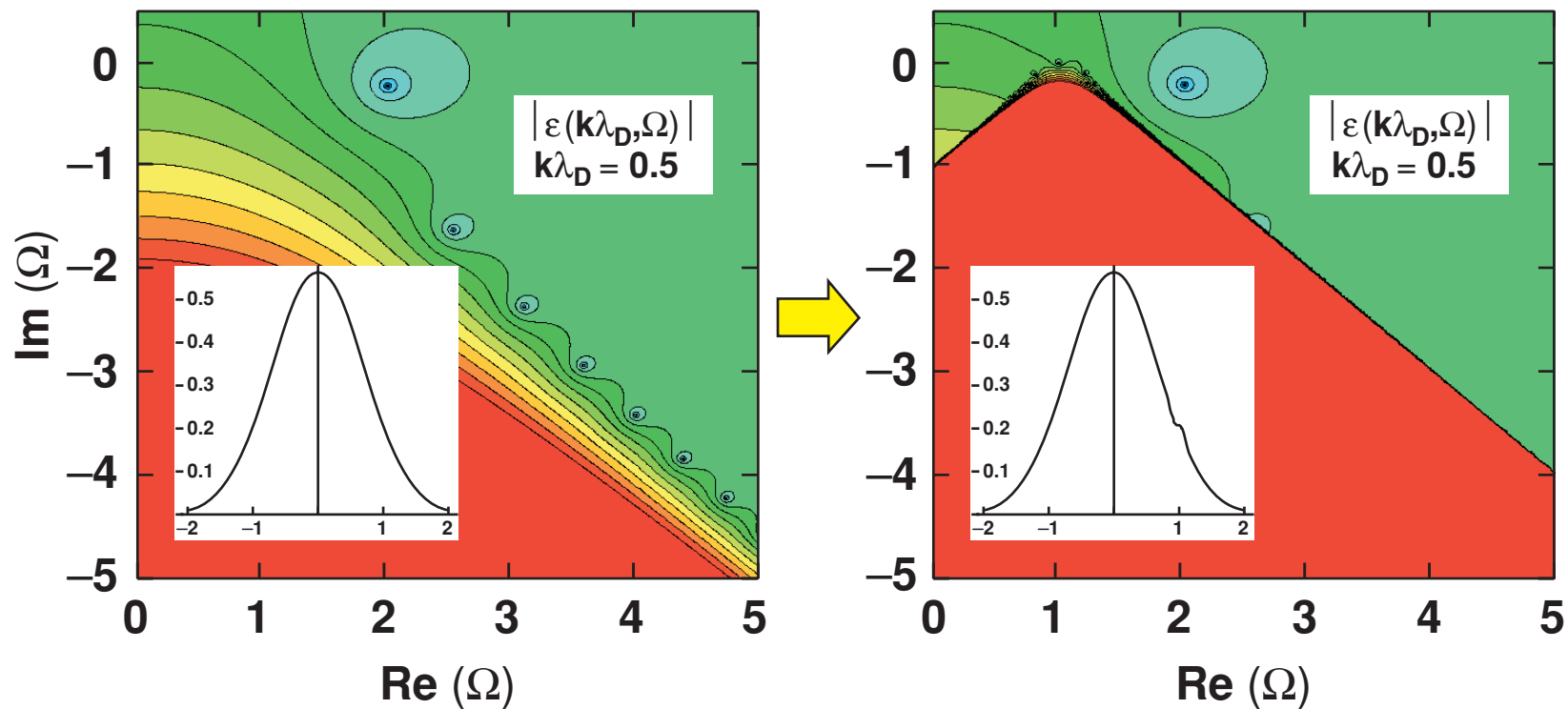# A Linear Model of Anomalous Stimulated Raman Scattering and Electron Acoustic Waves in Laser-Produced Plasmas

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Summary

Linear waves in locally flattened distribution functions can account for both anomalous SRS and SEAS

- Anomalous SRS: Strong Landau damping of plasma wave flattens distribution function at wave phase velocity, reducing damping.

- Electron acoustic waves: Local flattening arises from fluctuations that are maintained by damping if SRS matching conditions are met.

