Modeling Laser-Driven Ablative Magnetothermal Instability

L. S. LEAL,[1] A. V. MAXIMOV,[1] F. GARCÍA-RUBIO,[1] R. BETTI,[1] and V. V. IVANOV[2]

[1]University of Rochester, Laboratory for Laser Energetics

[2]University of Nevada, Reno

Abstract

The magnetothermal instability in plasmas is related to the interplay of heat flow and magnetic field transport. The instability growth rates depend on the transport coefficients used. Recent modifications to transport coefficients in magnetized plasmas have been suggested in the regime of Hall parameters of the order of 1 for magnetic-field diffusion, advection, and heat conduction. [1–3] A perturbation analysis is presented that studies the instability growth in laser-ablated plasmas from cylindrical wires driven with a 1-MA current. [4] The new instability growth rates are compared with the results of full hydrodynamic simulations.

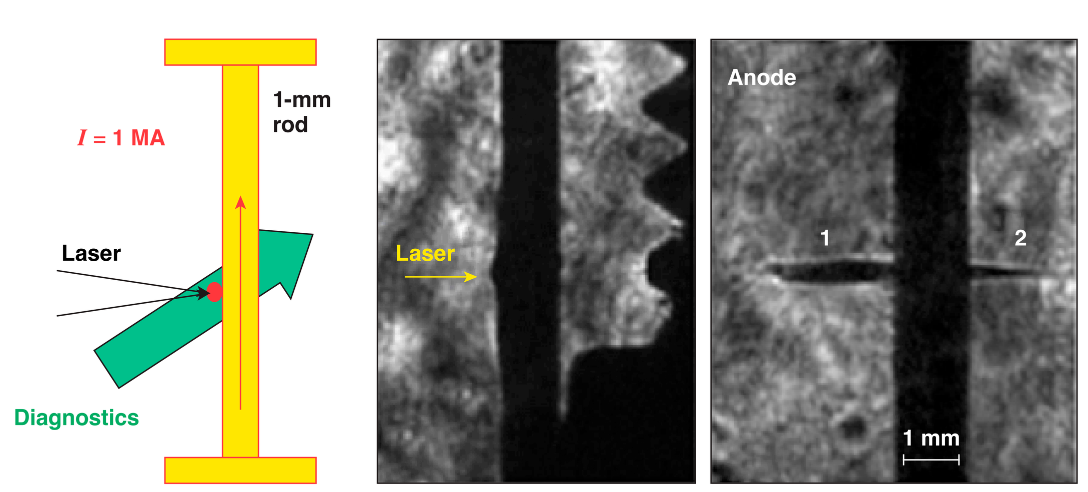
Acknowledgments

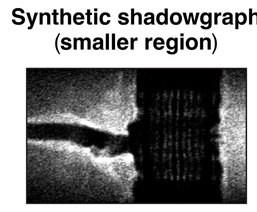
This material is based upon work supported by the Department of Energy under award number DOE Grant DE-SC0016500 and the Department of Energy National Nuclear Security Administration under Award Number DE-NA0003856, the University of Rochester, the New York State Energy Research and Development Authority, and LLNL computing resources.

Motivation

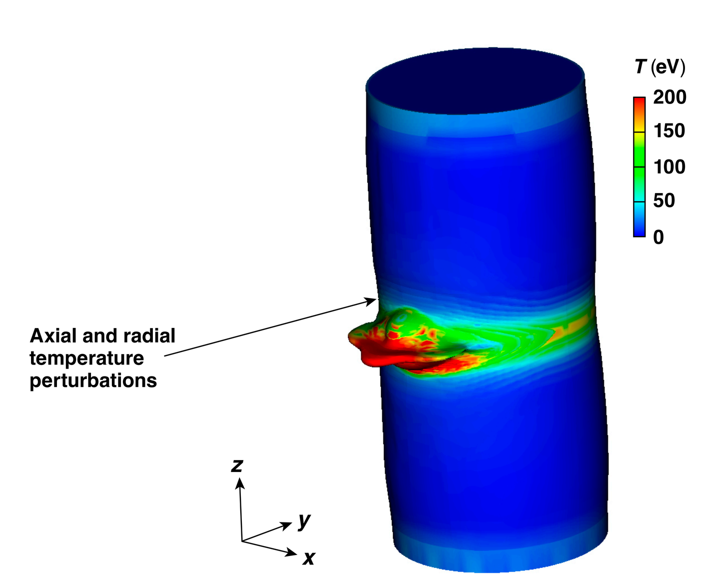
Experiments at University of Nevada, Reno using the Zebra Pulsed-Power Facility coupled with Leopard laser

- Wavelength: 1.06 μm
- 3×10^{15} W/cm^2
- Spot size: 30 μm
- Current 1 MA ⇔ field 3 MG = 300 T
- 1-mm-diam rod





Synthetic shadowgraphs with the most up-to-date transport coefficients [3] show some structure on the back end, but not to the extent seen in experiments. Simulations predict that the laser spot generates heat flow on the surface.



TC16123

Magnetized magnetothermal instability in an axial field [5]

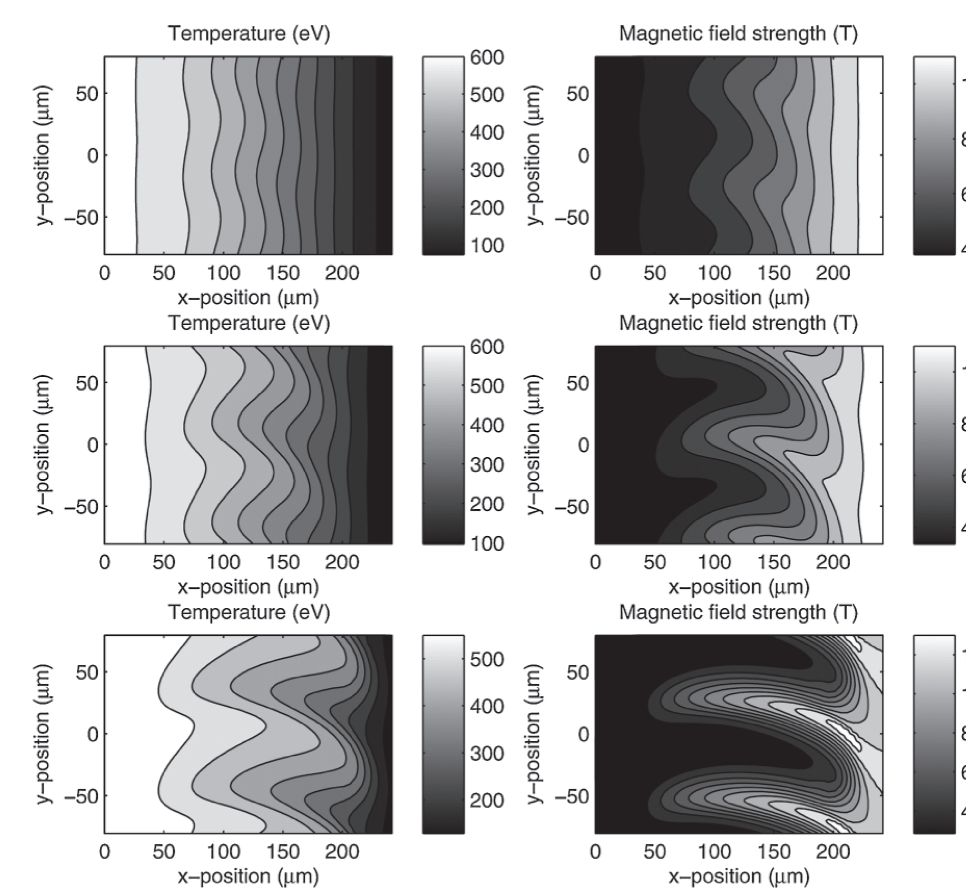
The magnetothermal instability is generated by the interplay of heat flow and magnetic field advection.

