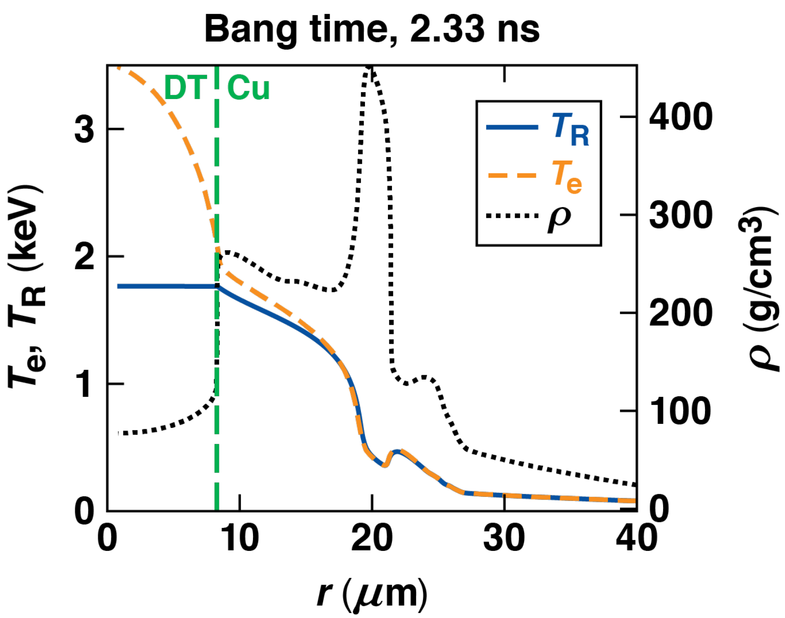
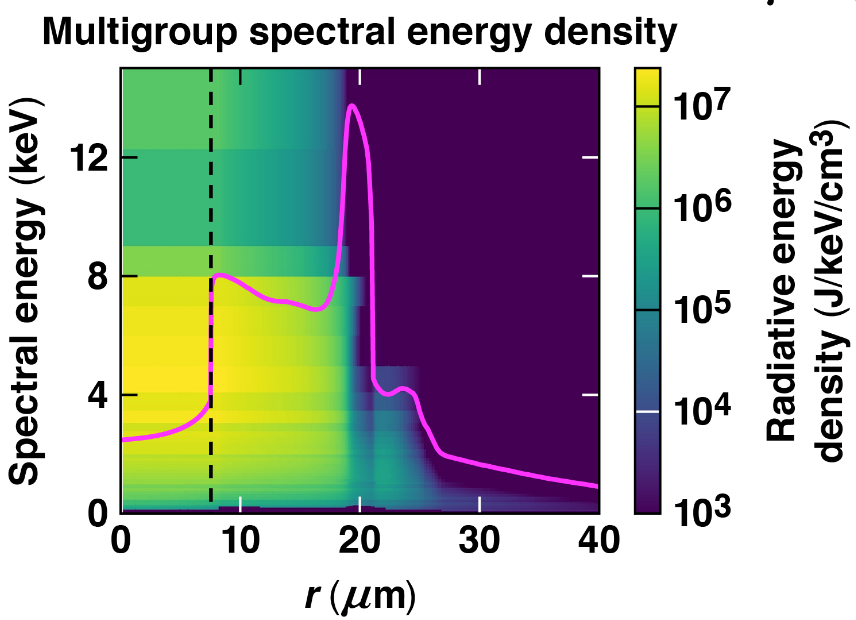
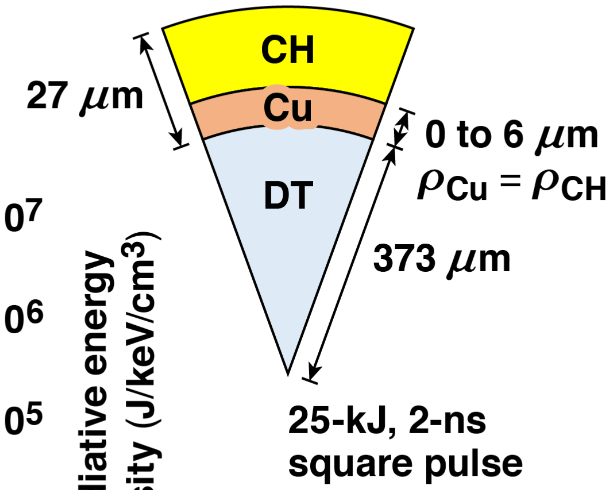
Assessment of Radiation Trapping in Inertial Confinement Fusion Implosion Experiments with High-Z–Lined, Single-Shell Targets







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Summary

Radiation trapping by an ICF pusher layer is being explored for new platforms for radiation transport and volume-ignition-related experiments

- Radiation trapping is most apparent in simulations through a characteristic Marshak waveform, where radiation and electron temperatures are equal ($T_R = T_e$), indicating the LTE atomic-radiative limit
- The Marshak wave model describes radiation trapping in pusher layers in terms of useful characteristic quantities
- The self-similar form and characteristic parameters of the Marshak wave model are obtained in cylindrical and spherical geometries
- The classic Marshak wave extends to a uniformly compressing pusher layer, preserving its self-similar analytic form with modified time scaling and modified parameter values

Volume-ignition capsule designs rely on radiation trapping.

