Two-Dimensional MHD Simulations of Tokamak Plasmas with Poloidal Flows

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Summary

- The theory of poloidally rotating Tokamak plasmas predicts the creation of a series of shocks along the poloidal angle.
- Shocks cannot exist at equilibrium unless a source of momentum is present in the system.
- We have carried out 2-D compressible MHD simulations that confirm the theoretical predictions.

The relevant compressible MHD equations

$$\partial_t \rho + \nabla \bullet (\rho \vec{\mathbf{v}}) = \mathbf{0}$$

Artificial viscosity

$$\rho(\partial_t \vec{\mathbf{v}} + \vec{\mathbf{v}} \bullet \nabla \vec{\mathbf{v}}) = -\nabla \mathbf{P} + \vec{\mathbf{J}} \times \vec{\mathbf{B}} + \nu \nabla \bullet \overline{\overline{\Pi}}$$

$$\partial_t \mathbf{p} + \vec{\mathbf{v}} \bullet \nabla \mathbf{p} + \gamma \mathbf{p} \nabla \bullet \vec{\mathbf{v}} = \nu \phi^{\mathbf{2}}$$ ← **Viscous heating**

$$\partial_t \vec{\mathbf{B}} = \nabla \times (\vec{\mathbf{v}} \times \vec{\mathbf{B}})$$

$$\nabla \bullet \vec{\mathbf{B}} = \mathbf{0}$$

The integration scheme

- Time-dependent equations are integrated using a predictor–corrector MacCormack scheme.
- Space derivatives are treated with standard finite-difference approximations.
- The scheme is second order accurate in both time and space.

Boundary conditions

- The following standard BC's are implemented:
  - Normal flow is zero at the (non-viscous) boundary.
  - Normal magnetic field is zero at the boundary.
  - Normal pressure gradient is zero at the boundary.
  - Normal density gradient is zero at the boundary.

Artificial terms

- The following numerical smoothing techniques are implemented in the code:
  - Artificial (numerical) viscosity is added to the system.
  - Artificial (numerical) heat conduction is added to the system.
  - In order to avoid numerical problems with the approximate BC's, artificial terms are enhanced near the boundary.
  - An additional diffusion equation is inserted to control the divergence of the magnetic field.

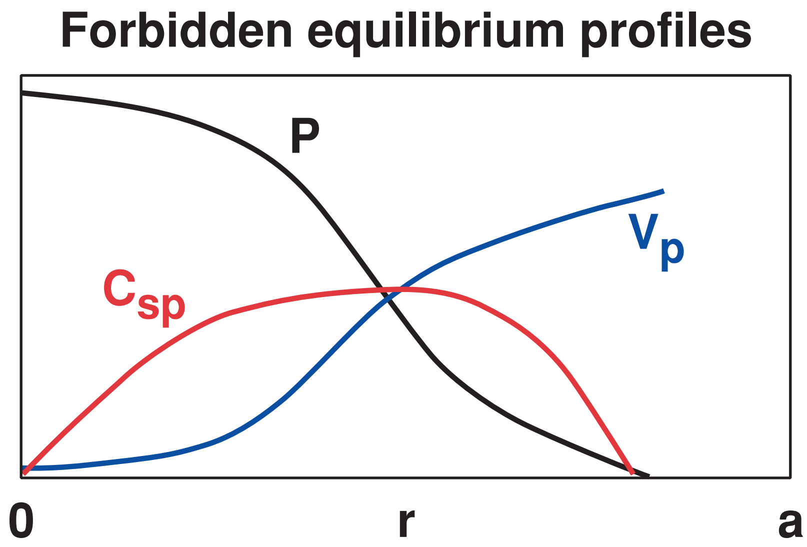
The predictions of theory: formation and evolution shocks

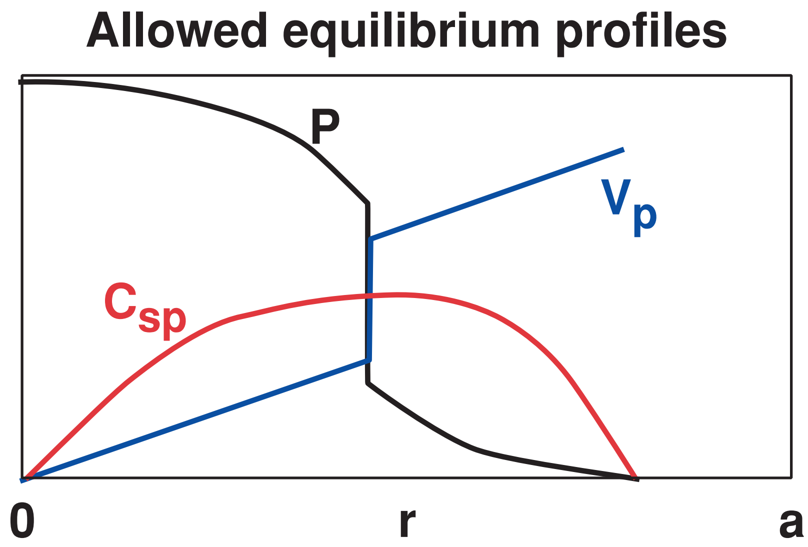
- The magnetic field acts as a de Laval nozzle for any initial poloidal velocity profile.
- In a toroidal plasma, an equilibrium with a continuous, transonic (with respect to the poloidal sound speed) poloidal velocity profile cannot exist.
- Any initially transonic poloidal velocity profile will evolve into a series of shocks distributed along the poloidal angle.
- Shocks at equilibrium can only exist if a momentum source is present in the system.