- Competing hypothesis: Nonlinear BGK waves represent a special case of local flattening, but require additional assumptions. The two models predict different ranges of density and wave number that give rise to scattering. No cutoff at $k\lambda_D > 0.53$ in the linear model.

- The linear model seems a better fit to observations so far.

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Outline

- **Landau damping of plasma waves is often much lower than predicted.**

- **Reduced damping implies a locally flattened distribution function.**

- **General theory of locally flattened distribution functions: conditions for linear undamped plasma waves**

- **Flattening resulting from Landau damping of the SRS plasma wave: accounting for anomalous SRS**

- **Evanescent spontaneous flattening due to plasma fluctuations: accounting for SEAS**

- **Comparison with experiment**

Landau damping is often much lower than expected in laser-produced plasmas

- **SRS is often dominated by scattering from low-phase-velocity plasma waves $(v_\phi \sim v_T)$ that should be strongly Landau damped.**

- **Electron-acoustic (EA) waves are solutions of $\text{Re}\,[\varepsilon(k,\omega)] = 0$ for real k and ω. In a Maxwellian plasma $\text{Im}\,[\varepsilon(k,\omega)] \neq 0$, so they should quickly dissipate, yet scattering from such waves has recently been identified in several experiments.**

- **Nonlinear BGK waves are undamped and at small amplitudes satisfy the EA dispersion relation, so they have been invoked to explain these results. But formation, superposition, and stability are problematic.**

- **BGK waves are a special form of locally flattened distribution function (LFDF). Only this flattening is required to suppress damping.**

- **Many aspects of SRS and EA waves involve only small amplitudes; suggests studying linear properties of LFDF's.**

Small-amplitude electrostatic perturbations in a collisionless plasma are studied using the linearized Vlasov-Poisson equations

- **Vlasov equation:** $\dfrac{\partial f}{\partial t} + v\dfrac{\partial f}{\partial x} - \dfrac{e}{m}E(x,t)\dfrac{\partial f_0(v)}{\partial v} = 0.$

- **Poisson equation:** $\dfrac{\partial E(x,t)}{\partial x} = -4\pi e\int_{-\infty}^{\infty} f(x,v,t)\,dv.$

- **The resulting dielectric function is given by**

$$\varepsilon(k,\omega) = 1 - \frac{1}{2(k\lambda_D)^2}\int_{-\infty}^{\infty}\frac{f_0'(u)}{u - \dfrac{\omega}{\sqrt{2}kv_T}}\,du,\ \textbf{where}\ \ u \equiv \frac{v}{\sqrt{2}v_T}.$$

- **For a Maxwellian the integral can be expressed in terms of the plasma dispersion function** Z**:**

$$\varepsilon(k,\omega) = 1 + \frac{1}{(k\lambda_D)^2}\left[1 + \Omega Z(\Omega)\right],\ \textbf{where}\ \ \Omega \equiv \frac{\omega}{\sqrt{2}kv_T}.$$

A model LFDF is analytic and can be arbitrarily close to a Maxwellian

- Landau damping is determined primarily by the first and second derivatives of the distribution function at the wave phase velocity.

- An analytic distribution function with zero slope at normalized velocity $u_0 = v_0/\sqrt{2}v_T$ and second derivative $f''(u_0) = \beta$ is given by

$$f(u) = f_0(u) + f_1(u) + f_2(u), \text{ where}$$

$$f_0(u) = \frac{1}{\sqrt{\pi}} e^{-u^2},$$

$$f_1(u) = -f_0'(u_0)(u - u_0)e^{-\frac{(u-u_0)^2}{(\Delta u)^2}}, \text{ and}$$

$$f_2(u) = \frac{1}{3}\left[\beta - f_0''(u_0)\right]\left[(u - u_0)^2 - \frac{1}{2}(\Delta u)^2\right]e^{-\frac{(u-u_0)^2}{(\Delta u)^2}}.$$

The resulting dielectric function and dispersion relation for SRS are also analytic

