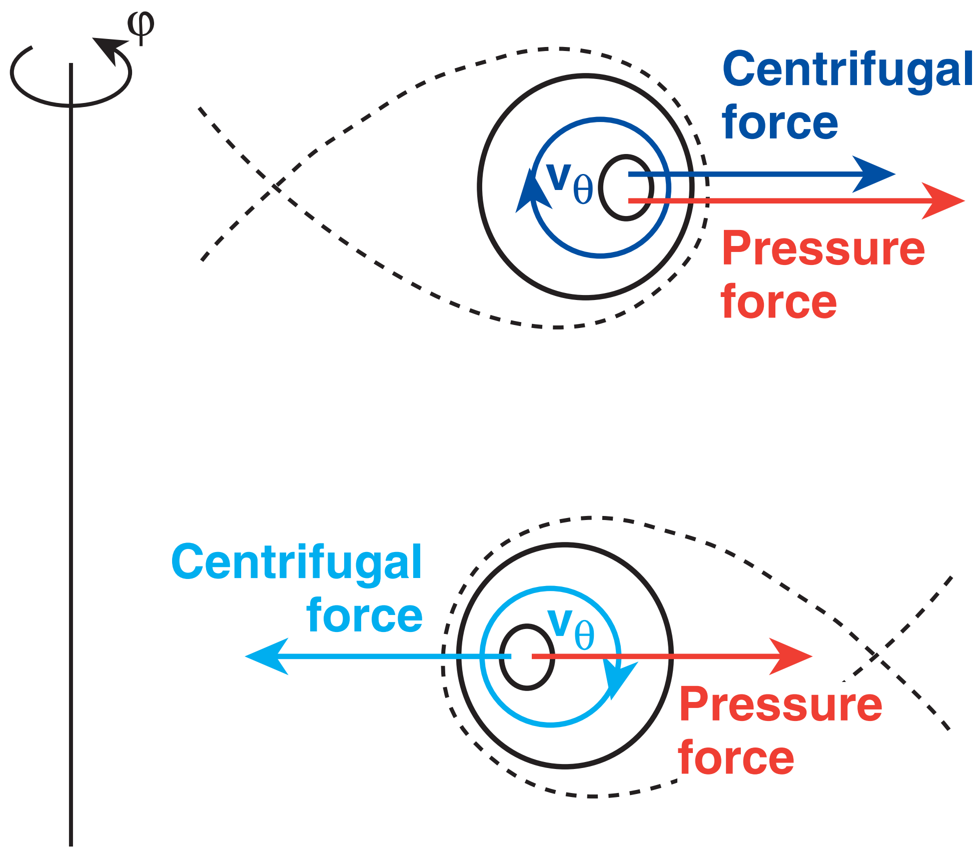


J. P. Freidberg, *Ideal Magnetohydrodynamics* (Plenum Press, New York, 1987), p. 138.
F. A. Haas, Phys. Fluids 15, 141 (1972).

TC6703

For inward shifted equilibria the pressure forces are balanced by the centrifugal forces

- Pressure forces cause outward Shafranov shift.
- Poloidal flow produces additional force.
- If the Shafranov shift is outward, pressure and centrifugal forces are aligned.
- If the Shafranov shift is inward, pressure and centrifugal forces are opposite.