ICF: inertial confinement fusion
LTE: local thermodynamic equilibrium

Collaborators

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The Marshak wave is based on a simple balance of electron thermal energy and radiative heating near local thermodynamic equilibrium (LTE)

Begin with radiation spectral energy density: $\frac{1}{c}\frac{\partial \phi_\nu}{\partial t} = \frac{\varepsilon_\nu}{c} - \kappa_\nu \phi_\nu + \frac{\partial}{\partial x}\left(\frac{1}{3\kappa_\nu}\frac{\partial \phi_\nu}{\partial x}\right)$ **and thermal energy:** $C_V \frac{\partial T_e}{\partial t} = \int \left(-\frac{\varepsilon_\nu}{c} + \kappa_\nu \phi_\nu\right) d\nu$

Assume quasi-static radiation: $\frac{1}{c}\frac{\partial \phi_\nu}{\partial t} \approx 0$

Define the radiation temperature: $E_R = \frac{4\sigma_{SB}}{c} T_R^{\,4} = \int \varphi_\nu \, d\nu$

Assume detailed balance: $\varepsilon_\nu \approx 4\pi \kappa_\nu B_\nu(T_e)$ **and LTE:** $\varphi_\nu \approx \frac{4\pi}{c} B_\nu(T_e)$

which gives: $\sigma_{SB} T_e^{\,4} = \pi \int B_\nu(T_e)\, d\nu$ **or** $T_R \approx T_e \equiv T$

Obtain the wave equation: $\boxed{C_V \frac{\partial T}{\partial t} \approx \frac{\partial}{\partial x}\left(\frac{c}{3\kappa_R}\frac{\partial}{\partial x}\left(\frac{4\sigma_{SB}}{c} T^4\right)\right)}$

Define the Rosseland mean opacity κ_R: $\frac{1}{\kappa_R}\int \frac{\partial B_\nu(T)}{\partial T} d\nu \equiv \int \frac{1}{\kappa_\nu}\frac{\partial B_\nu(T)}{\partial T} d\nu$

The appearance of LTE, e.g., $T_e = T_R$, is a sign of "trapped" radiation.

The constant-density planar Marshak wave problem has a self-similar temperature profile solution

$$\kappa = \kappa_0 \left(T/T_0\right)^{-n}$$

$$C_V = C_0 \left(T/T_0\right)^{q}$$

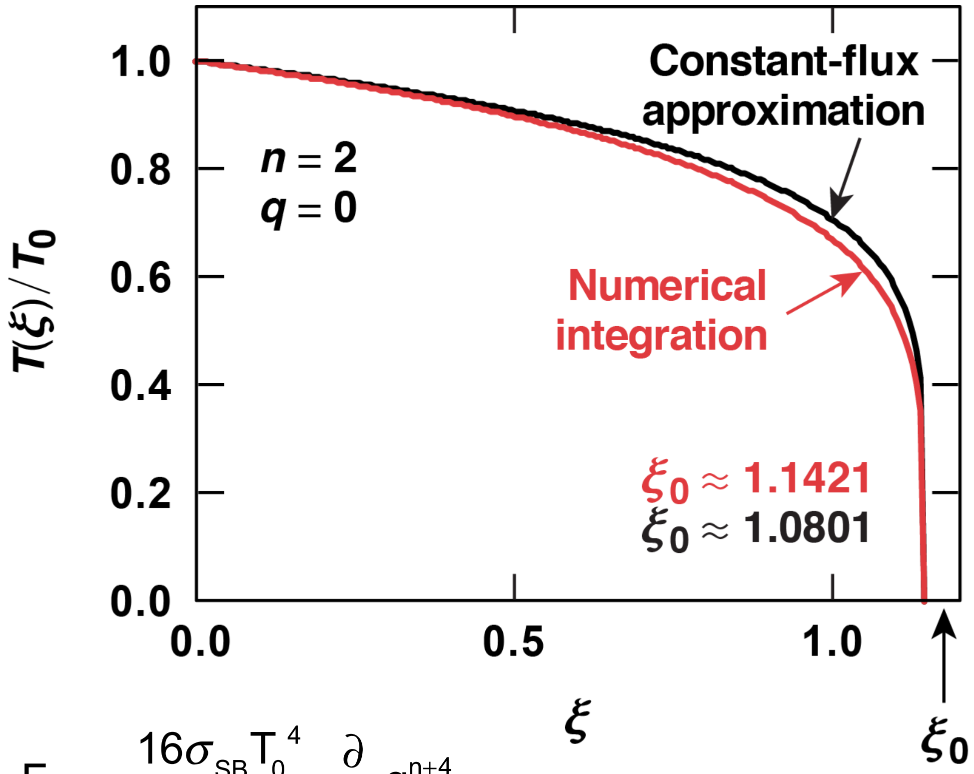
$$T(r,t) = T_0 g(\xi)$$

$$\boxed{\xi \frac{dg^{q+1}}{d\xi} - \frac{d^2}{d\xi^2} g^{n+4} = 0}$$

$$\xi = A \frac{x}{t^{1/2}}$$

$$A^2 = \frac{3}{32} \frac{(n+4)}{(q+1)} \frac{\kappa_0 C_0 \rho}{\sigma_{SB} T_0^3}$$

$$x_0(t) = \xi_0 \frac{t^{1/2}}{A}$$



$$F = -\frac{16\sigma_{SB} T_0^4}{3\kappa_0 (n+4)} \frac{\partial}{\partial x} g^{n+4}$$

Boundary conditions

At $\xi = 0$: conserve energy and $g(0) = 1$

At $\xi = \xi_0$: zero temperature

Constant-flux approximation

$$g(\xi) \approx \left(1 - \xi/\xi_0\right)^{\frac{1}{n+4}}$$

From conservation of energy

$$\xi_0^{\,2} \approx \frac{5+n+q}{4+n}$$

The "constant-flux" approximation is accurate and gives a useful expression for pretty much every quantity of interest.