- **As in the Maxwellian case, the dielectric function can be expressed in terms of the plasma dispersion function Z:**

$$\varepsilon(k,\omega) = 1 + \frac{1}{(k\lambda_D)^2}\left[1 + \Omega Z(\Omega)\right] + \frac{u_0 e^{-u_0^2}}{(k\lambda_D)^2}\left[2y + \left(2y^2 - 1\right)Z(y)\right]$$

$$+ \frac{\Delta u}{(k\lambda_D)^2}\left[\frac{\sqrt{\pi}}{2}\beta + \left(1 - 2u_0^2\right)e^{-u_0^2}\right]\left[\frac{2}{3}\left(y^2 - 1\right) + \left(\frac{2}{3}y^3 - y\right)Z(y)\right],$$

where $\Omega \equiv \dfrac{\omega}{\sqrt{2}\,k v_T}$ **and** $y \equiv \dfrac{\Omega - u_0}{\Delta u}$.

- **The kinetic dispersion relation for SRS is then**

$$\varepsilon(k,\omega)\left[(\omega - \omega_0)^2 - (k - k_0)^2 c^2 - \omega_p^2\right] - \left[1 - \varepsilon(k,\omega)\right]\frac{k^2 v_{osc}^2}{4} = 0.$$
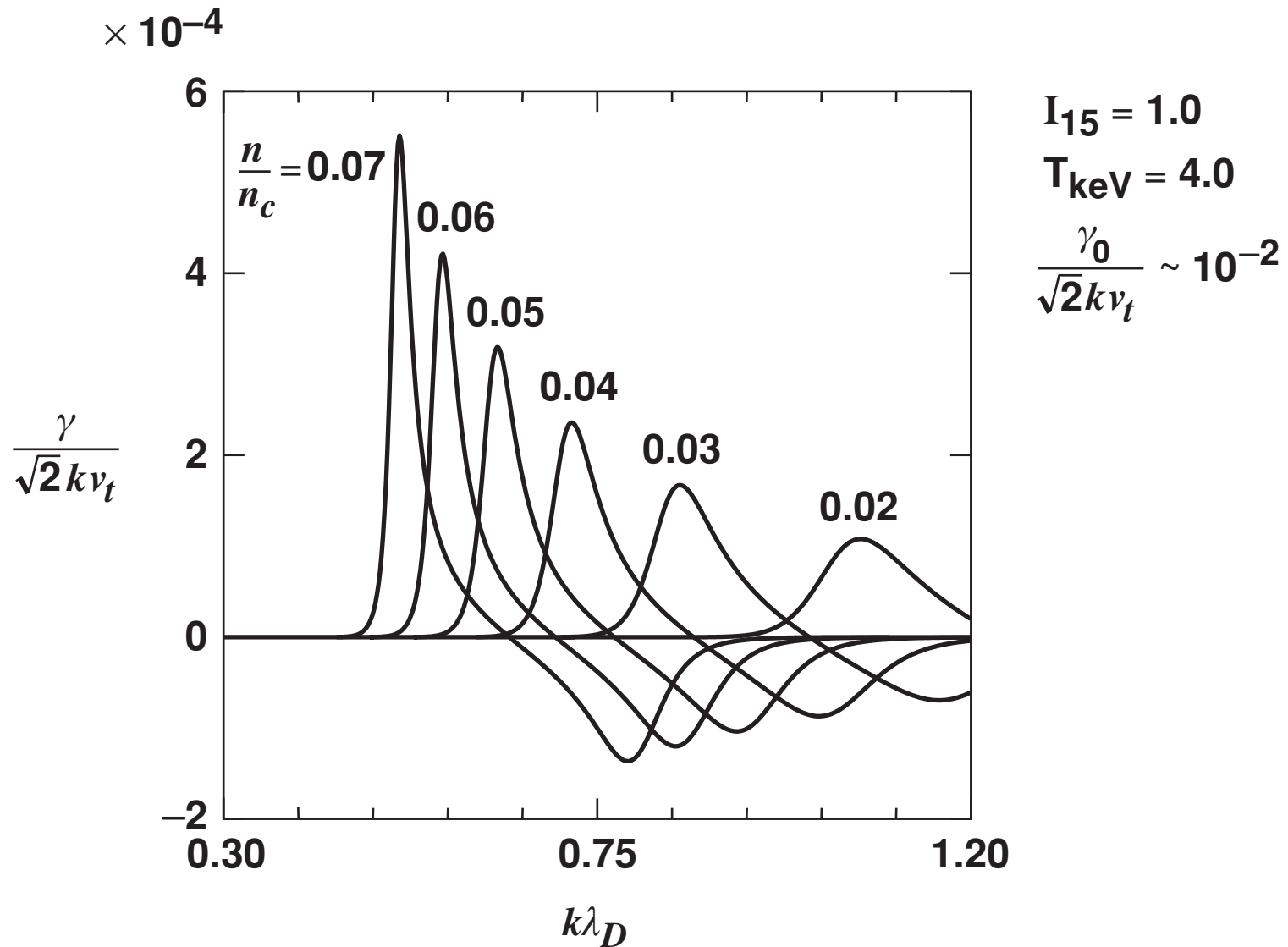
# There are two possible sources for the local flattening

