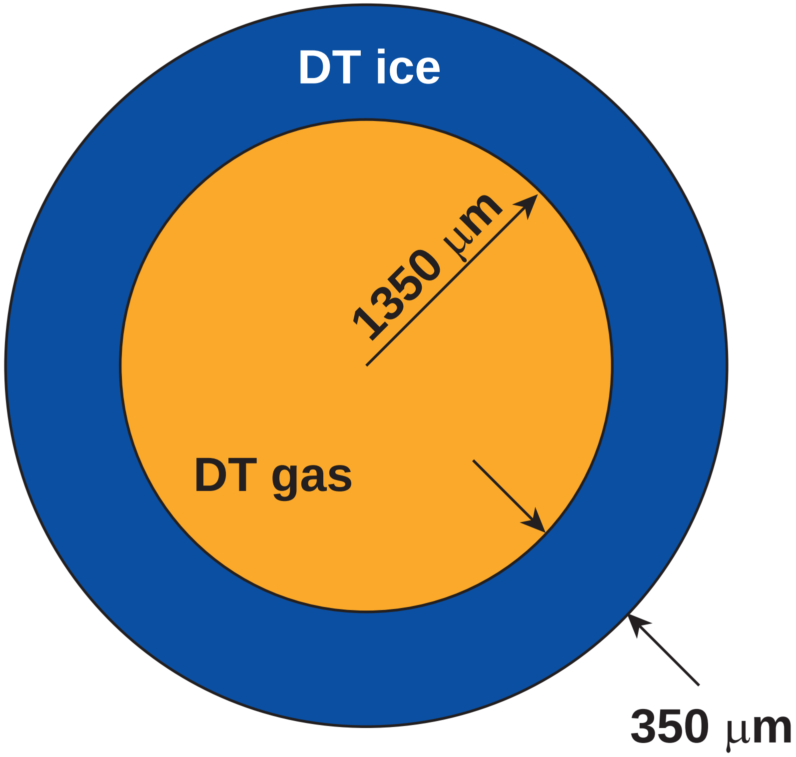
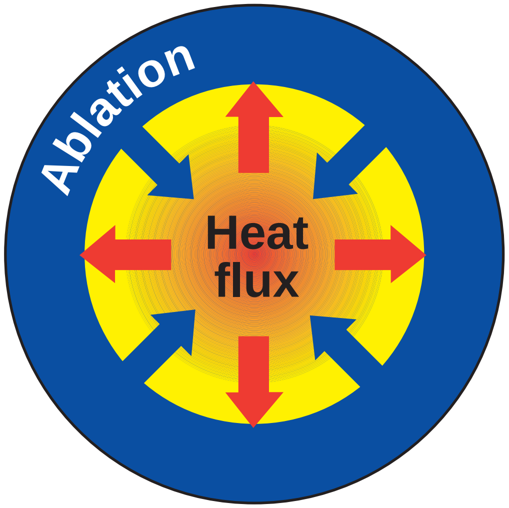
Theory of the Deceleration Phase Rayleigh–Taylor Instability





R. Betti, V. Lobatchev, and M. Umansky
University of Rochester
Laboratory for Laser Energetics

42nd Annual Meeting of the
American Physical Society
Division of Plasma Physics
Québec City, Canada
23–27 October 2000