The Marshak wave model yields several useful characteristic quantities, particularly $t_{\tau=1}$, the formation time of a one-optical-thickness wave

- **Optical thickness time scale: the time of formation of a τ_R = 1 trapping layer is the key time scale**

$$t_{\tau=1} = \frac{3}{2(n+4)(1+q)\xi_0^2}\frac{C_0\rho}{\sigma_{SB}T_0^3\kappa_0} = \frac{6}{(n+4)\xi_0^2}\frac{E_{Th}}{E_R}\frac{1}{\kappa_0 c} \qquad E_R = \frac{4\sigma_{SB}T_0^4}{c} \qquad E_{th} = \frac{C_0\rho T_0}{1+q}$$

- **The trapped flux F_R and the trapped energy E_R vary on this time scale**

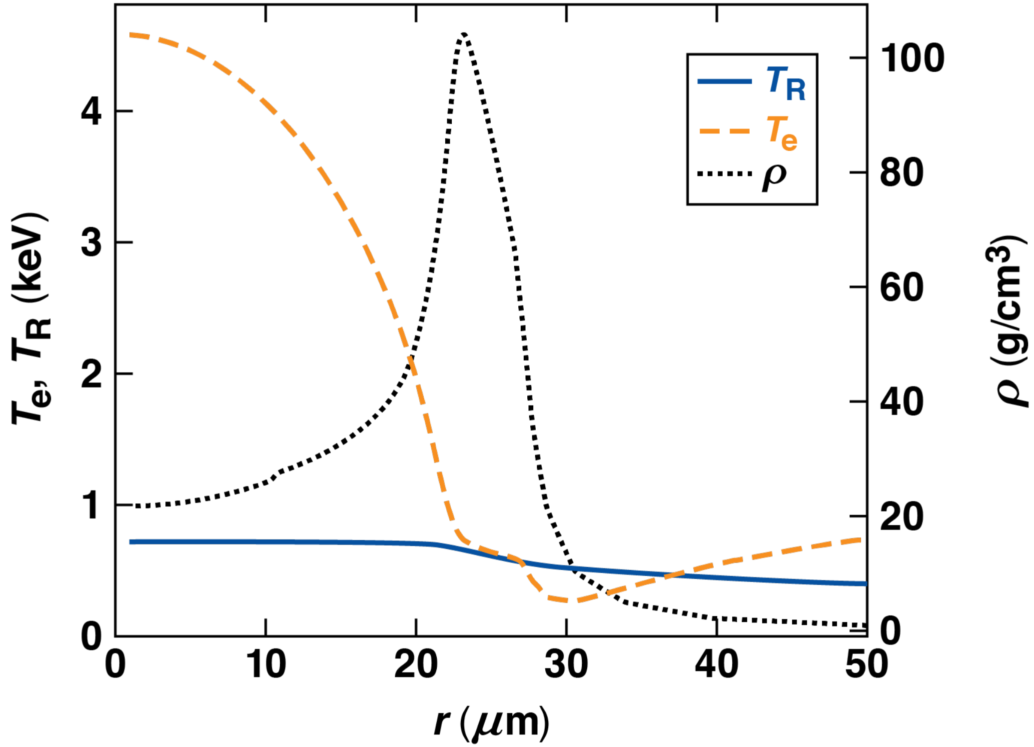
$$F_R \approx \frac{cE_R}{3}(t_{\tau=1}/t)^{1/2} \qquad E_R \approx 2F_R t \approx \frac{2cE_R}{3}(t_{\tau=1}t)^{1/2}$$

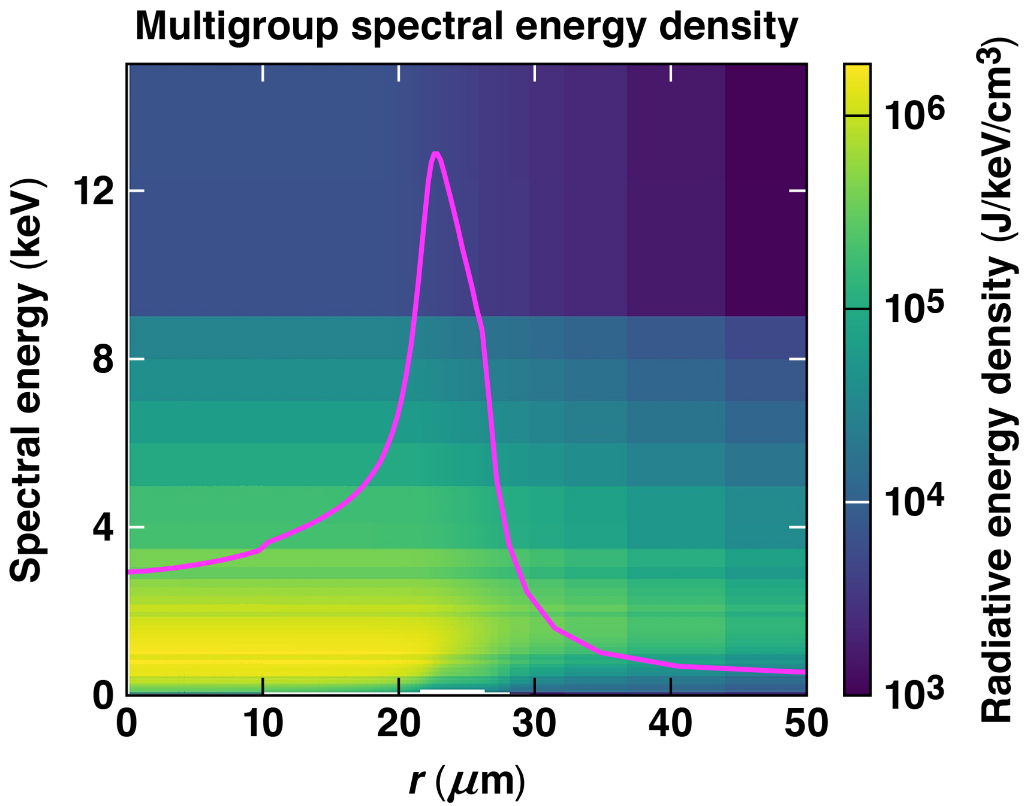
- **The wavefront, defined as $\xi = \xi_0$, decelerates**