- **Landau damping of the plasma waves associated with SRS leads to local flattening at the phase velocity of the wave and reduction in damping. These waves may have arbitrary values of $k\lambda_D$.**

- **Fluctuations (thermal or other) can result in transient reductions of the slope. This makes $\mathrm{Im}\,[\varepsilon(k,\omega)] = 0$, so that waves satisfying $\mathrm{Re}\,[\varepsilon(k,\omega)] = 0$ can propagate; this corresponds to Stix's original proposal of EA waves. EA waves are limited to $k\lambda_D < 0.53$.**

- **In the absence of a driver, the distribution function would quickly revert to a Maxwellian, and these waves would damp. But if they satisfy the matching conditions for SRS, they are driven and amplify, and their damping maintains the local flattening of the distribution function.**

In a Maxwellian, at low densities the SRS growth peak is broader and occurs at larger values of $k\lambda_D$

$\times 10^{-4}$

$I_{15} = 1.0$

$T_{keV} = 4.0$

$\dfrac{\gamma_0}{\sqrt{2}kv_t} \sim 10^{-2}$

vertical axis: $\dfrac{\gamma}{\sqrt{2}kv_t}$

horizontal axis: $k\lambda_D$

curve labels: $\dfrac{n}{n_c} = 0.07$, 0.06, 0.05, 0.04, 0.03, 0.02

A flattening velocity u_0 corresponding to the maximum Maxwellian SRS growth rate increases growth; further distortion results in long-wavelength instability
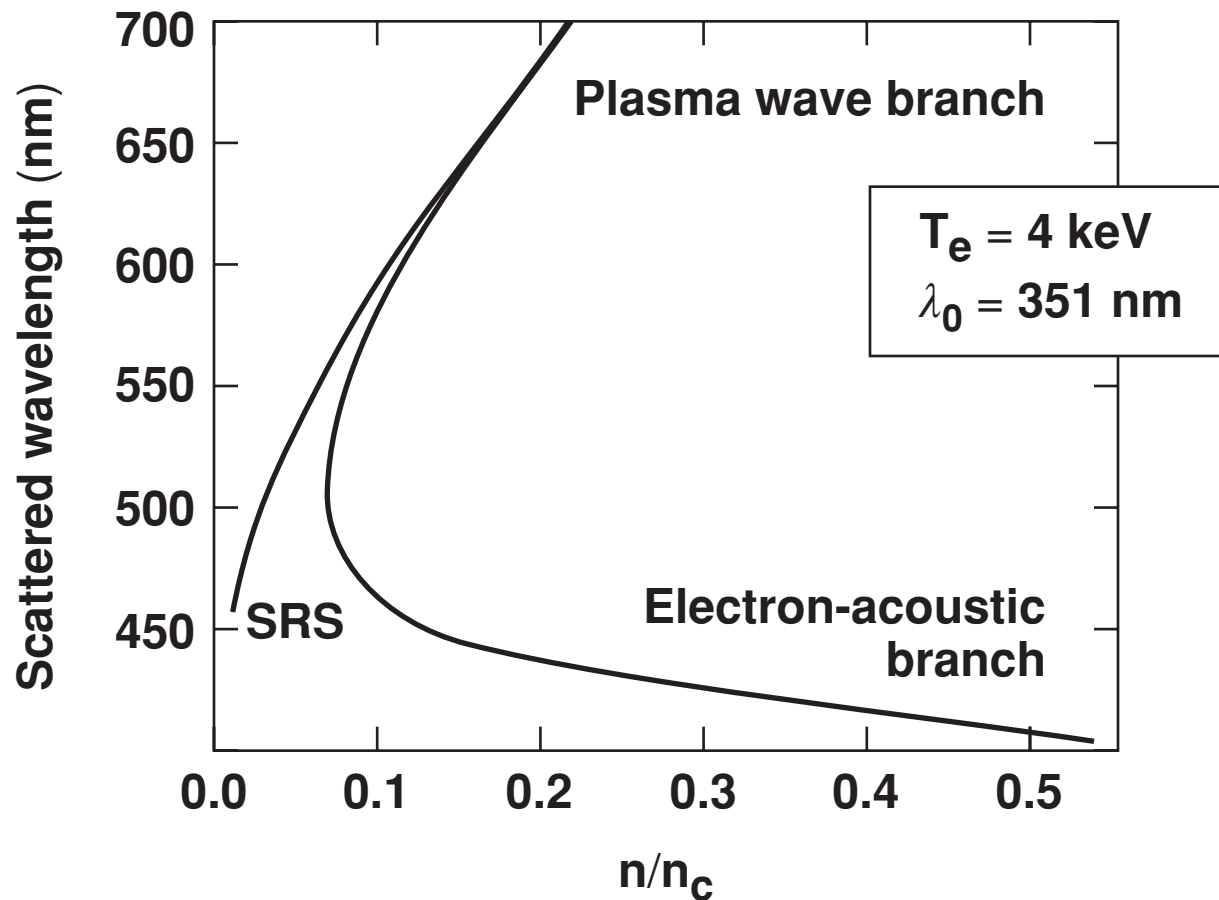
$\times 10^{-2}$

