Stability of the Resistive Wall Mode in the Presence of Nonsymmetric Differentially Rotating Walls

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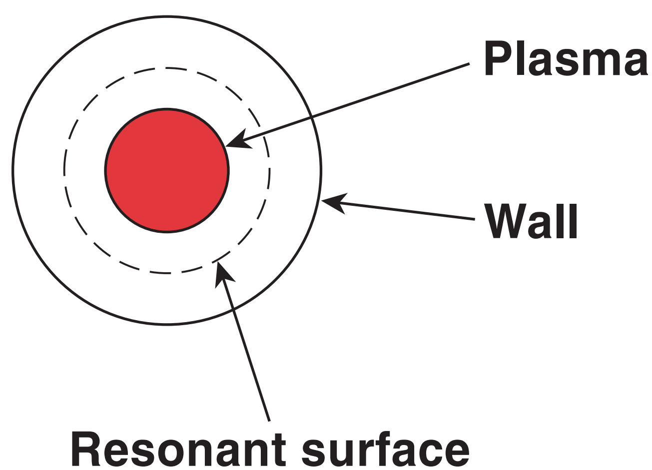
The effect of nonsymmetric differentially rotating walls on the stability of the resistive wall mode (RWM) is investigated for a cylindrical pinch configuration. In the simple case of two symmetric rotating walls with a large aspect ratio ($\in$ = wall thickness/wall radius <<1) and long magnetic diffusion times τ_w, the RWM has been found to be stabilized when the relative rotation frequency exceeds a critical value of the order of $1/\left(\tau_w\sqrt{\varepsilon}\right)$. If the two walls are made of stainless steel and liquid lithium, the typical rotation frequency is quite large and may be impractical in a realistic experiment. However, it has also been suggested that improved stability may be achieved if the two walls are not rotating symmetrically. Instead, the lithium flow around the metallic wall separates into two directions at the inlet, one clockwise and the other counter-clockwise, finally recombining at the outlet. The analysis is extended to include such a nonsymmetric flow pattern. This work was supported by the U.S. Department of Energy under contract DE-FG02-93ER54215.

Abstract

The effect of nonsymmetric differentially rotating walls on the stability of the resistive wall mode (RWM) is investigated for a cylindrical pinch configuration. In the simple case of two symmetric rotating walls with a large aspect ratio ($\varepsilon =$ wall thickness/wall radius $<< 1$) and long magnetic diffusion times τ_W, the RWM has been found to be stabilized when the relative rotation frequency exceeds a critical value of the order of $1/(\tau_W \, \varepsilon^{1/2})$. If the two walls are made of stainless steel and liquid lithium, the typical rotation frequency is quite large and may be impractical in a realistic experiment. However, it has also been suggested that improved stability may be achieved if the two walls are not rotating symmetrically. Instead, at the inlet, the lithium flow around the metallic wall separates into two directions, one clockwise and the other counterclockwise, finally recombining at the outlet. This analysis is extended to include such a nonsymmetric flow pattern.

The resistive wall kink—What is it?

- Start with an ideal MDH external kink
  - it is a leading order instability
- Driven by plasma current and pressure
  - sets limits on I_p and plasma β (in a torus)
- Add a perfectly conducting wall
  - within some critical R_{wall} the mode is stabilized
- Make the wall resistive
  - the mode remains unstable, slowly growing with $\gamma \sim 1/\tau_w$