- Nernst and Righi–Leduc terms source for growth
- Hall parameters $10^{-2} < \chi < 100$
- Restricted to $L_{T,B}^{-1} \ll k \ll \lambda_{mfp}^{-1}$

$$\frac{3}{2} n_e \frac{\partial T}{\partial t} - \nabla \cdot \left(\widetilde{\kappa} \cdot \nabla T - \widetilde{\beta} \cdot j\right)$$

$$\frac{\partial B}{\partial t} = -\nabla \times \left[\widetilde{\alpha} \cdot (\nabla \times B) - \widetilde{\beta} \cdot \nabla T\right]$$

$$\omega_{\pm} = \frac{1}{2}\left[s_B k - (d_R + d_T) i k^2\right] \pm \frac{1}{2}\left\{ s_B^2 k^2 + \left[s_P + 2 s_B \times (d_R + d_T)\right] i k^3 - \left[(d_R + d_T)^2 + s_E\right] k^4 \right\}^{1/2}$$



$$d_T = \frac{c_B \kappa_\perp \lambda_T^2}{3 \tau_T}$$

$$d_R = \frac{\alpha_\perp \delta^2}{c_B \tau_T}$$

$$s_E = \frac{4 \lambda_T^2 \delta^2}{3 \tau_T^2} \beta_\wedge \beta_\wedge$$

$$s_P = \frac{2 \beta_\wedge c_B^2 \lambda_T^4}{3 L_T \tau_T^2} \frac{\partial \kappa_\wedge}{\partial \chi} \sin\theta$$

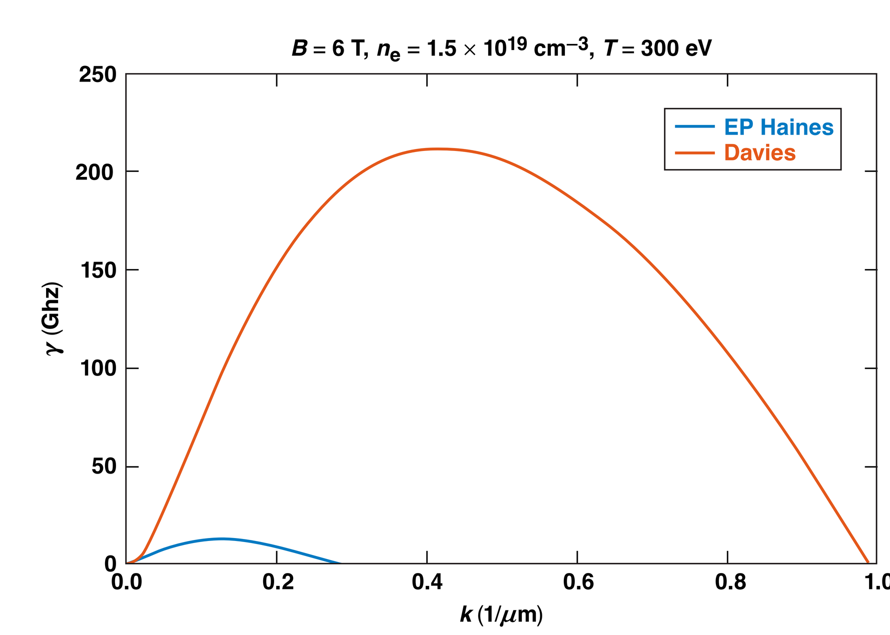
$$s_B = \frac{c_B \chi \lambda_T^2}{3 L_B \tau_T} \frac{\partial \kappa_\wedge}{\partial \chi} \sin\theta$$

[5] J. J. Bissell, C. P. Ridgers, and R. J. Kingham, Phys. Rev. Lett. 105, 175001 (2010).

Growth rate with updated coefficients

Recent studies have shown corrections to transport coefficients using calculated values from Fokker—Plank simulations. [2] major differences in polynomial fits

- $\alpha_\perp$ for $\chi \sim 1$
- $\kappa_\wedge$ functional form
- Z dependence



TC16125

Magnetized magnetothermal instability in azimuthal magnetic-field transport equations

Dispersion-relation coupling heat-flow equation and Ohm's law

- Azimuthal field
- Cylindrical geometry
- Transport coefficient derivatives

$$C_\ell = \chi\left(\frac{1}{L_b} + \frac{3}{2L_t}\right)$$

$$C_\kappa = \frac{2}{3n_e}\left\{-\kappa_\perp\left(-\frac{ik_r}{r} + k_r^2 + k_z^2\right) + \kappa_\wedge \frac{ik_z}{r}\, C_\ell \left[\frac{\partial \kappa_\perp}{\partial \chi}(ik_r + ik_z) + \frac{\partial \kappa_\wedge}{\partial \chi}(ik_z - ik_r)\right]\right\}$$

$$C_{\beta,q} = \frac{2}{3n_e}\left\{-\beta_{\wedge,q}\left(k_r^2 + k_z^2 - ik_r - \frac{ik_r}{r}\right) + C_\ell \left[\frac{\partial \beta_{\wedge,q}}{\partial \chi}\left(\frac{1}{r} + ik_r + ik_z\right) - \frac{\partial \beta_{\perp,q}}{\partial \chi}\left(ik_z - ik_z - \frac{1}{r}\right)\right]\right\}$$

$$C_{\beta,b} = -\left\{-\beta_\perp ik_r - \beta_\wedge ik_z + \beta_\perp ik_z - \beta_\wedge ik_r + C_\ell\left[-\frac{\partial \beta_\perp}{\partial \chi}(ik_r - ik_z) - \frac{\partial \beta_\wedge}{\partial \chi}(ik_z - ik_r)\right]\right\}$$