Theory of the Deceleration-Phase Rayleigh–Taylor Instability

R. Betti, M. Umansky, and V. Lobatchev

Laboratory for Laser Energetics, U. of Rochester

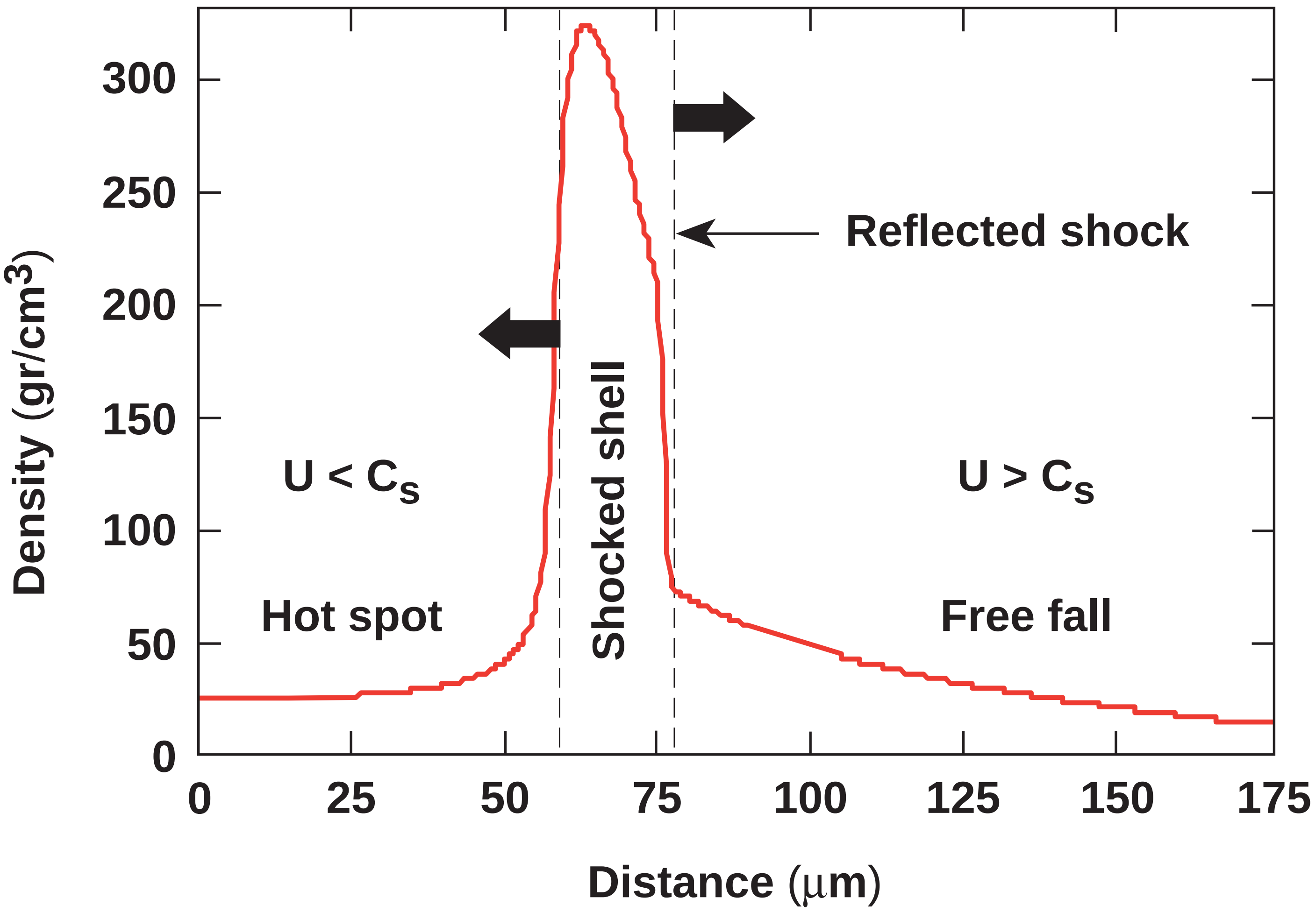
The theory of the deceleration phase of an imploding ICF capsule is carried out analytically by solving the conservation equations in Lagrangian coordinates inside the hot-spot region. The evolution of the hot-spot mass and temperature are calculated. The hot-spot mass increases in time because of the ablation flow off the shell's inner surface. Such a flow is caused by the heat flux leaving the hot spot and being deposited on the shell's inner surface. The resulting ablation velocity of the thermal front inside the shell is calculated and compared with the results of numerical simulations. The density-gradient scale length on the shell's inner surface is also determined as a function of the hot-spot radius, shell temperature, and hot-spot temperature. The shell acceleration, ablation velocity, and density-gradient scale length are used to determine the Rayleigh–Taylor growth rates, which are well below their classical value. This work was supported by the U.S. Department of Energy Office of Inertial Confinement Fusion under Cooperative Agreement No. DE-FC03-92SF19460.

Summary

A theoretical model for the deceleration is derived and compared with numerical simulations; the ablation velocity at the shell's inner surface leads to the suppression of short-wavelength RT modes

- All hot-spot profiles are determined analytically.
- The shell's properties are governed by four first-order ordinary differential equations.
- The heat flux leaving the hot spot and depositing on the shell's inner surface leads to a finite ablation velocity, which is the penetration velocity of the thermal front into the cold shell.
- Thermal conduction determines the density-gradient scale length at the shell's inner surface.
- Short-wavelength RT modes are stabilized by finite ablation velocity and density-gradient scale length.

The theory is developed assuming that the flow is subsonic in the hot spot and supersonic in the free-fall part of the shell



The model for the hot spot assumes subsonic flow and kσvl proportional to T^2

$$\frac{1}{\rho} = \frac{1}{3}\frac{\partial r^3}{\partial m}$$

mass

$$\cancel{\frac{\partial U}{\partial t}} = -r^2\frac{\partial P}{\partial m} \Rightarrow P \approx P_{hs}(t)$$

momentum

$$c_v \rho^{2/3}\frac{\partial}{\partial t}\frac{T}{\rho^{2/3}} = \frac{\partial}{\partial m}\kappa(T) r^4 \rho \frac{\partial T}{\partial m} + \frac{\rho}{4M_i^2}\overset{\text{3.5 MeV}}{\varepsilon_\alpha}\langle\sigma v\rangle$$

energy

$$m = \int_0^r \rho(r',t)\, r'^2\, dr'$$

Lagrangian coordinate

$U << C_{sound}$ $\quad$ $\langle\sigma v\rangle \approx \Sigma_\alpha T^2$

Assumptions

All the hot-spot parameters can be calculated analytically; they depend on the hot-spot pressure $P_{hs}(t)$ only (and the initial conditions)

Hot-spot mass $\rightarrow$ $M_{hs} = \left[M_{hs}(t_0)^{7/2} + 270\,\bar{\kappa}\, m_i \int_{t_0}^{t} P_{hs}(t')^{5/2} R_{hs}(t')^{17/2} dt' \right]^{2/7}$