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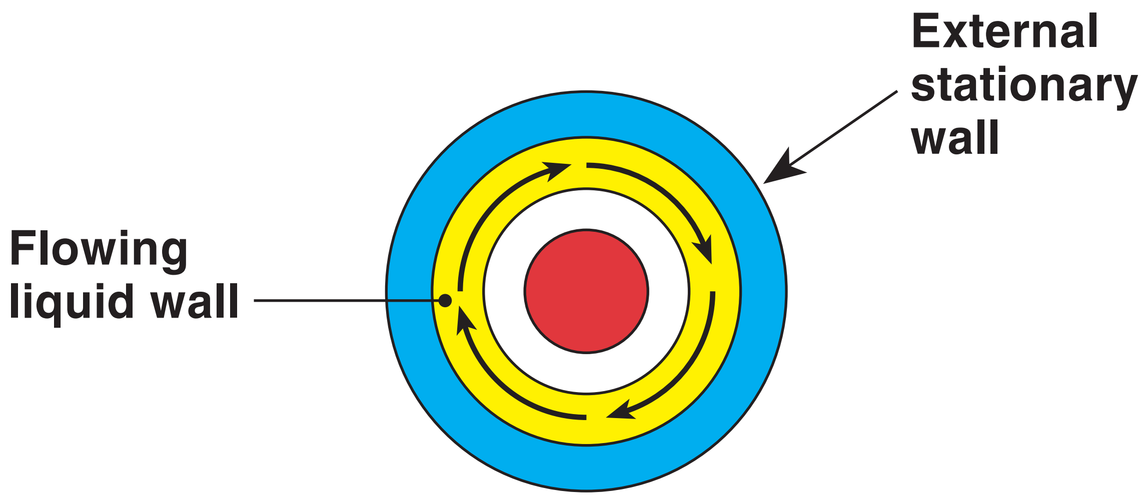
How to stabilize RWM?

- Feedback stabilization
- Plasma rotation
- Flowing liquid metal wall
  - If there are two walls with relative rotation, the unstable mode cannot lock on both of them $\Rightarrow$
  - If the mode rotates with intermediate angular velocity in either wall frame it has $\omega_{real} \Rightarrow$
  - Skin effect makes the wall behave like a perfect conductor
  - May also solve the first wall problem (heat load, neutron activation)

Differentially rotating walls*



- Stability analysis was conducted for a general diffusive pinch enclosed by two walls with relative rotation velocity.
- The resistive wall mode is stabilized when the rotation frequency exceeds a critical value Ω
- The critical Ω scales as $(1/\tau_W)\ (b/d)^{1/2}$, typically V ~ 100 m/s.
- A gap between the walls can reduce the critical Ω

