Modeling the Filamentation Instability of Relativistic Electron Beams for Fast Ignition

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37th Annual Anomalous Absorption Conference
Maui, HI
27–31 August 2007

Summary

Fluid approximations can differ significantly from kinetic results for relativistic beam filamentation

- Fluid models represent an algebraic approximation to the kinetic plasma-dispersion function.
- This approximation breaks down near threshold (low frequencies) or for calculations of spatial growth or absolute instability.
- Kinetic calculations show that in general the instability has somewhat larger growth rates and extends over a wider range of transverse wave numbers than indicated by fluid approximations.
- Inclusion of mobile ions also tends to extend the range of the instability.

Most fast-ignition scenarios require propagation of a relativistic electron beam through a plasma

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- Large-scale beam instabilities (kinking, pinching) develop slowly on the FI timescale.
- Microinstabilities grow faster and include beam–plasma (electrostatic) and filamentation (electromagnetic or mixed) instabilities.
- These instabilities require impedance.
  - reactive (electron inertia, Weibel and beam–plasma instability): dominant at low densities (few × critical).
  - resistive (collisional, resistive filamentation): dominant at high densities (compressed core).
  - a FI beam will transit both regions (reactive first).
- A fully relativistic treatment of the collisionless case can be carried out analytically; the collisional case is more difficult.

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Instabilities can be treated as a perturbed equilibrium provided the growth times are shorter than the beam slowing time

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- Assume that, in equilibrium, the charge densities, currents, and fields vanish and that all perturbed quantities have the space and time dependence $e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$.

- Maxwell's equations relate the current to the perturbed electric field.

$$\mathbf{j} = \mathbf{j}_b + \mathbf{j}_p = -\frac{ic^2}{4\pi\omega}\left(k^2 \mathbf{I} \underbrace{- \mathbf{k}\mathbf{k}}_{\substack{\text{Longitudinal}\\ \text{(electrostatic)}\\ \text{term}}} \underbrace{- \frac{\omega^2}{c^2}\mathbf{I}}_{\substack{\text{From}\\ \text{displacement}\\ \text{current}}}\right)\cdot \mathbf{E}$$

- The rest of the problem consists of using the plasma properties to derive the perturbed current as a response to *E* (the conductivity tensor).

The collisionless relativistic electron beam can be represented as a Maxwell–Boltzmann–Jüttner (MBJ) distribution

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- The MBJ distribution is a relativistic generalization of the Maxwellian

$$f_0(\mathbf{w}) = \frac{\xi n_0}{4\pi\Gamma K_2(\xi)} e^{-\xi\Gamma(\gamma - \boldsymbol{\beta}\cdot\mathbf{w})}, \text{ where } \xi \equiv \frac{mc^2}{k_B T_R} = \frac{c^2}{\upsilon_T^2},\ \mathbf{w} \equiv \frac{\mathbf{p}}{mc} = \gamma\frac{\mathbf{v}}{c},$$

$\boldsymbol{\beta}$ is the average beam $\boldsymbol{\beta}$, and $\Gamma \equiv \left(1 - \beta^2\right)^{-1/2}$.

- When the thermal spread is small compared to the beam velocity, this can be approximated as a drifting Maxwellian

$$f_0(\mathbf{p}) = \frac{1}{(2\pi)^{3/2}} \frac{1}{(m\nu_T)^3\Gamma} n_b e^{-\frac{\left(\frac{p_z}{\Gamma} - mc\beta\right)^2 + p_\perp^2}{2(m\upsilon_T)^2}}.$$

- These forms are also used to represent the return current in the collisionless case.

In the collisionless case, the perturbed currents are calculated from the relativistic Vlasov equation

- The linearized relativistic Vlasov equation can be written as

$$\frac{\partial f_{1\alpha}}{\partial t} + \frac{c\mathbf{w}}{\gamma} \cdot \frac{\partial f_{1\alpha}}{\partial \mathbf{x}} = -\frac{q_\alpha}{m_\alpha c}\left(\mathbf{E} + \frac{\mathbf{w}}{\gamma} \times \mathbf{B}\right) \cdot \frac{\partial f_{0\alpha}}{\partial \mathbf{w}}.$$

- Solving for the perturbed current gives

$$\mathbf{j}_b = \frac{-ie^2}{m\omega}\left[\int \frac{\mathbf{p}}{\gamma}\frac{\partial f_0}{\partial \mathbf{p}} d^3p + \int \frac{\left(\mathbf{k} \cdot \frac{\partial f_0}{\partial \mathbf{p}}\right)\mathbf{p}\mathbf{p}}{\gamma\left(\gamma m\omega - \mathbf{k} \cdot \mathbf{p}\right)} d^3p\right] \cdot \mathbf{E}.$$

- In the drifting Maxwellian approximation, the integrals can be expressed in terms of the usual plasma *Z* function.

