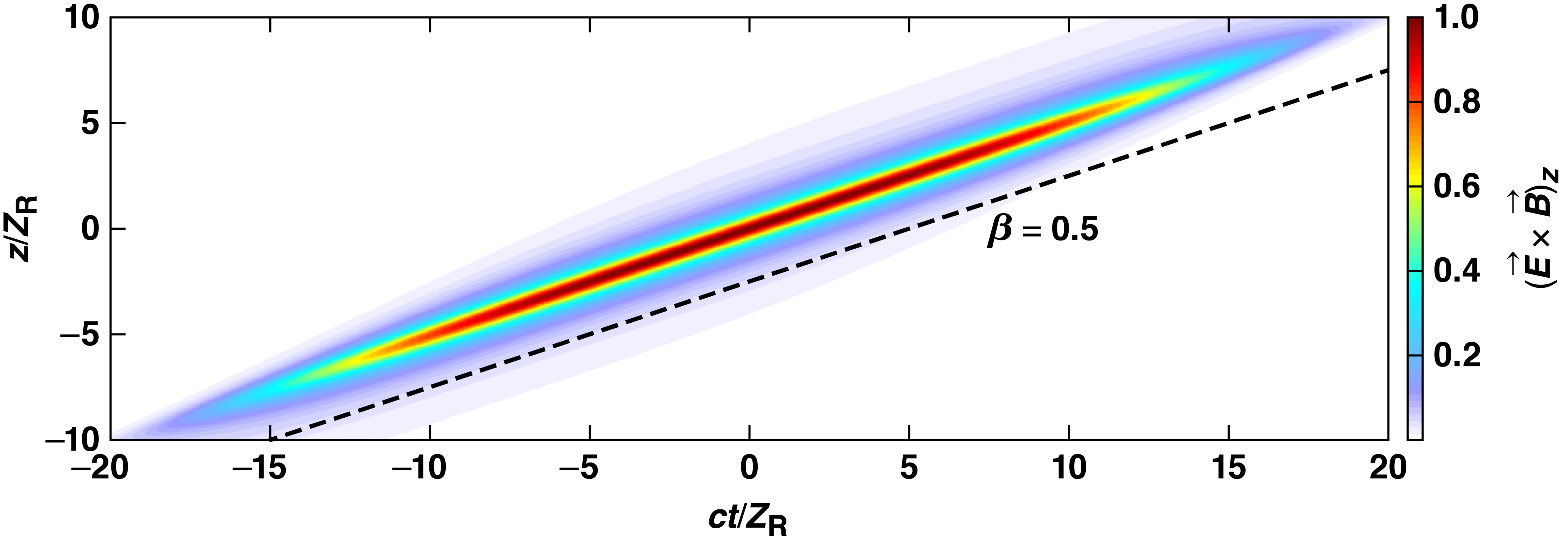
Exact Analytic Solutions Yielding Flying Focus Pulses (EASYFFP)



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Exact solutions to Maxwell's equations have been derived for flying focus pulses

- Flying focus pulses provide velocity control and an extended range of high intensity that can enable or enhance a wide-range of laser-based applications

- The exact fields were derived by combining the complex source-point method with the invariance of Maxwell's equations under a Lorentz transformation

- Propagating the fields backwards in space reveals the space-time profile that an optical assembly must produce to realize these solutions in the laboratory

The exact solutions can be used for accurate calculations of charged particle motion for advanced accelerators or radiation sources

Collaborators

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A. Di Piazza, M. Formanek

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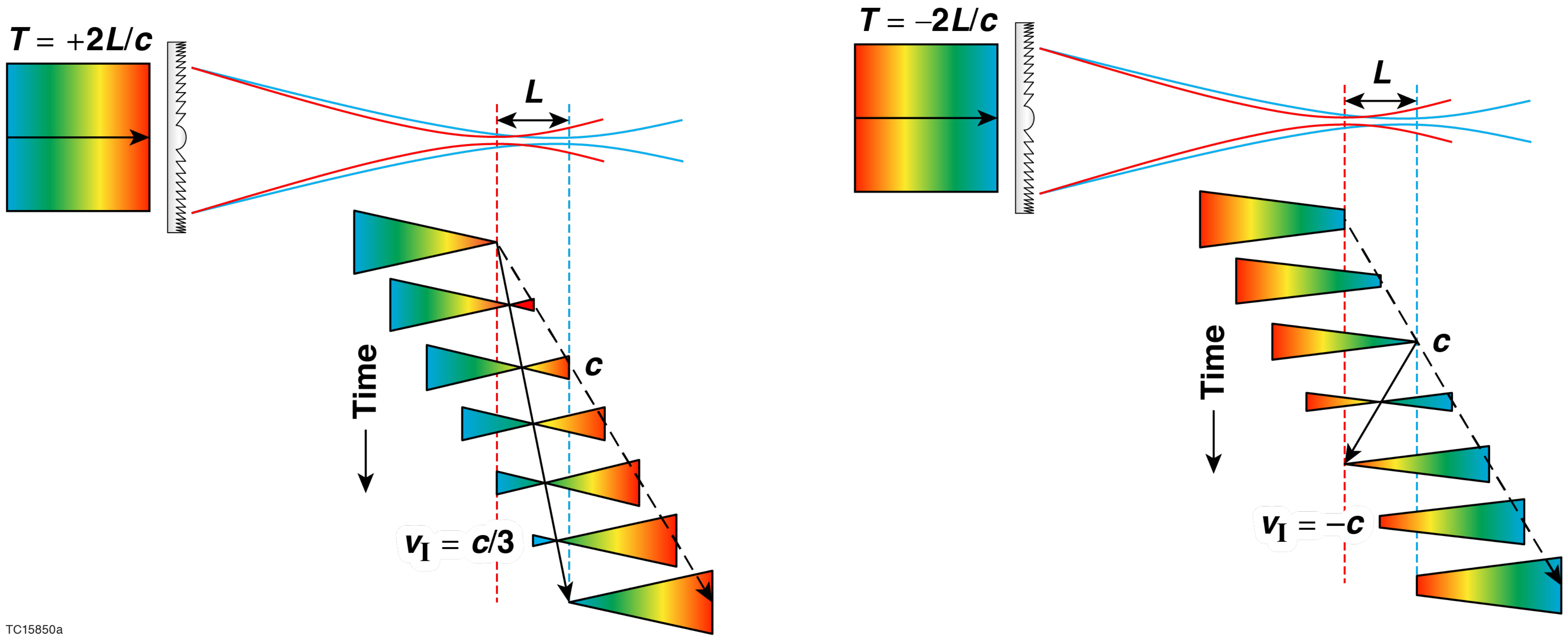
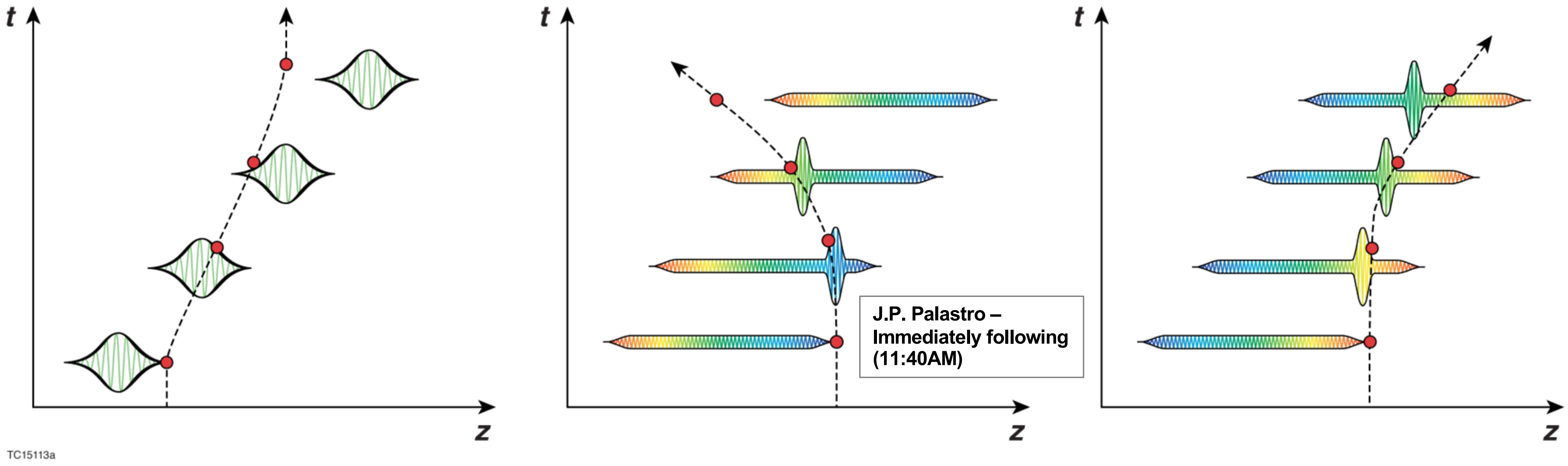
Spatiotemporal pulse shaping can produce a moving, or “flying,” focus over distances much greater than a Rayleigh range