TC6470a

FLOW confirms the existence of inward shifted equilibria for $M_{A\theta} > 1$

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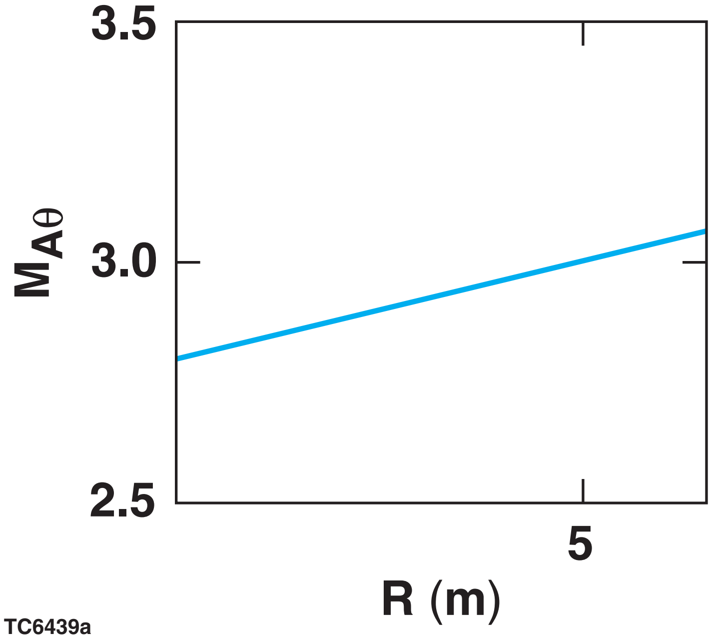
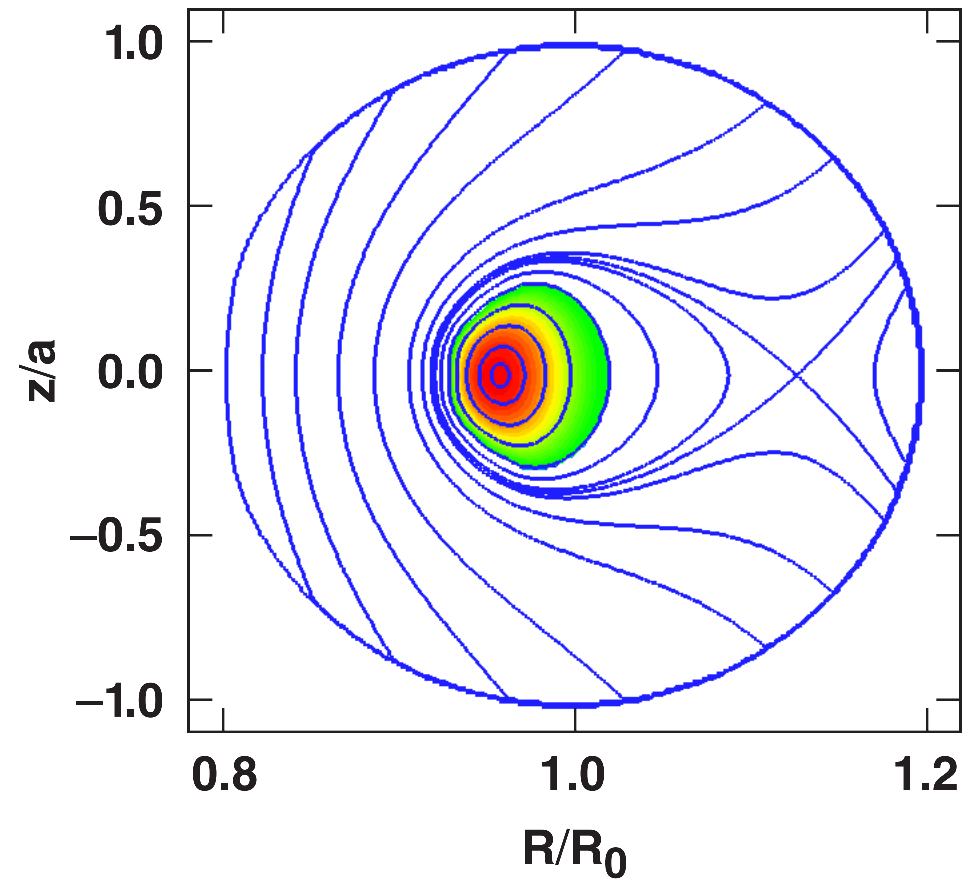
Numerical example:

- $M\varphi\ (\Psi) = 0$
- Flat density
- Peaked pressure





TC6439a

FLOW confirms the existence of inward shifted equilibria for $M_{A\theta} > 1$

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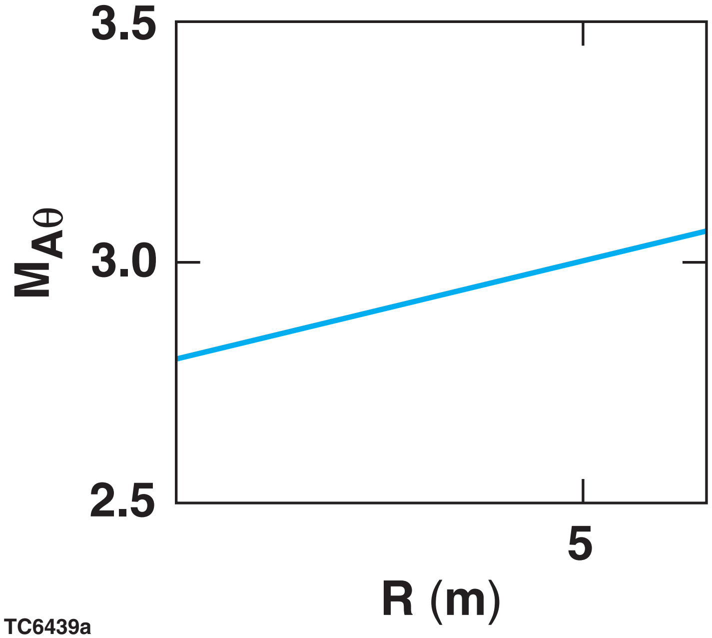
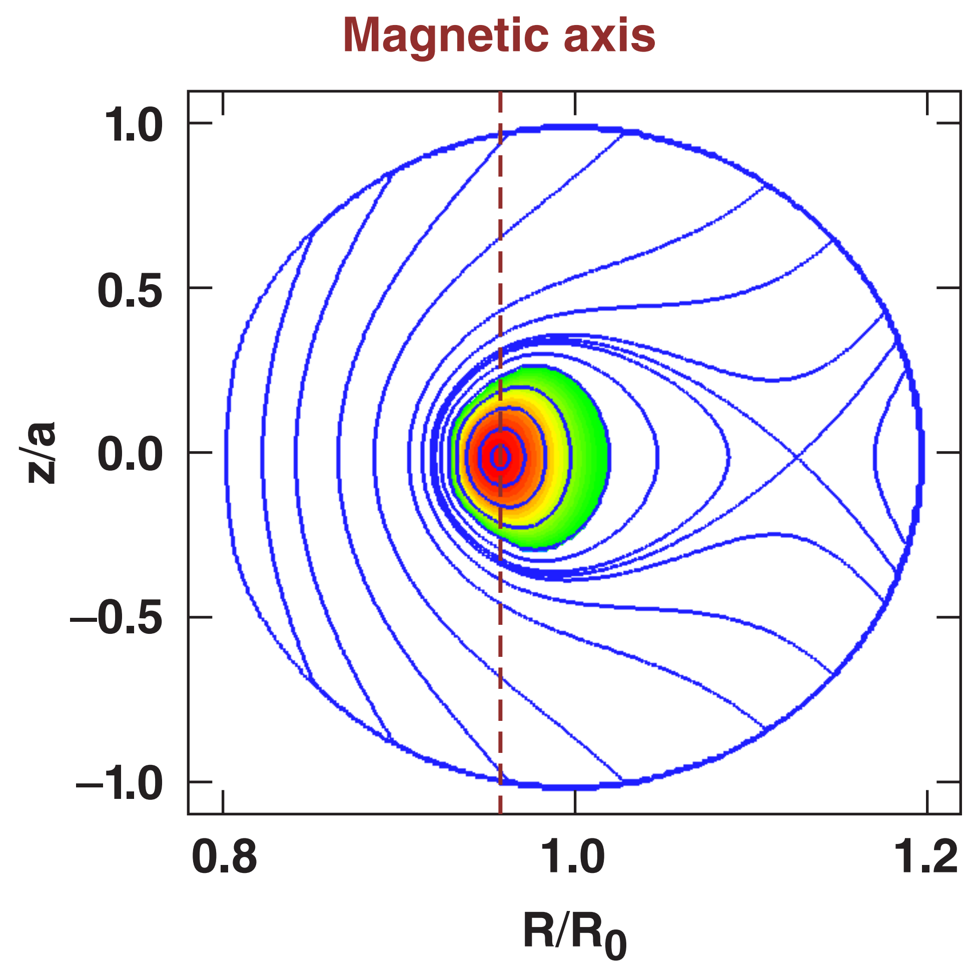




TC6439a

Quasi-omnigenous $[|B| = |B|(\Psi) + O(\epsilon^2)]$ equilibria with fast poloidal flow

- The solution uses an ϵ expansion, assuming the ordering:

$$B_\theta \sim \epsilon B_\varphi \qquad \left(M_{A\theta}^2 - 1\right) \sim 1 \qquad \beta \sim \epsilon$$

$$\downarrow$$

$$|B|^2 = B_\varphi^2 + B_\theta^2 \approx B_\varphi^2 + O\left(\epsilon^2\right)$$

- B_φ is imposed to be a function of Ψ only up to $O(\epsilon^2)$ corrections.

