Two-Dimensional MHD Simulations of Tokamak Plasmas with Poloidal Flow

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Abstract

Recent ideal MHD calculations have shown that poloidal flow in a tokamak can result in a pedestal structure across which the velocity and pressure vary strongly. In a low-β tokamak the effective sound speed in the poloidal direction is the sound speed C_s scaled by the ratio of the poloidal to total magnetic field strength, $C_{sp} = C_s(B_p/B)$. This is a measure of the time scale necessary for a sound wave to propagate from the outer to the inner radial edge of the plasma. The poloidal sound speed goes to zero at the center of the plasma by symmetry and gets quite small near the outer radial edge. Hence, even small poloidal rotational velocities can result in a supersonic flow. Owing to the finite inverse aspect ratio, the toroidal geometry of the flow behaves as a de Laval nozzle with a throat at the inner midplane. Under the right conditions, a subsonic flow can become supersonic near the inner midplane and remain supersonic as it completes a poloidal revolution. This in turn results in a shock wave that propagates from the outer to the inner midplane, irreversibly heating the plasma and generating a poloidal shear flow. We present time-dependent numerical simulations detailing the formation and evolution of this flow pattern and a comparison with analytic results.

Poloidal flow generates a pedestal-like structure

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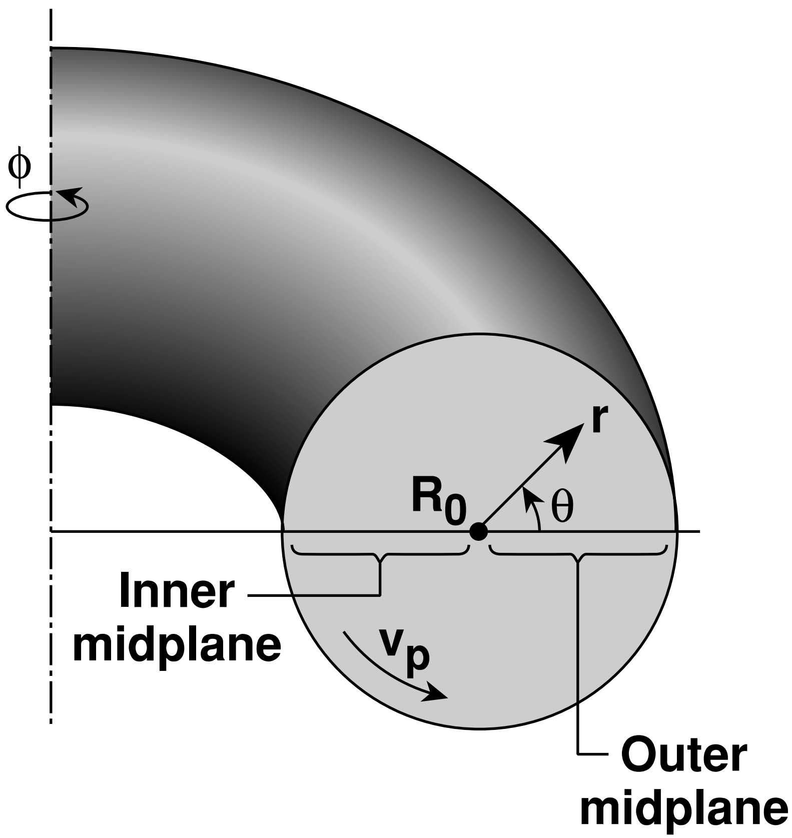
- Experiments find that tokamak plasmas can rotate in the poloidal as well as toroidal directions.
- Large velocity shear is an important ingredient for inhibiting energy transport.
- Recent ideal MHD calculations show that poloidal flow can lead to a pedestal-like structure.
- The relevent signal speed for a low-β plasma is the poloidal sound speed $C_{sp} = C_s(B_p/B)$, where C_s is the sound speed.
- In terms of ideal MHD modes, this is $\approx$ the slow-mode speed in the poloidal direction.
- A pedestal-like structure generally forms when the poloidal velocity is transonic, $v_p \geq C_{sp}$.
- This pedestal has the following properties:
  - a shock-generated pressure jump,
  - a transonic poloidal shear flow, and
  - a toroidal shear flow.
- The poloidal flow and shock formation is an analog of a periodic, shocked transonic de Laval nozzle.