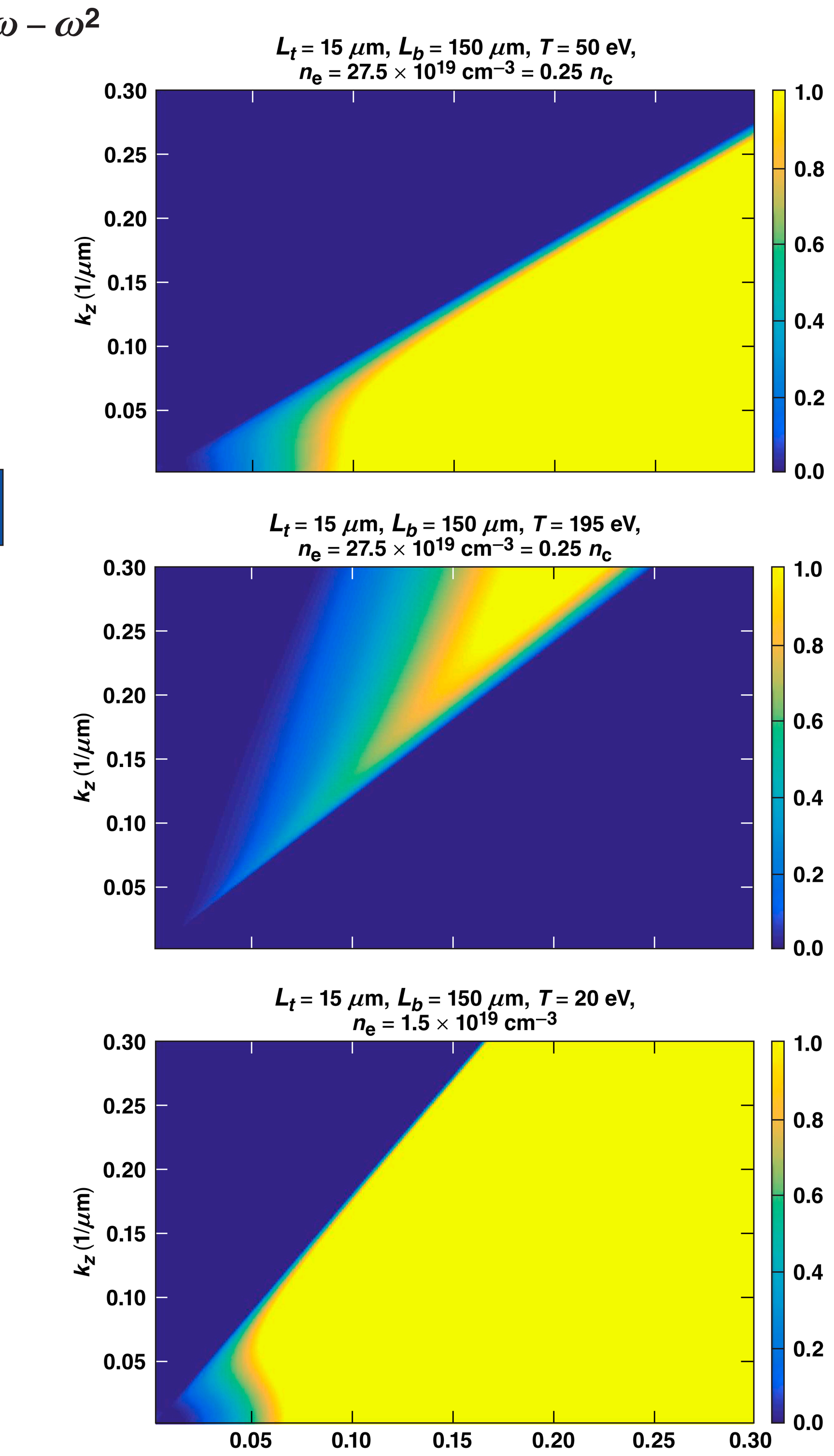
$$C_\eta = \left\{\eta_\perp k_z^2 - \eta_\wedge \frac{ik_z}{r} - \eta_\perp \frac{ik_r}{r} - \eta_\wedge k_z k_r + C_\ell\left[-\frac{\partial \eta_\perp}{\partial \chi}\left(ik_z + \frac{1}{r}\right) - \frac{\partial \eta_\wedge}{\partial \chi}\left(\frac{1}{r} - ik_z\right)\right]\right\}$$

Dispersion relation

$$0 = C_\eta C_\kappa - C_{\beta,b} C_{\beta,q} - i\left(C_\eta + C_\kappa\right)\omega - \omega^2$$

Axial and radial perturbations can cause unstable

- Time scales from 1 to 4 ns
- Length scales 30 to 10 μm



Conclusion

Simulations show that heat flow during laser ablation can lead to temperature perturbations in the axial and radial directions. The field compressing magnetothermal instability growth rate and range of wavelengths differ when using updated transport coefficients. A similar instability can exist in wire experiments seeded by perturbations generated by laser ablation.

TC16126

References

[1] J.-Y. Ji and E. D. Held, Phys. Plasmas 20, 042114 (2013).

[2] J. R. Davies et al., Phys. Plasmas 28, 012305 (2021).

[3] J. D. Sadler, C. A. Walsh, and H. Li, Phys. Rev. Lett. 126, 075001 (2021).

[4] L. S. Leal et al., Phys. Plasmas 27, 022116 (2020).

[5] J. J. Bissell, C. P. Ridgers, and R. J. Kingham, Phys. Rev. Lett. 105, 175001 (2010).

[6] E. M. Epperlein and M. G. Haines, Phys. Fluids 29, 1029 (1986).

Abstract

The magnetothermal instability in plasmas is related to the interplay of heat flow and magnetic field transport. The instability growth rates depend on the transport coefficients used. Recent modifications to transport coefficients in magnetized plasmas have been suggested in the regime of Hall parameters of the order of 1 for magnetic-field diffusion, advection, and heat conduction. [1–3] A perturbation analysis is presented that studies the instability growth in laser-ablated plasmas from cylindrical wires driven with a 1-MA current. [4] The new instability growth rates are compared with the results of full hydrodynamic simulations.