R. Betti and J. P. Freidberg, Phys. Plasmas 7, 2439 (2002).

Equilibrium profiles

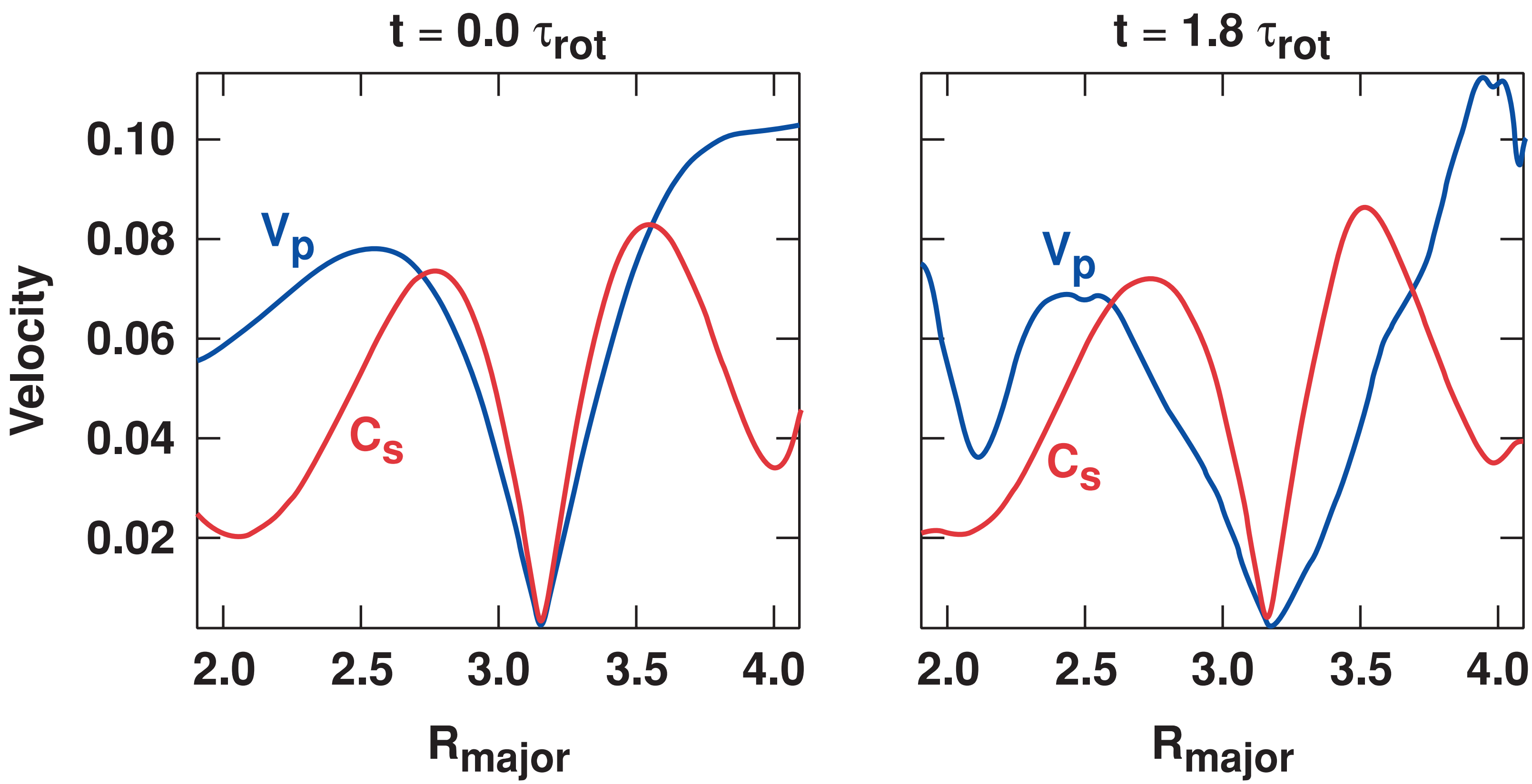
- If no source of momentum is present in the system, the series of shocks will evolve into a radial discontinuity of the physical quantities at equilibrium.