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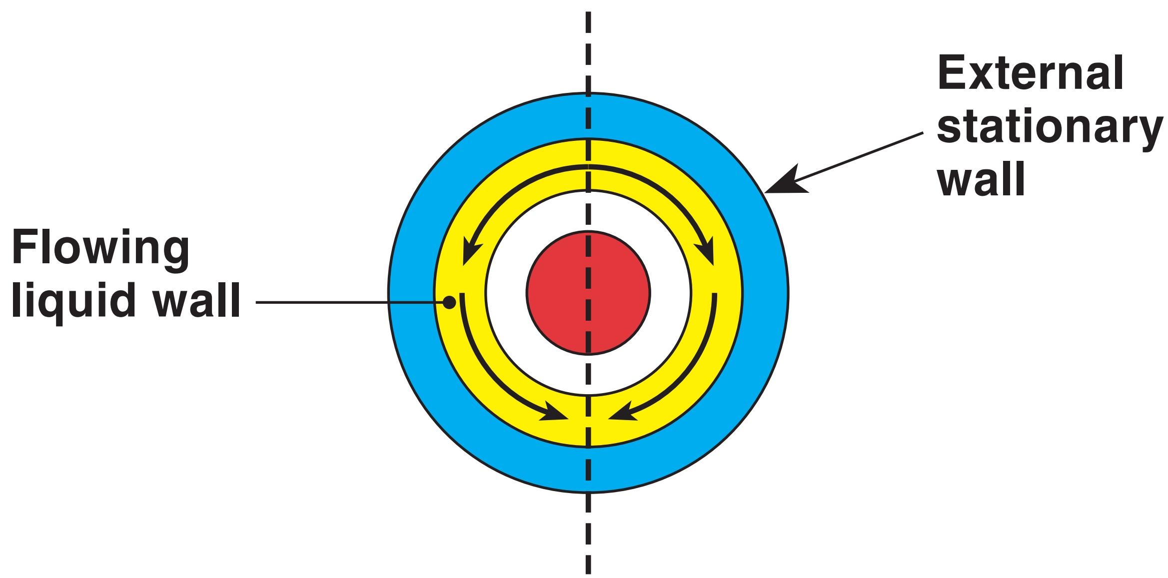
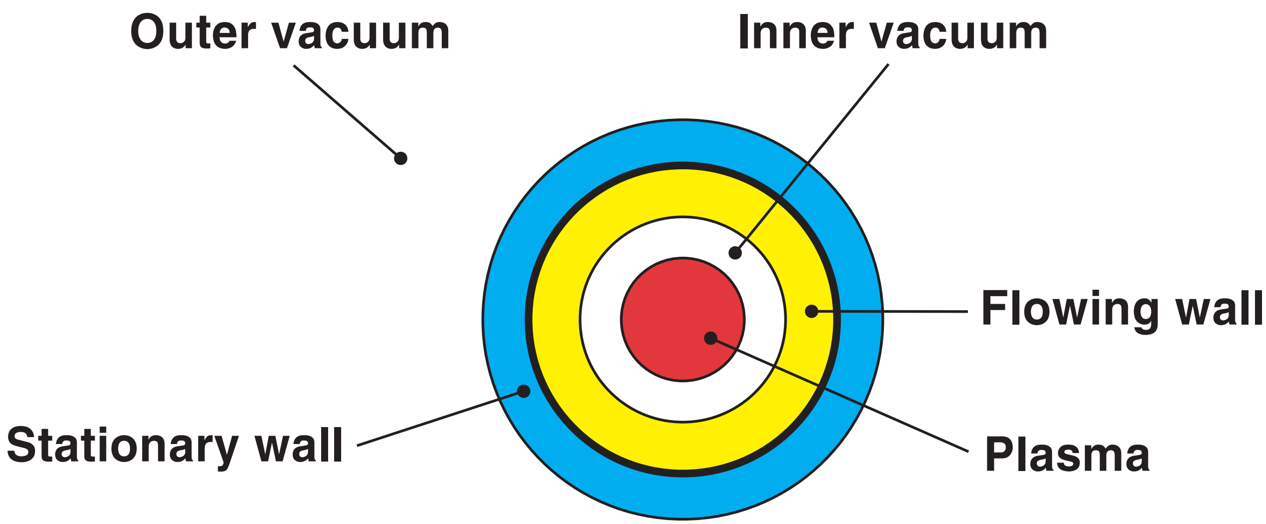
*Freidberg and Betti (submitted to Phys. Plasmas)

Nonsymmetrically rotating walls



- An interesting flow configuration is two opposite streams separating at the top and merging at the bottom
- Since the flow pattern is not poloidally symmetric a usual single mode ~exp (imθ) solution is not possible
- Present stability analysis is extended to this case

Governing equations



- **In vacuum domains:** $\mathbf{B} = -\nabla \phi$
- **In stationary wall:** $\dfrac{\partial \mathbf{B}}{\partial t} = D_M \nabla^2 \mathbf{B}$
- **In flowing wall:** $\dfrac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \mathbf{J})$

General solution for B field [assuming $\exp(\gamma t + ikz)$]

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- **External vacuum:** $\sum_m e^{im\theta}\left[C_{5m}\, k\, K'_m(kr)\right]$
- **Inner vacuum:** $\sum_m e^{im\theta}\left[C_{1m}\, k\, I'_m(kr) + C_{2m}\, k\, K'_m(kr)\right]$
- **Stationary wall:** $\sum_m e^{im\theta}\left[d_{3m} \exp(\delta x) + d_{4m} \exp(-\delta x)\right]$