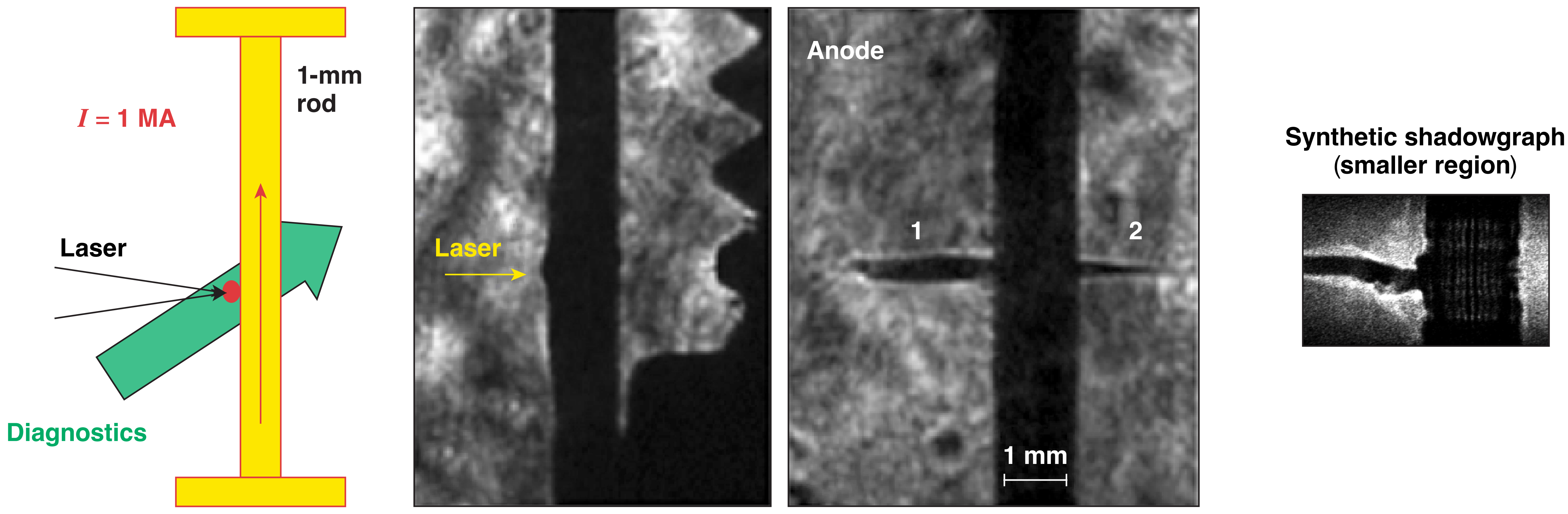
Acknowledgments

This material is based upon work supported by the Department of Energy under award number DOE Grant DE-SC0016500 and the Department of Energy National Nuclear Security Administration under Award Number DE-NA0003856, the University of Rochester, the New York State Energy Research and Development Authority, and LLNL computing resources.

Motivation

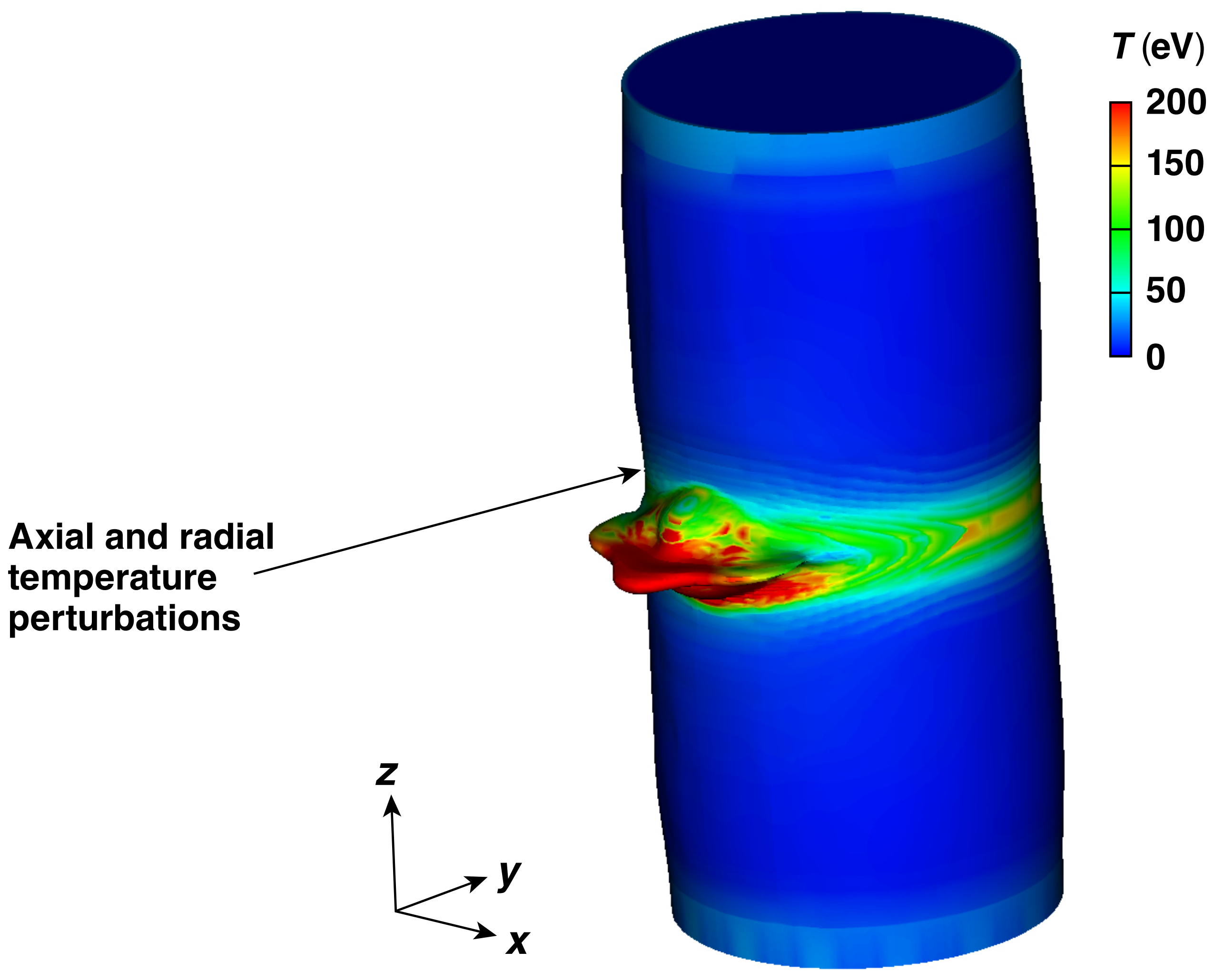
Experiments at University of Nevada, Reno using the Zebra Pulsed-Power Facility coupled with Leopard laser

- Wavelength: 1.06 μm
- 3×10^{15} W/cm^2
- Spot size: 30 μm
- Current 1 MA $\Leftrightarrow$ field 3 MG = 300 T
- 1-mm-diam rod



TC14531a

Synthetic shadowgraphs with the most up-to-date transport coefficients [3] show some structure on the back end, but not to the extent seen in experiments. Simulations predict that the laser spot generates heat flow on the surface.