Hot-spot radius $\rightarrow$ $R_{hs}(t) = R_{hs}(0) \left[\frac{P_{hs}(0)}{P_{hs}(t)} \right]^{1/5} \exp\left[\frac{1}{3} D_{\alpha} \int_{t_0}^{t} P_{hs}(t')\, dt' \right]$

Hot-spot ρR $\rightarrow$ $\rho R \equiv \int_{0}^{R_{hs}} \rho dr = 0.18 \frac{M_{hs}(t)}{R_{hs}^{2}(t)}$

Hot-spot temperature $\rightarrow$ $T(r,t) \approx T_0(t) \left[\frac{1 - (r/R_{hs})^2}{1 - 0.17(r/R_{hs})^2} \right]^{2/5}$

t_0 = beginning of deceleration phase; $\kappa_{Spitz} = \bar{\kappa}\, T^{5/2}$; $D_{\alpha} = 0.046$ Gbar^{-1}ns^{-1}

The hot-spot pressure depends on the shell dynamics

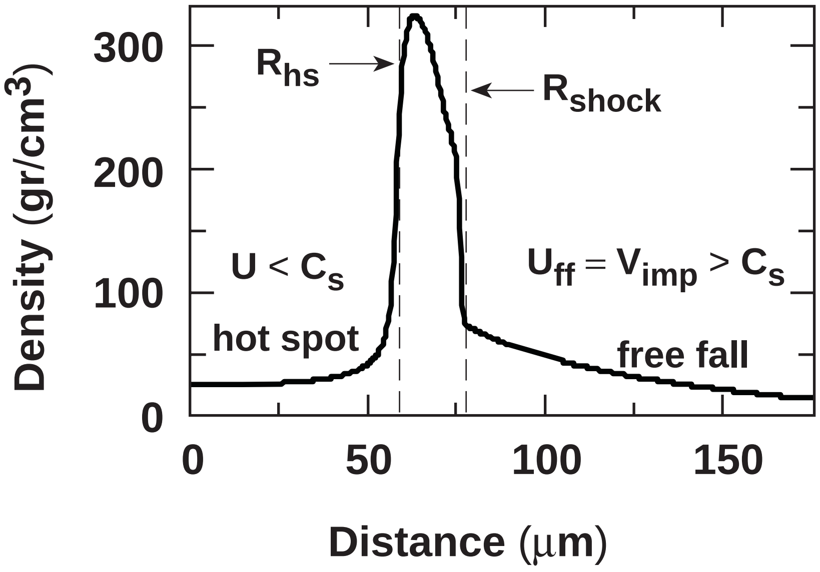
Newton's law for shocked shell:
$$\frac{d}{dt}\left[M_{ss}(t)\dot{R}_{hs}(t)\right] = 4\pi R_{hs}^{2}(t)P_{hs}(t) + \dot{M}_{ss}U_{ff}$$

Mass conservation for shocked shell:
$$\dot{M}_{ss} = 4\pi R_{shock}^{2}\rho_{ff}\left(\dot{R}_{shock} - U_{ff}\right)$$

Hugoniot relation:
$$\dot{R}_{shock} = \frac{4}{3}\dot{R}_{hs} - \frac{1}{3}U_{ff}$$

