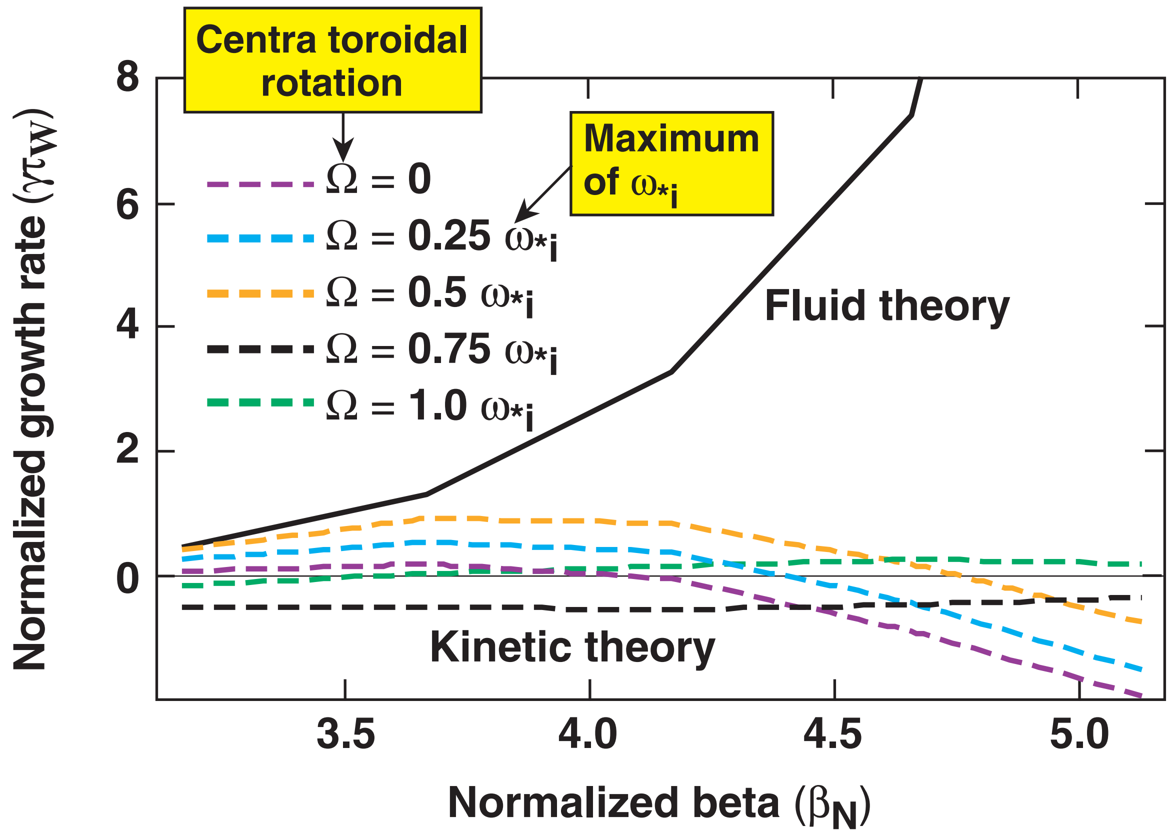
Application of the Low-Frequency Energy Principle to Wall Modes



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Summary

Marshall's contribution to the kinetic-energy principle has been instrumental in our understanding of the interaction between particles and MHD modes

- The low-frequency energy principle[1,2] is applied to resistive wall modes.
- When all the kinetic species (alphas, ions, and electrons) are included, the RWM growth is strongly reduced or fully suppressed in the low-rotation regime for ITER-like plasmas.

[1]M. N. Rosenbluth and N. Rostoker, Phys. Fluids 2, 23 (1959).
[2]J. W. Van Dam, M. N. Rosenbluth, and Y. C. Lee, Phys. Fluids 25, 1349 (1982).

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Outline

- Marshall's contribution to the low-frequency energy principle.
- The low-frequency energy principle application to wall modes.
- The qualitative picture of RWM interaction with trapped particles.
- The PEST kinetic postprocessor and the RWM stability in ITER-like plasmas.

The trapped particle contribution to the macroscopic plasma stability is included in the kinetic energy principle (KEP); Marshall's contribution to the KEP dates as early as 1959

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- M. N. Rosenbluth and N. Rostoker [Phys. Fluids 2, 23 (1959)]; first derivation of the kinetic energy principle from the solution of the "transport" equation. Thermodynamic derivation in M. D. Kruskal and C. R. Oberman [Phys. Fluids 1, 275 (1958)].

$$\delta W = \delta W_F + \delta W_K$$

$$\delta W_K = -\frac{1}{2}\int dV \left\{ \left(\delta P_{\|} - \delta P_{\perp}\right)\kappa \cdot \xi + \delta P_{\perp} \nabla \cdot \xi_{\perp} \right\}$$

$$\delta P_{\|,\perp} = \frac{1}{2} m \int_{\text{trapped}} d\vec{v} \left(v_{\|,\perp}^2 \delta f\right)$$

From the solution of the Vlasov equation in the limit of small Larmor radius and $\omega >> \omega_*$.

$$\boxed{\delta W_{Kj} \sim \int dV \beta_j \int_{\Lambda_{trap}} d\Lambda \hat{\tau}_{bounce} \left|\langle \kappa \cdot \xi \rangle\right|^2}$$

Positive-definite (stabilizing)

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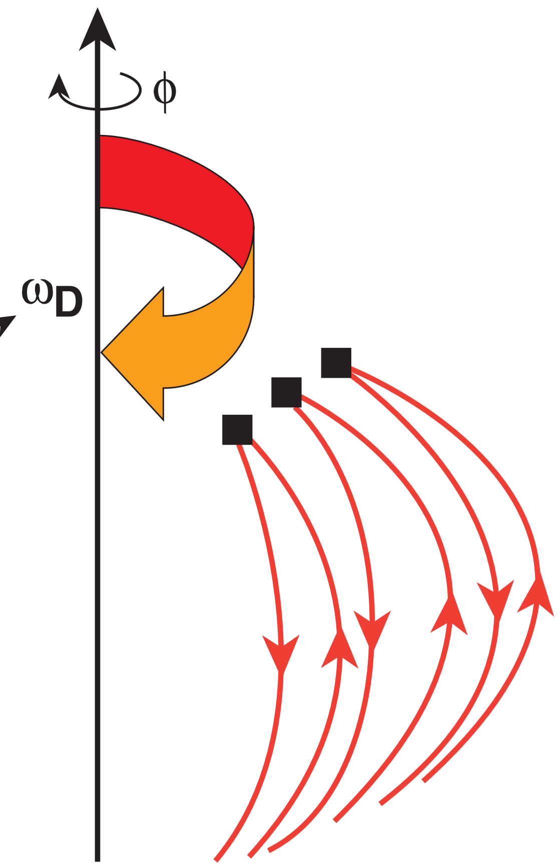
The "slow" trapped-particle orbit-drift motion was not included in the early formulations of the KEP

Precession or magnetic-drift frequency

$$\omega_D = \langle [V_{\nabla B} + V_\kappa] \cdot \nabla S \rangle$$

$$\omega_D \sim \frac{V_{th}}{R}\frac{r_L}{r} \sim \in \omega_*$$



Precession motion of the trapped-particle banana orbits

Van Dam, Rosenbluth, and Lee[3] (1982) generalized the energy principle to include the precession motion of trapped particles

- The Rosenbluth–Rostoker (and Kruskal–Oberman) KEP was equivalent to requiring that both the trapped-particle magnetic moment and longitudinal action are conserved.