The chromatic optic and chirp determine the focal location and time of each color, respectively, resulting in a peak intensity with a dynamic trajectory

Analysis of flying focus applications often use a simplified representation of the electromagnetic field structure



Assessing the validity of the simplified representations requires an accurate description of the electromagnetic field

D. Ramsey *et al.* Phys. Rev. E (2020) D. Ramsey *et al.* Phys. Rev. E (2022)

The Hertz vectors provide a natural, closed form representation for waves driven by dipole sources

Consider magnetic and electric dipole sources,

$$(\nabla^2 - \partial_{tt})\Pi_e \hat{\mathbf{e}} = \delta(\mathbf{r}) e^{-i\kappa t} \qquad (\nabla^2 - \partial_{tt})\Pi_m \hat{\mathbf{m}} = \delta(\mathbf{r}) e^{-i\kappa t}$$

$$\mathbf{\Pi}_e = \frac{e^{i\kappa(r-t)}}{4\pi r}\hat{\mathbf{e}} \qquad \mathbf{\Pi}_m = \frac{e^{i\kappa(r-t)}}{4\pi r}\hat{\mathbf{m}}$$

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The four potentials,

$$\Phi = -\nabla \cdot \mathbf{\Pi}_e$$

$$\mathbf{A} = \partial_t \mathbf{\Pi}_e + \nabla \times \mathbf{\Pi}_m$$

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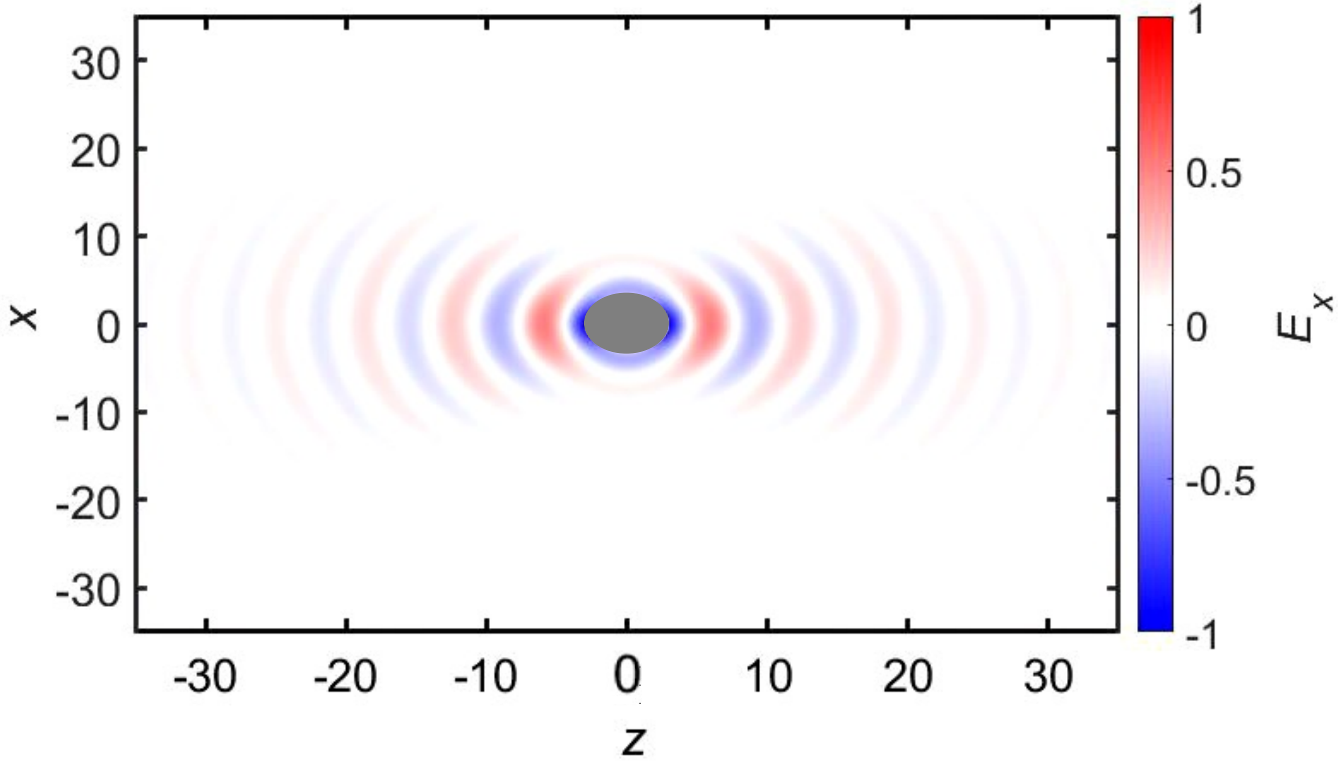