$$A^2 = \frac{3(n+4)}{32(1+q)}\frac{\kappa_0 C_0\rho}{\sigma_{SB}T_0^3} = \frac{3(n+4)}{8}\frac{\kappa_0}{c}\frac{E_{th}}{E_R} \qquad x_0(t) = \xi_0\frac{t^{1/2}}{A} = \xi_0\frac{2\sqrt{2}}{\sqrt{3(n+4)}}\sqrt{\frac{c}{\kappa_0}\frac{E_R}{E_{th}}}t^{1/2} = \frac{4}{(n+4)\kappa_0}\left(\frac{t}{t_{\tau=1}}\right)^{1/2}$$

$$\xi_0^2 \approx \frac{5+n+q}{4+n}$$

Pure-CH OMEGA-scale imploded shells do not trap radiation



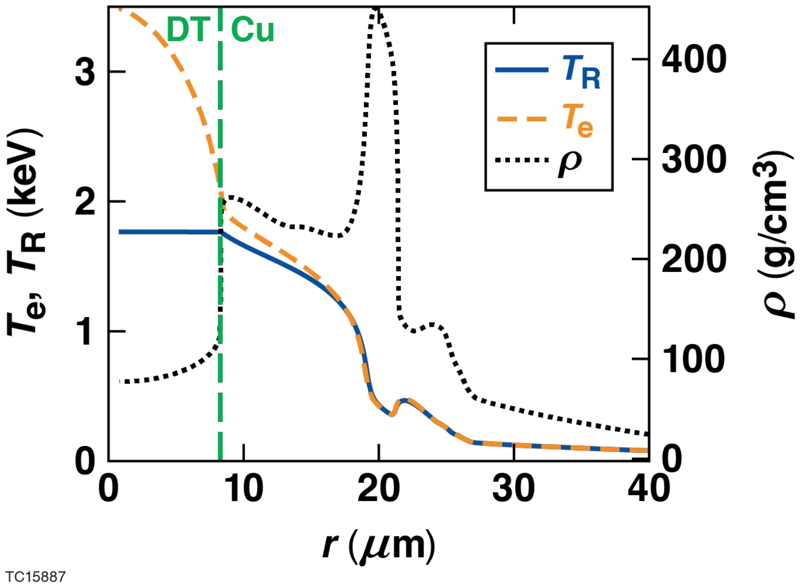


TC15886

- High-yield shot 90288
- Radiation source $r < 22$ μm
- Near-free escape of radiation through the DT shell

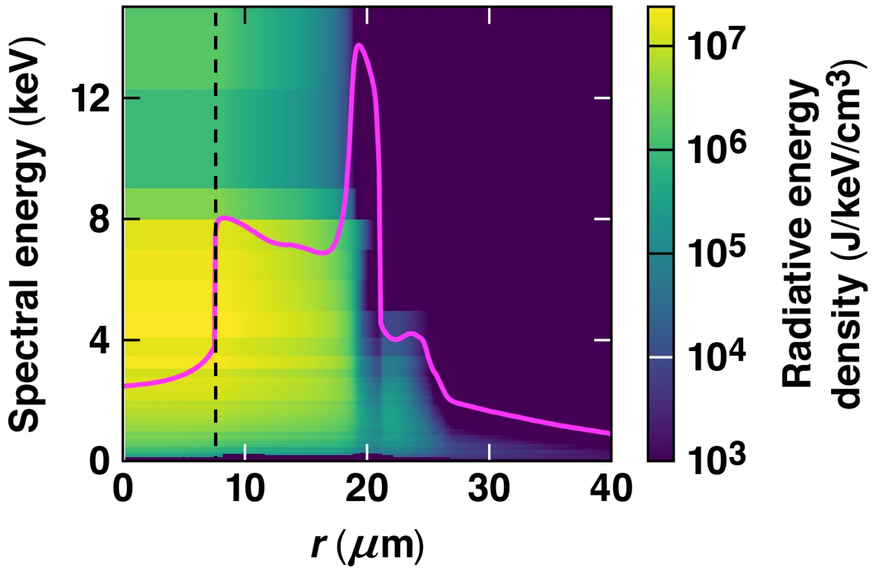
A 6-μm Cu inner pusher layer traps radiation through bang time as seen in a *LILAC* simulation where $T_R = T_e$ indicates optically thick LTE