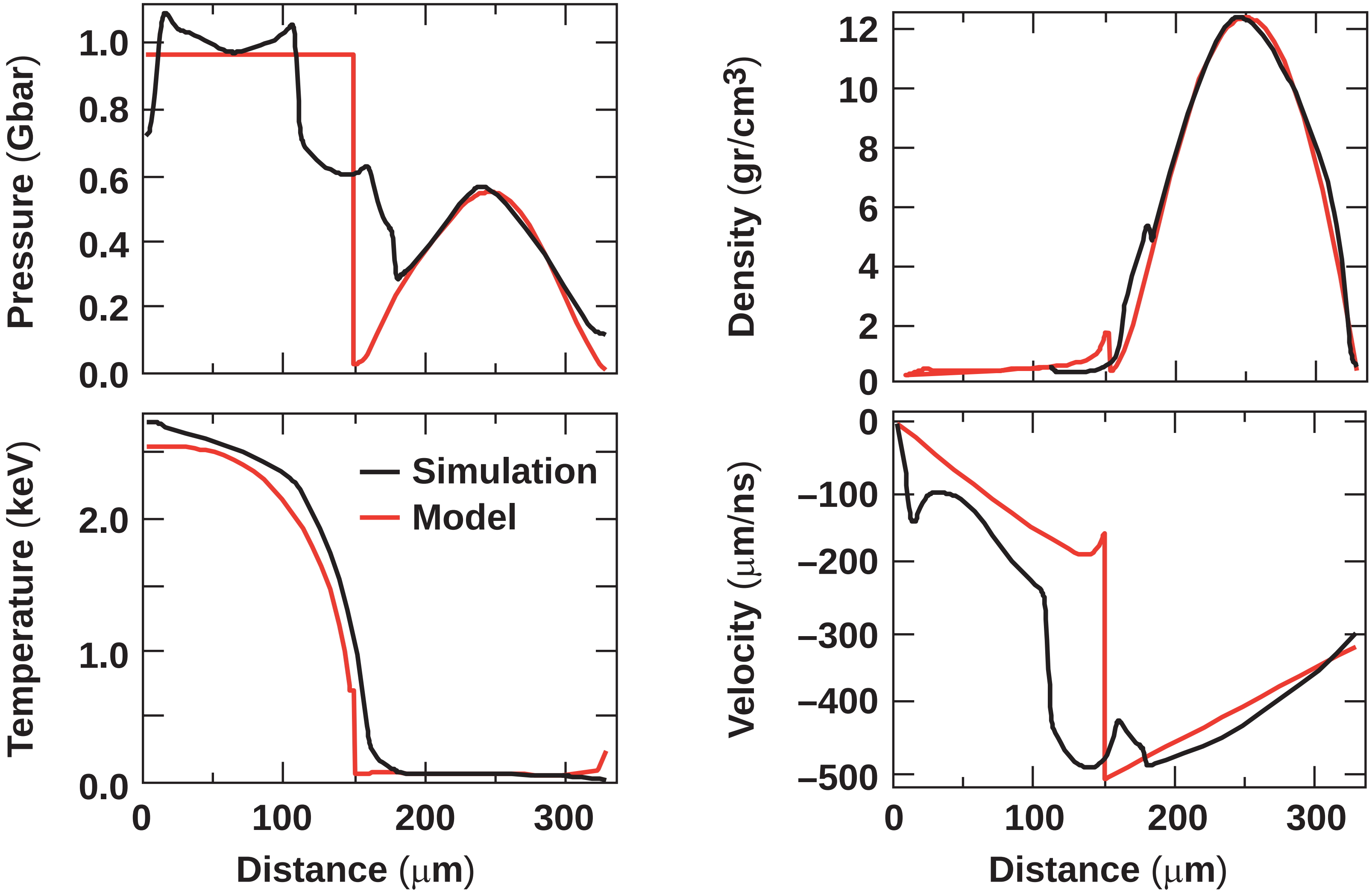
Hot-spot energy conservation:
$$\frac{d}{dt}\left(P_{hs}R_{hs}^{3}\right) = \frac{5}{3}D_{\alpha}\underset{\substack{\uparrow\\ \alpha\text{ heating}}}{P_{hs}^{2}R_{hs}^{3}} - \underset{\substack{\uparrow\\ \text{pdV}}}{2P_{hs}R_{hs}^{2}\dot{R}_{hs}}$$

$$\boxed{D_{\alpha} = 0.046\ \text{Gbar}^{-1}\ \text{ns}^{-1}}$$



TC5474

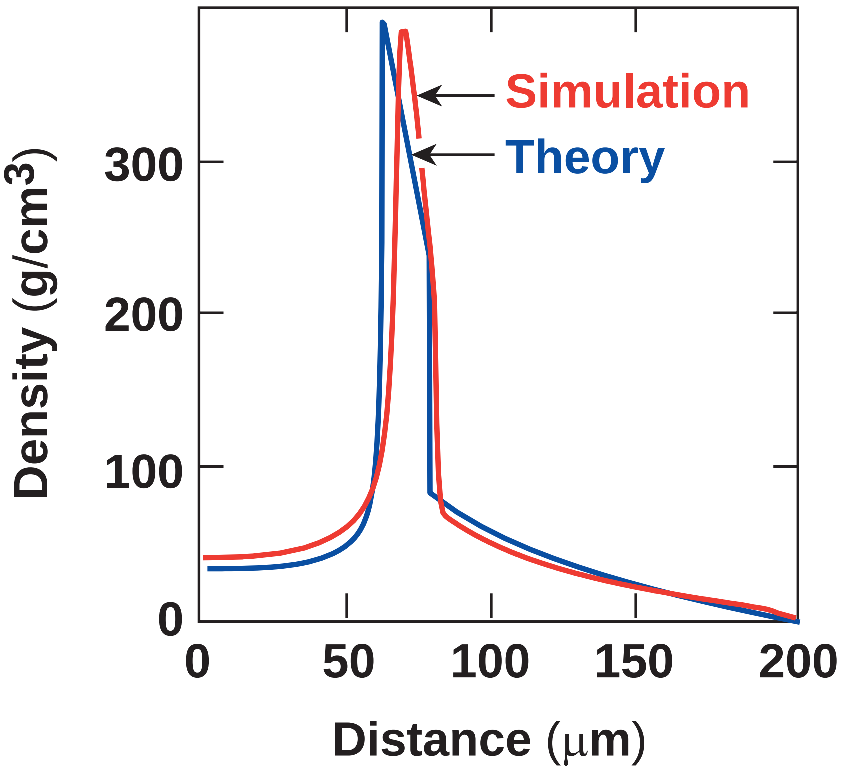
The initial conditions of the model are taken from the simulations at the beginning of the deceleration phase