- Van Dam, Rosenbluth, and Lee[3] recognized that if the mode frequency $\omega << \omega_D$, then the "magnetic flux passing through the precessional drift orbit is adiabatically conserved."

- The condition $\omega << \omega_D$ was easily satisfied for energetic particle species, such as in the microwave-heated poloidal ring of 100 to 500 keV electrons in the Elmo Bumpy Torus.

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[3] J. W. Van Dam, M. N. Rosenbluth, and Y. C. Lee, Phys. Fluids 25, 1349 (1982).

Using the third invariant connected with the fast precession motion, Van Dam, Rosenbluth, and Lee derive the generalized kinetic energy principle

Hot particle contribution for $\omega << \omega_{Dh}$

$$\delta W_{Kh} \sim \int dV \beta_h \int_{\Lambda_{trap}} d\Lambda \hat{\tau}_{bounce} \left| \langle \kappa \cdot \xi \rangle \right|^2 \frac{\omega_{*h}}{\omega_{Dh}}$$

- A similar form of the KEP was derived independently (submitted five months earlier) by T. M. Antonsen, Jr., B. Lane, and J. J. Ramos [Phys. Fluids 24, 1465 (1981)].
- This form of the KEP had extremely important applications in Tokamak physics.

The low-frequency KEP has improved our understanding of Tokamak physics; fishbones, sawtooth suppression by fast ions, and many resonant interactions with MHD modes

- When applied to the m = 1 internal kink, the KEP including fast ions show the existence of a new branch with the frequency $\sim\omega_{Dh}$, which is destabilized by the hot-ion pressure (fishbones) [L. Chen, R. B. White, and M. N. Rosenbluth, Phys. Rev. Lett. 52, 1122 (1984)].

- The same KEP also revealed the existence of stable regimes for the m = 1 mode (sawtooth suppression by fast ions) [Coppi *et al*., Phys. Rev. Lett. 63, 2733 (1989)].

- The KEP had many more applications, mostly in the area of interaction between trapped particles and MHD modes [TAE interaction: G. Y. Fu and C. Z. Chang, Phys. Fluids B 4, 3722 (1992); Energetic Particle Modes: S.-T. Tsai and L. Chen, Phys. Fluids B 5, 3284 (1993)].

The low-frequency energy principle can also be applied to wall modes

$$\gamma\tau_W = -\frac{\delta W_{MHD}^{\infty} + \delta W_K}{\delta W_{MHD}^{b} + \delta W_K}$$

$$\delta W_K = \delta W_K^i + \delta W_K^e + \delta W_K^{\alpha}$$

$$\delta W_K = \Re[\delta W_K] + i\Im[\delta W_K]$$

Instability condition

$$-\delta W_{MHD}^{b}\delta W_{MHD}^{\infty} > |\delta W_K|^2 + \Re[\delta W_K]\left(\delta W_{MHD}^{b} + \delta W_{MHD}^{\infty}\right)$$

MHD energy principle for RWM: S. W. Haney and J. P. Freidberg, Phys. Fluids B 1, 1637 (1989).

Without kinetic effects, the RWM is unstable between the wall and no-wall beta limits

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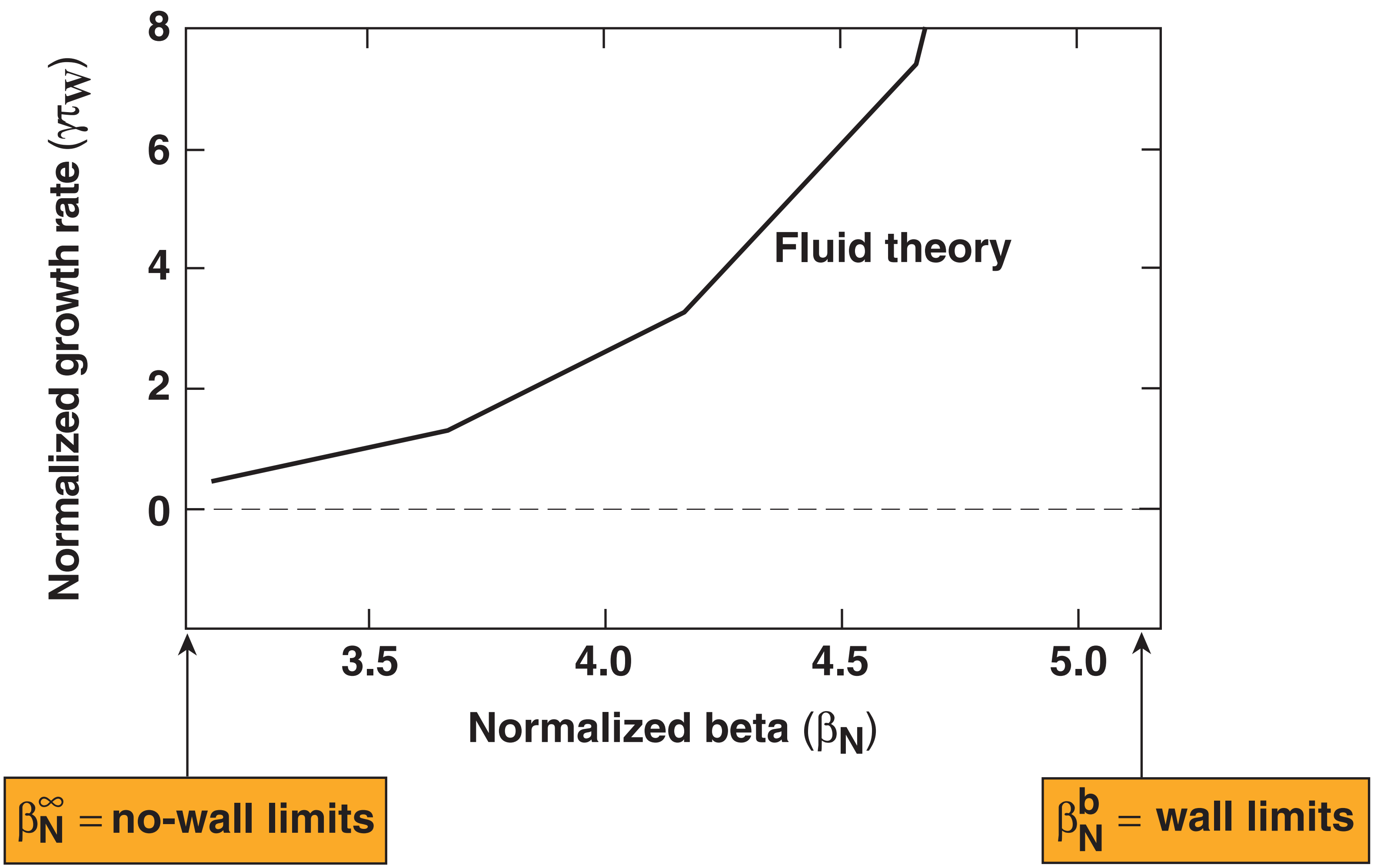
- Normalized RWM growth rate without kinetic effects.