The four potentials,

$$\Phi = -\nabla \cdot \mathbf{\Pi}_e$$

$$\mathbf{A} = \partial_t \mathbf{\Pi}_e + \nabla \times \mathbf{\Pi}_m$$

The fields,

$$\mathbf{E} = -\partial_t \mathbf{A} - \nabla \Phi$$

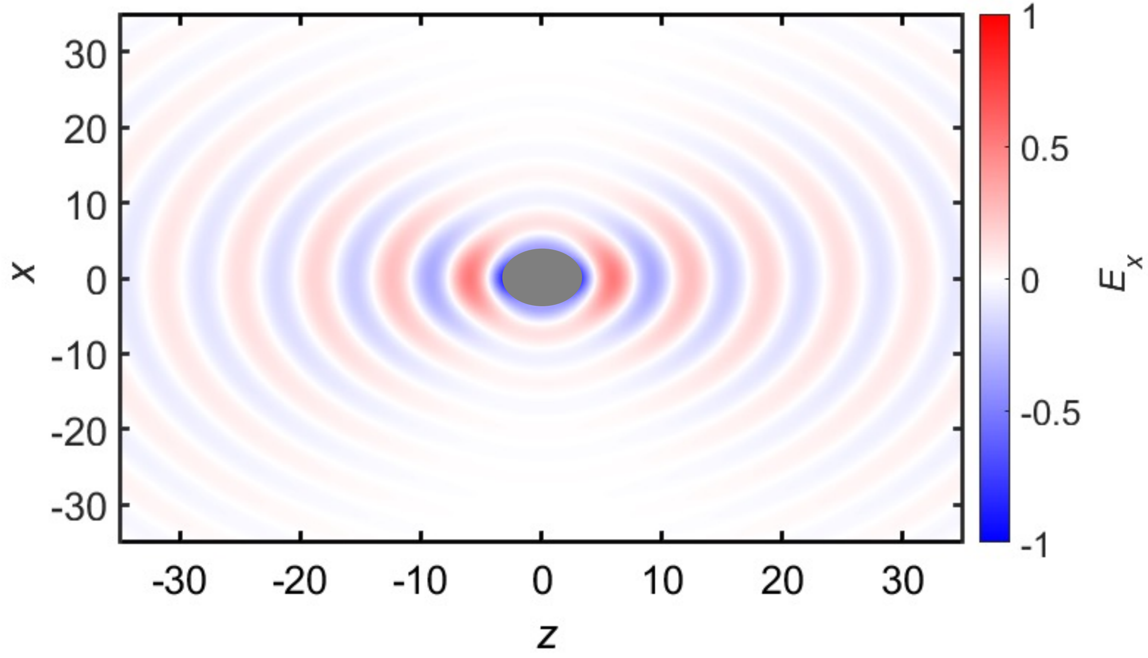
$$\mathbf{B} = \nabla \times \mathbf{A}$$

The complex source-point method transforms spherical wave solutions into beam solutions

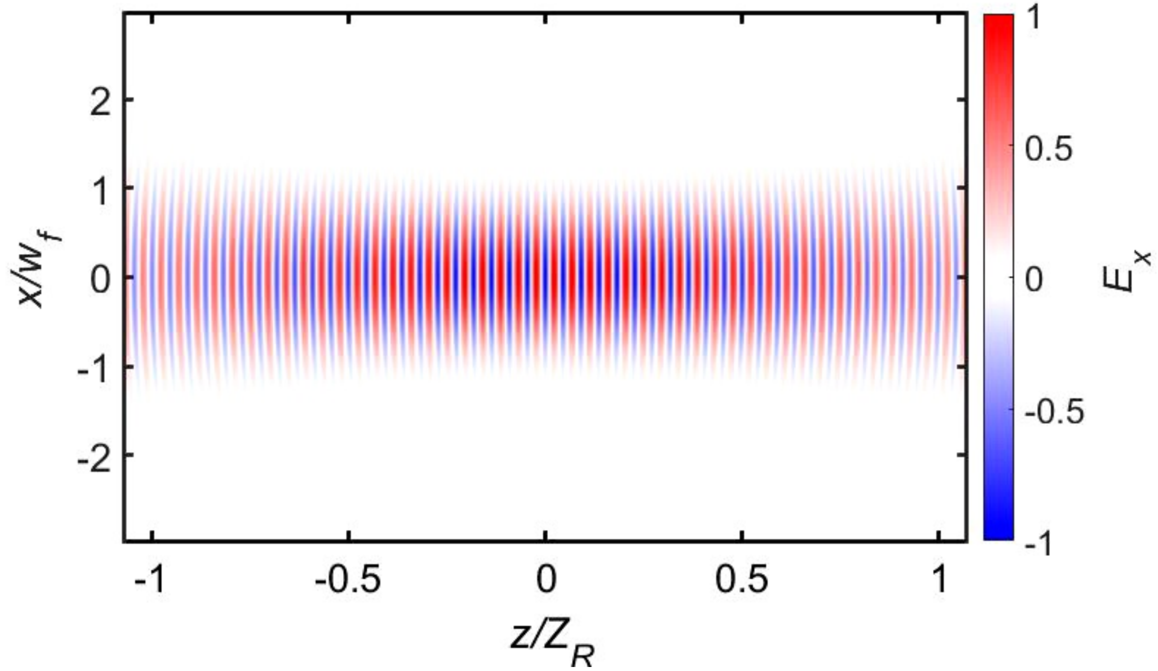
Solutions to Maxwell's equations remain solutions under coordinate translations, real or *imaginary*

$$z \to z - iZ_R$$

Spherical wave



Focused continuous wave



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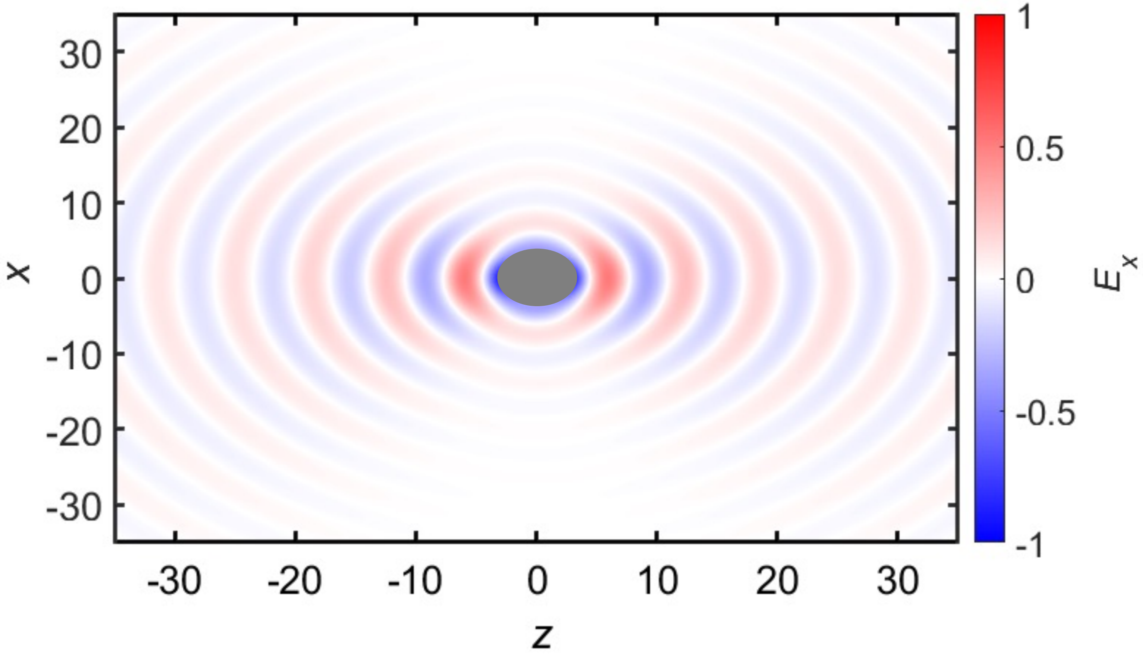
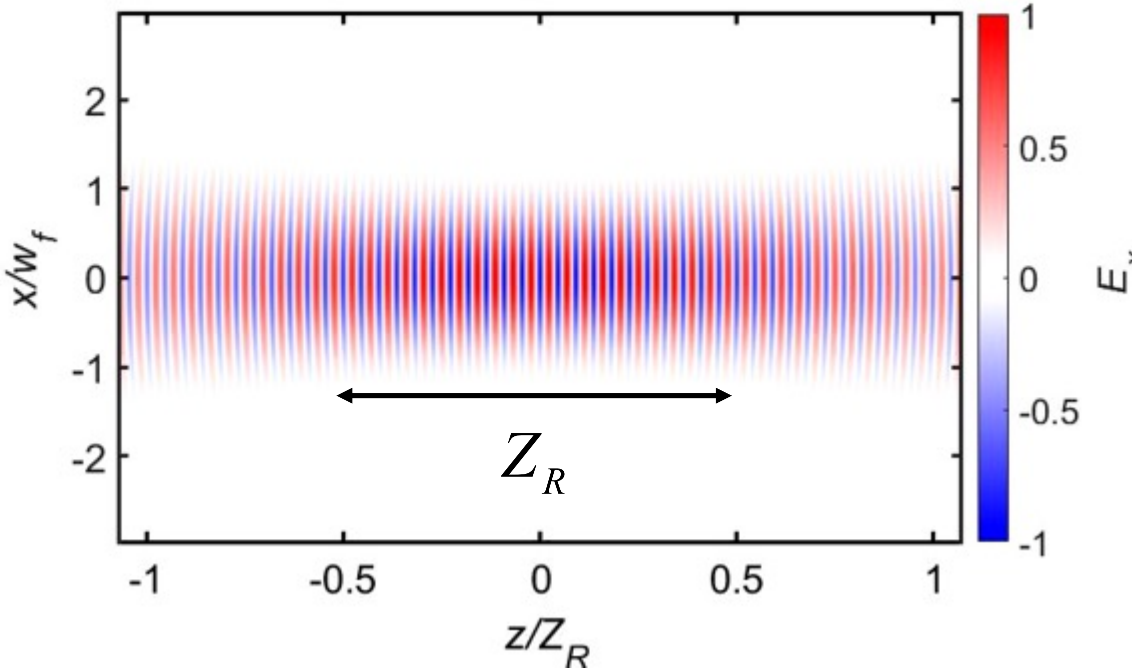
Spherical wave → Focused continuous wave