TC16123

Magnetized magnetothermal instability in an axial field [5]

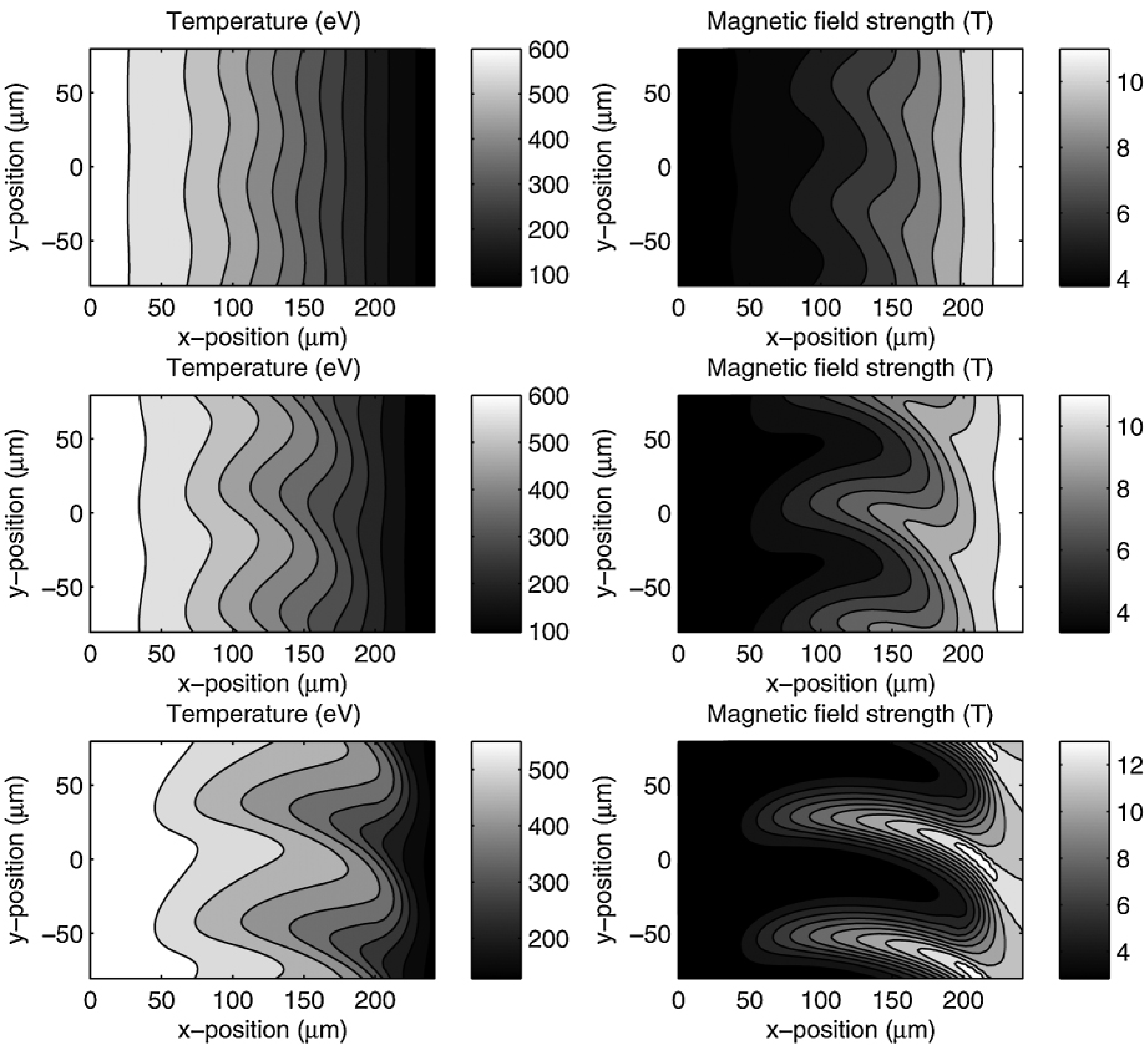
The magnetothermal instability is generated by the interplay of heat flow and magnetic field advection.

- **Nernst and Righi–Leduc terms source for growth**
- **Hall parameters $10^{-2} < \chi < 100$**
- **Restricted to $L_{T,B}^{-1} \ll k \ll \lambda_{mfp}^{-1}$**

$$\frac{3}{2} n_e \frac{\partial T}{\partial t} - \nabla \cdot \left(\widetilde{\kappa} \cdot \nabla T - \widetilde{\beta} \cdot j \right)$$

$$\frac{\partial B}{\partial t} = -\nabla \times \left[\widetilde{\alpha} \cdot (\nabla \times B) - \widetilde{\beta} \cdot \nabla T \right]$$

$$\omega_{\pm} = \frac{1}{2}\left[s_B k - (d_R + d_T) i k^2 \right] \pm \frac{1}{2}\left\{ s_B^2 k^2 + \left[s_P + 2 s_B \times (d_R + d_T) \right] i k^3 - \left[(d_R + d_T)^2 + s_E \right] k^4 \right\}^{1/2}$$



$$d_T = \frac{c_B \kappa_{\perp} \lambda_T^2}{3 \tau_T}$$

$$d_R = \frac{\alpha_{\perp} \delta^2}{c_B \tau_T}$$

$$s_E = \frac{4 \lambda_T^2 \delta^2}{3 \tau_T^2} \beta_{\wedge} \beta_{\wedge}$$

$$s_P = \frac{2 \beta_{\wedge} c_B^2 \lambda_T^4}{3 L_T \tau_T^2} \frac{\partial \kappa_{\wedge}}{\partial \chi} \sin\theta$$