- The exact relativistic integrals can be expressed in terms of integrals of the form $\int_{-1}^{1} ds \frac{\gamma}{\omega - cks} \frac{K_{n/2}(z)}{z^{n/2}}$, where $\gamma = \left(1 - s^2\right)^{-1/2}$ and

$$z \equiv \xi\Gamma\sqrt{\left(1 - \beta_3 s\right)^2 \gamma^2 - \left(\beta_1^2 + \beta_2^2\right)}.$$

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The resulting dispersion relations are complicated algebraically but readily evaluated numerically

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- The dispersion relation is obtained from

$$\left[\left(c^2k^2-\omega^2+\frac{\omega_p^2}{\Gamma}-\frac{4\pi i\omega}{\eta}\right)\underset{\sim}{I}-c^2kk-\frac{\omega_p^2}{\Gamma}\underset{\sim}{R}\right]\cdot E=0.$$

- A typical R component in the drifting Maxwellian approximation with the beam propagating in the z direction: $R_{zz}\equiv\left(k_y^2\upsilon_{T\perp}^2+\Gamma^2k_z^2\upsilon_{Tz}^2\right)^{-2}$

$$\times\left[\begin{array}{l}[1+\zeta Z(\zeta)]\left\{\begin{array}{l}\Gamma^2\left(k_y^2+k_z^2\right)\left[\left(\sqrt{2}\,\Gamma k_z\upsilon_{Tz}^2\zeta+\beta c\sqrt{k_y^2\upsilon_{T\perp}^2+\Gamma^2k_z^2\upsilon_{Tz}^2}\right)^2+k_y^2\upsilon_{Tz}^2\upsilon_{T\perp}^2\right]\\ \quad+2k_y^2k_z^2\upsilon_{Tz}^2\left(\upsilon_{T\perp}^2-\Gamma^2\upsilon_{Tz}^2\right)\end{array}\right\}\\ +k_z\Gamma\left[\sqrt{2}\,\beta ck_y^2\sqrt{k_y^2\upsilon_{T\perp}^2+\Gamma^2k_z^2\upsilon_{Tz}^2}\left(\upsilon_{T\perp}^2-\Gamma^2\upsilon_{Tz}^2\right)Z(\zeta)+\Gamma^3\left(k_y^2+k_z^2\right)k_z\upsilon_{Tz}^4\right]\end{array}\right]$$

where $\zeta\equiv\dfrac{\Gamma(\omega-\beta ck_z)}{\sqrt{2}\sqrt{\left(k_y\upsilon_{T\perp}\right)^2+\left(\Gamma k_z\upsilon_{Tz}\right)^2}}$

Generalizing the dispersion relation to complex *k* allows the study of spatial growth and absolute instability

- **Roots of the dispersion relation with real *k* and complex ω indicate instability (pure temporal growth), usually convective.**

- **Start with a temporally growing mode and decrease $\mathrm{Im}(\omega)$; if one of the complex k_z roots crosses the real axis and acquires $\mathrm{Im}(k_z) < 0$ for $\mathrm{Im}(\omega) = 0$, it represents spatial growth.**

- **Perturbations introduced where the beam originates grow as it propagates into the plasma; spatial growth is most appropriate to the FI problem.**

- **If two k_z roots merge across the real axis to a double root with $\mathrm{Im}(k_z) < 0$ as $\mathrm{Im}(\omega)$ decreases to 0, absolute instability is indicated. Absolute modes grow at a fixed point independently of the original perturbation amplitudes and so eventually dominate.**

Fluid models represent an algebraic approximation to the kinetic plasma-dispersion function $Z(\zeta)$

- For small arguments $Z(\zeta) \cong i\sqrt{\pi}\, e^{-\zeta^2} - 2\zeta\left(1 - 2\frac{\zeta^2}{3} + 4\frac{\zeta^4}{15} - 8\frac{\zeta^6}{105} + \ldots\right).$