$$x = r - r_b\,, \quad \delta^2 = \gamma / D_M$$

General solution for B field in the flowing wall

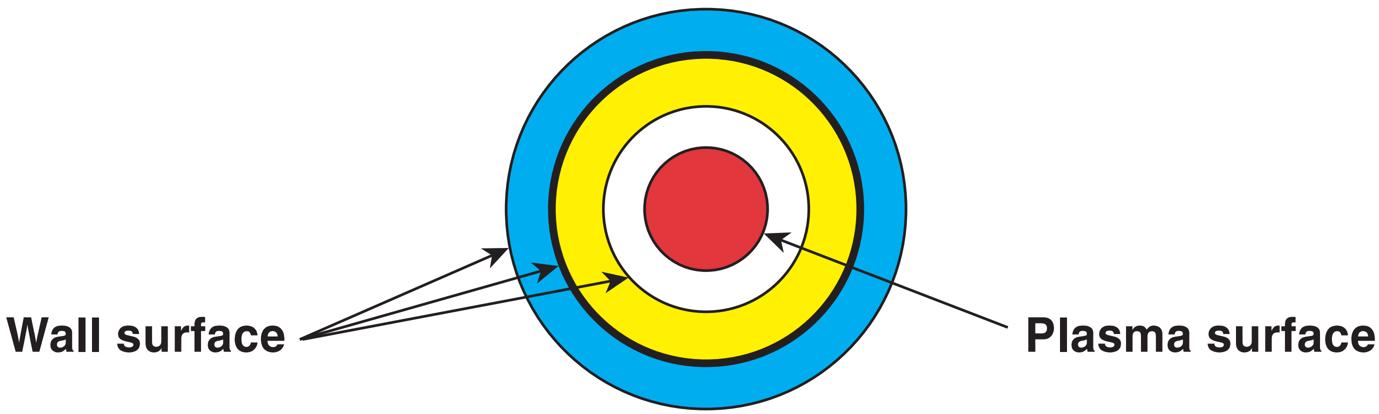
$$\Re_{m,l}\, e^{im\theta}\, \vec{V}_m^{(\ell)} \left[C_{3l} \exp(\lambda_l x) + C_{4l} \exp(-\lambda_l x)\right]$$

$$\lambda_k^2 = \left[\gamma + (2\Omega\Lambda_k/\pi)\right]/D_M$$

Λ_k and $\overline{V}^{(k)}$ – k^{th} eigenvalue and eigenvector of $\hat{G}\overline{v} = \Lambda\overline{v}$

$\hat{G}$ – matrix defined by the flow profile

At each interface boundary conditions must hold



$$[[B_r]] = 0$$

$$[[B_r]] = 0$$

$$[[B_\tau]] = 0 \Rightarrow [[(B_r)' r]] = 0$$

$$[[(p + B^2/2\mu_0)_\perp]]$$

Plasma model

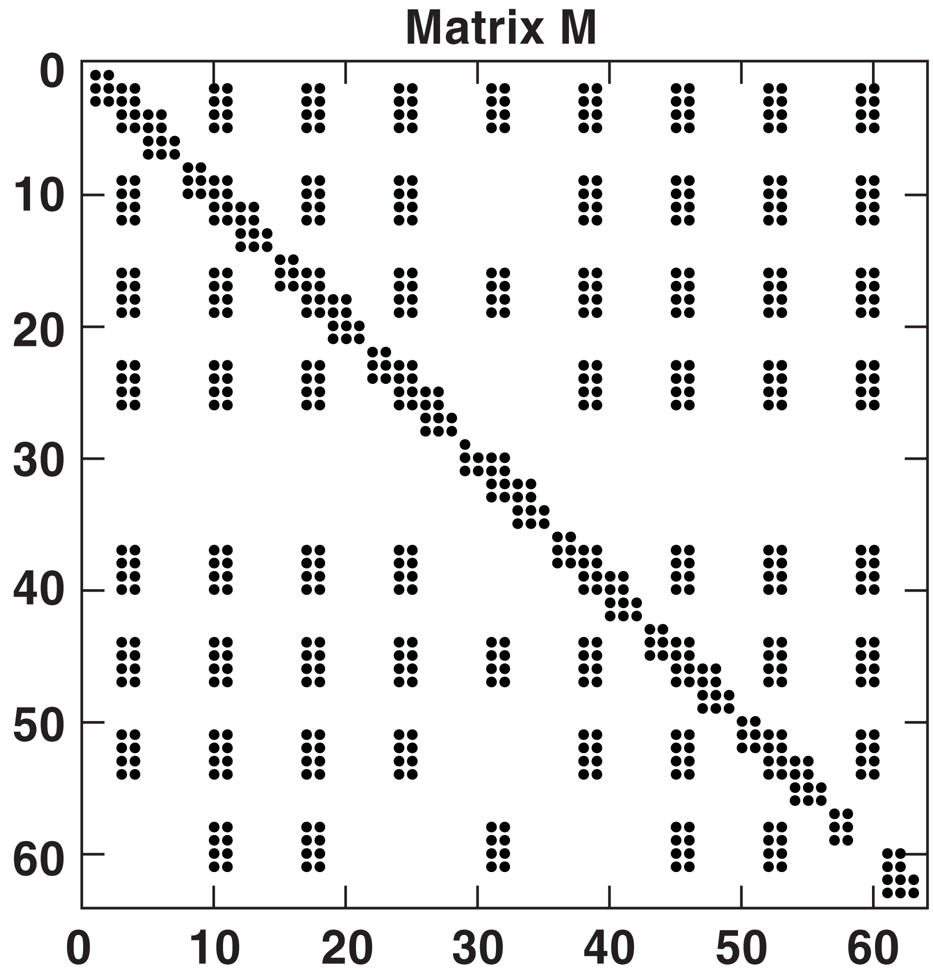
- Uniform current model is a standard test-bed for MHD stability analysis:

$$j(r) = \text{const} \Rightarrow q(r) = q_a$$

- Potential energy of perturbation

$$\frac{\delta W_F + \delta W_{v\infty}}{\delta W_{v\infty}} = 2|m| \frac{n q_a - m + 1}{m - n q_a}$$

Matching solutions across domain boundaries results in a matrix equation $\hat{M}\,\bar{C} = \bar{0}$



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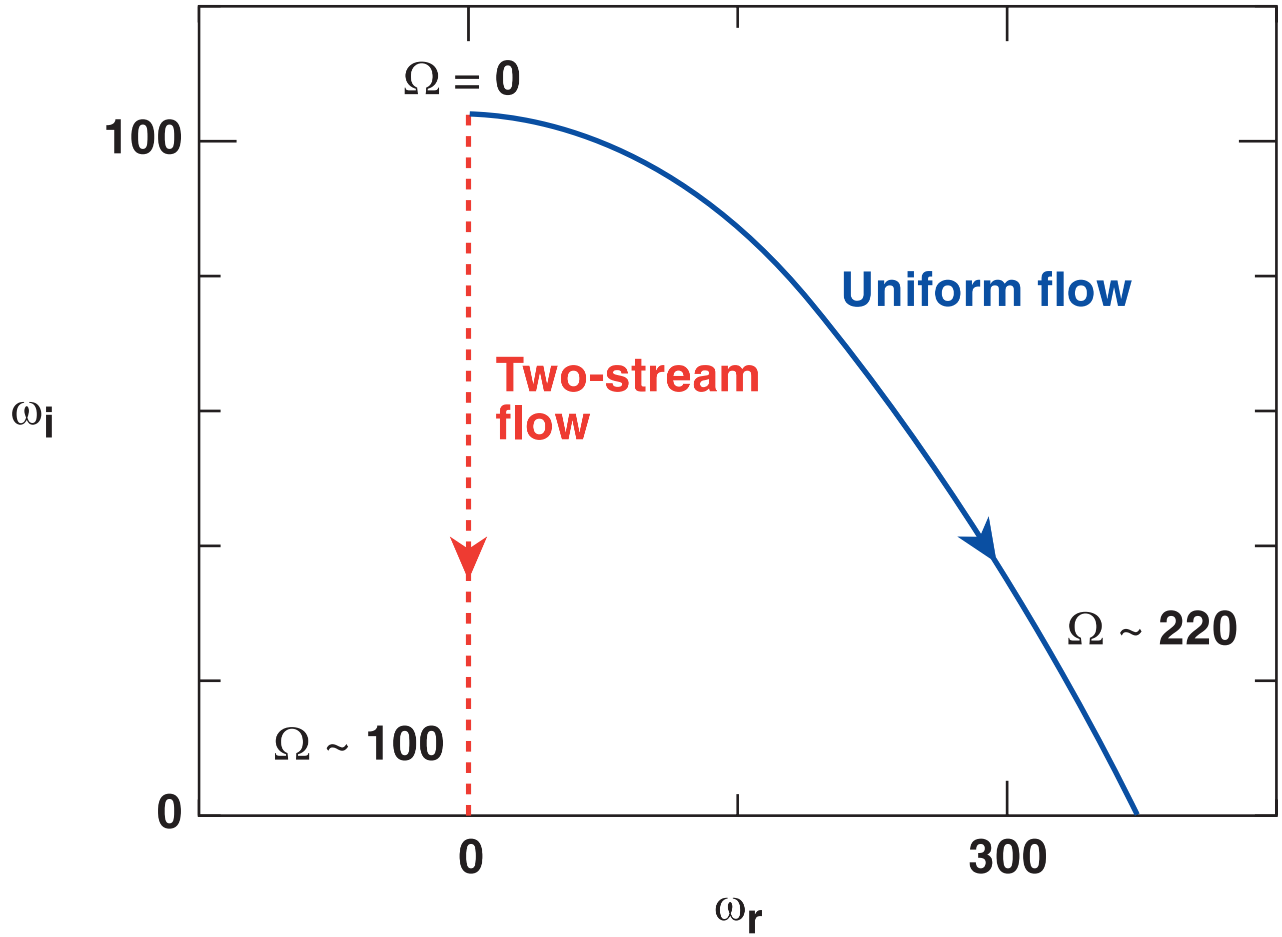
Unstable root of equation det (M) = 0 is traced in complex ω plane as the flow angular velocity is increased