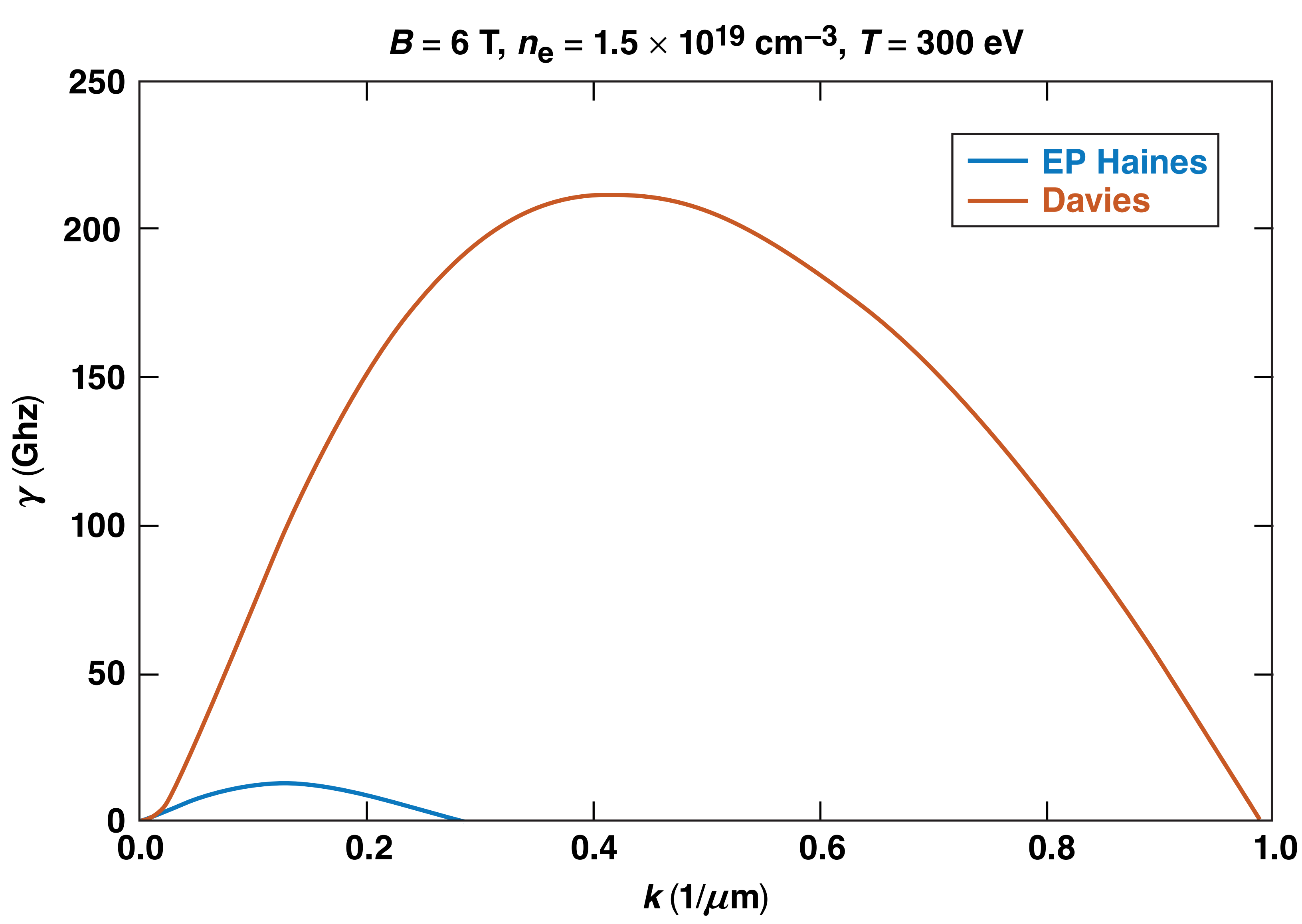
$$s_B = \frac{c_B \chi \lambda_T^2}{3 L_B \tau_T} \frac{\partial \kappa_{\wedge}}{\partial \chi} \sin\theta$$

[5] J. J. Bissell, C. P. Ridgers, and R. J. Kingham, Phys. Rev. Lett. 105, 175001 (2010).

Growth rate with updated coefficients

Recent studies have shown corrections to transport coefficients using calculated values from Fokker—Plank simulations. [2] major differences in polynomial fits

- **$\alpha_{\perp}$ for $\chi \sim 1$**
- **$\kappa_{\wedge}$ functional form**
- **Z dependence**



TC16125

Magnetized magnetothermal instability in azimuthal magnetic-field transport equations

Dispersion-relation coupling heat-flow equation and Ohm's law

- **Azimuthal field**
- **Cylindrical geometry**
- **Transport coefficient derivatives**

$$C_\ell = \chi\left(\frac{1}{L_b} + \frac{3}{2L_t}\right)$$

$$C_\kappa = \frac{2}{3n_e}\left\{-\kappa_\perp\left(-\frac{ik_r}{r} + k_r^2 + k_z^2\right) + \kappa_\wedge\frac{ik_z}{r}\, C_\ell\left[\frac{\partial\kappa_\perp}{\partial\chi}\left(ik_r + ik_z\right) + \frac{\partial\kappa_\wedge}{\partial\chi}\left(ik_z - ik_r\right)\right]\right\}$$

$$C_{\beta,q} = \frac{2}{3n_e}\left\{-\beta_{\wedge,q}\left(k_r^2 + k_z^2 - ik_r - \frac{ik_r}{r}\right) + C_\ell\left[\frac{\partial\beta_{\wedge,q}}{\partial\chi}\left(\frac{1}{r} + ik_r + ik_z\right) - \frac{\partial\beta_{\perp,q}}{\partial\chi}\left(ik_z - ik_z - \frac{1}{r}\right)\right]\right\}$$

$$C_{\beta,b} = -\left\{-\beta_\perp ik_r - \beta_\wedge ik_z + \beta_\perp ik_z - \beta_\wedge ik_r + C_\ell\left[-\frac{\partial\beta_\perp}{\partial\chi}\left(ik_r - ik_z\right) - \frac{\partial\beta_\wedge}{\partial\chi}\left(ik_z - ik_r\right)\right]\right\}$$

$$C_\eta = \left\{\eta_\perp k_z^2 - \eta_\wedge\frac{ik_z}{r} - \eta_\perp\frac{ik_r}{r} - \eta_\wedge k_z k_r + C_\ell\left[-\frac{\partial\eta_\perp}{\partial\chi}\left(ik_z + \frac{1}{r}\right) - \frac{\partial\eta_\wedge}{\partial\chi}\left(\frac{1}{r} - ik_z\right)\right]\right\}$$

