The Effect of Weak Collisions on Plasma Oscillations

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Lenard *et al.*[1] studied this using an approximate collision operator. Observation of plasma echoes led Su *et al.*[2] to study the effect of weak collisions on the propagation of f_1 from a localized source. They used boundary layer theory to obtain the v-dependence of f_1 and found that the spatial echo is attenuated as $\exp(-ßx^3)$. Recently, this problem has been revisited by Ng *et al*.[3] They use a f.t. in x and t and obtain eigenvalues of the velocity equation by a numerical procedure and state that the results in Ref. 2 are in error. We show that this is not the case: they recover instead the collective oscillations (identical to those in Ref. 1) but not the dominant f_1 behavior. In addition, it is incorrect to state that Ref. 2 assumed that the Van Kampen spectrum is preserved in the presence of collisions. We use the b.l. method with complex ω. Our results are in essential agreement with those of Ref. 2 (and Landau). This work was supported by the U.S. DOE Office of ICF under Coop. Agreem. DE-FC03-92SF19460.

1. A. Lenard *et al.*, Phys. Rev. **112**, 1456 (1958).
2. C. H. Su *et al.*, Phys. Rev. Lett. **20**, 427 (1968).
3. C. S. Ng *et al.*, Phys. Rev. Lett. **83**, 1974 (1999).

The effect of small collisions on plasma waves has been studied, using a model operator

- Lenard and Bernstein (1958) looked at the effect on electron plasma waves. There was only a small correction to the decay rate of the least-damped Landau mode.
- Motivated by plasma echo experiments, Su and Oberman (1968) looked at spatial variation of $f_1(x,v,\omega_a)$ downstream from a steady-state oscillator. They found that

$$f_1 \propto \exp\left(-\frac{\nu x^3 \omega_a^2 v_t^2}{3v^5}\right),$$

where ν is the collision rate and ω_a is the oscillator frequency.

Recently, Skiff *et al.* measured the ion distribution function in a weakly collisional plasma

- **Skiff *et al*. (1998) measured the perturbed ion velocity distribution function $f_1(x,v)$ associated with ion-acoustic waves arising from a localized antenna at a fixed frequency ω. There is a static uniform magnetic field in the x (and v) direction.**

- **Spatial Fourier analysis of the data showed two dominant amplitude groups, $|f_1(k,v)|$, one at the ion-acoustic phase velocity, and the other at a higher phase velocity.**

- **Comparison was made with numerically determined eigenvalues of the ion kinetic equation (sans antenna). Here too there are two groups of complex k_j eigenvalues. However, there was poor agreement between the measured and calculated second groups as far as spatial decay and velocity shape are concerned. The authors concluded that the Su–Oberman theory must be incorrect.**

Following up on this, Ng *et al*. returned to the effect of weak collision on electron plasma oscillations

- Ng *et al*. (1999) calculated the eigenvalues of the 1-D electron-kinetic equation (again sans antenna). For a given real ω they obtained complex eigenvalues k_j and eigenfunctions g(v).
- The least damped k is very close to the Landau value. More heavily damped roots differ considerably from the collisionless analysis. All roots are distinct, and there is no continuum.
- They conclude that Su and Oberman are in error and ascribe this to an improper perturbation analysis about the Van Kampen continuum.
- We show that (1) the Su and Oberman result remains correct, (2) it has no relation to the Van Kampen modes, and (3) the starting equation for the Skiff or the Ng analysis is inadequate for antenna propagation analysis.

Let us sketch a procedure for solving the antenna response problem

- Consider the driven response to a localized antenna:

$$E_e = \left|E_a\right| \delta(x)\ \exp\left(i\omega_a t\right)$$

- Use the Lenard–Bernstein collisional operator:

$$\frac{\partial f_1}{\partial t} + v\frac{\partial f_1}{\partial x} - \frac{e}{m} E_1 \frac{dF_0}{dv} = \nu \frac{\partial}{\partial v}\left(v f_1 + u_0^2 \frac{\partial f_1}{\partial v}\right)$$

$$\frac{\partial E_1}{\partial x} = \frac{\partial E_e}{\partial x} - 4\pi e \int f_1 dv$$