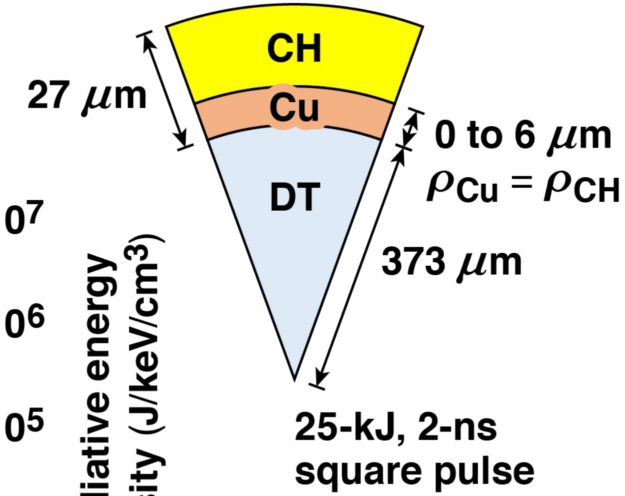
T_R, T_e, ρ at bang time, 2.33 ns



TC15887

Multigroup spectral energy density



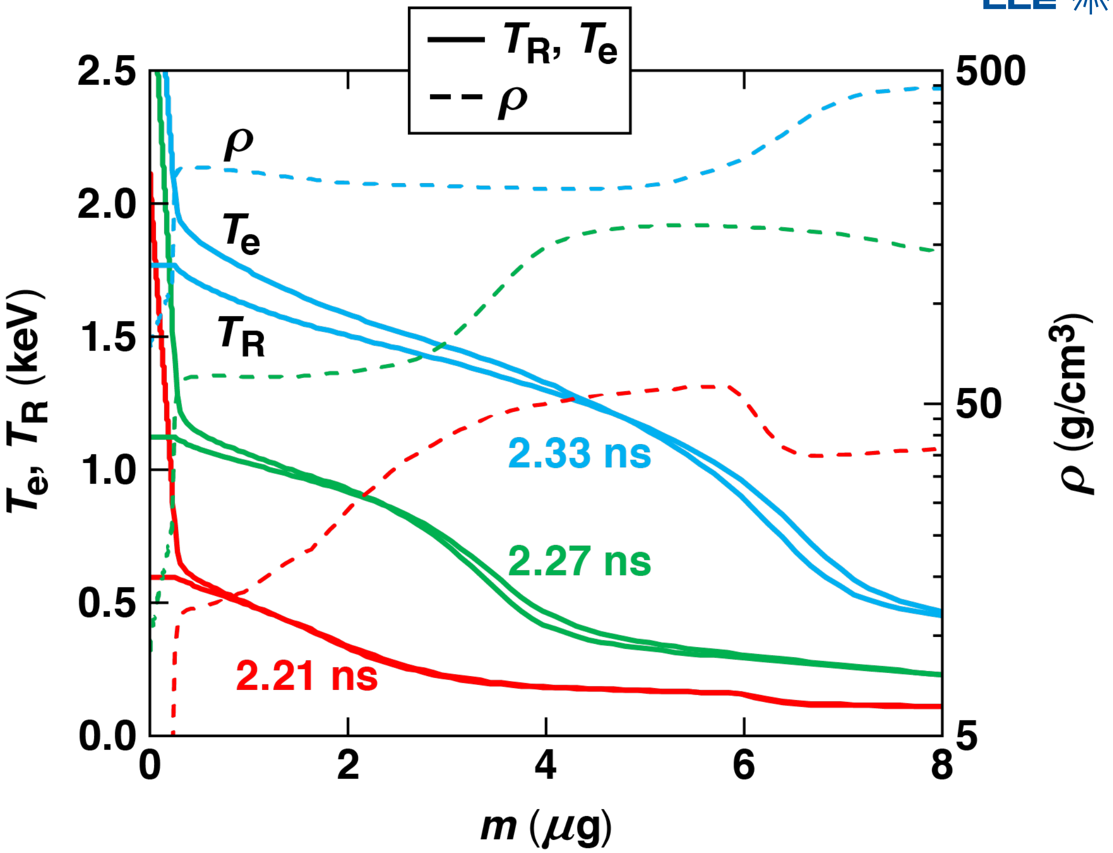


$T_R = T_e$ indicates LTE and a Planck spectrum, key assumptions underlying the Marshak wave model.

Marshak wave "τ = 1" formation times for Cu are short relative to the pusher hydro time, but far too long for a pure-CH shell

$$t_{\tau=1} = \frac{6}{(n+4)\xi_0^{\,2}} \frac{E_{Th}}{E_R} \frac{1}{\kappa_0 c}$$

	t sim (ns)	T_0 (keV)	ρ (g/cm^3)	t (τ = 1) (ps)
Cu	2.21	0.60	12.8	5.5
Cu	2.27	1.12	60.5	3.6
Cu	2.33	1.77	239.0	2.7
CH	2.33	1.12	57.0	66.8



TC15889

The "τ = 1" formation time is a parameter that anticipates the effectiveness of radiation trapping in an imploding pusher layer.

Marshak waves have nearly identical wavefronts in planar, cylindrical, and spherical geometries, but very different $g(\xi) = T/T_0$ profiles