There exists a frame of reference in which the focus moves at a constant velocity

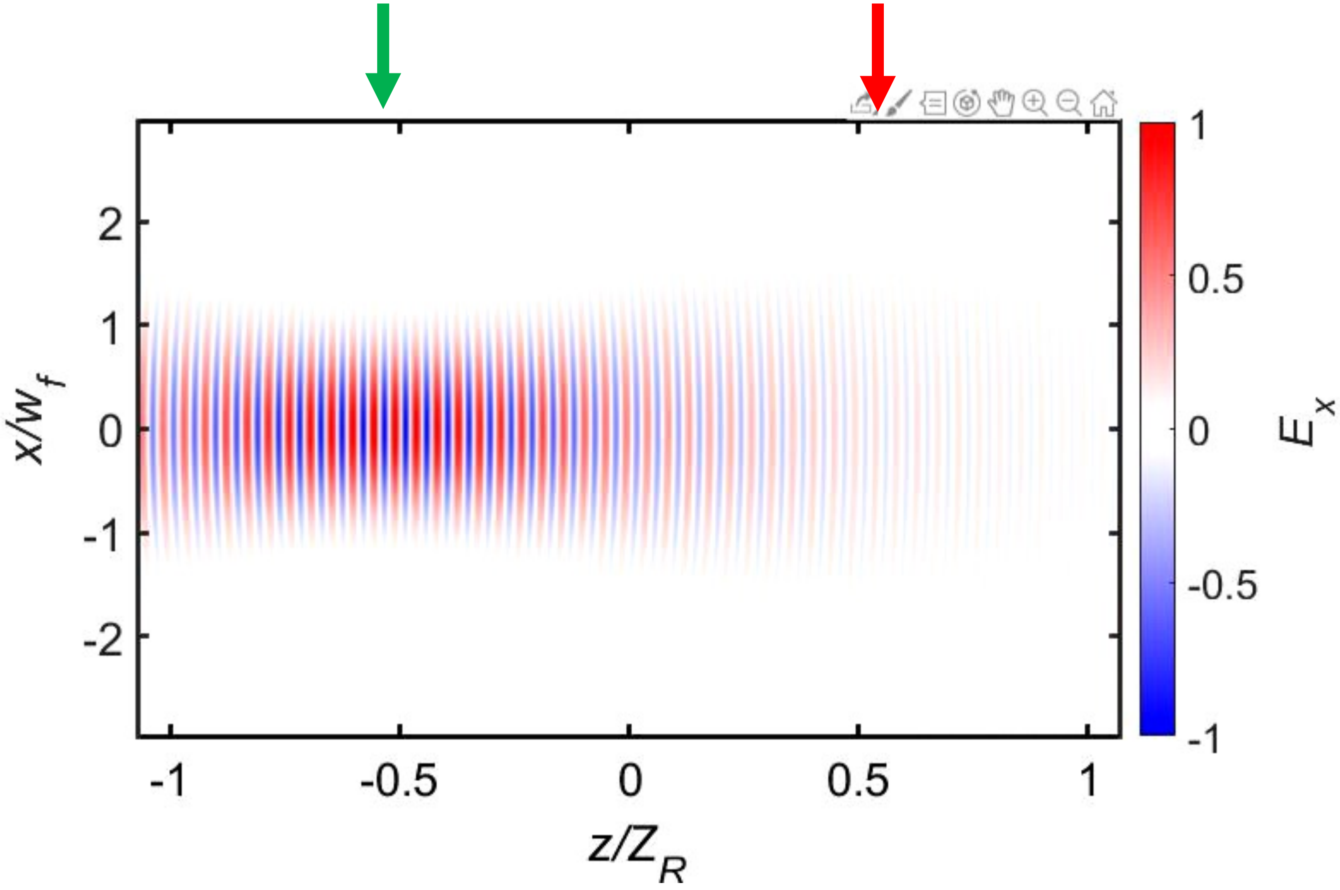
Maxwell's equations are invariant under a Lorentz coordinate transformation

$$t' = \gamma(t - \beta z)$$
$$z' = \gamma(z - \beta t) - iZ_R$$

β is the velocity of the focus

with the 4-potential transformation

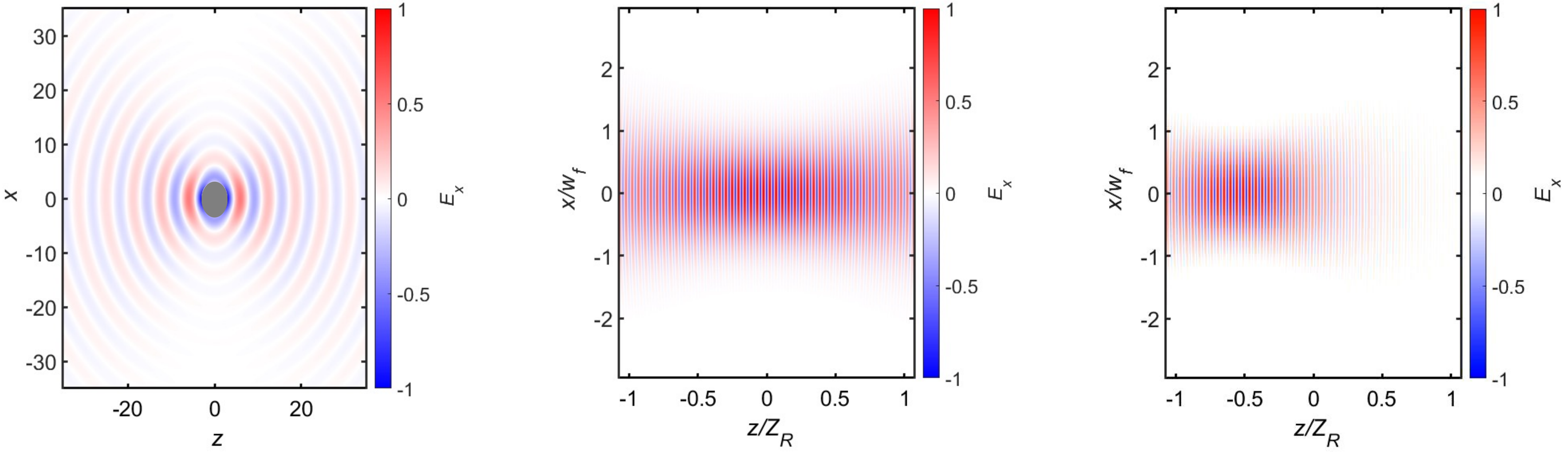
$$\Phi = \gamma(\Phi' + \beta A_z')$$
$$A_z = \gamma(A_z' + \beta\Phi')$$