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Qualitative analysis of the instability condition with kinetic effects

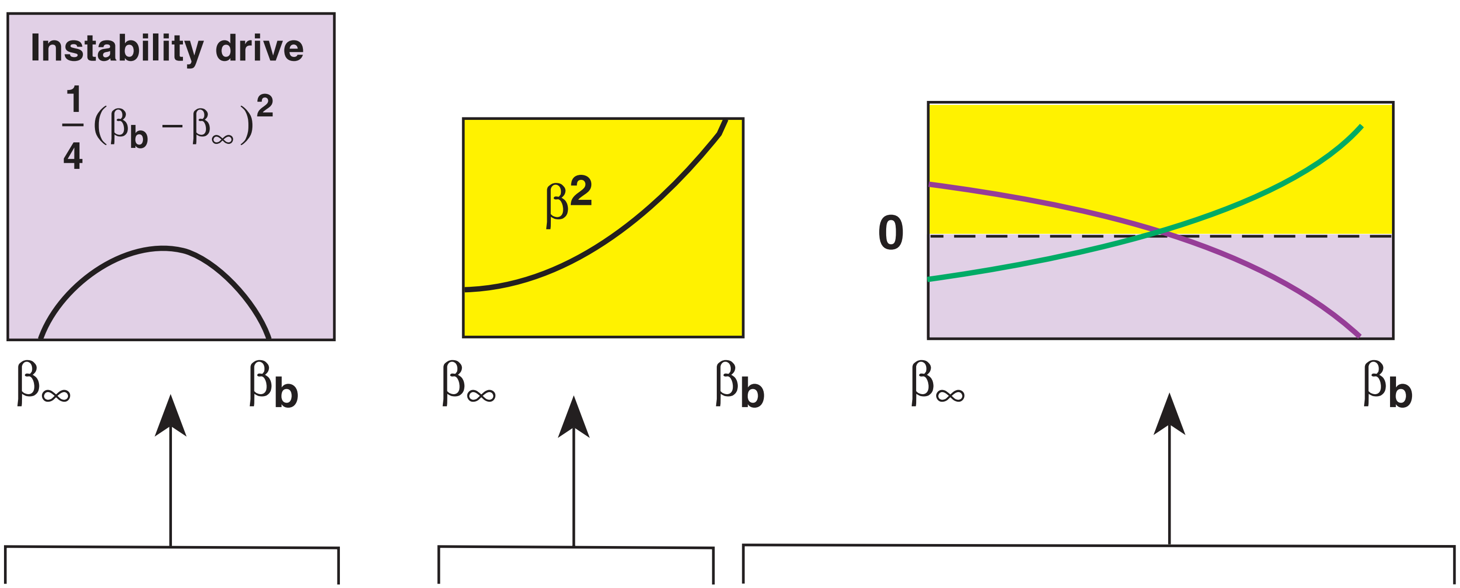
$$\delta W_{MHD}^{\infty} \sim \beta_\infty - \beta$$

$$\delta W_{MHD}^{b} \sim \beta_b - \beta$$

$$\delta W_K \sim \beta(\mathbf{X} + i\mathbf{Y})$$

Stabilizing

Destabilizing



$$-\delta W_{MHD}^{b}\delta W_{MHD}^{\infty} > |\delta W_K|^2 + \Re[\delta W_K]\left(\delta W_{MHD}^{b} + \delta W_{MHD}^{\infty}\right)$$

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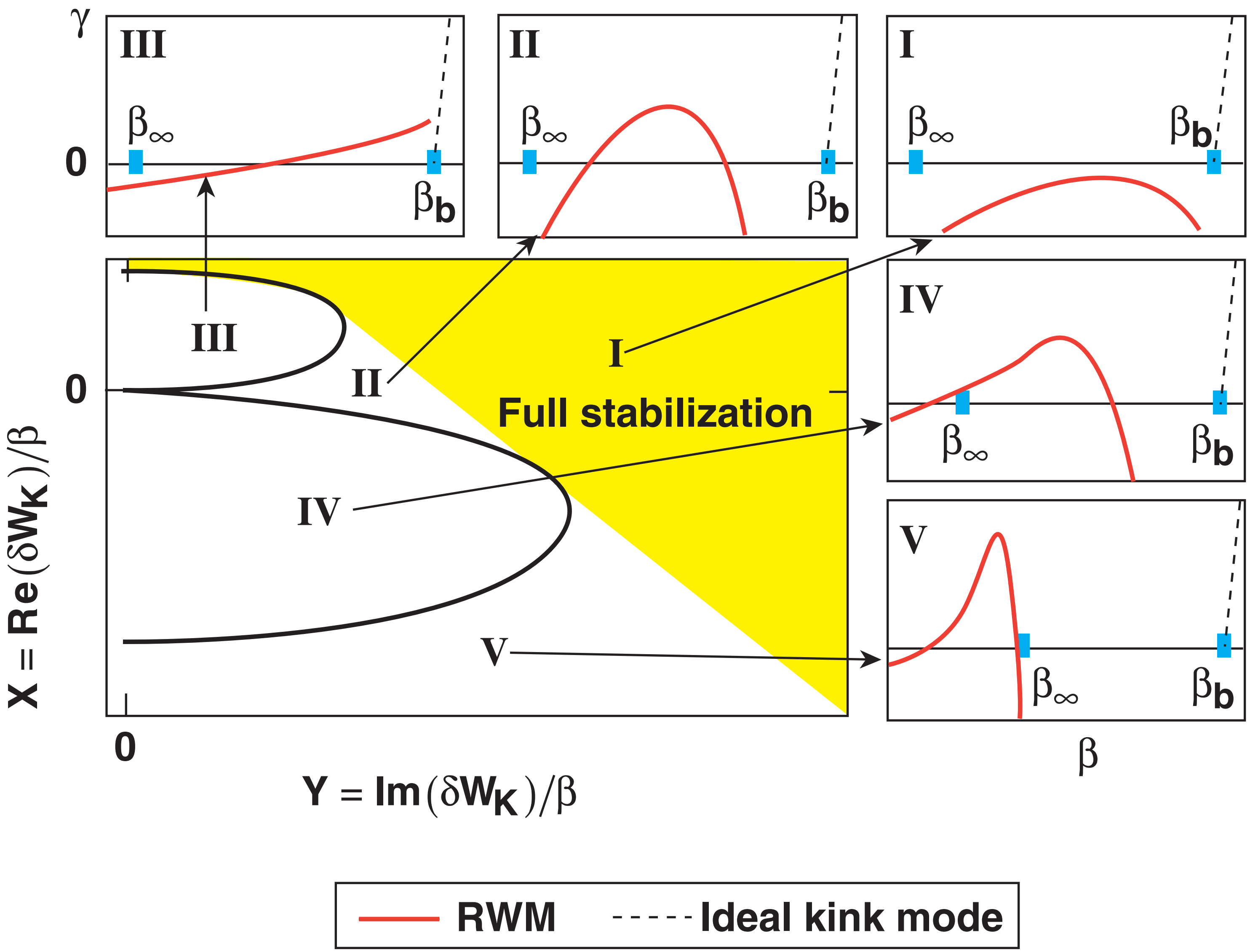
Five regimes of RWM stability/instability