Betti and Freidberg (2000) recently studied the evolution of weak poloidal shocks



- Coordinate system (r, θ, ϕ) and major radius R_0. Flow is assumed to be axi-symmetric, $\partial_\phi = 0$.
- In terms of the inverse aspect ratio ($\epsilon = a/R_0$), $B_p/B_\phi \sim \epsilon$, $\beta \sim \epsilon^2$.
- Letting $B_p = \nabla\psi \times e_\phi/R$, it can be shown that $\psi = \psi_0(r) + \epsilon\psi_1(r, \theta, t) + ...$
- In the presence of weak shocks, $B_p \sim$ independent of time.
- Adjacent poloidal flux surfaces play the role of a rigid duct with a nozzle at the inner midplane.

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System of equations and numerical method

- We solve the equations of ideal magnetohydrodynamics using an eight-wave formulation [Powell (1998) and Toth (2000)].
- In this approach, “magnetic charge” is advected with the fluid (the eighth wave):

$$\frac{\partial \rho}{\partial \mathbf{t}} + \nabla \bullet (\rho \mathbf{v}) = \mathbf{0}$$

$$\frac{\partial \rho \mathbf{v}}{\partial \mathbf{t}} + \nabla \bullet (\rho \mathbf{vv} - \mathbf{BB}) + \nabla\left(\mathbf{P} + \mathbf{B}^2/2\right) = -\mathbf{B}(\nabla \bullet \mathbf{B})$$

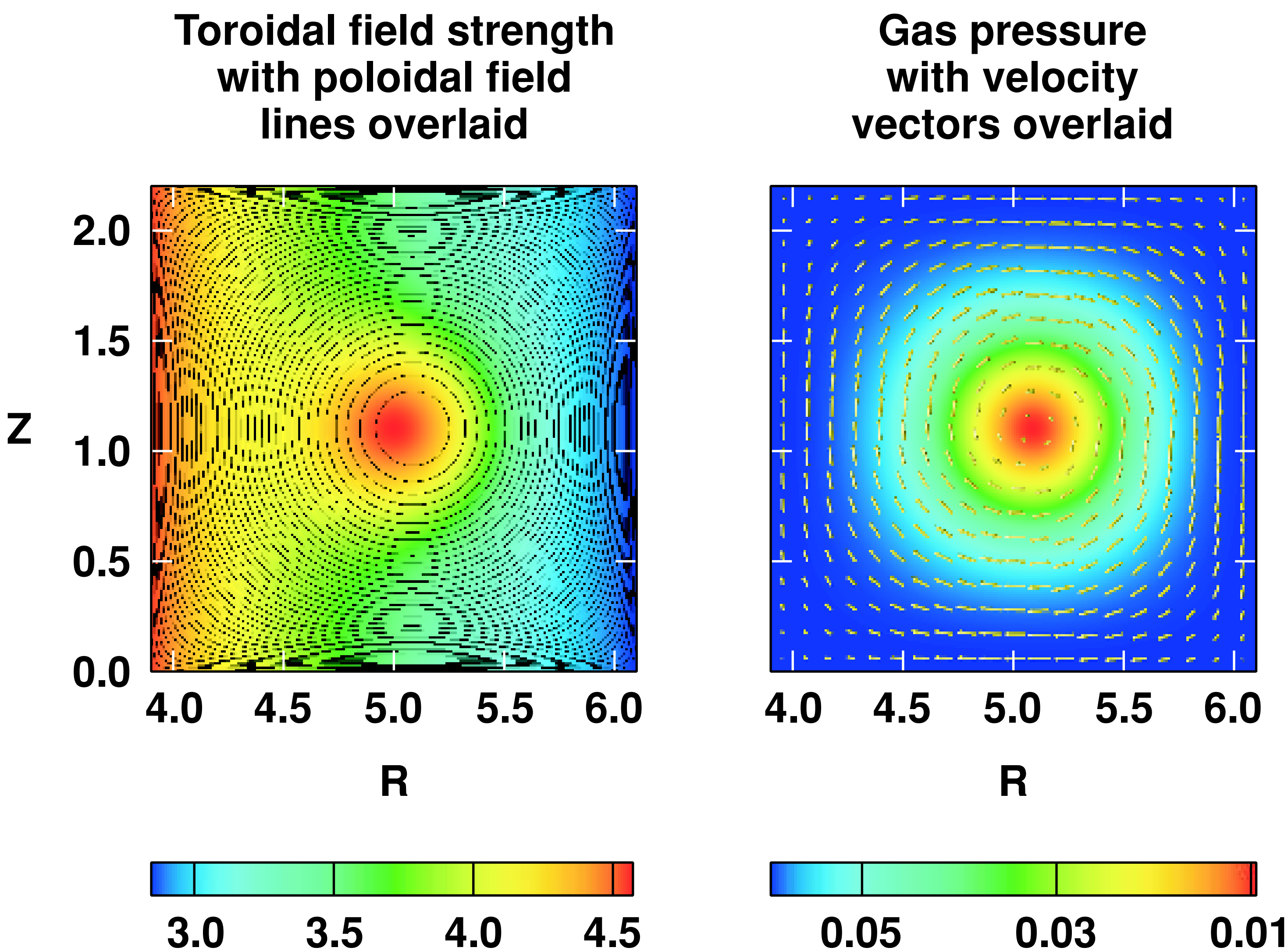
$$\frac{\partial \mathbf{B}}{\partial \mathbf{t}} + \nabla \bullet (\mathbf{vB} - \mathbf{Bv}) = -\mathbf{v}(\nabla \bullet \mathbf{B})$$

