Axisymmetric MHD Equilibria with Arbitrary Flow and Applications to NSTX

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Outline

- The code FLOW:
  - the system of equations
  - the numerical solution
- NSTX-like equilibria with toroidal flow
- NSTX-like equilibria with poloidal flow
- Conclusions

The relevant equations

- **Continuity:**

$$\nabla \bullet (\rho \vec{v}) = \mathbf{0}$$

- **Momentum:**

$$\rho \vec{v} \bullet \nabla \vec{v} = \vec{\mathbf{J}} \times \vec{\mathbf{B}} - \nabla \bullet \overleftrightarrow{\mathbf{P}}$$

$$\overleftrightarrow{\mathbf{P}} \equiv \mathbf{p}_\perp \overleftrightarrow{\mathrm{I}} + \Delta \vec{\mathbf{B}}\vec{\mathbf{B}} \qquad \Delta \equiv (\mathbf{p}_{||} - \mathbf{p}_\perp) / \mathbf{B}^2$$

- **Maxwell equations:**

$$\nabla \times (\vec{v} \times \vec{\mathbf{B}}) = \mathbf{0} \qquad \nabla \bullet \vec{\mathbf{B}} = \mathbf{0}$$

The previous system of equations can be reduced to a "Bernoulli" and a "Grad–Shafranov" equation

- Faraday's law yields the plasma flow:

$$\vec{v} = \mathbf{M}_{\mathbf{A}\theta}\vec{v}_{\mathbf{A}} + \mathbf{R}\Omega(\Psi)\hat{\phi} \qquad \mathbf{M}_{\mathbf{A}\theta} = \Phi(\Psi)/\sqrt{\rho}$$

- The φ-component of the momentum equation gives an equation for the toroidal component of the magnetic field:

$$\mathbf{B}_{\varphi}\mathbf{R} = \frac{\mathbf{I}(\Psi) - \mathbf{R}^2\mathbf{M}_{\mathbf{A}\theta}\sqrt{\rho}\,\Omega(\Psi)}{1 - \mathbf{M}_{\mathbf{A}\theta}^2 - \Delta}$$

- The B-component of the momentum equation reduces to a "Bernoulli-like" equation for the total energy along the field lines:

$$\frac{1}{2}\frac{(\mathbf{M}_{\mathbf{A}\theta}\mathbf{B})^2}{\rho} - \frac{1}{2}[\mathbf{R}\Omega(\Psi)]^2 + \mathbf{W} = \mathbf{H}(\Psi)$$

The modified Grad–Shafranov equation

- Finally, the $\nabla\Psi$-component of the momentum equation gives a "GS-like" equation:

$$\nabla \bullet \left[\left(1 - M_{A\theta}^2 - \Delta\right)\left(\frac{\nabla\Psi}{R^2}\right)\right]$$

$$= -\frac{\partial p_{||}}{\partial\Psi} - \frac{B_\varphi}{R}\frac{dI(\Psi)}{d\Psi} - \vec{v} \bullet \vec{B}\frac{d\Phi(\Psi)}{d\Psi} - R\rho v_\varphi \frac{d\Omega(\Psi)}{d\Psi} - \rho\frac{dH(\Psi)}{d\Psi} + \rho\frac{dW}{d\Psi}$$

- W (ρ, B, Ψ) is the enthalpy of the plasma, and its definition depends on the description of the plasma (MHD, CGL, or kinetic).
- $I(\Psi)$, $\Phi(\Psi)$, $\Omega(\Psi)$, $H(\Psi)$, $(\partial p_{||}/\partial\Psi)$, $(\partial W/\partial\Psi)$ are free functions of Ψ.

R. Iacono *et al.*, Phys. Fluids B (1990).

The code input requires a "user-friendly" set of free functions

- The input is a set of free functions representing quasi-physical variables.
- Functions can be supplied as analytical expressions or numerical tables.

$D(\Psi)$ → quasi-density

$P_{||}(\Psi)$ → quasi-parallel pressure

$P_{\perp}(\Psi)$ → quasi-perpendicular pressure

$B_0(\Psi)$ → quasi-toroidal magnetic field

$M_{\theta}(\Psi)$ → quasi-poloidal sonic Mach number

$M_{\varphi}(\Psi)$ → quasi-toroidal sonic Mach number

The numerical algorithm: the multi-grid solver

- The Bernoulli equation is solved for ρ.
- The Grad-Shafranov equation is solved for Ψ using a red-black algorithm.
- If the system is anisotropic, the equation for B_φ is also solved.
- The procedure is repeated until convergence; then the solution is interpolated onto the next grid.