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Necessary condition for stabilization requires relatively small δW_K

$\delta W_{MHD}^{\infty/b} \sim \beta_{\infty/b} - \beta$

Instability drive δW_{MHD}^{Drive}

$\delta W_K \sim \beta(X + iY)$

Kinetic effects

- Kinetic stabilization enhanced for strong dissipation. Full suppression for $\beta_b \sim 2\,\beta_\infty$ requires

$$|\delta W_K| > 0.29\,\delta W_{MHD}^{Drive} \Rightarrow \delta W_K^{Minimum} = \beta(0.08 + i\,0.28)$$

- Kinetic stabilization in the absence of dissipation ($Y = 0$) requires $X > 0.5$:

$$\delta W_K > 0.5\,\beta \Rightarrow |\delta W_K| > 0.5\,\delta W_{MHD}^{Drive}$$

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Low-frequency kinetic theory of the RWM: approximations

- **RWM frequency:** $\omega \sim 1/\tau_W - 50/\tau_W$ (τ_W = **wall time**)
- $\omega \ll \omega_D, \omega_{*i} \Leftarrow$ **zero-mode frequency**
  - ω_D **magnetic-drift frequency**
  - ω_{*i} **ion-diamagnetic-drift frequency**
- $\nu_{eff} \ll \omega_D, \omega_{*i} \Leftarrow$ **collisionless ions (and electrons?)**
- $\Omega_{rot} \sim \omega_{*i} \Leftarrow$ **quasi-stationary plasma**
- **For large rotation frequencies** ($\Omega_{rot} \gg \omega_{*i}$)**, the resonance with the trapped-particle bounce motion becomes important.**[4,5] (**Not included here**)
- **In the presence of sufficient dissipation, the wall mode can be suppressed by fast rotation.**[6–9] (**Not included here**)

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[4]A. Bondeson and M. S. Chu, Phys. Plasmas 3, 3013 (1996).
[5]Y. Liu *et al*., Nuc. Fusion 44, 232 (2004).
[6]A. D. Turnbull *et al*., Phys. Rev. Lett. 74, 718 (1995).
[7]A. Bondeson and D. J. Ward, Phys. Rev. Lett. 72, 2809 (1994).
[8]A. M. Garofalo *et al*., Phys. Plasmas 9, 1997 (2002).
[9]R. Fitzpatrick, Phys. Plasmas 9, 3459 (2002).

A resonance occurs between the precession frequency and the E $\times$ B Doppler-shifted mode frequency

$$\delta W_{Kj} \sim \int dV \beta_j \int_{\Lambda_{trap}} d\Lambda \hat{\tau}_{bounce} \left| \langle \kappa \cdot \xi \rangle \right|^2 I_\varepsilon (\Lambda, \vec{r})$$

$$\hat{\varepsilon} = \frac{\varepsilon}{T_j} \qquad I_\varepsilon = \int_0^\infty \hat{\varepsilon}^{5/2} e^{-\hat{\varepsilon}} \frac{\omega_{*j}\left(\hat{\varepsilon} - 3/2\right) + \omega_E}{\bar{\omega}_{Dj}\hat{\varepsilon} - \omega_{Doppler}^E} d\hat{\varepsilon}$$

$$\boxed{\omega_{Doppler}^E \equiv \omega_{Lab} - \omega_E}$$

$$\boxed{\omega_{Doppler}^E \equiv \bar{\omega}_{Dj}\hat{\varepsilon}}$$

Resonance condition

Ions can resonate in the absence of plasma rotation

Mode-particle resonance:
Doppler-shifted frequency = magnetic-drift frequency

$$\omega^{E\times B}_{\text{doppler}} \equiv \omega_{Di}$$

$$\omega^{E\times B}_{\text{doppler}} \equiv \cancel{\omega} - \omega_{E\times B}$$

$$\omega_{E\times B} = -\omega_{Di}$$

Zero-frequency approximation

$$\omega_{Di} = \omega_{*i} - \cancel{\Omega}_{\text{rot}}$$

$$\omega_{E\times B} = \Omega_{\text{rot}} - \omega_{*i}$$

Ion-force balance equation

Ions can resonate in the absence of plasma rotation

The strongest resonance comes from suprathermal particles (Ω_{rot} = 0 case)