These laser fields are a flying focus

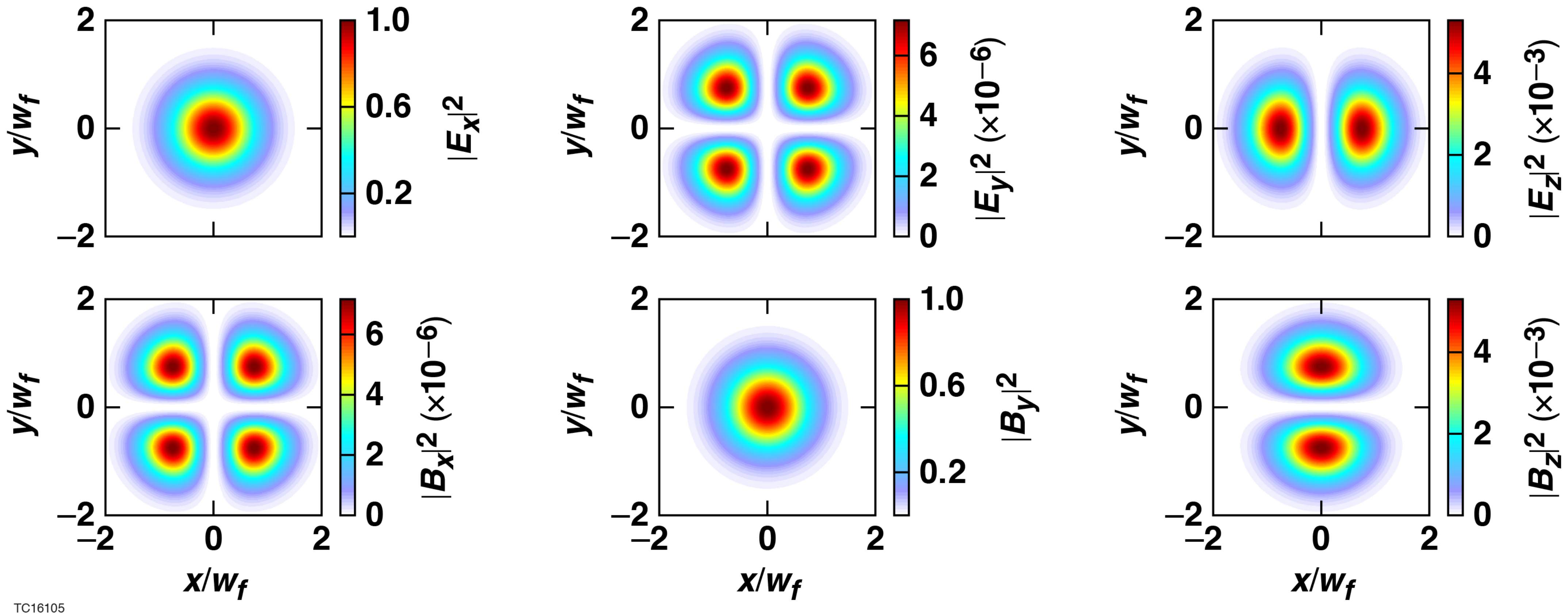
A stationary focus in one frame of reference is a flying focus in another frame of reference

Spherical wave → Complex coordinate translation → Focused wave → Lorentz transformation → Flying focus



The prescribed method can generate all six components of the electromagnetic field for arbitrary polarization and orbital angular momentum

Forward subluminal ($\beta = 0.5$), linearly polarized ($\hat{\mathbf{x}}$), $\ell = 0$



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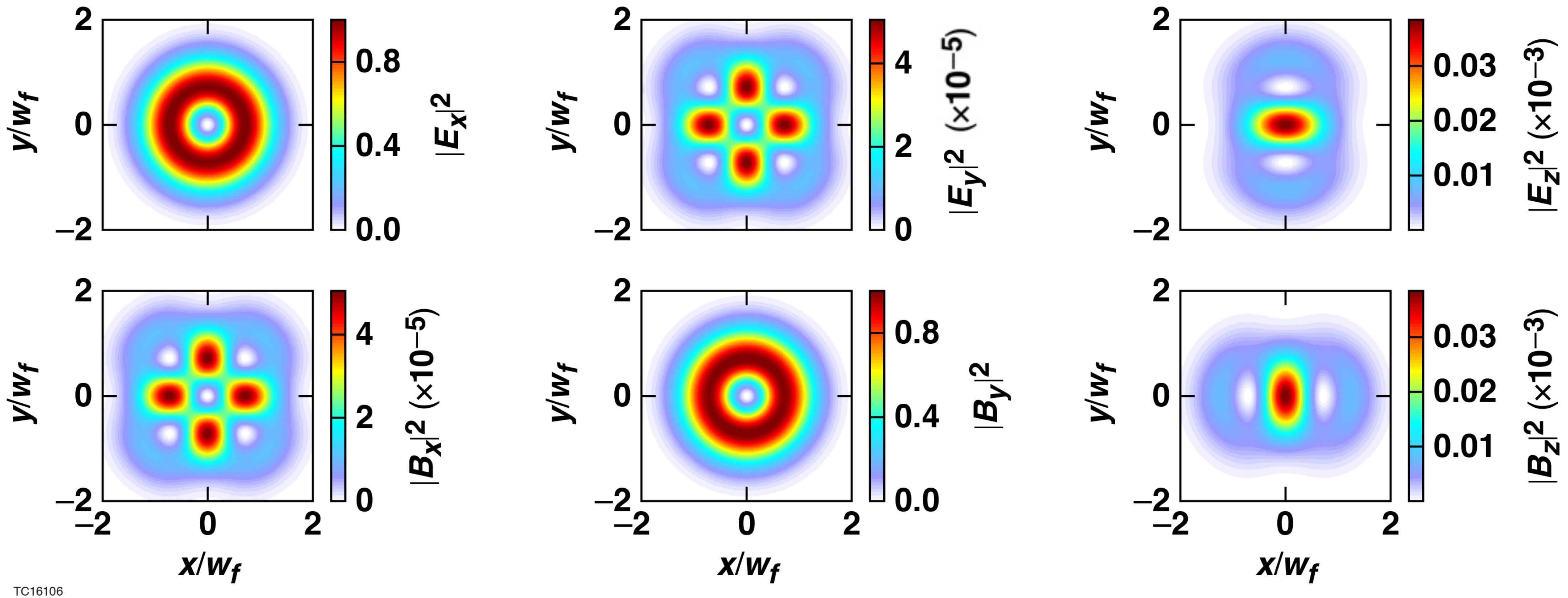
The prescribed method can generate all six components of the electromagnetic field for arbitrary polarization and orbital angular momentum

Backwards subluminal ($\beta = -0.99$), linearly polarized ($\hat{\mathbf{x}}$), $\ell = 1$