NSTX-like equilibria with toroidal flow

- The following set of free functions is used as input to compute anisotropic equilibria with toroidal flow:

$$D(\Psi) = D^C \sqrt{\Psi}$$

$$P_{||}(\Psi) = P_{||}^C \Psi^2$$

$$B_0(\Psi) = \delta B_0 \Psi^3 + B_{vacuum}$$

$$P_{\perp}(\Psi) \leq P_{||}(\Psi)$$

$$M_{\theta}(\Psi) = 0$$

$$M_{\varphi}(\Psi) = M_{\varphi}^C \sqrt{\Psi}$$

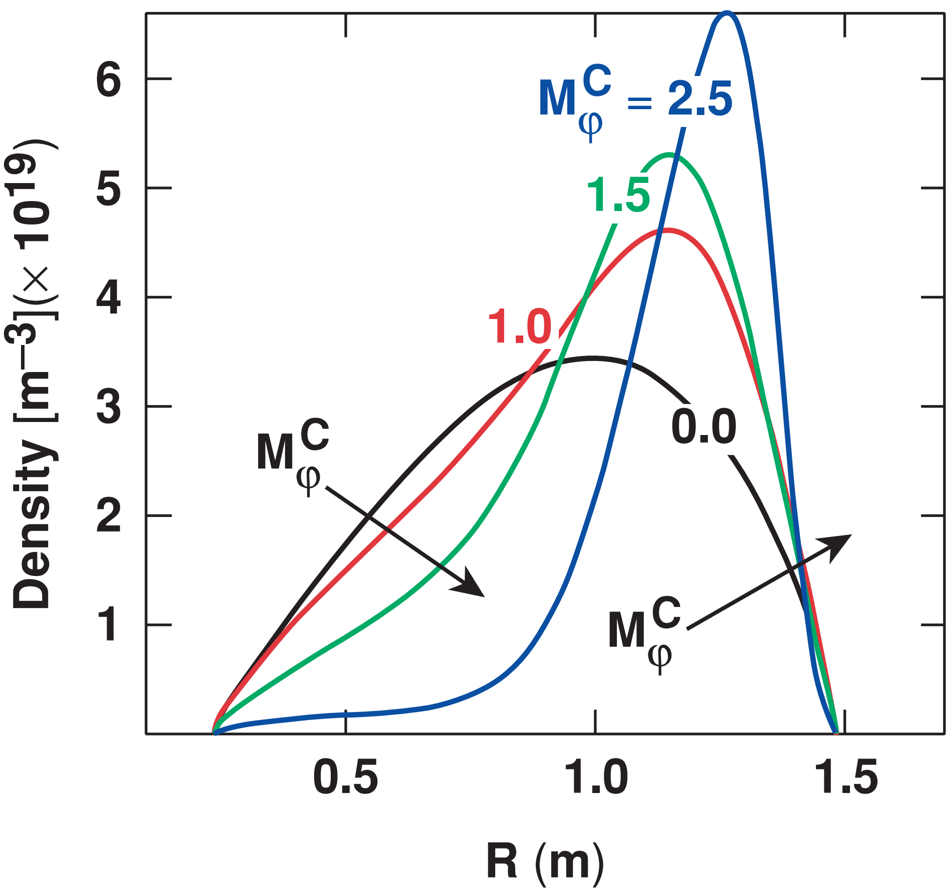
$D^C = 3.4 \times 10^{19}\ (m^{-3})$ | $B_0^C = 0.29\ (T)$ | $0 \leq M_{\varphi}^C \leq 2.5$