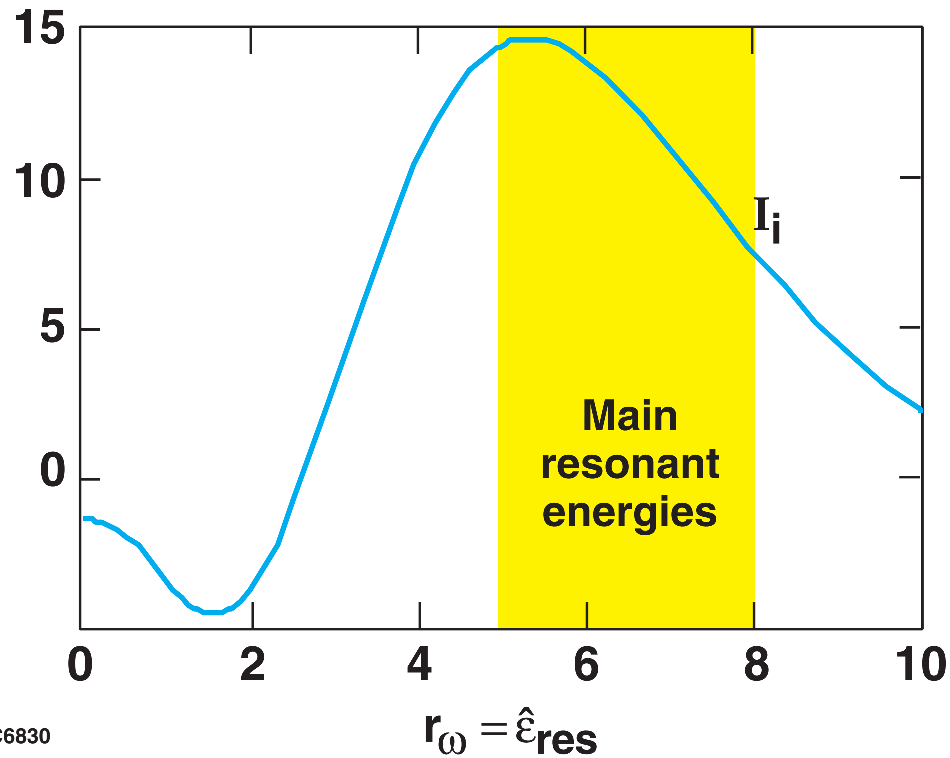
$$I_\varepsilon = \int_0^\infty \frac{\hat{\varepsilon}^{5/2} e^{-\hat{\varepsilon}} (\hat{\varepsilon} - 5/2)}{\frac{\hat{\varepsilon}}{r_\omega} - 1} d\hat{\varepsilon} = I_r + iI_i$$

$$\hat{\varepsilon} = \frac{\varepsilon}{T_j}$$

Large-aspect-ratio approximation

$$\hat{\varepsilon}_{res} = r_\omega \equiv \frac{\omega_{*i}}{\bar{\omega}_{Di}^{th}} \sim 5 - 8$$

$$I_i = \pi r_\omega^{7/2} (r_\omega - 5/2) \exp(-r_\omega)$$



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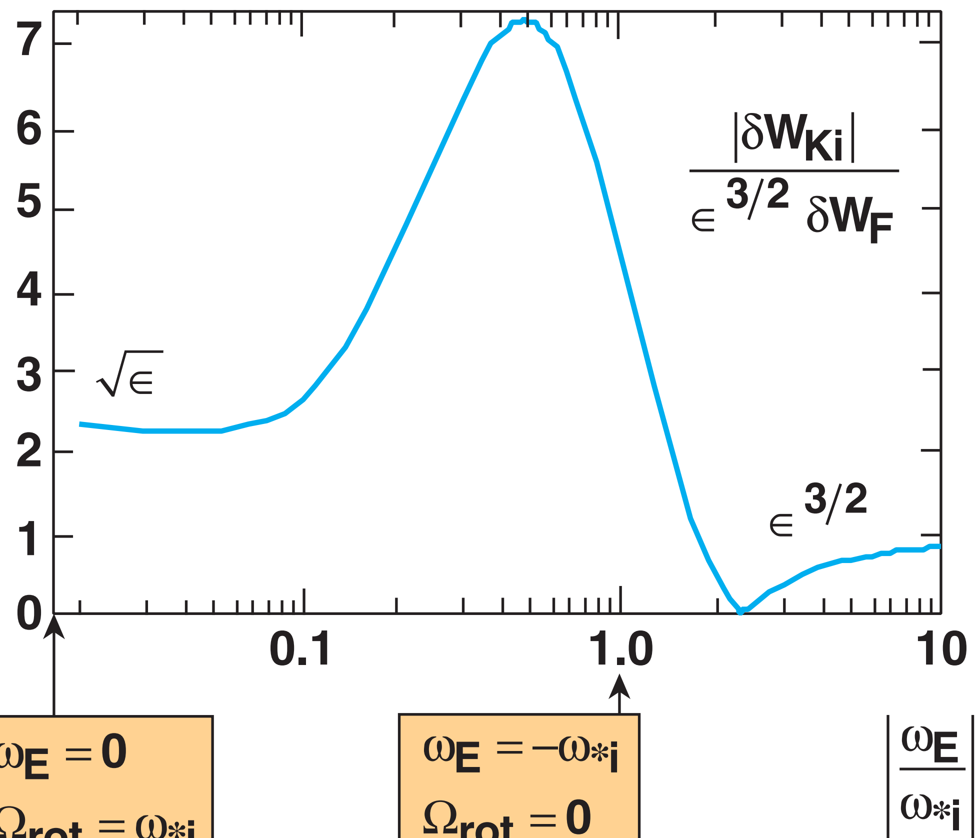
Large-aspect-ratio ordering underestimates the size of ion-kinetic terms

$$\frac{\delta W_{Ki}\left(\omega_E \ll \omega_D\right)}{\delta W_{Fluid}} \sim \sqrt{\epsilon} \quad \longleftrightarrow \quad \frac{\delta W_{Ki}\left(\omega_E \gg \omega_D\right)}{\delta W_{Fluid}} \sim \epsilon^{3/2}$$



$$\frac{\left|\delta W_{Ki}\left(\omega_E \sim 3-8\,\omega_{Di}^{th}\right)\right|}{\delta W_{Fluid}} > \sqrt{\epsilon}$$