Planar

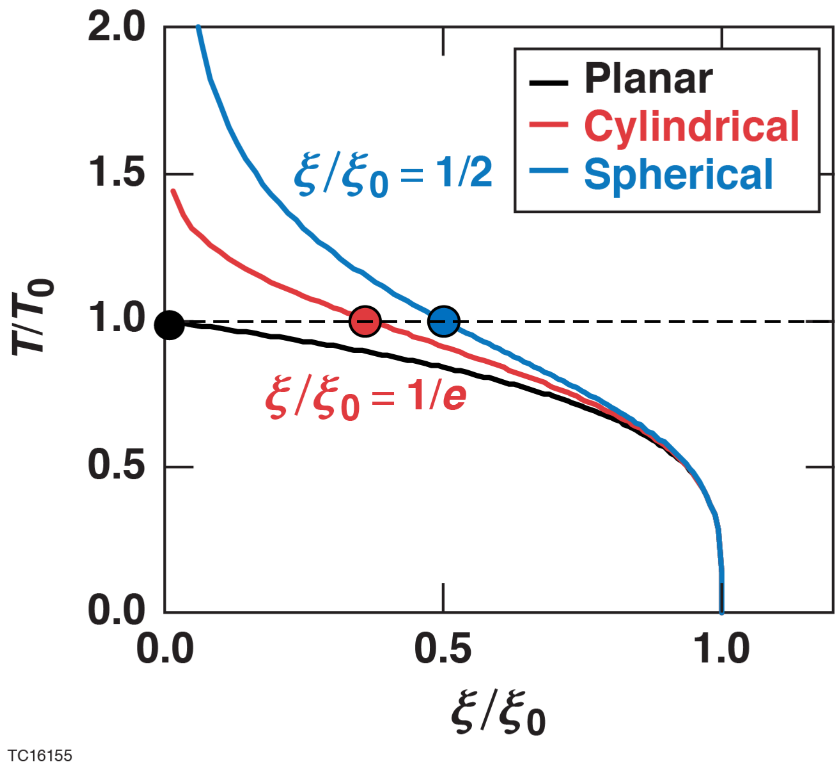
$$\xi \frac{dg^{q+1}}{d\xi} - \frac{d^2}{d\xi^2} g^{n+4} = 0$$

Cylindrical

$$\xi \frac{dg^{q+1}}{d\xi} - \frac{1}{\xi}\frac{d}{d\xi}\left(\xi \frac{d^2}{d\xi^2} g^{n+4}\right) = 0$$

Spherical

$$\xi \frac{dg^{q+1}}{d\xi} - \frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2 \frac{d^2}{d\xi^2} g^{n+4}\right) = 0$$



- In cylindrical and spherical geometry, T_0 is not a good fuel/pusher boundary condition parameter; it is fixed at a definite point in ξ, but not in space or time
- A better boundary condition is the total radiated power at or near $\xi = 0$

Marshak waves have the same trajectory and flux trapping parameter in all three geometries, all expressed in terms of the optical thickness formation time t_τ

Common to all three geometries:

$$\xi = A\frac{x}{t^{1/2}}$$

$$A^2 = \frac{3(n+4)}{8}\frac{\kappa_0}{c}\frac{E_{Th}}{E_R}$$

$$F = -\frac{16\sigma_{SB}T_0^{\,4}}{3\kappa_0(n+4)}\frac{\partial}{\partial x}g^{n+4}$$

Radiated power:

Planar: $$F = \frac{16}{3(n+4)}\frac{\sigma_{SB}T_0^{\,4}}{\kappa_0}\frac{A}{\xi_0 t^{1/2}} = \frac{cE_R}{3}\left(\frac{t_\tau}{t}\right)^{1/2}$$

Cylindrical: $$2\pi r F = \frac{32\pi}{3(n+4)}\frac{\sigma_{SB}T_0^{\,4}}{\kappa_0} = \frac{8\pi}{(n+4)}\frac{cE_R}{3}\frac{1}{\kappa_0}$$

Spherical: $$4\pi r^2 F = \frac{64\pi}{3(n+4)}\frac{\sigma_{SB}T_0^{\,4}}{\kappa_0}\frac{\xi_0 t^{1/2}}{A} = \frac{64\pi}{(n+4)^2}\frac{cE_R}{3}\frac{1}{\kappa_0^{\,2}}\left(\frac{t}{t_\tau}\right)^{1/2}$$

Flux trapping ratio:

$$F_{SB} = \sigma_{SB}T_0^{\,4} = \frac{cE_R}{4}$$

$$\frac{F}{F_{SB}} = \frac{4}{3}\left(\frac{t_\tau}{t}\right)^{1/2}$$

Still a key parameter: $$t_{\tau=1} = \frac{3}{2(n+4)(1+q)\xi_0^{\,2}}\frac{C_0\rho T_0}{\sigma_{SB}T_0^{\,4}\kappa_0} = \frac{6}{(n+4)\xi_0^{\,2}}\frac{E_{th}}{c\kappa_0 E_R}$$

$$E_R = \frac{4\sigma_{SB}T_0^{\,4}}{c} \qquad E_{th} = \frac{C_0\rho T_0}{1+q}$$

All geometries have the same wavefront trajectory: $x_0(t) = \xi_0\frac{t^{1/2}}{A}$.

The Marshak wavefront has nearly identical trajectories and optical thicknesses in planar, cylindrical, and spherical geometries