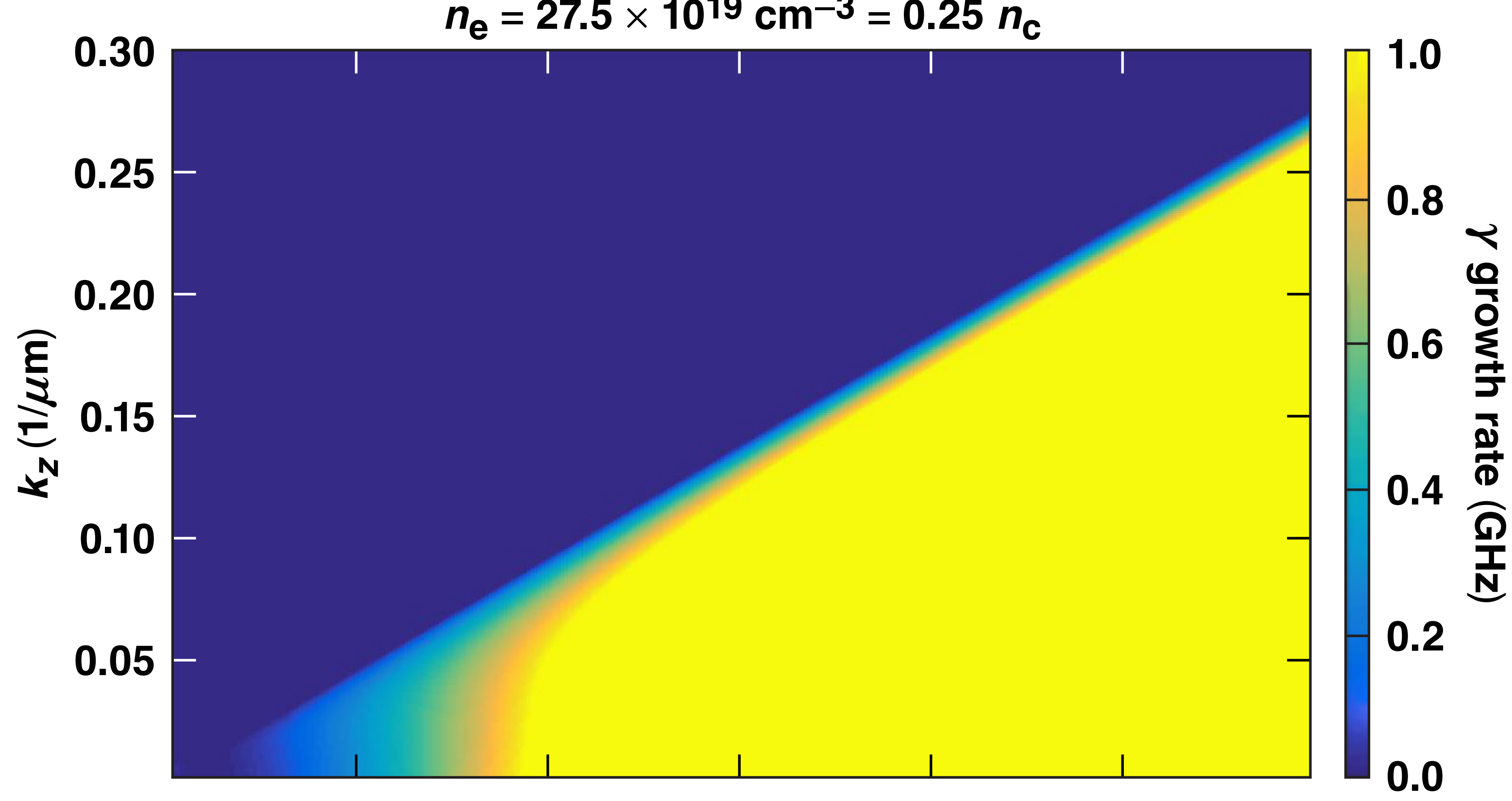
Dispersion relation

$$0 = C_\eta C_\kappa - C_{\beta,b}C_{\beta,q} - i\left(C_\eta + C_\kappa\right)\omega - \omega^2$$

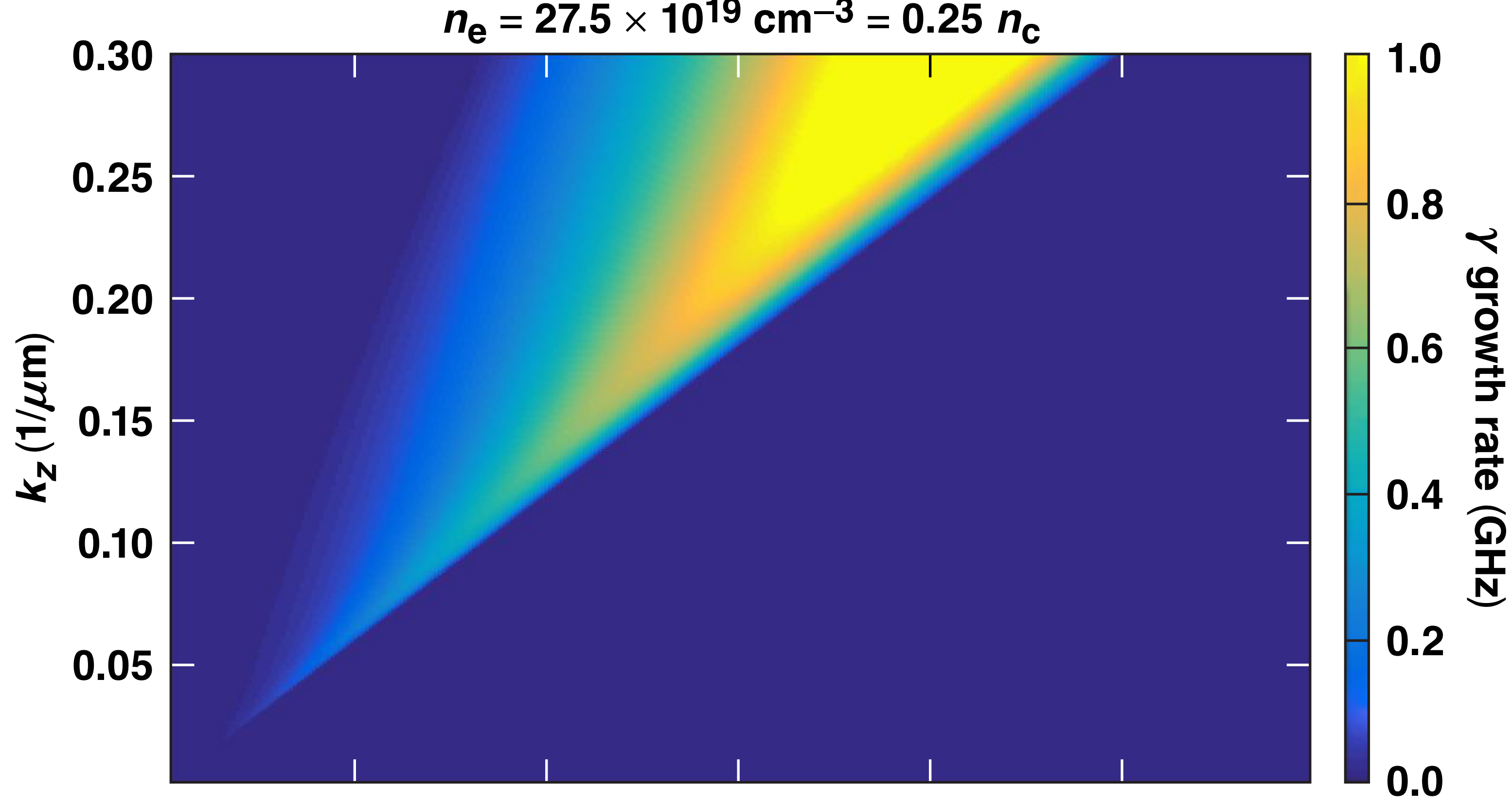
Axial and radial perturbations can cause unstable