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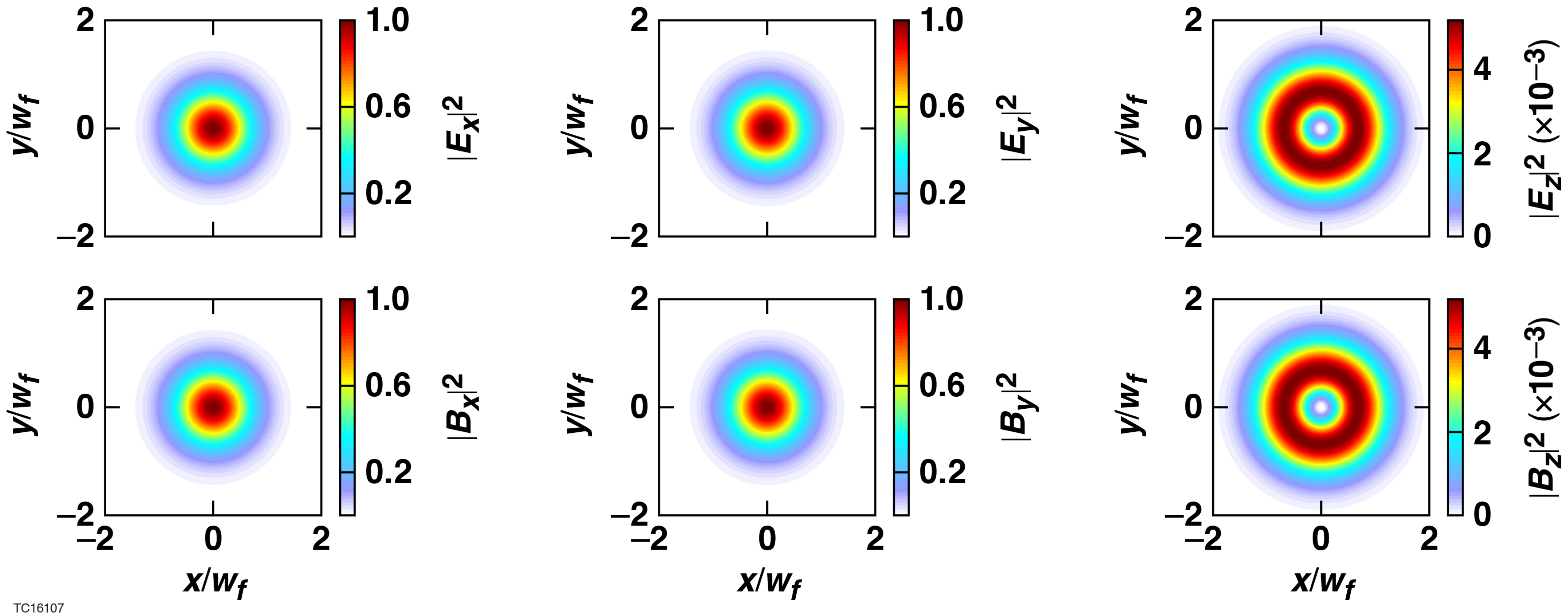
The prescribed method can generate all six components of the electromagnetic field for arbitrary polarization and orbital angular momentum

Forward superluminal ($\beta = 2$), **circularly polarized**, $\ell = 0$



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Finite energy, i.e., pulse, solutions can be found by superposing modal solutions

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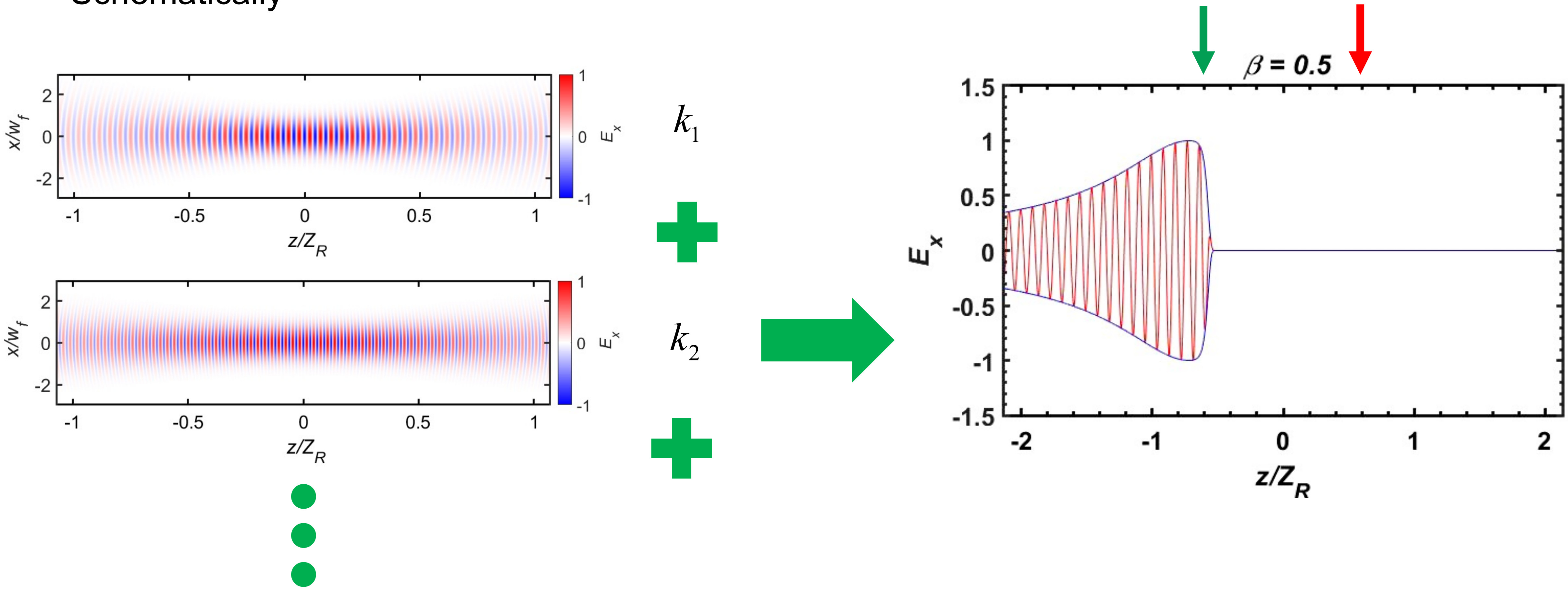
Schematically