Common to all three geometries

$$\xi = A\frac{x}{t^{1/2}}$$

$$A^2 = \frac{3(n+4)}{32(1+q)}\frac{\kappa_0 C_0 \rho}{\sigma_{SB} T_0^3}$$

$$A^2 = \frac{3(n+4)}{8}\frac{\kappa_0}{c}\frac{E_{Th}}{E_R}$$

$$x_0(t) = \xi_0 \frac{t^{1/2}}{A}$$

$$\kappa = \kappa_0 \left(T/T_0\right)^{-n}$$

Planar

$$\xi_0^2 \approx \frac{(1+m)}{m}$$

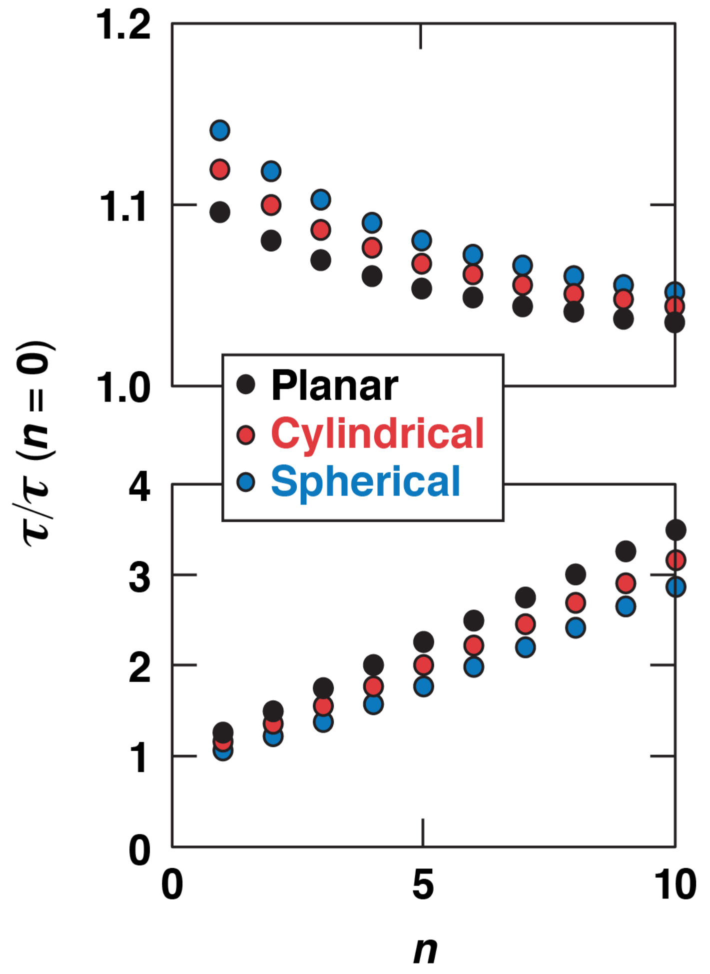
Cylindrical

$$\xi_0^2 \approx \frac{2^{1/m}}{\Gamma(1+1/m)}$$

Spherical

$$\xi_0^2 \approx \frac{1}{\pi}\frac{2m^3 \sin(\pi/m)}{(2m-1)(m-1)}$$

$$m = \frac{4+n}{1+q}$$



TC16142

The Marshak wave* is a self-similar solution to the Euler fluid energy equation including thermal radiation transport and adiabatic compression

Energy equation: $$\frac{1}{c}\frac{DE}{Dt}-\frac{P}{\rho^2}\frac{D\rho}{Dt}-\frac{\partial}{\partial m}\left(\frac{4}{3\kappa}\frac{\partial}{\partial m}\left(\sigma_{SB}T^4\right)\right)=0$$

Properties of matter:** $\kappa=\kappa_0\left(\rho/\rho_0\right)^r\left(T/T_0\right)^{-n}$ $C_V=C_0\left(\rho/\rho_0\right)^s\left(T/T_0\right)^q$ $P=\left(\gamma-1\right)\rho E$

New compression terms are in red

Uniform adiabatic compression: $\rho(t)=\rho_0\left(t/t_0\right)^{\alpha}$ **New parameters t_0 and α**

Marshak wave: $T(m,t)=T_0 g\left(m,t\right)h(t)$ $h(t)=\left(\rho(t)/\rho_0\right)^{\gamma-1-s}$

$$\xi\frac{dg(\xi)^{q+1}}{d\xi}-\frac{d^2}{d\xi^2}g(\xi)^{n+4}=0 \quad 0\le\xi\le\xi_0 \quad g(0)=1 \quad g(\xi_0)=0$$

Self-similar space–time *m,t* dependence through the variable ξ: $$\xi=(1+\nu)^{1/2}A\frac{m}{t^{1/2}}\left(\frac{t_0}{t}\right)^{\nu/2} \qquad A^2=\frac{3(4+n)}{32(1+q)}\frac{\kappa_0 C_0 T_0}{\sigma_{SB}T_0^{\,4}} \qquad \nu=\alpha\left(\frac{(\gamma-1)(3+n-q)}{(q+1)}-s-r\right)$$

Wave front position $m_0(t)$ at $\xi=\xi_0$: $$m_0(t)=\frac{\xi_0}{(1+\nu)^{1/2}}\frac{t^{1/2}}{A}\left(\frac{t}{t_0}\right)^{\nu/2}$$

The Marshak wave profile and trajectory are described approximately but completely.

* R. E. Marshak, Phys. Fluids 1, 24 (1958).
** A. P. Cohen *et al.*, Phys. Rev. Research 2, 023007 (2020).

The Marshak wave model yields several useful characteristic quantities

- **Optical thickness time scale: The time of formation of a τ_R = 1 trapping layer is the key time scale**

$$t_{\tau=1} = \left[\frac{6(1+\nu)}{\xi_0^{\,2}(4+n)}\frac{E_{th}}{E_R}\frac{1}{c\kappa_0}t_0^{\alpha\psi}\right]^{1/(1+\alpha\psi)} \qquad E_R = \frac{4\sigma_{SB}T_0^{\,4}}{c} \qquad E_{th} = \frac{C_0T_0}{1+q} \qquad \psi = \frac{(\gamma-1-s)(3-n-q)}{(1+q)} + r - s$$

- **The trapped flux varies on this time scale more slowly:**

$$F_R(t) \approx \frac{cE_R}{3}\left(t_{\tau=1}/t\right)^{1/2}\left(t/t_0\right)^{\frac{\alpha}{2}\left[\frac{(\gamma-1)(5+n+q)}{(1+q)}-r-s\frac{(4+n)}{(1+q)}\right]}\left(t_{\tau=1}/t_0\right)^{\frac{\alpha}{2}\left[\frac{(\gamma-1)(3-n-q)}{(1+q)}+r-s\frac{(4-n)}{(1+q)}\right]}$$