- **Time scales from 1 to 4 ns**
- **Length scales 30 to 10 μm**

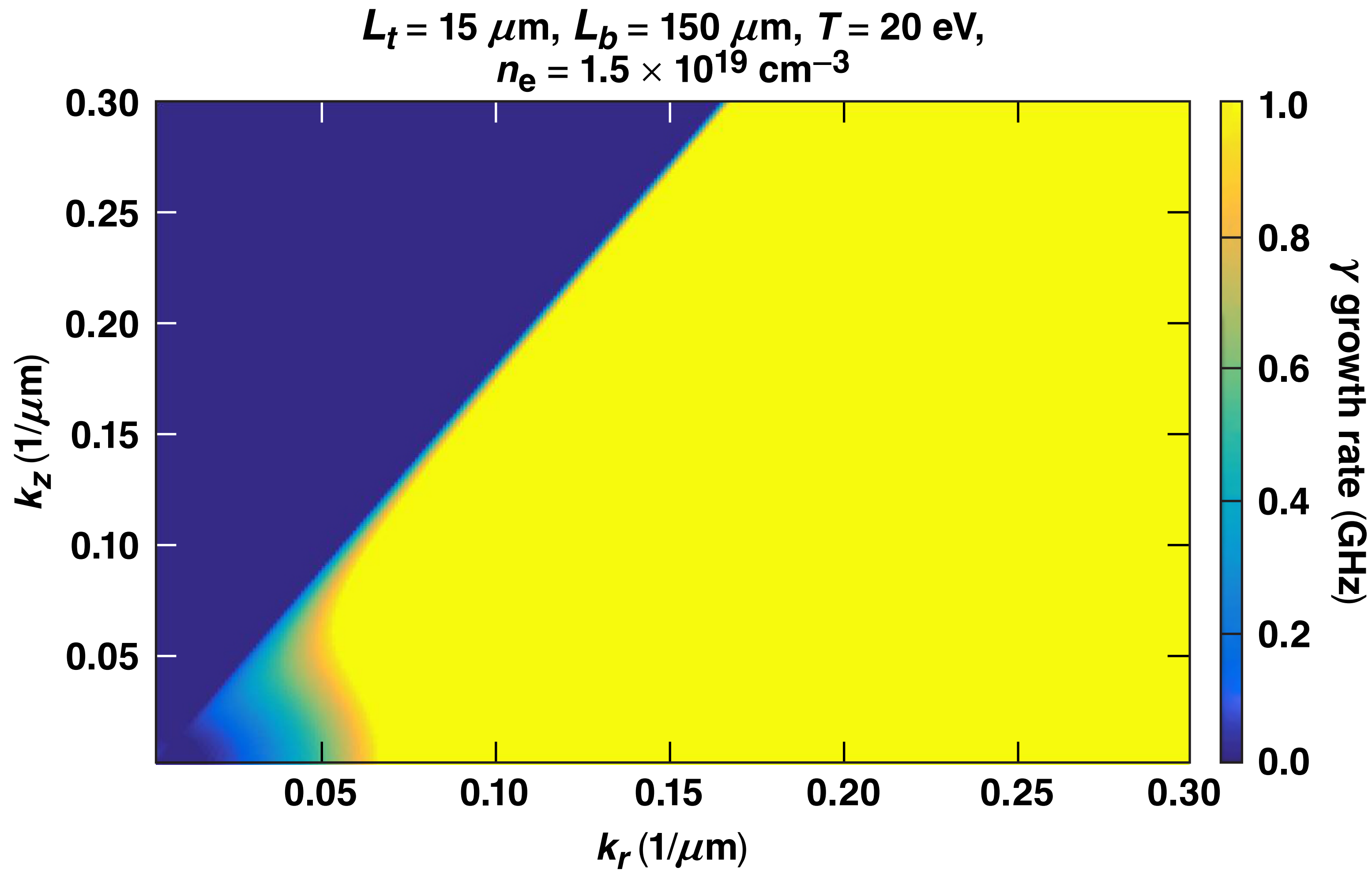
$L_t = 15\ \mu$m, $L_b = 150\ \mu$m, $T = 50$ eV, $n_e = 27.5 \times 10^{19}$ cm^{-3} = 0.25 n_c



$L_t = 15\ \mu$m, $L_b = 150\ \mu$m, $T = 195$ eV, $n_e = 27.5 \times 10^{19}$ cm^{-3} = 0.25 n_c



$L_t = 15\ \mu$m, $L_b = 150\ \mu$m, $T = 20$ eV, $n_e = 1.5 \times 10^{19}$ cm^{-3}



Conclusion

Simulations show that heat flow during laser ablation can lead to temperature perturbations in the axial and radial directions. The field compressing magnetothermal instability growth rate and range of wavelengths differ when using updated transport coefficients. A similar instability can exist in wire experiments seeded by perturbations generated by laser ablation.

TC16126

References

[1] J.-Y. Ji and E. D. Held, Phys. Plasmas 20, 042114 (2013).

[2] J. R. Davies *et al.*, Phys. Plasmas 28, 012305 (2021).

[3] J. D. Sadler, C. A. Walsh, and H. Li, Phys. Rev. Lett. 126, 075001 (2021).

[4] L. S. Leal *et al.*, Phys. Plasmas 27, 022116 (2020).

[5] J. J. Bissell, C. P. Ridgers, and R. J. Kingham, Phys. Rev. Lett. 105, 175001 (2010).

[6] E. M. Epperlein and M. G. Haines, Phys. Fluids 29, 1029 (1986).