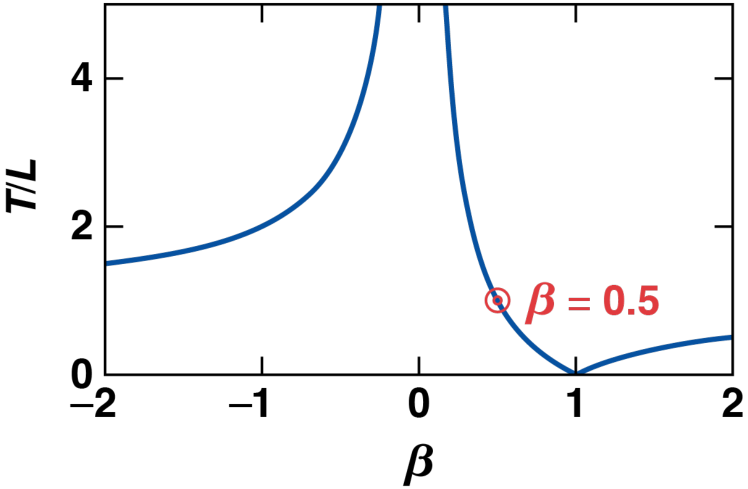


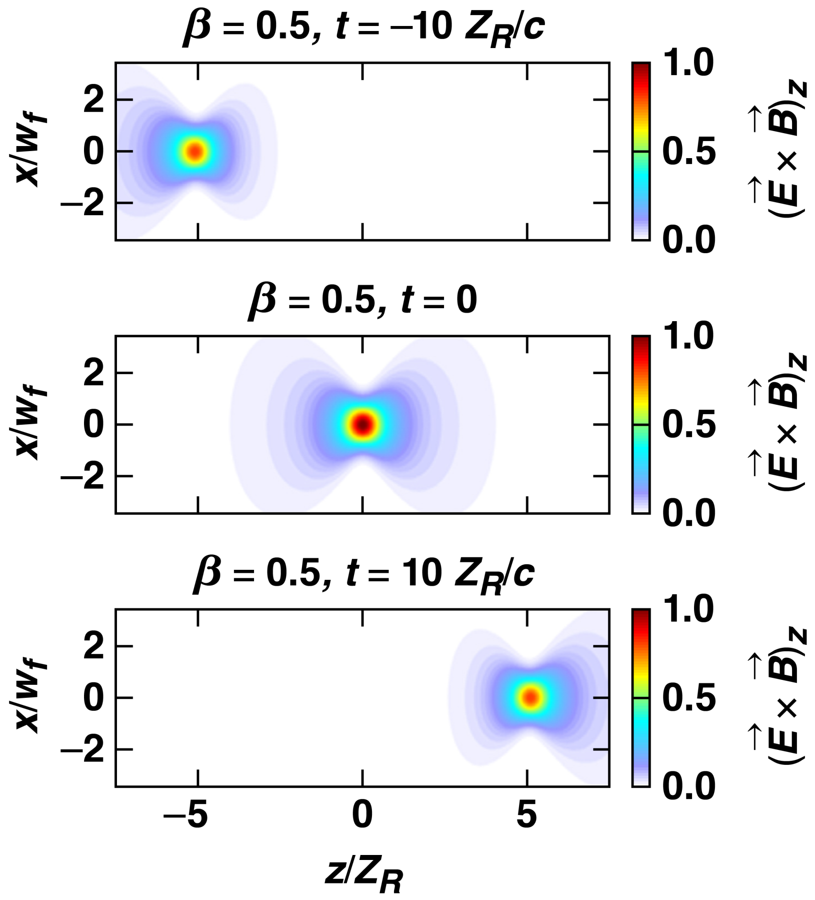
Pulse like solutions limit the range over which the intensity is nearly constant, i.e., the “focal range”

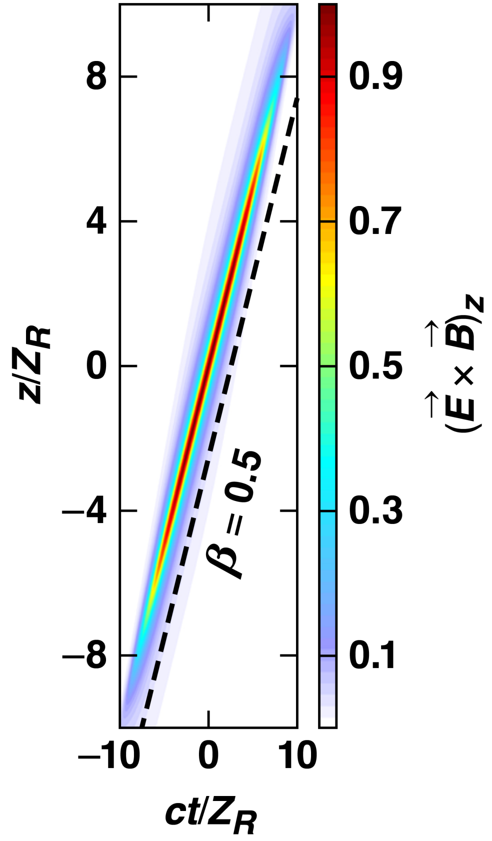
The pulse duration, T sets the focal range, L

$$L = T\left|\frac{\beta}{1-\beta}\right|$$

Maintaining a fixed focal range requires changing the pulse duration with the velocity

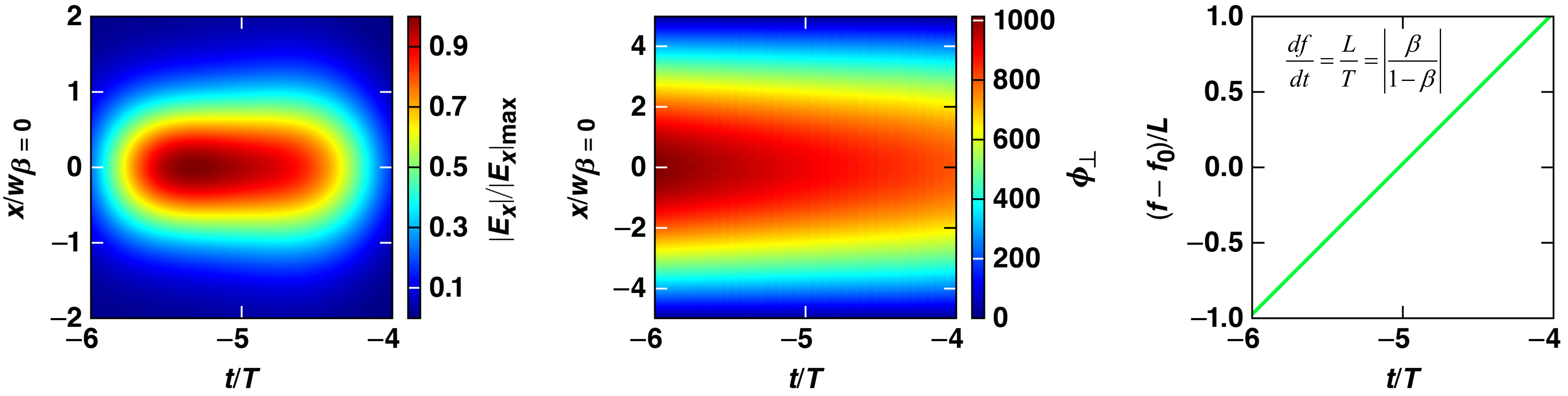






Propagating the field backwards in space provides the initial amplitude and phase required to create a flying focus pulse

$\beta = 0.5$, linearly polarized ($\hat{\mathbf{x}}$), $z/L = -5$



TC16109

The flying focus is formed by a time-dependent focal length

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- The exact fields were derived by combining the complex source-point method with the invariance of Maxwell's equations under a Lorentz transformation

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The exact solutions can be used for accurate calculations of charged particle motion for advanced accelerators or radiation sources

Selecting Hertz vectors that satisfy the wave equation results in electromagnetic fields that satisfy Maxwell's equations

Consider a crossed magnetic and electric dipole source,

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$$(\nabla^2 - \partial_{tt})\mathbf{\Pi}_{e,m} = 0$$