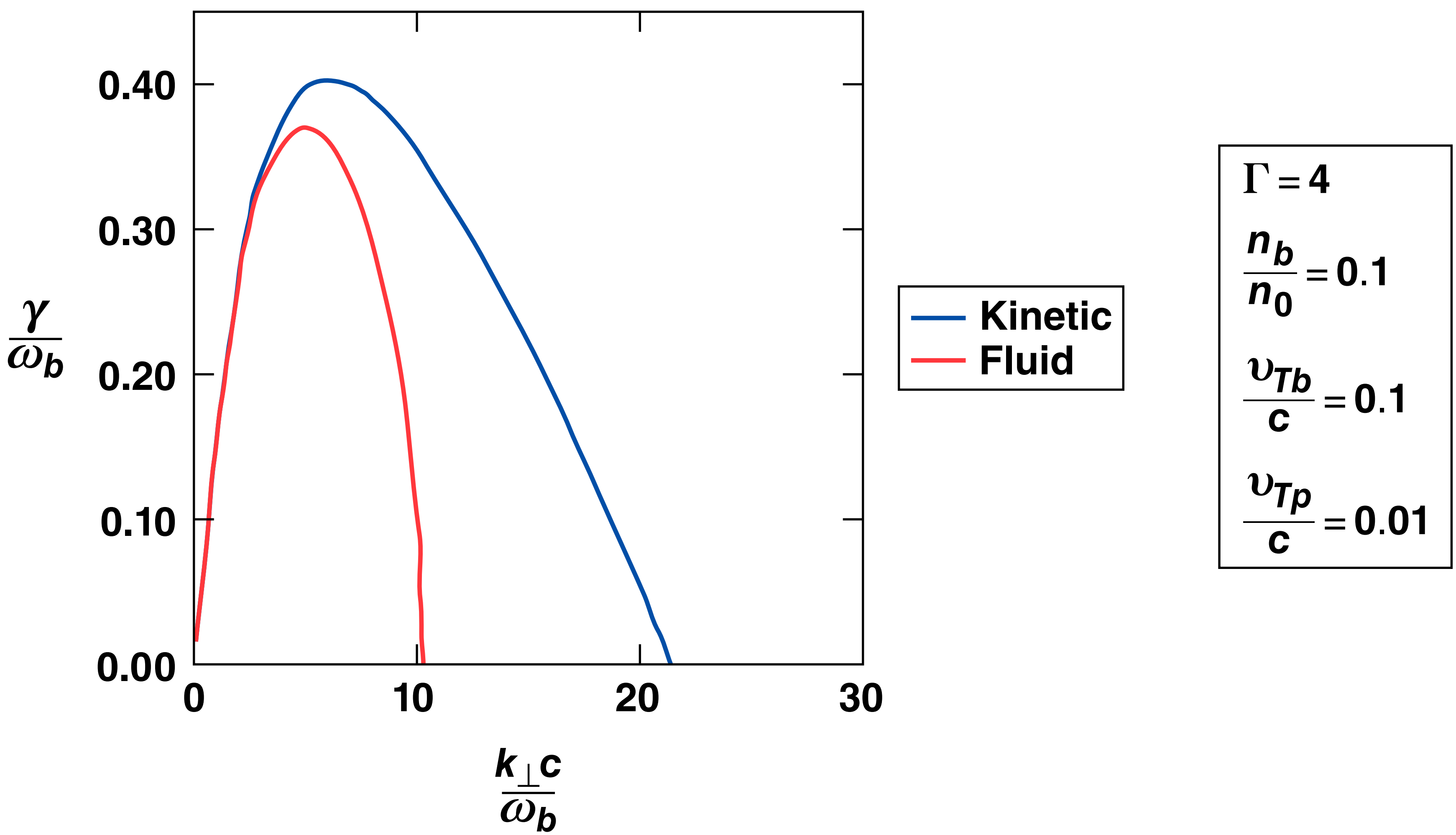
- For large arguments $Z(\zeta) \sim i\sqrt{\pi}\,\sigma e^{-\zeta^2} - \frac{1}{\zeta}\left(1 + \frac{1}{2\zeta^2} + \frac{3}{4\zeta^4} + \frac{15\zeta^6}{8\zeta^6} + \ldots\right)$

  where $\sigma = \begin{cases} 0, \mathrm{Im}(\zeta) > 1/\left|\mathrm{Re}(\zeta)\right| \\ 1, \mathrm{Im}(\zeta) < 1/\left|\mathrm{Re}(\zeta)\right| \\ 2, -\mathrm{Im}(\zeta) > 1/\left|\mathrm{Re}(\zeta)\right| \end{cases}$

- $\zeta \equiv \dfrac{\Gamma(\omega - \beta c k_z)}{\sqrt{2}\sqrt{\left(k_y \upsilon_{T\perp}\right)^2 + \left(\Gamma k_z \upsilon_{Tz}\right)^2}}$,

  so the asymptotic approximation fails in several instances of interest for the filamentation instability.