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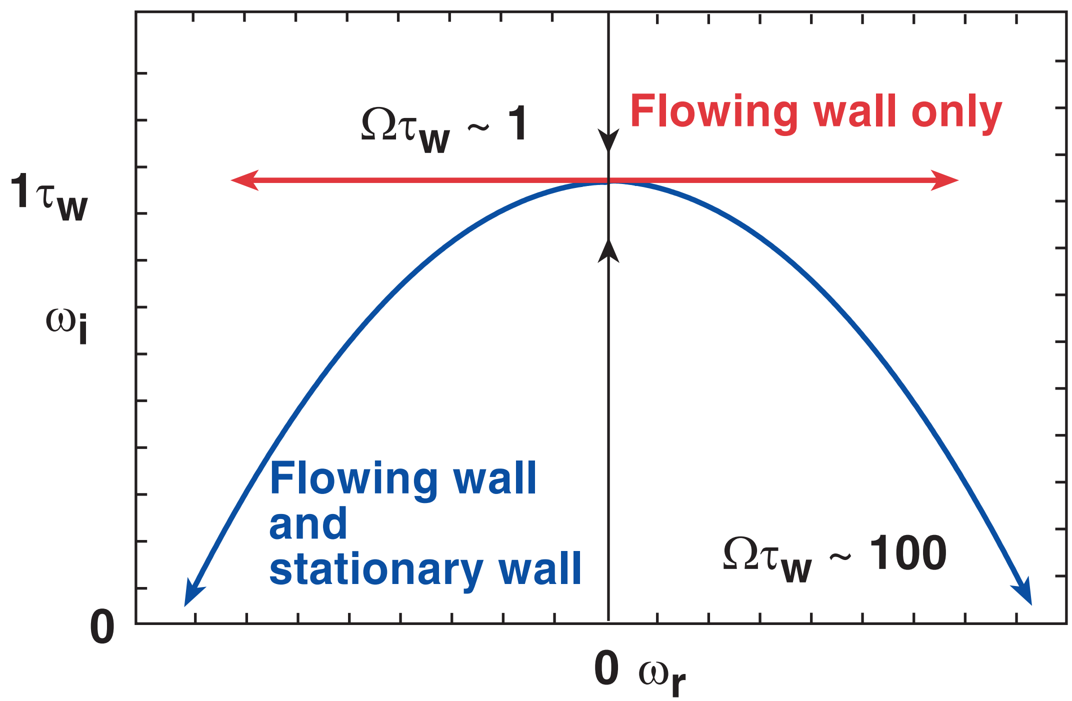
Single-stream locked modes can exist in the two-stream flow case

- SLM's were found in the two-stream configuration.
- Without stationary walls SLM's can be unstable for any Ω.
- With stationary walls SLM's are stabilized by the flow.