$$\frac{\partial \mathbf{e}}{\partial \mathbf{t}} + \nabla \bullet \left[\mathbf{v}\left(\mathbf{e} + \mathbf{P} + \mathbf{B}^2/2\right) - \mathbf{B}(\mathbf{B} \bullet \mathbf{v})\right] = -\mathbf{B} \bullet \mathbf{v}(\nabla \bullet \mathbf{B})$$

- Periodically the $\nabla \bullet \mathbf{B}$ errors are cleaned with a Poisson solver.
- The equations are linearized following Roe (1981), Roe and Balsara (1996), and Cargo and Gallice (1997).
- Integration is unsplit, using the second order accurate, TVD Runge–Kutta method of Shu and Osher (1989).
- In 1-D the linearized system of equations retains Galilean invariance and takes the following form:

$$\frac{\partial \mathbf{q}}{\partial \mathbf{t}} + \tilde{\mathbf{A}} \frac{\partial \mathbf{q}}{\partial \mathbf{x}} = \mathbf{0}$$

Initial conditions for a poloidally rotating plasma



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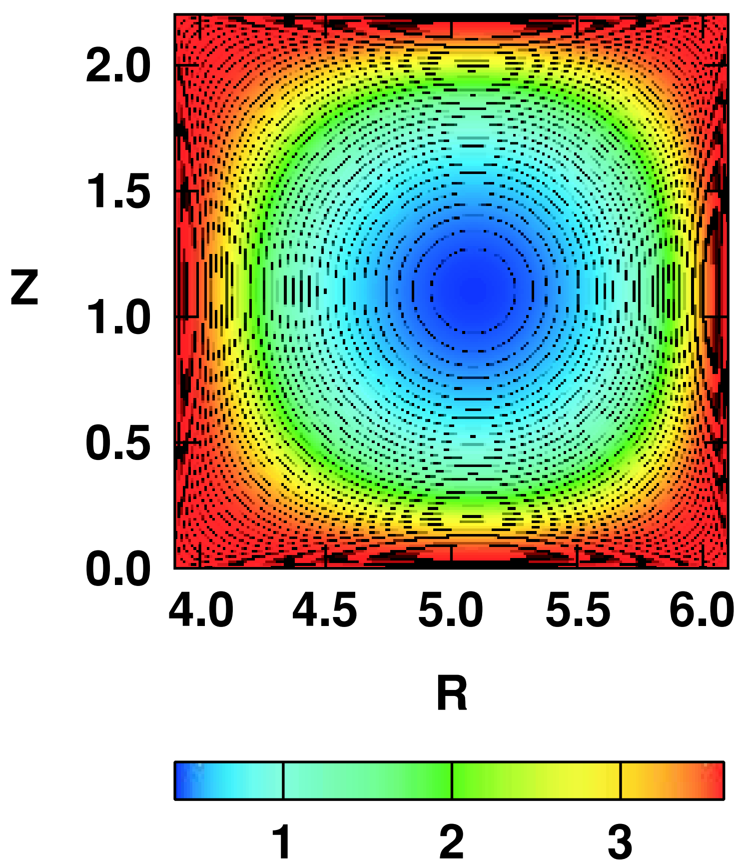
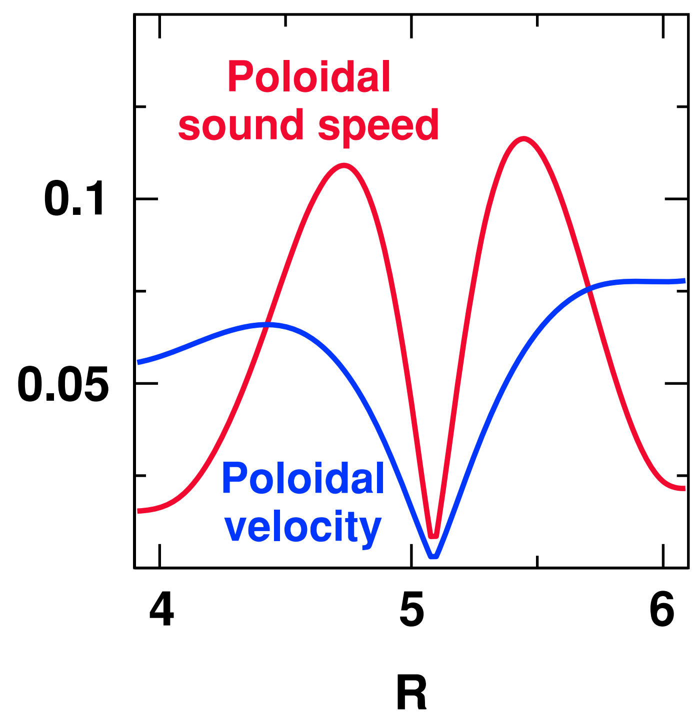
The initial conditions contain a transonic surface

Initial poloidal Mach number and field lines



Poloidal velocity and sound speed showing a transonic surface



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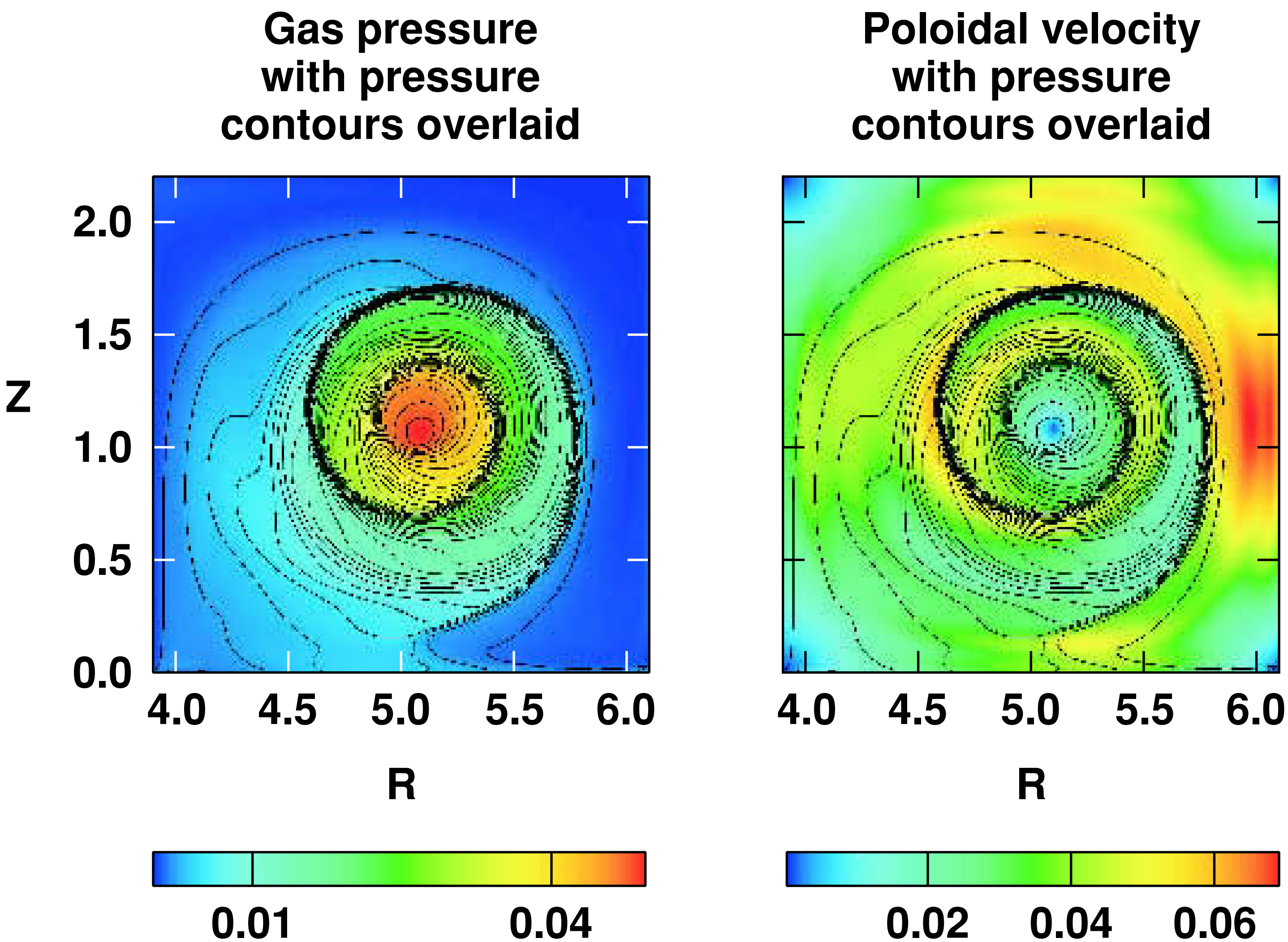
Poloidal flow develops a spiral shock and poloidal shear flow