Electron-acoustic mode

SRS mode

$\beta = 2.54$

$\dfrac{\gamma}{\sqrt{2}\,k v_t}$

$k\lambda_D$

$k\lambda_D$
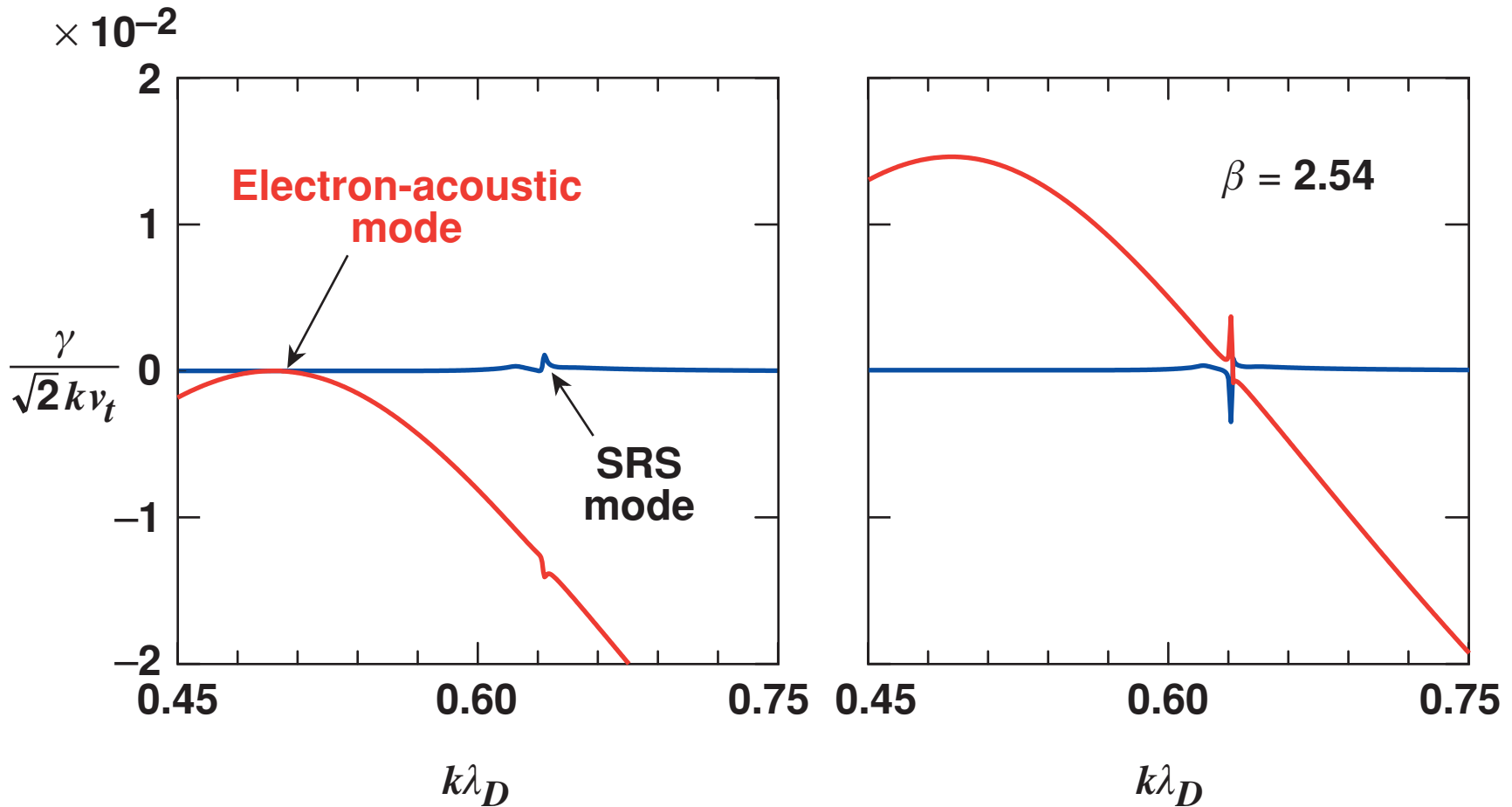
$\dfrac{n}{n_c} = 0.05$

Local flattening of the distribution function results in realization of the electron-acoustic mode

- **Electron-acoustic mode (Stix, 1962) is a real frequency root of $\mathbf{Re}\ [\varepsilon\ (\omega,k)] = 0$. Solution of Vlasov (not Landau) dispersion relation; not an actual mode in a Maxwellian plasma.**

- **Local flattening at u_0 makes $\mathbf{Im}\ [\varepsilon\ (\omega = ku_0,k)] = 0$, so the electron-acoustic root becomes a true mode.**

- **In the previous analysis, SRS drives a true (Landau) mode, *not* the electron-acoustic mode.**

- **But an arbitrarily small flattening range (Δu) results in an EA mode.**

- **So evanescent local flattening resulting from fluctuations could allow an EA mode satisfying the SRS matching conditions to propagate and amplify (SEAS).**

- **SEAS has recently been observed (Montgomery 2001).**

Unlike SRS, SEAS is limited to $k\lambda_D < 0.53$, which limits the densities at which it occurs

Plasma wave branch

$T_e = 4$ keV
$\lambda_0 = 351$ nm

SRS

Electron-acoustic branch

Scattered wavelength (nm)

n/n_c

Comparison of solid targets and exploding foils indicates anomalous SRS arises from low densities