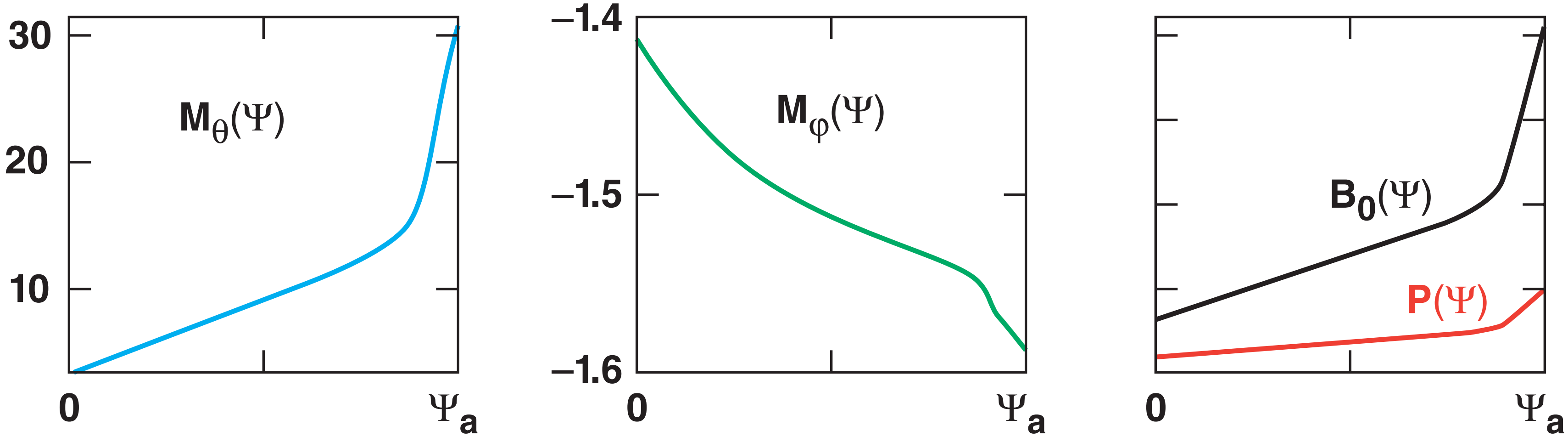
Fast poloidal flows are used to make the magnetic field quasi-omnigenous

In the static case $B_\varphi = \frac{F(\Psi)}{R}$ $\longrightarrow$ $B_\varphi = B_\varphi(\Psi) + O(\in)$.

With flow:

$$B_\varphi = \frac{1}{1 - M_{A\theta}^2}\left[\frac{F(\Psi)}{R} + R M_{A\theta}\sqrt{\rho}\,\Omega(\Psi)\right] \longrightarrow$$

adjust flow to impose $B_\varphi = B_\varphi(\Psi) + O(\in^2)$



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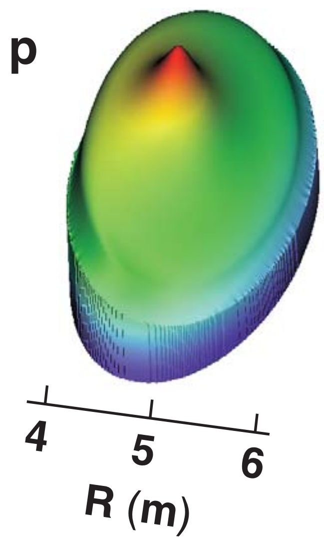
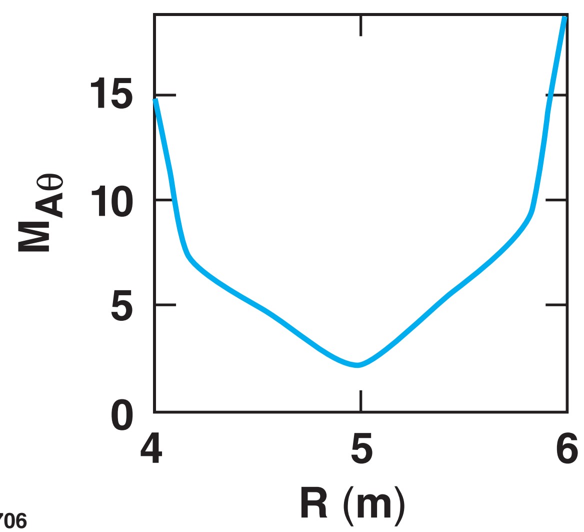
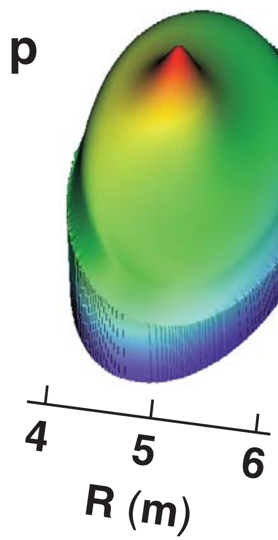
FLOW is used to compute the quasi-omnigenous equilibria of arbitrary shape