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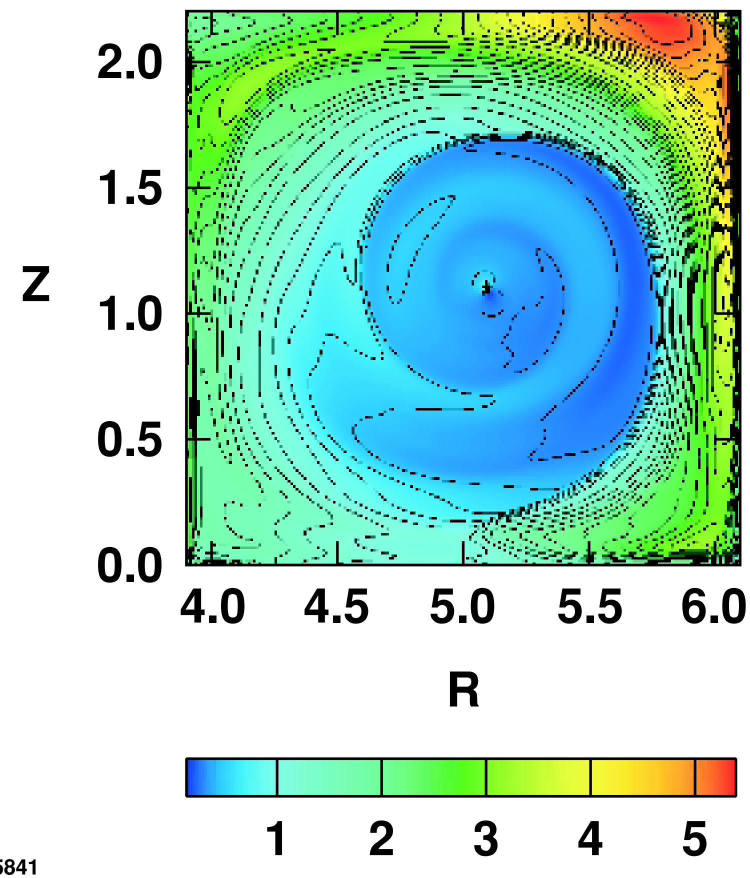
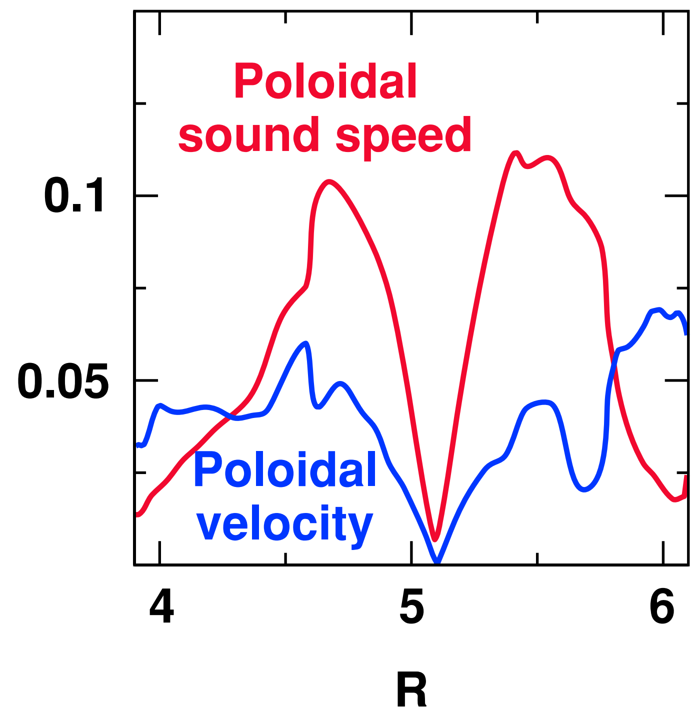
The poloidal shear flow is transonic