- **Again, compression sets the time scale t_0 and the power-law index α**

$$\rho(t) = \rho_0\left(\frac{t}{t_0}\right)^{\alpha} \qquad \nu = \alpha\left(\frac{(\gamma-1)(3+n-q)}{(1+q)} - s - r\right)$$

- **Optical thickness grows faster:**

$$\tau(t) = \left(t/t_{\tau=1}\right)^{\frac{1}{2}+\frac{\alpha}{2}\left[\frac{(\gamma-1-s)(3-n-q)}{(1+q)}+2n\right]}$$

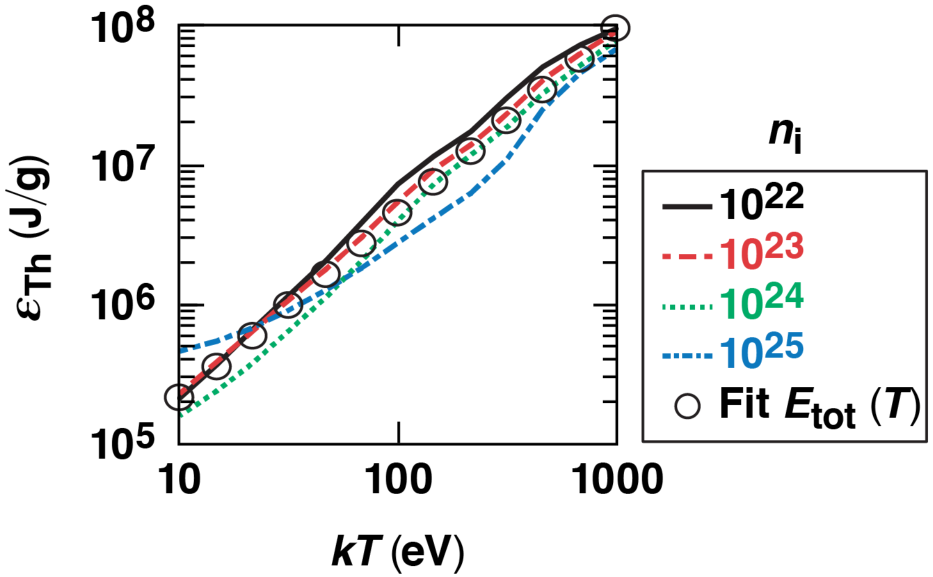
Radiation trapping by an ICF pusher layer can be characterized in terms of a Marshak wave model

- **Radiation trapping is most apparent in simulations through a characteristic Marshak waveform, where radiation and electron temperatures are equal ($T_R = T_e$), indicating the LTE atomic-radiative limit**
- **The Marshak wave model describes radiation trapping in pusher layers in terms of useful characteristic quantities**
- **The self-similar form and characteristic parameters of the Marshak wave model are obtained in cylindrical and spherical geometries**
- **The classic Marshak wave extends to a uniformly compressing pusher layer, preserving its self-similar analytic form with modified time scaling and modified parameter values**

Volume-ignition capsule designs rely on radiation trapping.

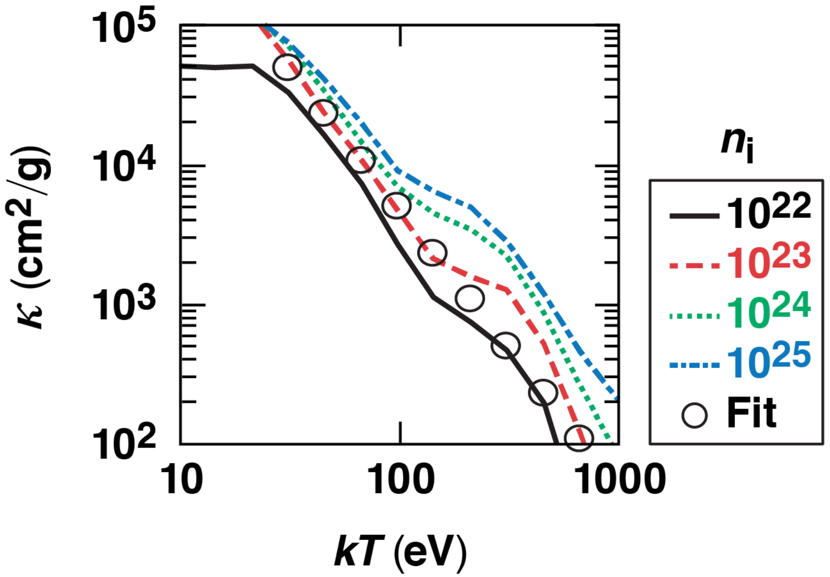
Rosseland mean opacity and specific heat for Cu is obtained from PrOpacEOS* and fit with an expression suitable for the Marshak wave model

$$C_V = C_0 \left(\rho/\rho_0\right)^s \left(T/T_0\right)^q$$



TC16143

$$\kappa = \kappa_0 \left(\rho/\rho_0\right)^r \left(T/T_0\right)^{-n}$$



Fit: $C_V = 1.33 \times 10^4 \times (T/\mathrm{eV})^{1/3}$ (J/g/eV)

Fit: $\kappa_R = 5.0 \times 10^7 \times (T/\mathrm{eV})^{-2}$ (cm^2/g)

$r \neq 0$ and $s \neq 0$ anticipates time-varying ρ

* PrOpacEOS: Prism Computational Sciences, Inc.