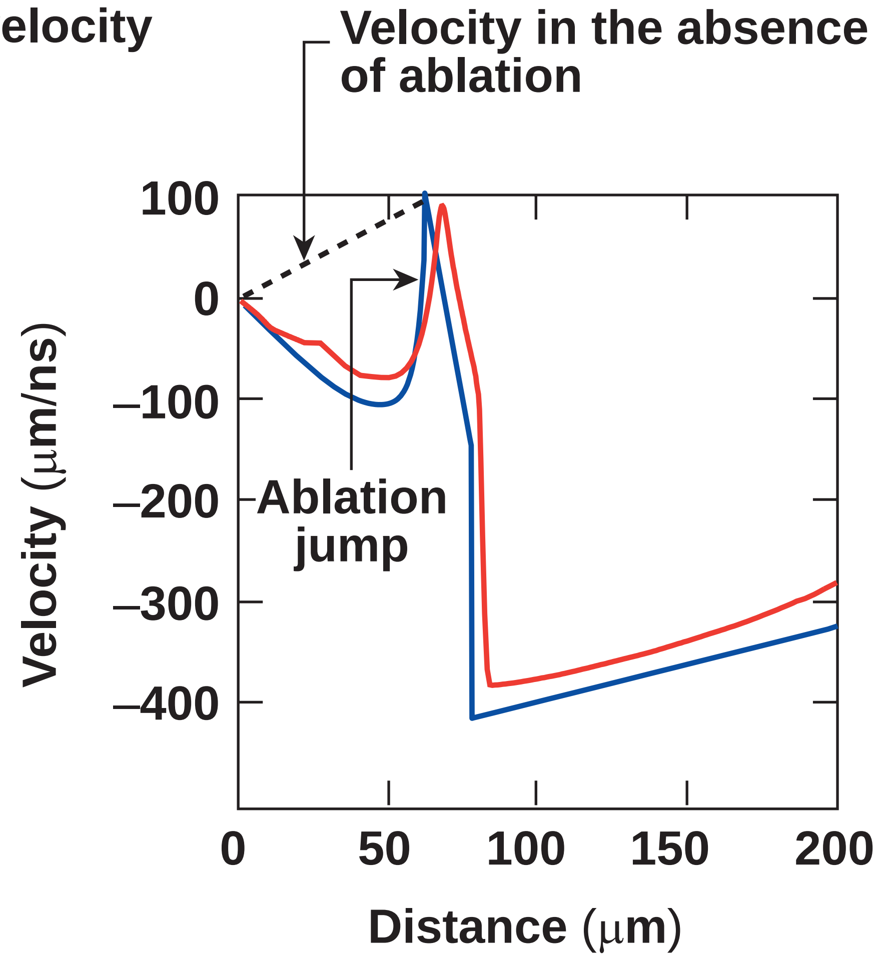


TC5447

The theoretical profiles have been compared with *LILAC* simulations of a NIF-like capsule at stagnation

- *LILAC* also shows a large ablation velocity





The deceleration of a direct-drive, NIF-like capsule at stagnation is approximately 4000 μm/ns^2

$$g_{\text{stag}} = 4\pi \frac{P_{\text{hs}}(t_0) R_{\text{hs}}^2(t_0)}{M_{\text{shell}}} \left[\frac{2E_{\text{shell}}^{\text{kinetic}}(t_0)}{3E_{\text{hs}}^{\text{internal}}(t_0)} \right]^{3/2} (1 + C_\alpha)$$

$$E_{\text{shell}}^{\text{kinetic}}(t_0) = \left(\frac{1}{2}\right) M_{\text{shell}} V_{\text{imp}}^2$$

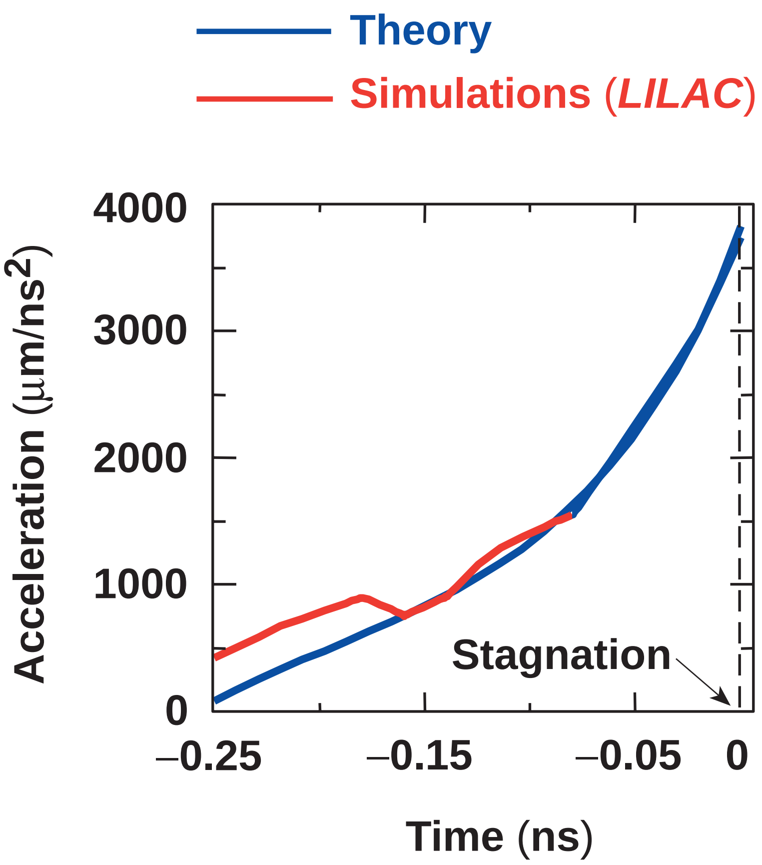
$$E_{\text{hot spot}}^{\text{internal}}(t_0) = \frac{4\pi}{3} P_{\text{hs}}(t_0) R_{\text{hs}}(t_0)^3$$

t_0 = beginning of deceleration phase

C_α = alpha amplification ~ 1

NIF direct drive:

- M_{shell} ~ 1 mg
- $P(t_0)$ ~ 900 Mbar
- R_0 ~ 240 μm
- g_{stag} ~ 4200 μm/ns^2 for $C_\alpha = 1$

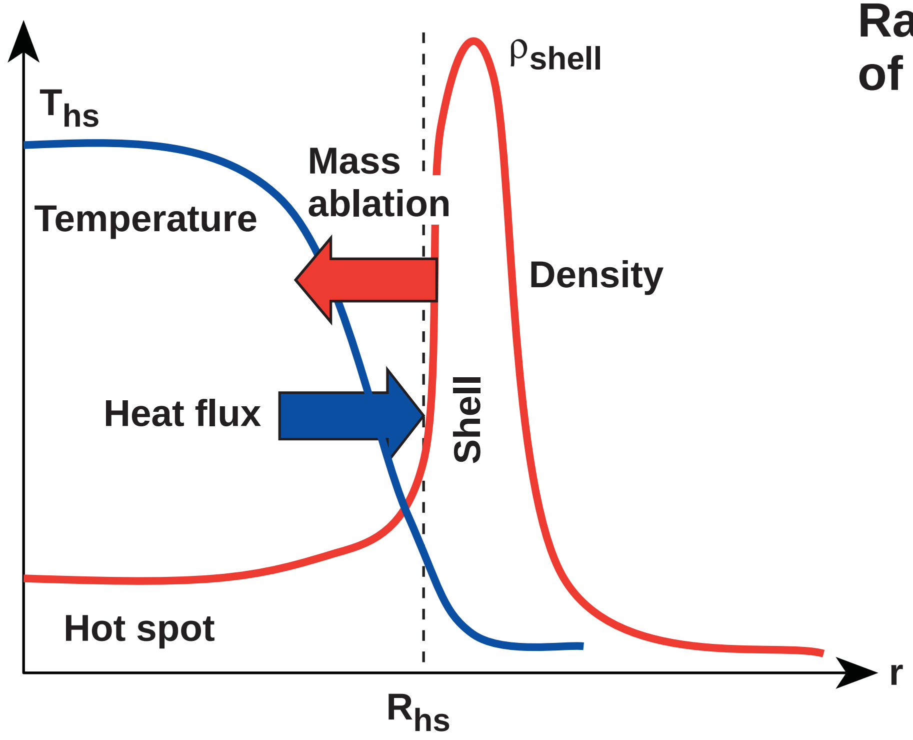


The heat flux leaving the hot spot is deposited onto the shell surface, causing mass ablation from the shell into the hot spot; the hot-spot mass increases in time