$\omega_E = 0$
$\Omega_{rot} = \omega_{*i}$

$\omega_E = -\omega_{*i}$
$\Omega_{rot} = 0$

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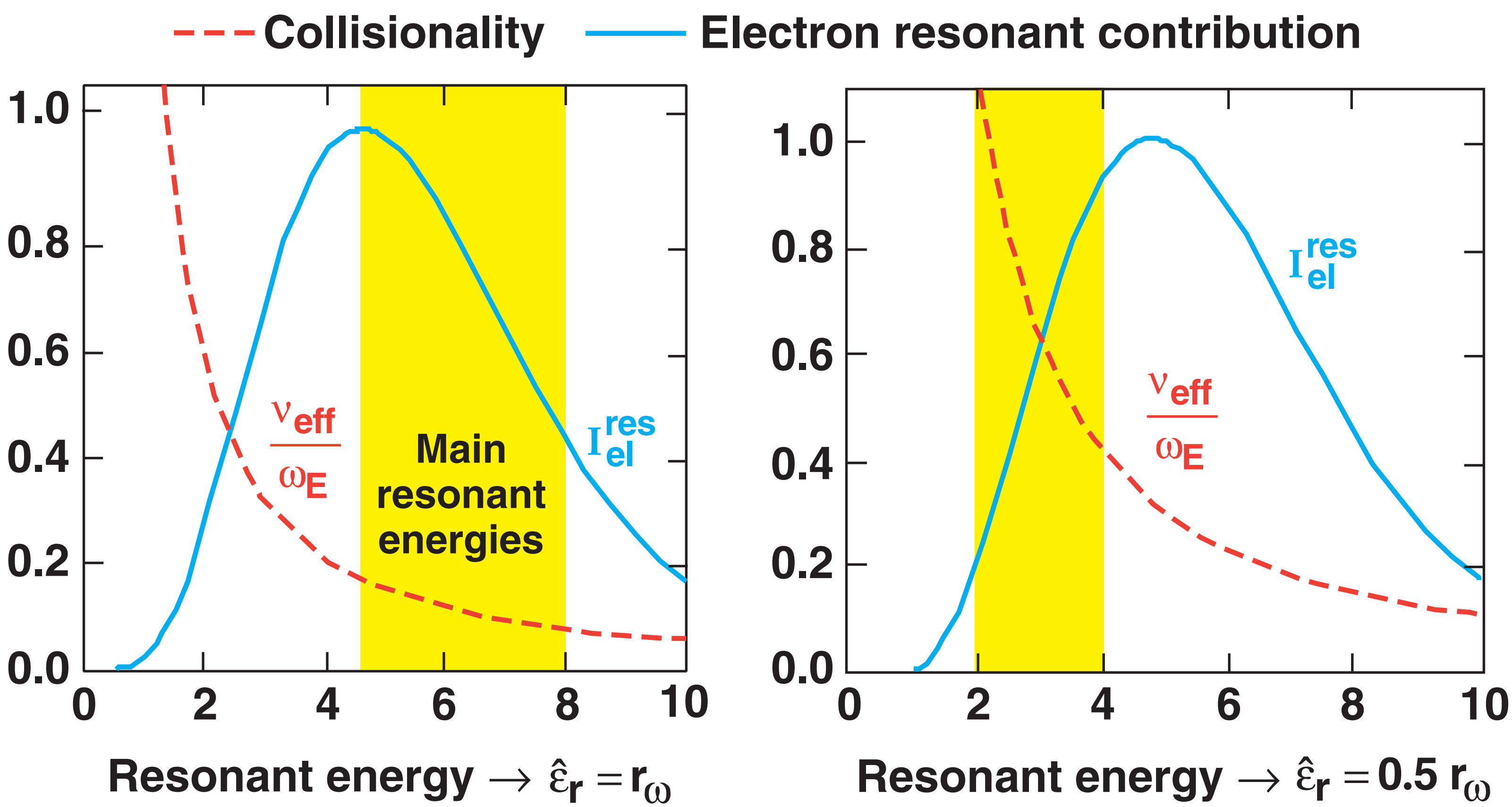
Electrons can resonate for $\Omega_{rot} > \omega_{*i}$; collisionality reduces the resonant interaction but does not eliminate it



$\omega_E = \omega_{*i}$
$\Omega_{rot} = 2\,\omega_{*i}$

$r_\omega \equiv \dfrac{\omega_{*i}}{\bar{\omega}_{De}^{th}} \sim 5-8, \quad \hat{\varepsilon} = \dfrac{\varepsilon}{T_e}$

$\omega_E = 0.5\;\omega_{*i}$
$\Omega_{rot} = 1.5\;\omega_{*i}$

TC6832

In a burning plasma, nonresonant and resonant α particles contribute to the stabilization

$$\delta W_{K\alpha} \sim \int dV \int_{\Lambda_{trap}} d\Lambda \hat{\tau}_{bounce} \left| \langle \kappa \bullet \xi \rangle \right|^2 \beta_\alpha \frac{\omega_{*\alpha}}{\bar{\omega}_{D\alpha}} I_\varepsilon (\Lambda, \vec{r})$$

$$I_\varepsilon = \int_0^1 \frac{\hat{\varepsilon}}{\hat{\varepsilon} - (\omega_E / \bar{\omega}_{D\alpha})} d\hat{\varepsilon} \qquad \hat{\varepsilon} = \frac{\varepsilon}{\varepsilon_\alpha}.$$

- In high-β plasmas, ω_D can be significantly smaller than large-aspect-ratio prediction $\rightarrow$ strong resonance can occur between ω_E and $\omega_{D\alpha}$
- Nonresonant contribution always stabilizing and enhanced by ratio

  $\omega_{*_\alpha} / \omega_{D\alpha} >> 1$

- α contribution is significant since ∇p_α is large where the RWM eigenfunction is large

Analytic predictions are not reliable based on the variability of ω_D, with respect to ω_E (drift reversal, rotation profile, eigenfunction shape); quantitative assessment requires a stability code

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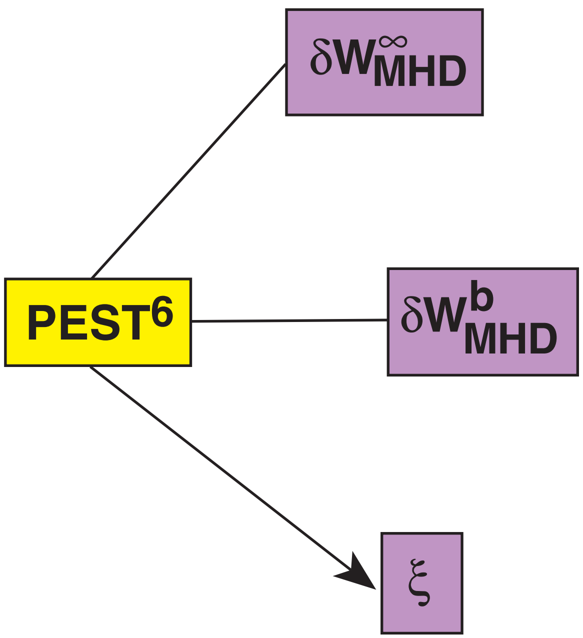


- ξ from PEST is used to compute

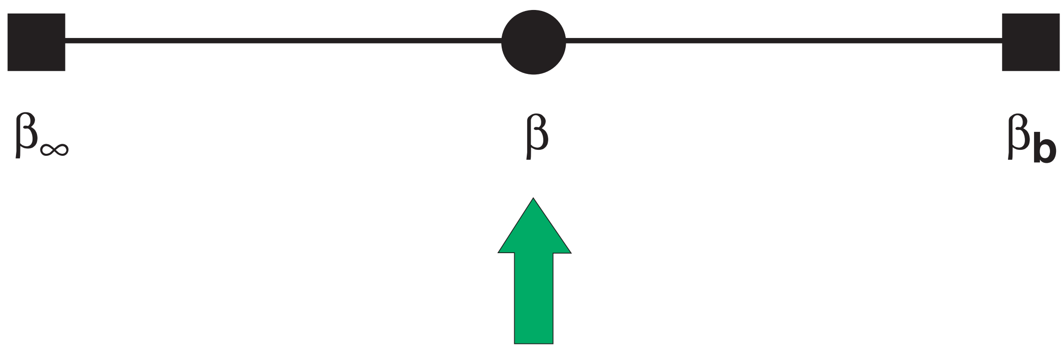
$$\delta W_K = \delta W_K^{i} + \delta W_K^{e} + \delta W_K^{\alpha}$$

- The RWM energy principle is used to compute the RWM growth rate.

$$\gamma \tau_w = -\frac{\delta W^{\infty}_{MHD} + \delta W_K}{\delta W^{b}_{MHD} + \delta W_K}$$

[6]S. Preische, J. Manickam, and J. L. Johnson, Comput. Phys. Commun. 76, 318 (1993).
R. C. Grimm, J. M. Greene, and J. L. Johnson, Methods Comput. Phys. 16, 253 (1976).