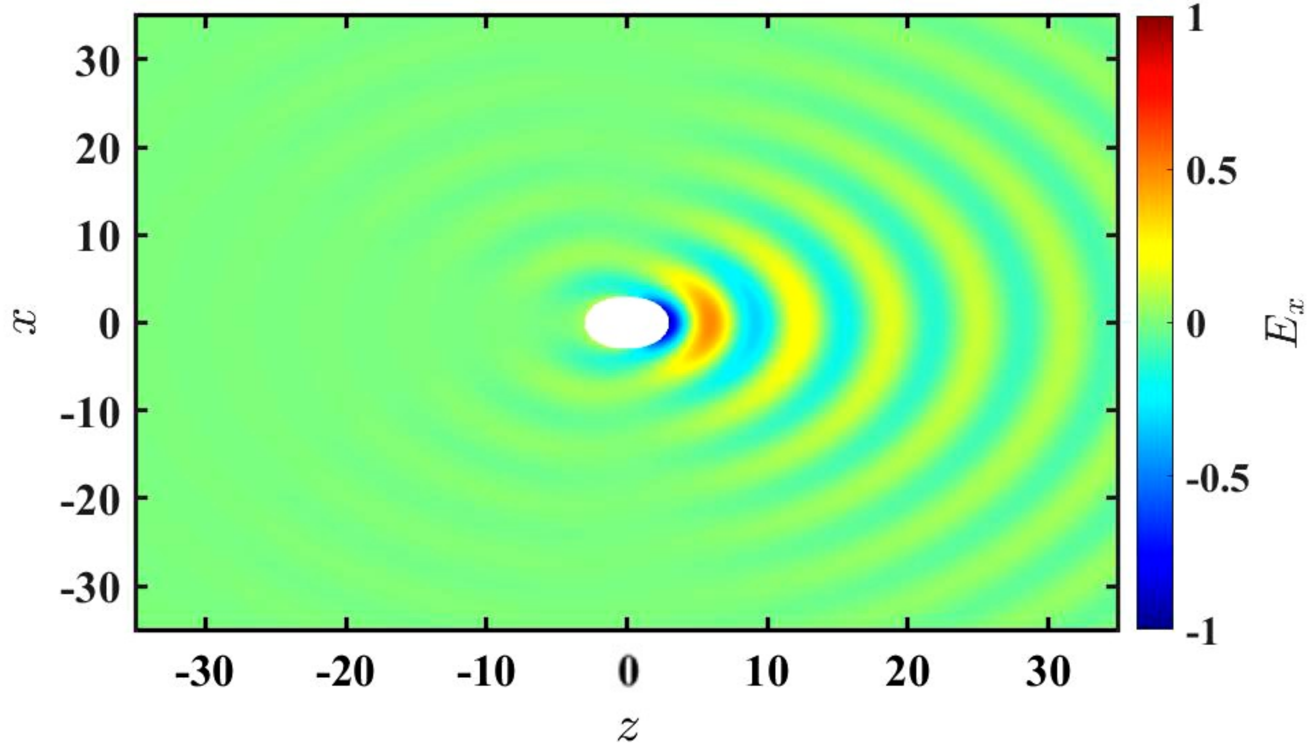
$$\downarrow$$

$$(\nabla^2 - \partial_{tt})A^\mu = 0 \qquad \partial_t\Phi + \nabla\cdot\mathbf{A} = 0$$

$$\downarrow$$

$$\nabla\cdot\mathbf{E} = 0 \qquad \nabla\cdot\mathbf{B} = 0$$

$$\nabla\times\mathbf{E} = -\partial_t\mathbf{B} \qquad \nabla\times\mathbf{B} = \partial_t\mathbf{E}$$



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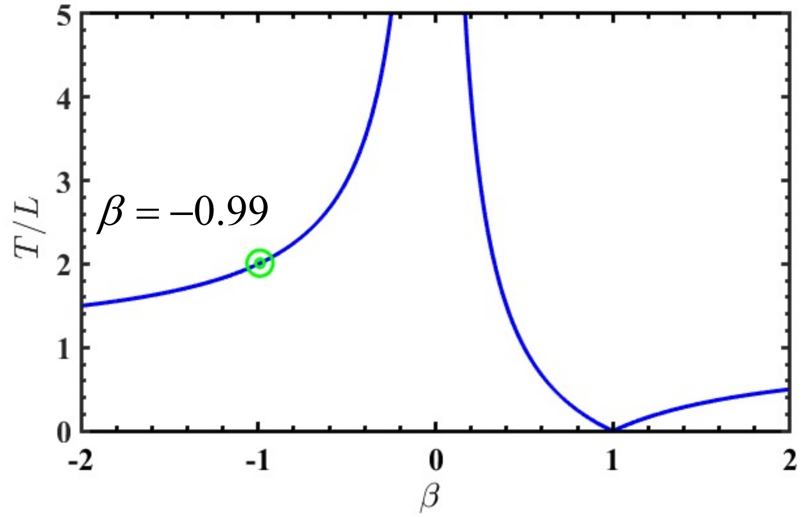

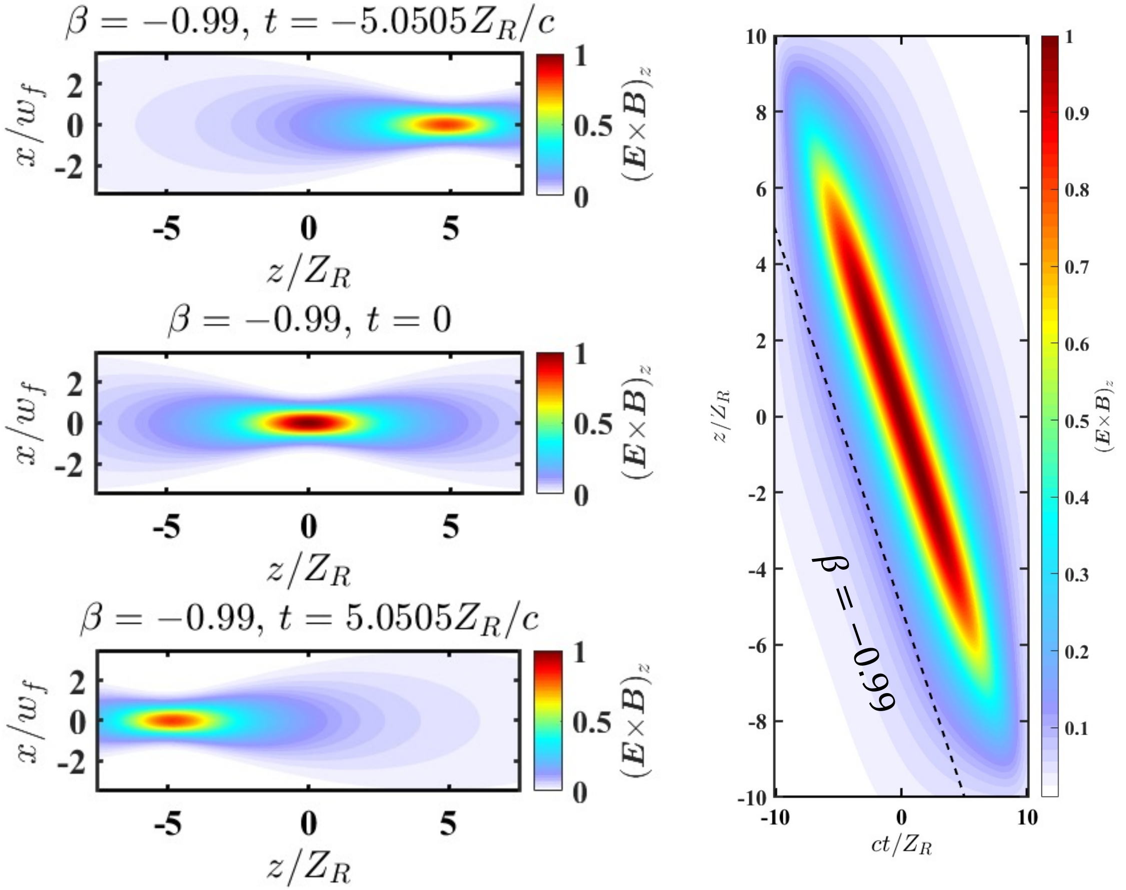

Why does the pulse duration drop for negative focal velocities?

While the time in which the pulse body coincides with the focus is less,

$$\Delta t \propto \left|\frac{1}{1-\beta}\right|$$

the faster focus is faster

$$|\beta|\,\Delta t \propto \left|\frac{\beta}{1-\beta}\right|$$

The pulse duration does not need to be as long to support the focal range of faster moving foci