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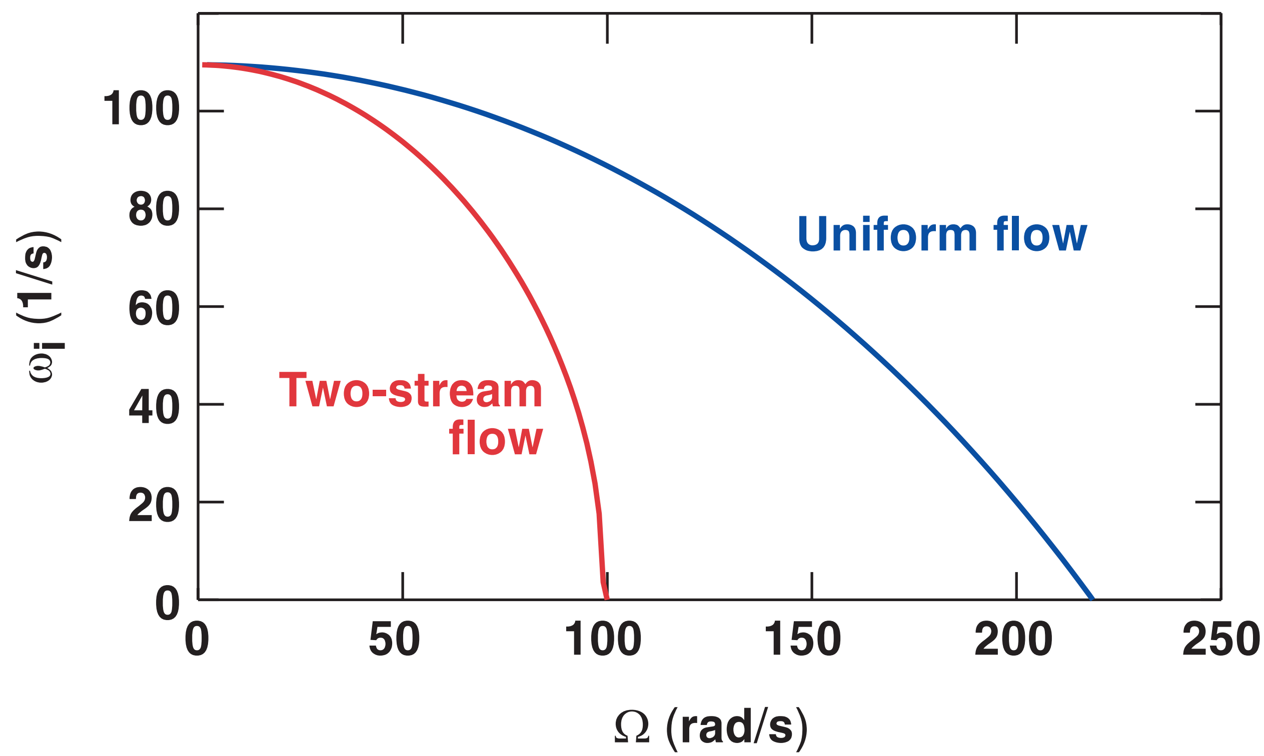
Some numerical results

- Uniform j model, q_a = 2.2, m = 3, n = 1 mode unstable
- R_{plasma} = 52 cm, R_{wall} = 55 cm, Li wall 2 cm + stainless steel wall 2 cm



- Stabilization by two-stream flow is more efficient than by uniform flow

Conclusions

- Stability analysis of the resistive wall mode is conducted for a general plasma pinch in the presence of a nonsymetric flow of liquid metal around the plasma
- With nonsymmetric flow, the velocity required for stabilizing the RWM is two to four times smaller than that with uniform flow
- In a realistic experiment, one needs a flow velocity of a few tens of meters per second to stabilize the RWM