The RWM plasma-eigenfunction ξ is approximated by the ideal-MHD ξ from PEST for marginally stable wall position



- **For each equilibrium with $\beta_\infty < \beta < \beta_b$, the wall is moved closer to the plasma until marginal stability is reached.**
- **Since the RWM is essentially zero frequency, the corresponding plasma eigenfunction computed with PEST is used to approximate the RWM eigenfunction.**
- **Such an eigenfunction is used to compute fluid and kinetic contributions to δW.**

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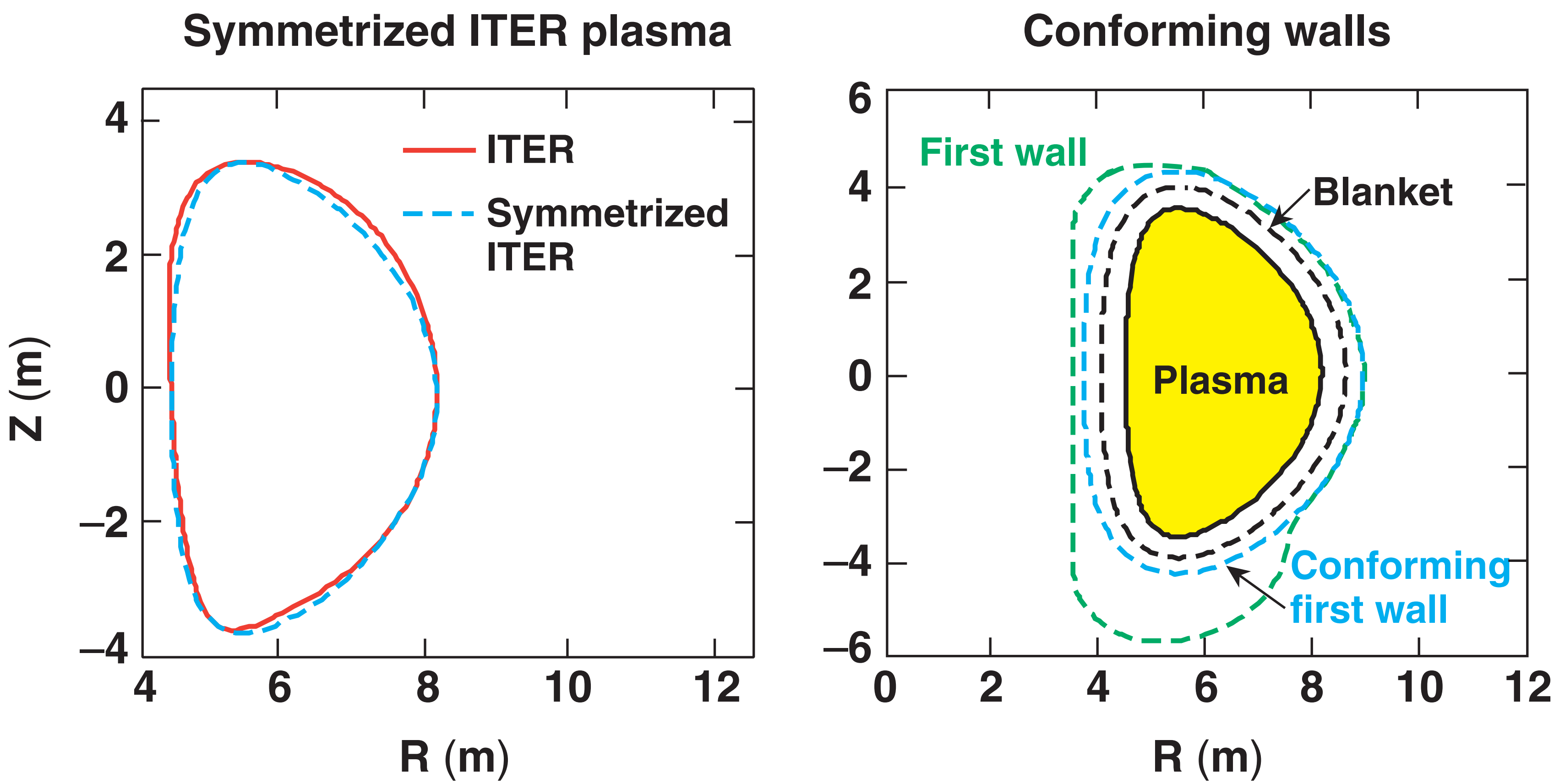
The RWM stability of an ITER-advanced Tokamak scenario is studied using a symmetrized plasma and conforming wall



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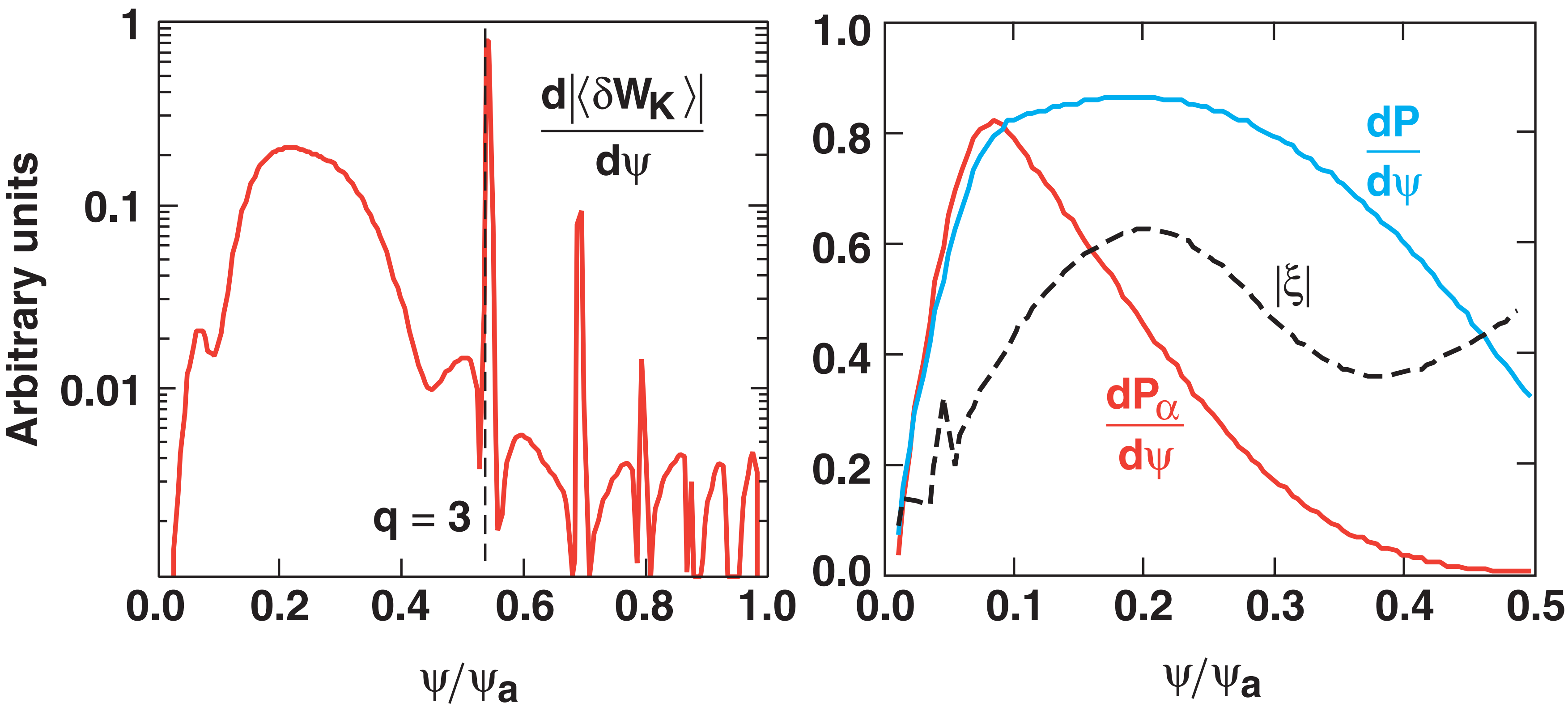
Most of the kinetic contribution is produced within the q = 3 surface