Velocity profile evolution

- The natural time scale for the system is the poloidal rotation period τ_{rot}.



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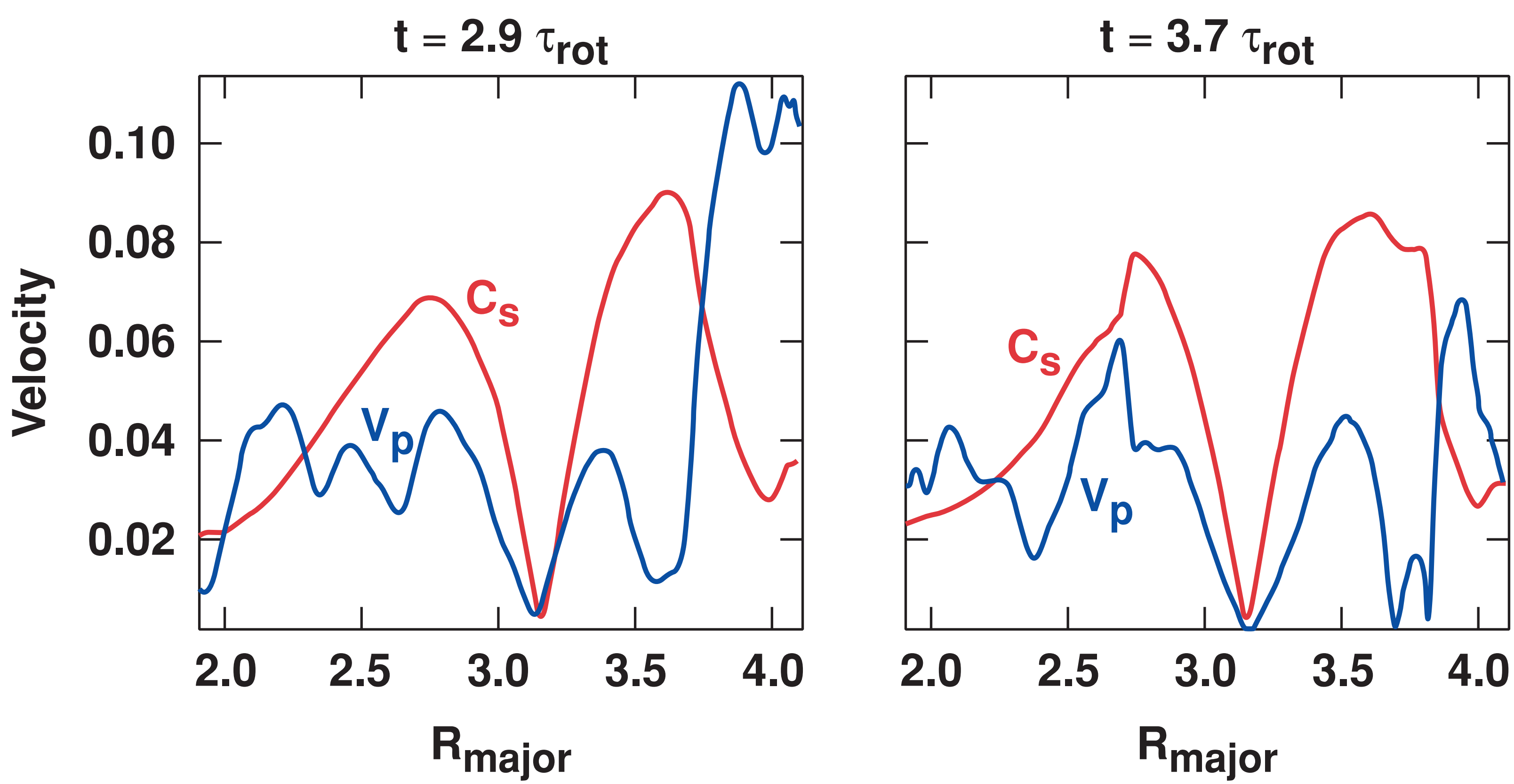
Velocity profile evolution (2)

- The time evolution of the system shows the formation of a radial discontinuity in the poloidal velocity profile.



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Conclusions

- A two-dimensional, time-dependent code for the analysis of the behavior of toroidal plasma with poloidal flow has been developed.
- In accordance with the predictions of the theory, the formation and evolution of a set of shocks has been observed in the poloidal direction.
- Shocks can be present in a steady-state system only if sustained by a momentum source.