Poloidal Mach number with contours



Radial plot of the poloidal velocity and sound speed at z = 1.1 showing a transonic shear flow



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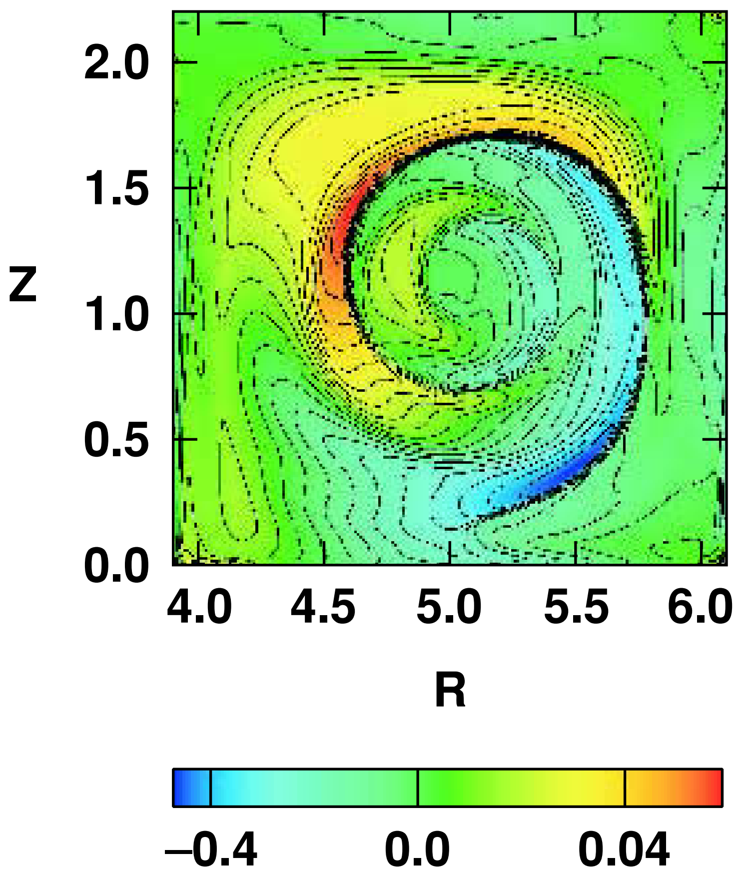
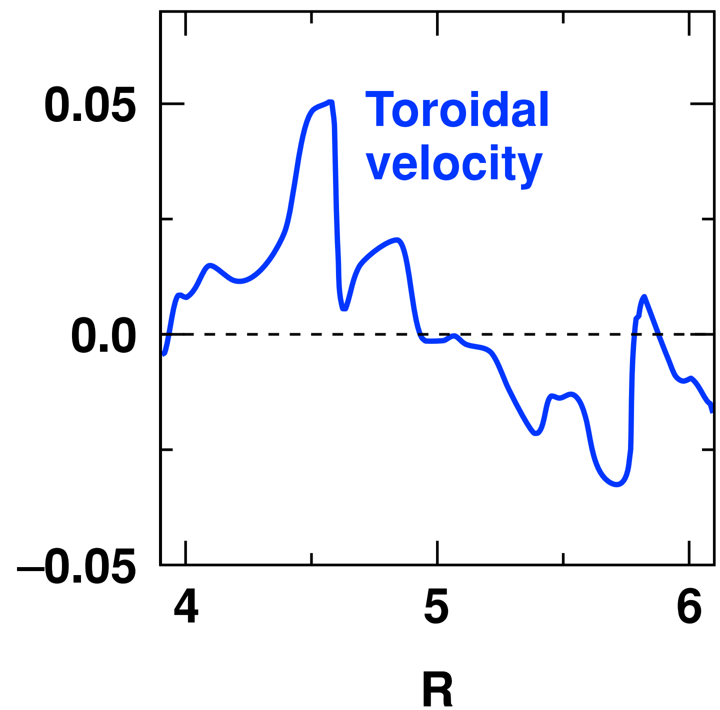
Spiral shock generates a toroidal shear flow