- Now take a F.T. in space and keep only the driven time response:

$$i\left(\omega_a + kv\right)\tilde{f}_1 - \nu \frac{\partial}{\partial v}\left(v\tilde{f}_1 + u_0^2 \frac{\partial}{\partial v}\tilde{f}_1\right) = \frac{e}{m}\tilde{E}_1 \frac{d}{dv} F_0$$

$$ik\tilde{E}_1 = ik\left|E_a\right| - 4\pi e \int \tilde{f}_1 dv$$

The expression for the spatial dependence of f_1 is the key to the problem

- Solve the kinetic equation for f_1 in terms of E_1:

$$\tilde{f}_1(k,v) = \frac{e}{m}\tilde{E}_1 \frac{dF_0}{dv} g(k,v).$$

Here $g(k,v)$ can be obtained in several ways: Su and Oberman use the boundary-layer theory. The resultant $g(k,v)$ is highly peaked in a narrow $(\sim\nu^{1/3})$ layer at $v = -\omega_a/k$. Another approach is to take a F.T. in velocity to solve for g (Lenard–Bernstein, Karpman, Short, Betti).

- Further, E_1 can be expressed in terms of $|E_a|$ through the Poisson equation:

$$\tilde{E}_1 = \frac{|E_a|}{D(k,\omega)} \qquad D(k,\omega) = 1 + \frac{\omega_p^2}{k^2}\int g(k,v)\frac{dF_0}{dv}dv.$$

The expression for the spatial dependence of f_1 is the key to the problem (continued)

- Substitute this expression for E1 back into the solution for $f_1(k,v)$ and then invert the spatial F.T. to get

$$f_1(x, v, t) = \exp(i\omega_a t)\,\frac{|E_a|}{2\pi}\int_{-\infty}^{\infty}\frac{e^{ikx}g(k, v)}{D(k, \omega_a)}\,dk\,.$$

- This form of the result enables us to explain the differences between Su–Oberman and Skiff–Ng.

- The eigenvalues of the Skiff–Ng integral equation (which is the kinetic equation sans antenna term) are identical to the solutions of $D(k,\omega_a) = 0$. This is easy to show analytically and has also been verified numerically by Short. Short (see poster) has also obtained an analytic expression for $D(k,\omega)$ in the form of an incomlete gamma function. An earlier expression, in series form, is in Lenard–Bernstein.

The "collective" roots have no connection to the dominant term in $f_1(x,k,t)$

- The corresponding contributions to the k-integral above can be described as "collective" effects and decay in space as the Landau solutions. (Su–Oberman explicitly neglect these as uninteresting.)

- The dominant response in $f_1(x,v,t)$ [beyond a few Debye lengths from the antenna] comes from the $g(k,v)$ contribution to the inversion integral. One readily obtains

$$f_1(x,v,t) \approx \exp\left(\frac{\nu\omega_a^2 v_t^2 \left|x^3\right|}{3v^t}\right).$$

- The importance of this result for plasma echoes is obvious.

What is the relation of all this to the Van Kampen continuum?

- There is no relation! Su and Oberman did not refer to the Van Kampen continuum either directly or implicitly. Perhaps the confusion arose because they treated ω as real throughout (as they should for a driven-oscillator problem).

- We agree with Ng that "the Van Kampen continuum is eliminated, even as $\nu \to 0$;" however, this was also apparent in the results of Lenard–Bernstein and explicitly stated by Karpman (1966).

What about the discrepancies between the theory and experiment noted by Skiff?

- The eigenvalues of the (ion sound) integral equation (sans antenna) represent only the "collective response" parts of the $f_1(x,v)$ behavior.
- One must also solve the inhomogeneous ion kinetic equation (again, a boundary layer problem) for $f_1(x,v)$ and compare this with the measured date (Fig. 4 in Skiff).
- Fourier inversion of this to yield the spatial dependence should also be carried out and then compared with the raw data $f_1(x,v)$.
- Future collaboration between Iowa and LLE? I hope so—it could be useful.

Summary

- The Su–Oberman result is correct. It does not use the Van Kampen continuum in any way.
- The dominant behavior of $f_1^{ion}(x,v)$ downstream from a localized oscillator requires solution of a boundary-layer-type inhomogeneous kinetic equation.
- There is, as yet, no obvious discrepancy between experiment and theory in the ion sound case. Further analysis is needed.