Exploding foil

Solid target

SRS streak (12594)
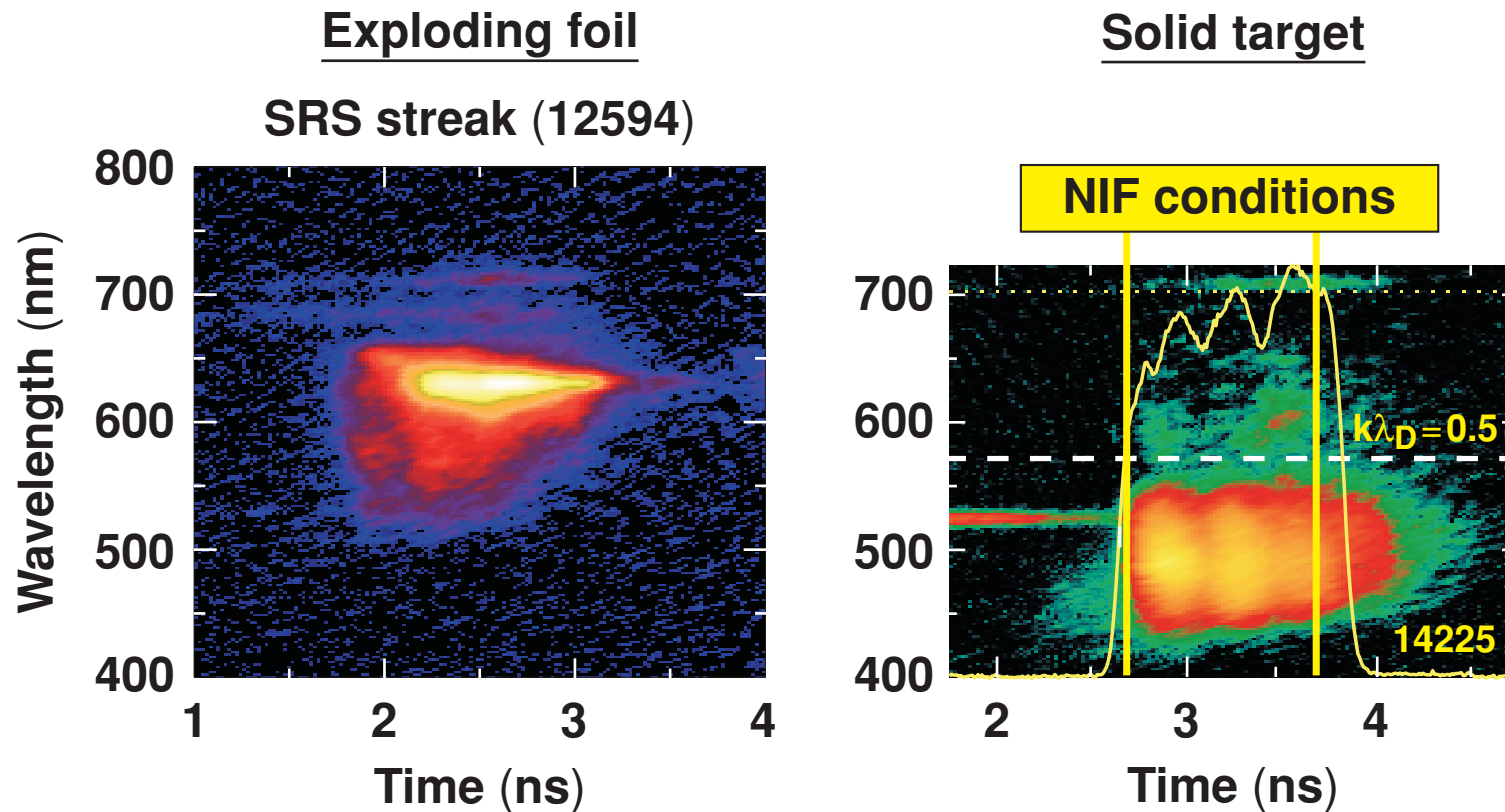
$I = 1.5 \times 10^{15}$ W/cm^2

SSD (0.25 THz)

$T_e \sim 4$ keV

Linear model differs from BGK interpretation in several observable aspects

- In linear theory anomalous SRS arises from linear undamped plasma waves at low density, not BGK waves at higher densities. Wave location can be detected by Thomson scattering.

- Linear theory does not predict sharp cutoff in SRS for $k\lambda_D > 0.53$ as does the BGK model. Proposals to suppress SRS in NIF by modifying plasma parameters so $k\lambda_D > 0.53$ may not be effective.

- In linear theory SRS and SEAS can coexist, as seen in experiments; this is difficult to reconcile with BGK model (nonlinear waves cannot be superposed.)

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