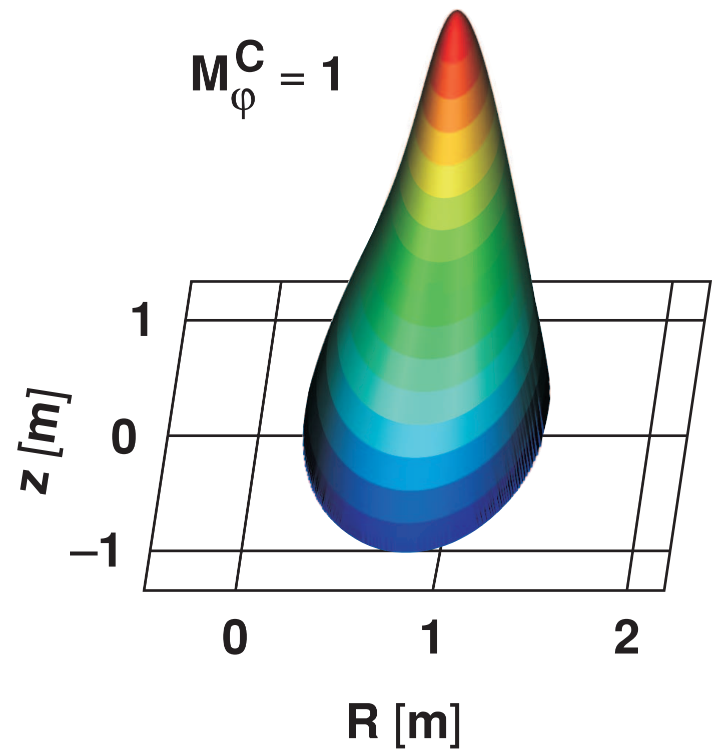
$\beta_T = 9\%$ | $I = 0.9\ (MA)$

$R_0 = 0.86\ (m)$ | $a = 0.69\ (m)$ | $k = 1.9$

M. Ono *et al.*, Nucl. Fusion (2001).

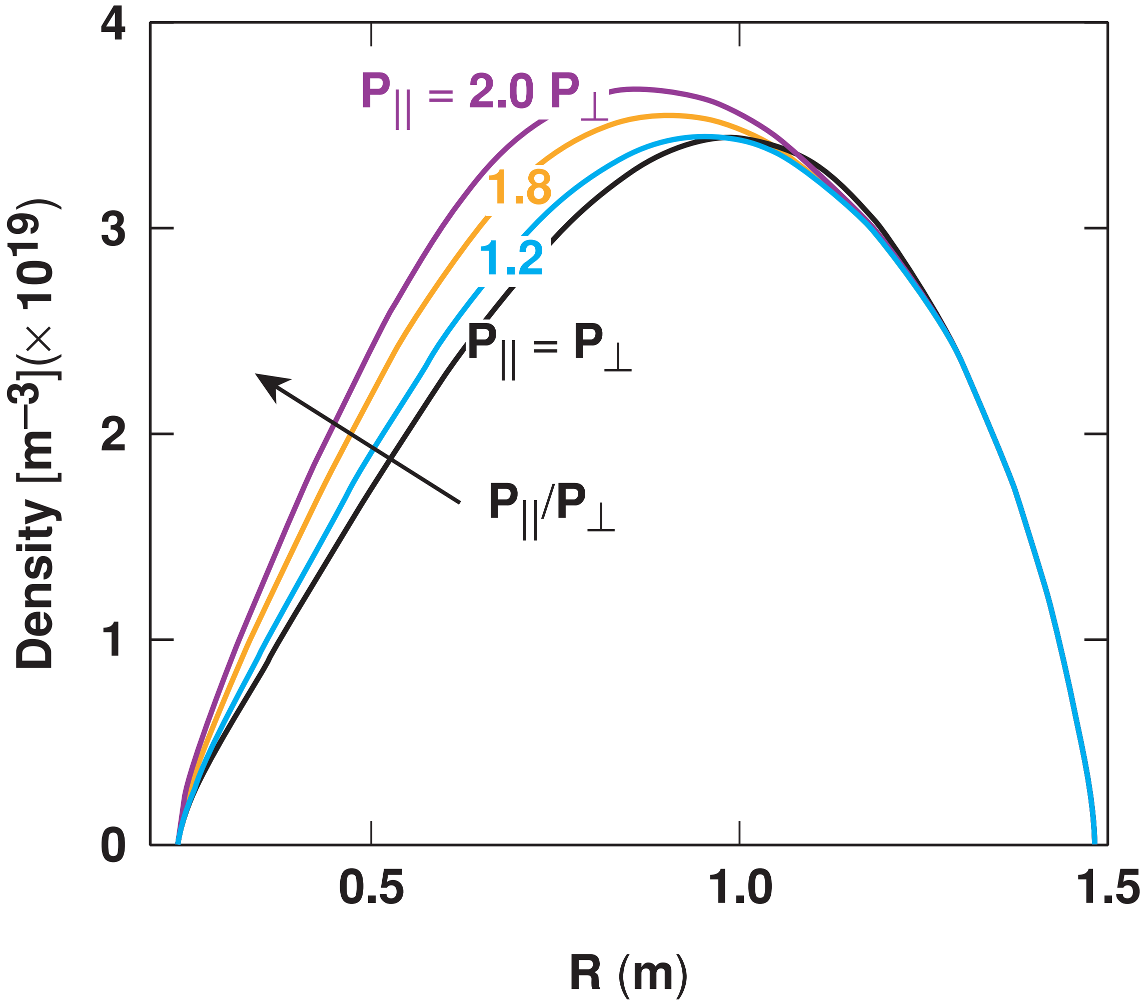
The centrifugal force causes an outward shift of the plasma