$$\frac{dM_{hs}}{dt} = 4\pi R_{hs}^{2}\,\rho_{shell}\,V_{a}$$

Rate of change of hot-spot mass

Mass flow through hot-spot surface

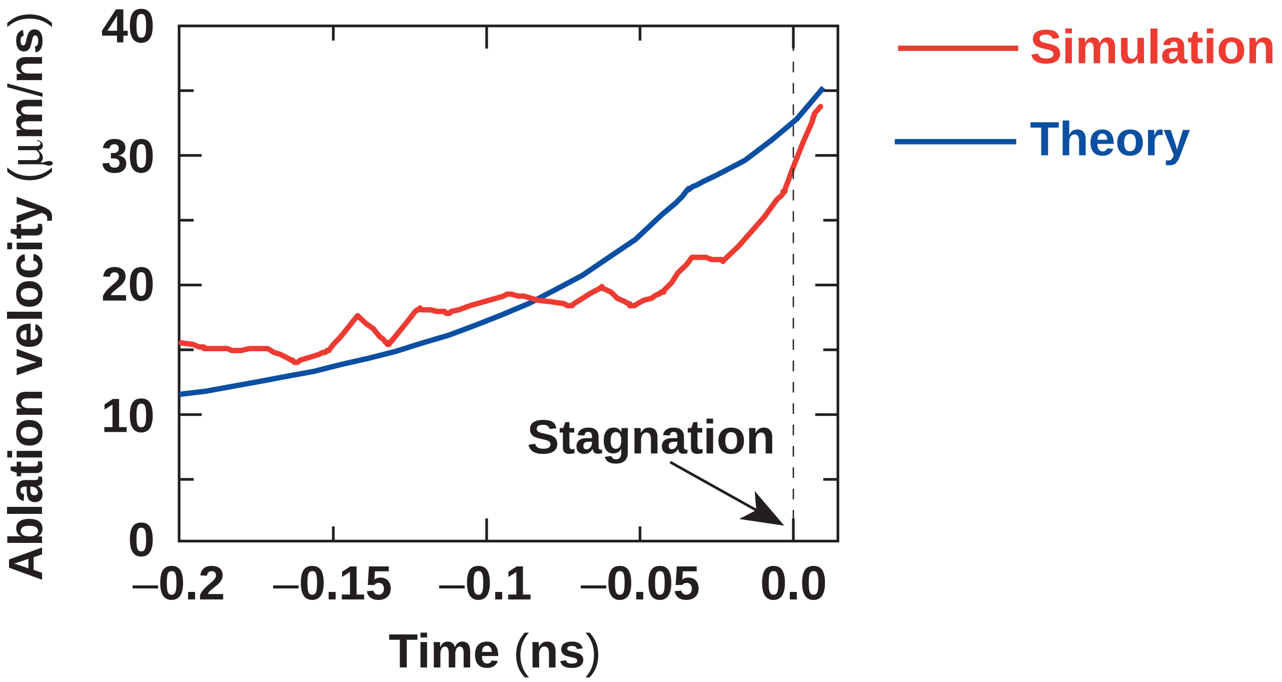


Ablation velocity:

$$V_a = 0.21\frac{M_i\,\kappa_{Spitz}\left[T_{hs}^{keV}(0,t)\right]}{\rho_{shell}\,R_{hs}}$$

V. Lobatchev and R. Betti, Phys. Rev. Lett. (in press).

The ablation velocity varies between 12 and 30 μm/ns during the 200-ps interval before stagnation



Ablation velocity from theory

$$V_a(\mu m/ns) = 6000 \frac{\left(T^{\text{hot spot}}_{\text{keV}}\right)^{5/2}}{\rho^{\text{shell}}_{\text{gr/cm}^3} R^{\text{hot spot}}_{\mu m} \Lambda^{\text{coulomb}}_{\text{hot spot}}}$$

The density-gradient scale length at the shell’s inner surface is derived for an isobaric ablation front

- Balance of heat flux to the shell and internal energy flux leaving the shell:

$$\frac{5}{2}pV_a \approx -\kappa\left(T_{sh}\right)\frac{dT_{sh}}{dr} \approx \kappa\left(T_{sh}\right)T_{sh}\,\frac{1}{\rho_{sh}}\,\frac{d\rho_{sh}}{dr}$$

- The density-gradient scale length is found using the formula for the ablation velocity:

$$L_m = \left(\frac{1}{\rho}\frac{d\rho}{dr}\right)^{-1}_{min} \approx 1.6\,\frac{M_i\kappa\left(T_{sh}\right)}{\rho_{sh}V_a} = 6.8\ R_{hs}\left[\frac{T_{shell}}{T_{hs}(0,t)}\right]^{5/2}$$

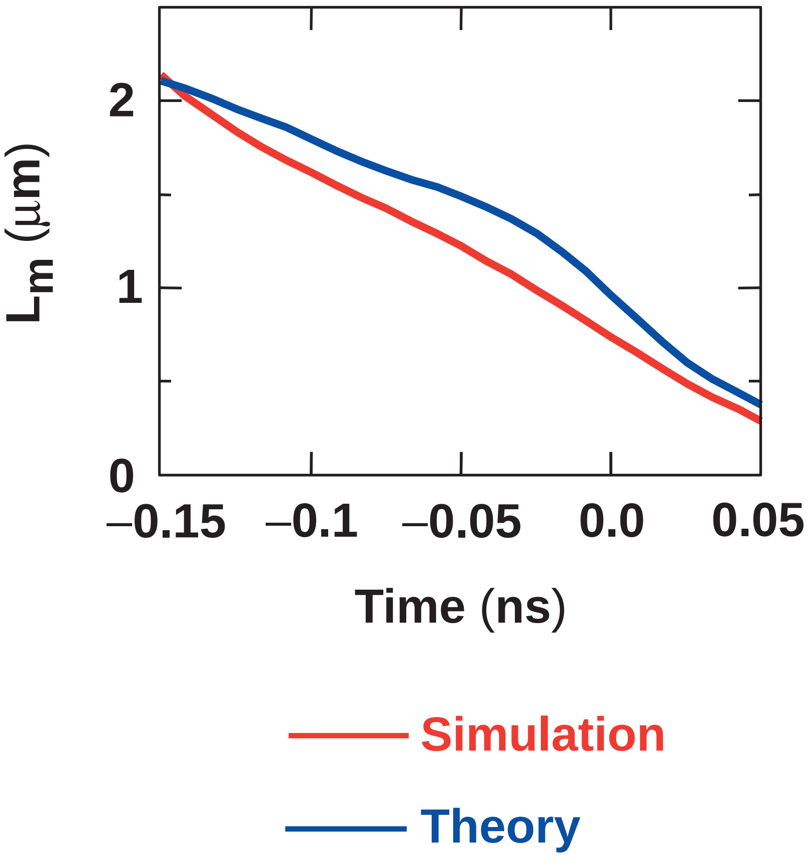
$$= 6.8\ R_{hs}\left[\frac{0.5\,M_i\,P_{hs}(t)}{\rho_{shell}(t)\,T_{hs}(0,t)}\right]^{5/2}$$