- Alpha contribution is significant since ∇p_α is large where the RWM eigenfuction is large.
- Alpha contribution can be comparable to ions and electrons for $\langle \beta_\alpha \rangle = 0.15 \langle \beta \rangle$.

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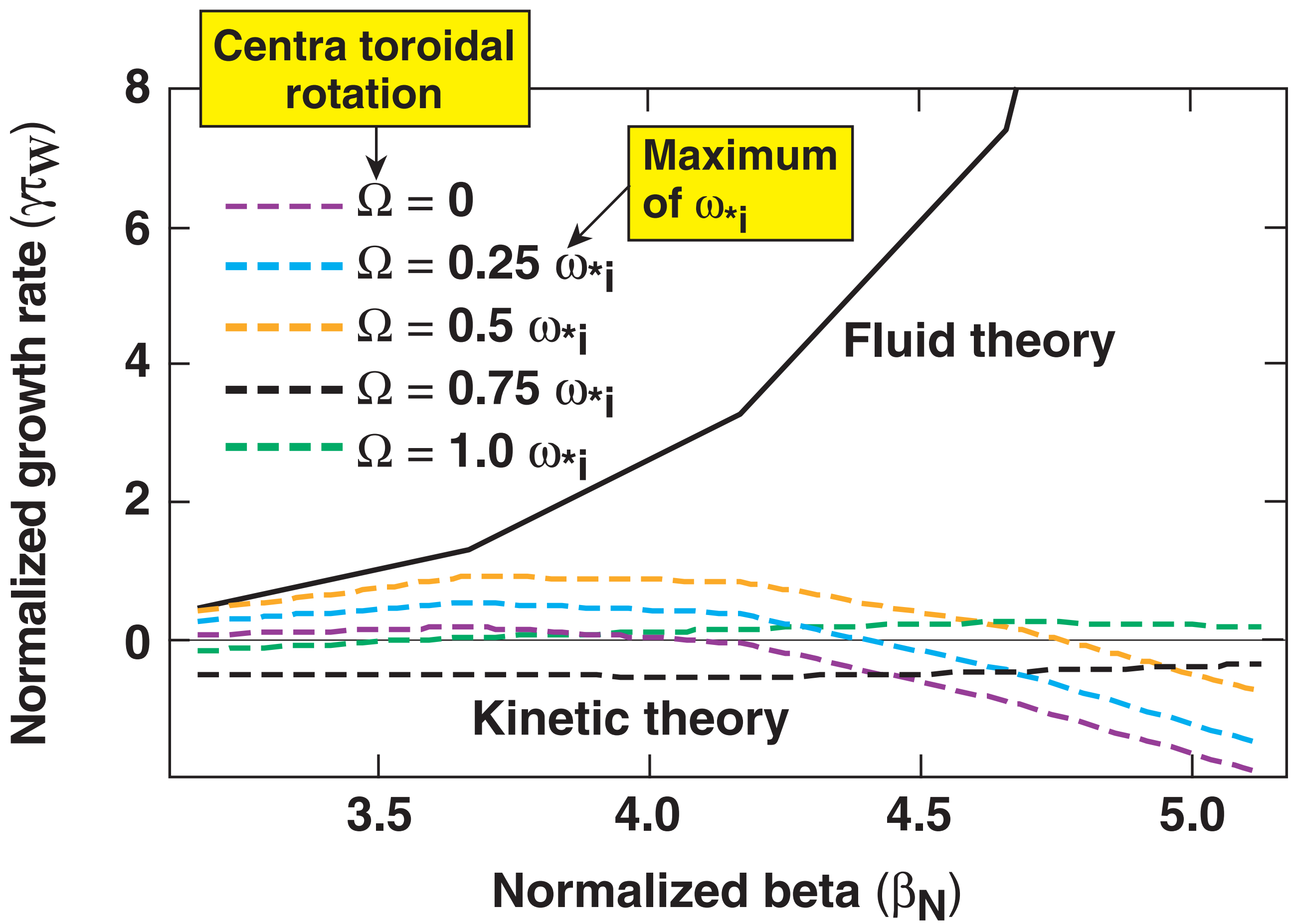
The growth of the RWM is strongly reduced or fully suppressed by the kinetic effects in slow-rotating ITER-like plasmas

- RWM normalized growth rate with and without kinetic effects and varying plasma rotation frequencies $\Omega(0)$.



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Summary/Conclusions

The low-frequency energy principle[1,2] applied to wall modes shows the possibility of RWM growth reduction/ suppression at low rotation frequencies

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- Marshall's contribution to the kinetic-energy principle has been instrumental in our understanding of the interaction between particles and MHD modes.
- The low-frequency energy principle is applied to wall modes.
- When all the kinetic species (alphas, ions, and electrons) are included, the RWM growth is strongly reduced or fully suppressed in the low-rotation regime for ITER-like plasmas.

[1] M. N. Rosenbluth and N. Rostoker, Phys. Fluids 2, 23 (1959).
[2] J. W. Van Dam, M. N. Rosenbluth, and Y. C. Lee, Phys. Fluids 25, 1349 (1982).

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