TC6070

The parallel anisotropy ($p_{||} > p_{\perp}$) causes an inward shift



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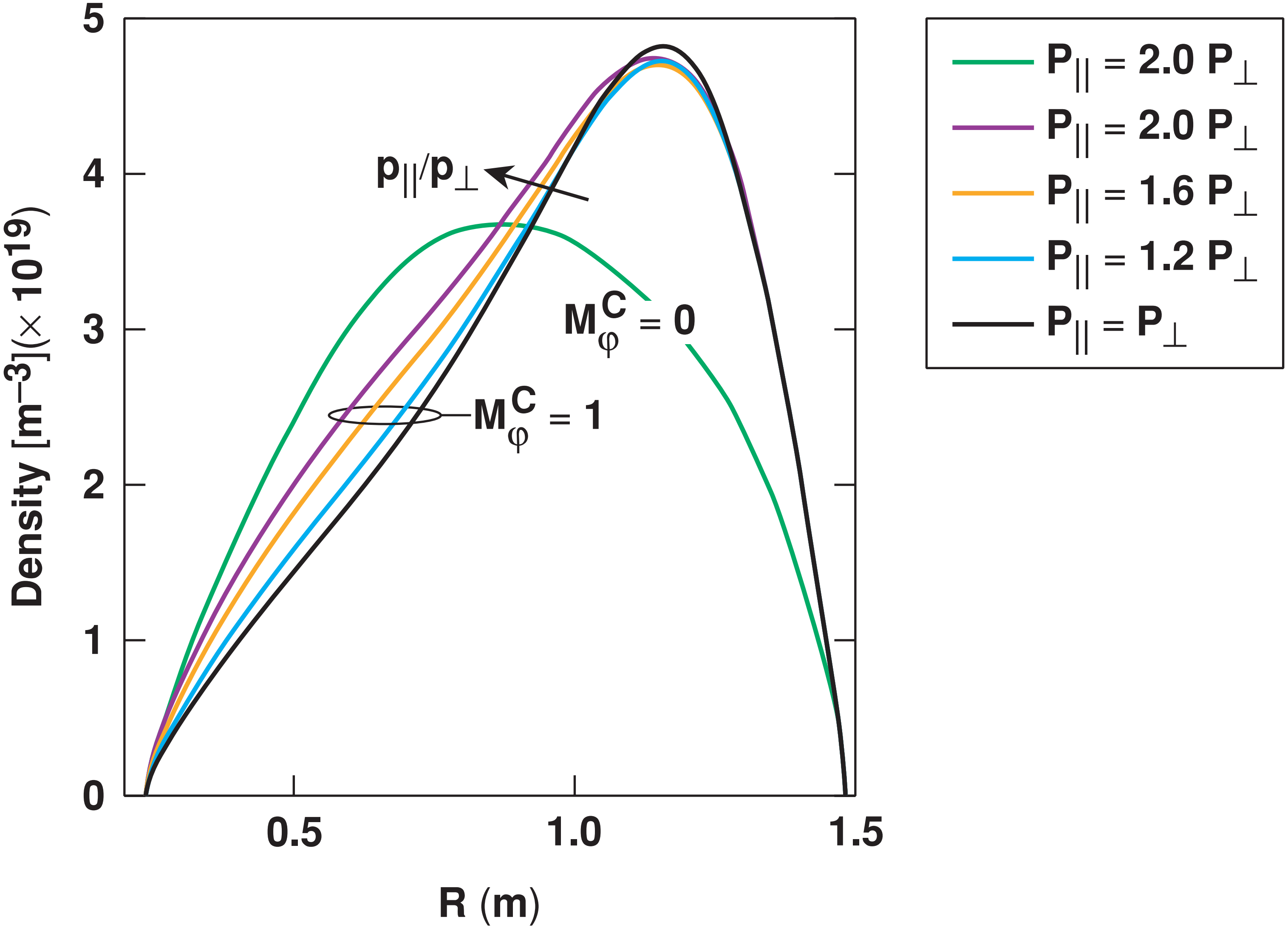
Flow and parallel anisotropy have opposite effect