Toroidal velocity
with contours overlaid



Radial plot of the toroidal
velocity at z = 1.1 showing
a shock-generated shear flow



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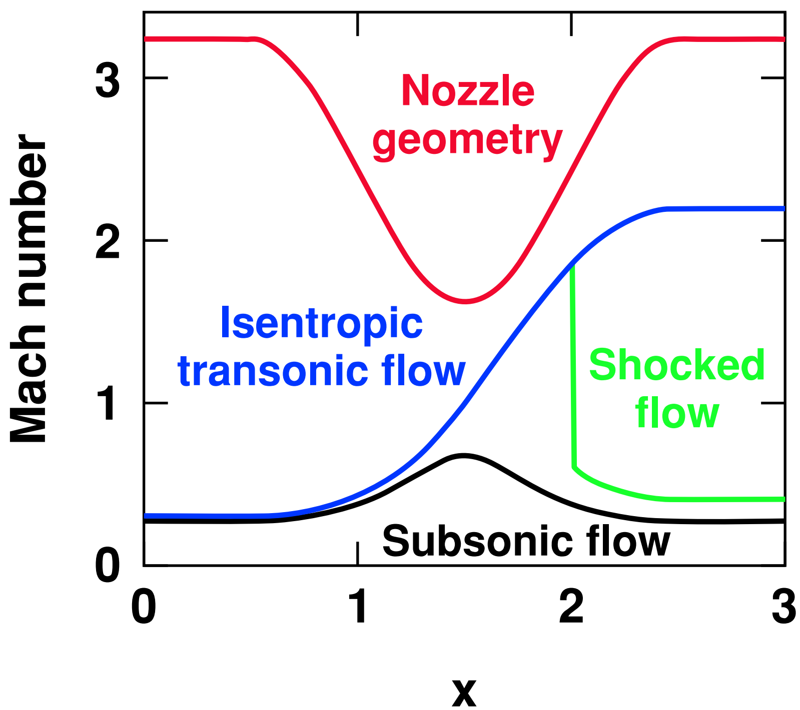
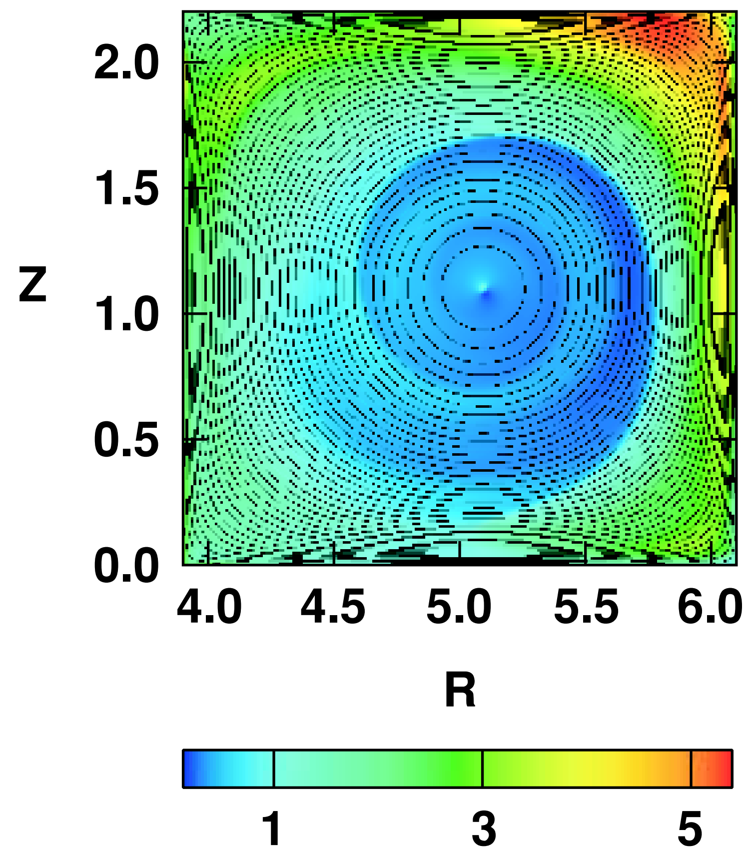
Evolution is similar to a de Laval nozzle