The density-gradient scale length at the shell’s inner surface of a direct-drive, NIF-like capsule is ~1.5 μm

Density-gradient scale length:

$$L_m = \min\left(\frac{\rho}{d\rho/dr}\right)$$

$$L_m = 6.8\,R_{hs}\left(\frac{0.5\,M_i\,P_{hs}(t)}{\rho_{shell}(t)\,T_{hs}(0,t)}\right)^{5/2}$$



The growth rates of the deceleration-phase RT are reduced by ablation off the shell's inner surface; short-wavelength modes are suppressed

$\langle g \rangle = 3100\ \mu m/ns^2$

$\langle V_a \rangle = 18\ \mu m/ns$

$\langle L_m \rangle = 1.5\ \mu m$

$k = l/R_{hs}$

$R_{hs} = 65\ \mu m$

$$\Rightarrow \text{Froude} = \frac{8\, V_a^2}{g L_m} \approx 0.5 \Rightarrow \text{Ref. }1 \Rightarrow \alpha = 0.9,\ \beta = 1.4$$

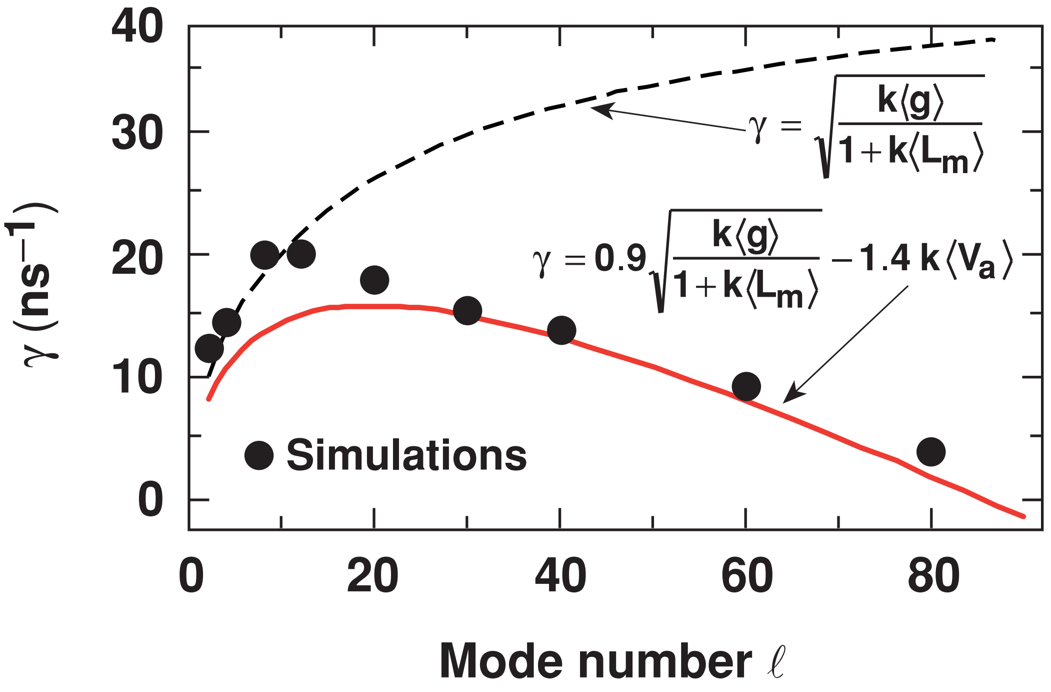
$$\gamma = 0.9 \sqrt{\frac{k\langle g \rangle}{1 + k\,\langle L_m \rangle}} - 1.4\, k \langle V_a \rangle$$

$$l_{cutoff} \approx 90$$

---

^{1}R. Betti *et al*., Phys. Plasmas <u>5</u>, 1446 (1998).

Two-dimensional, high-resolution Eulerian simulations confirm the theoretical results*



- $k = \ell/R_{hs}$, $R_{hs} \approx 65\ \mu m$, $\langle g\rangle \approx 3100\ \mu m/ns^2$, $\langle V_a\rangle \approx 18\ \mu m/ns$, $\langle L_m\rangle \approx 1.5\ \mu m$

---

*See poster JP1.037 by Lobatchev and Umansky.
V. Lobatchev and R. Betti, Phys. Rev. Lett. (in press).

Conclusions

The growth rates of the deceleration-phase RT instability are significantly reduced by mass ablation at the shell's inner surface

- For a direct-drive, NIF-like capsule during the 200 ps before stagnation,
  - $g \sim 1000$ to 4000 μm/ns^2
  - $V_a \sim 12$ to 30 μm/ns
  - $L_m \sim 2$ to 1 μm/ns
  - maximum growth rate at $\ell \sim 10$ and $\gamma \sim 20$ ns^{-1}
  - maximum RT growth factor ~ 8
  - cutoff mode number at $\ell_{cutoff} \sim 90$