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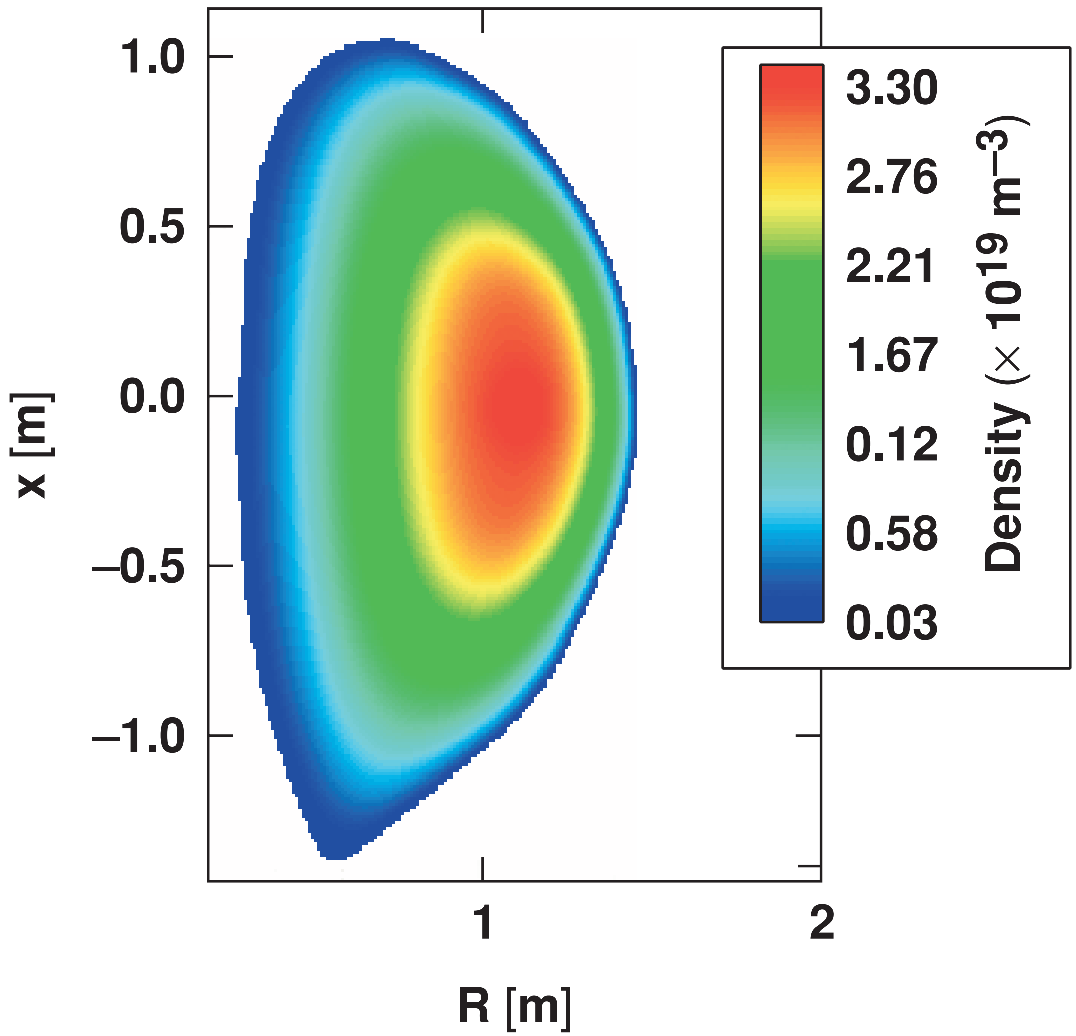
User-supplied tables can be used as input

- **Input functions from numerical data:**

  $\Omega(\Psi)$

  $P(\Psi)$

$M_{\varphi}^{max} \cong 0.4$



A strongly shaped, NSTX-like equilibrium

FLOW can also be applied to equilibria with poloidal flow

- The free function determining the poloidal flow is $M_\theta(\Psi)$ representing (approximately) the sonic poloidal Mach number (poloidal velocity/poloidal sound speed). Poloidal sound speed = $C_S B_\theta / B$.
- Radial discontinuities in the equilibrium profiles develop when the poloidal flow becomes transonic [$M_\theta(\Psi) \sim 1$].
- Since the poloidal sound speed is small at the plasma edge, transonic flows may develop near the edge.
- FLOW can describe MHD or CGL equilbria with poloidal flow.