Example of nozzle geometry and Mach number for subsonic, isentropic transonic, and transonic shocked flow



Poloidal Mach number and field lines showing periodic transonic shocked flow



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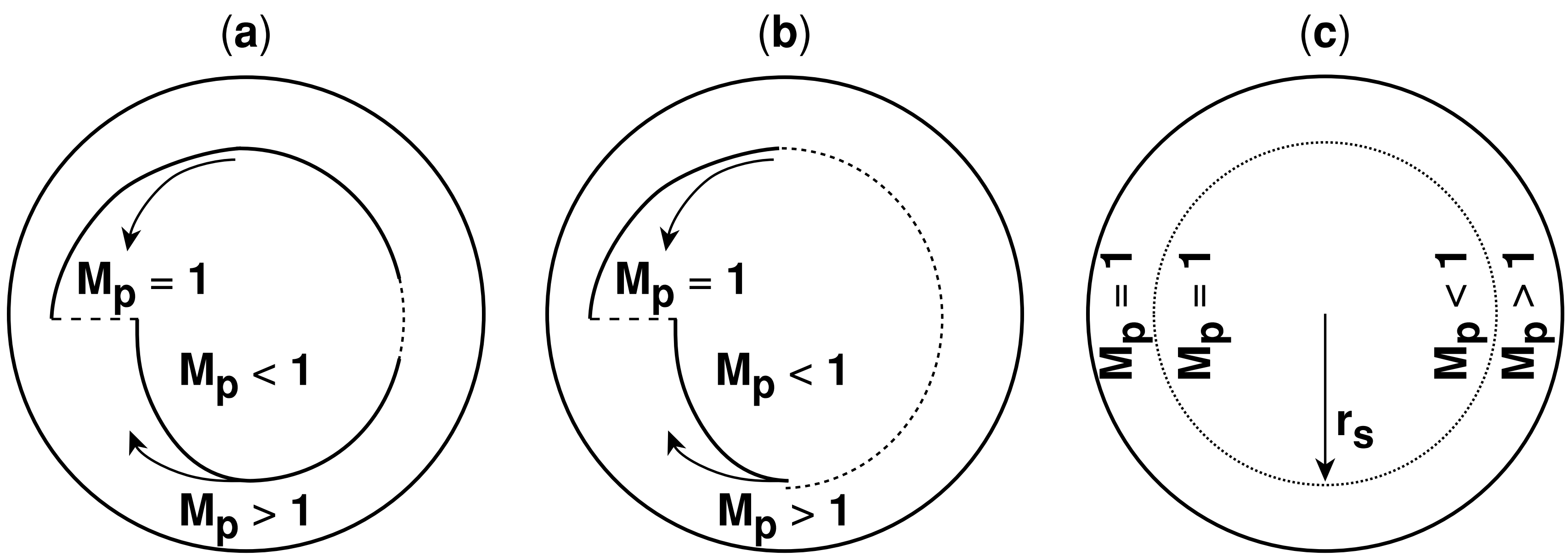
Future studies will address long-term evolution



- Betti and Freidberg (2000) showed that weak shocks propagate from the outer midplane to the inner midplane.
- On a long time scale the spiral shocks in figure (a) will decay to figure (b) and ultimately (c).
- The final state is predicted to be a shockless equilibrium with a discontinuous Mach number.

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