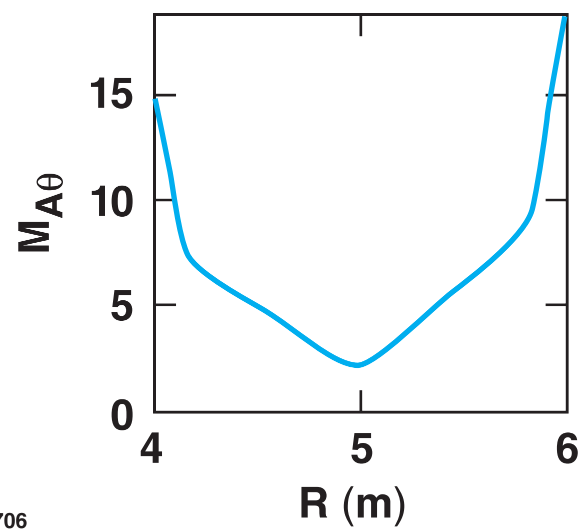


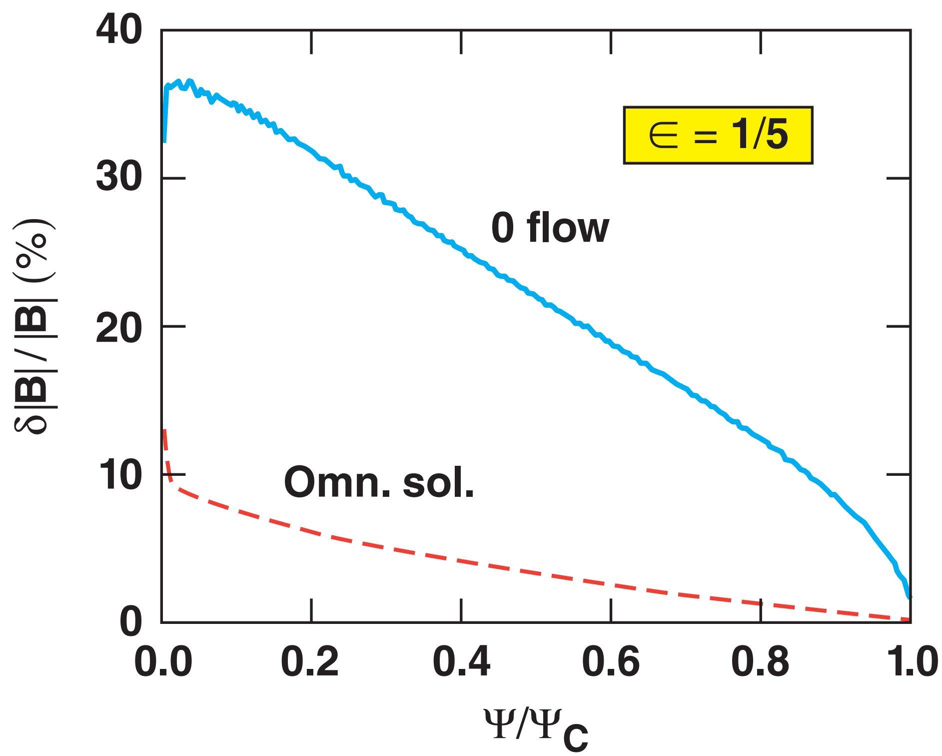
TC6706

FLOW is used to compute the quasi-omnigenous equilibria of arbitrary shape



Magnetic field relative variations are computed for a quasi-omnigenous and a static equilibrium.





TC6706

Conclusions

Tokamak equilibria with flow are very different from static equilibria

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- Equilibria with macroscopic flows have been studied analytically. The numerical results of the code FLOW confirm the results of theory.
- Equilibria with poloidal flow in the range of the poloidal sound speed $C_s B_\theta / B$ develop a pedestal structure due to the transition from subsonic to supersonic regime.
- Equilibria with poloidal flow in the super-Alfvénic regime show inverted Shafranov shift. The existence of a new class of quasi-omnigenous equilibria in such regime has been discussed.