R. Betti and J. P. Freidberg, Phys. Plasmas (2000).

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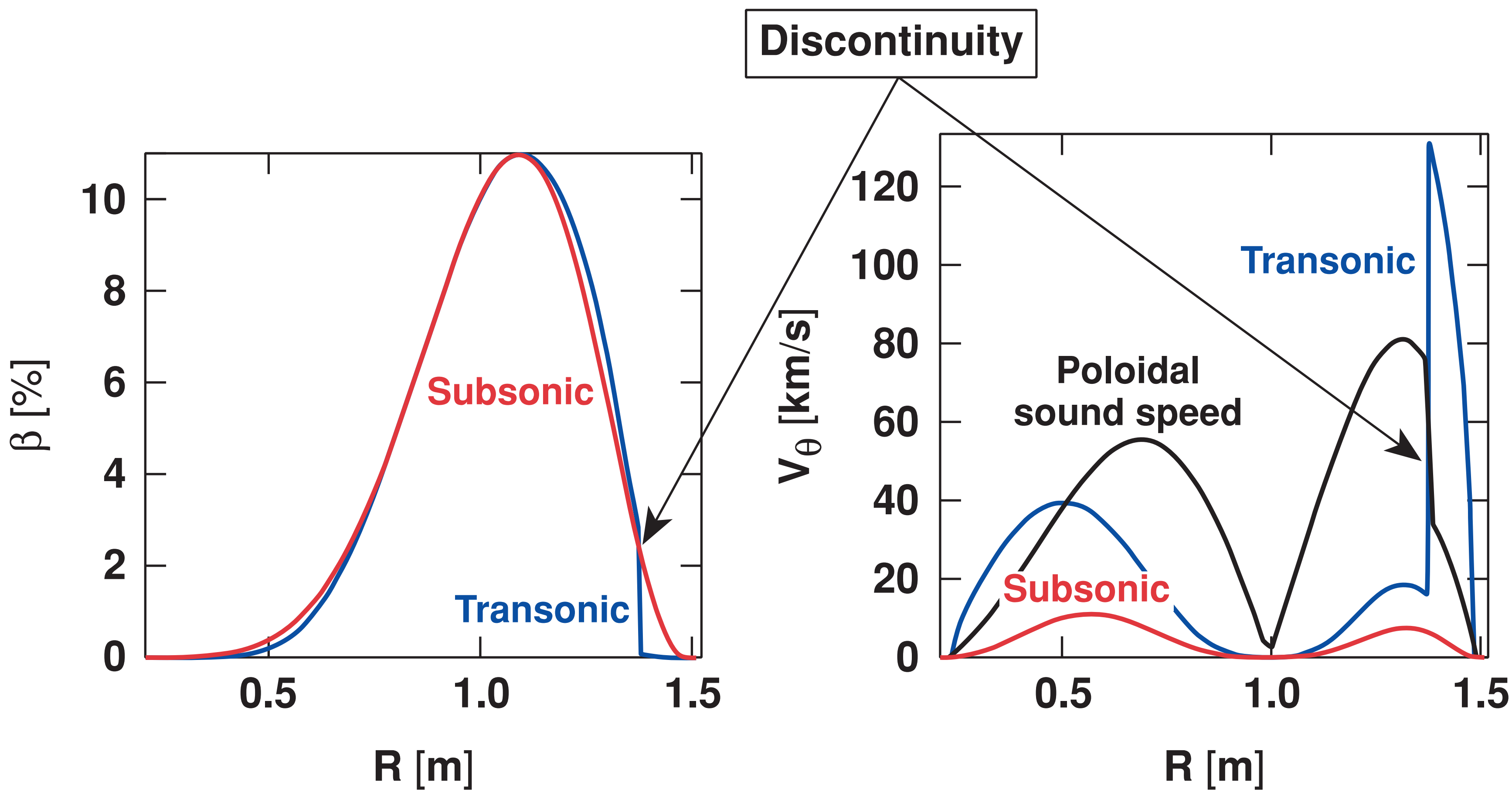
NSTX-like equilibria with poloidal flow can exhibit a pedestal structure at the edge



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Conclusions

- The code FLOW can compute axysymmetric anisotropic equilibria with arbitrary flow.
- MHD, CGL, and kinetic closures are implemented. At the moment, poloidal flow is described only by the MHD and CGL closures.
- Fast-rotating anisotropic NSTX-like equilibria have been computed with FLOW.