Fluid model underpredicts instability at large transverse wave numbers



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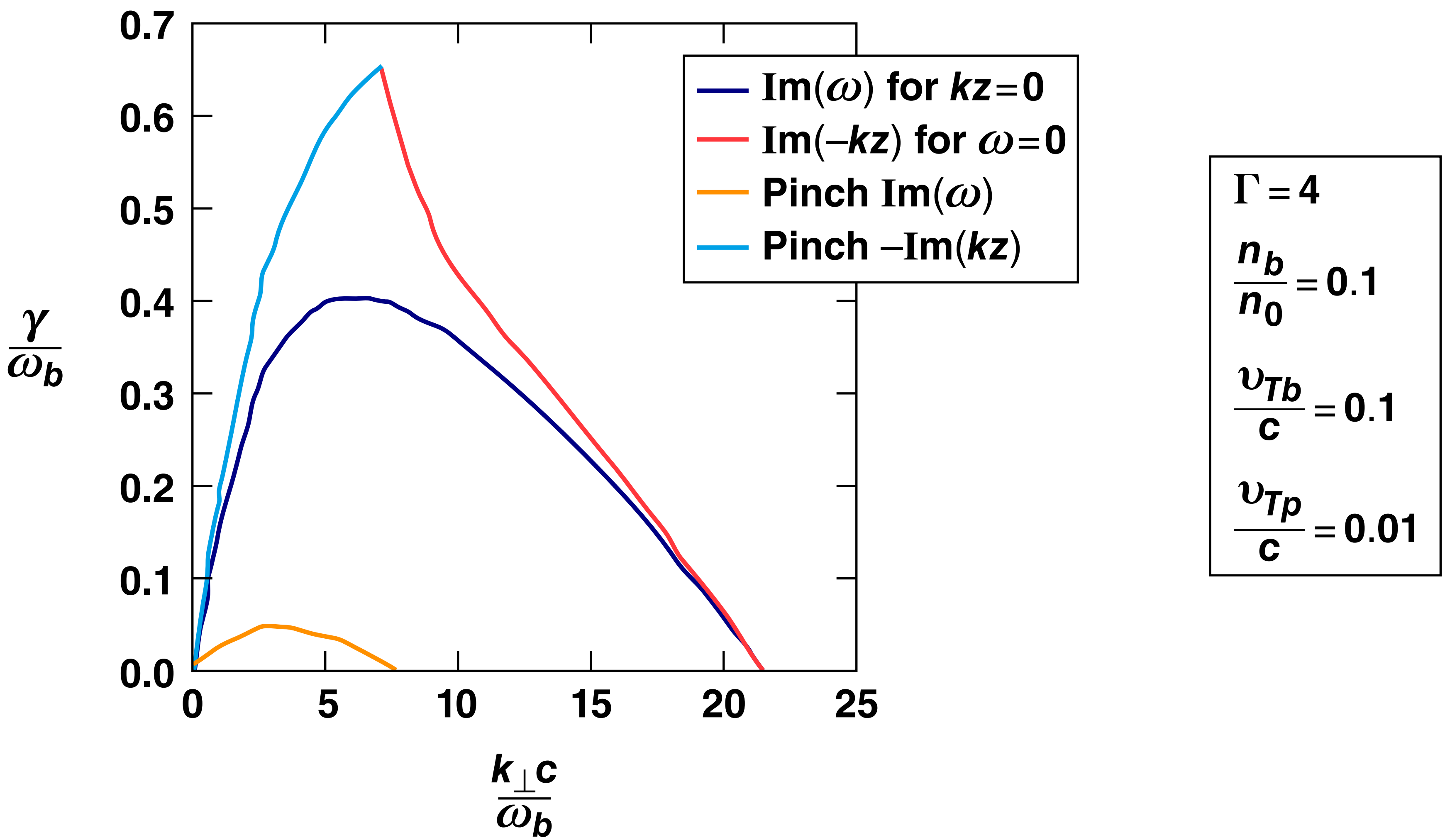
Absolute mode appears only in kinetic model



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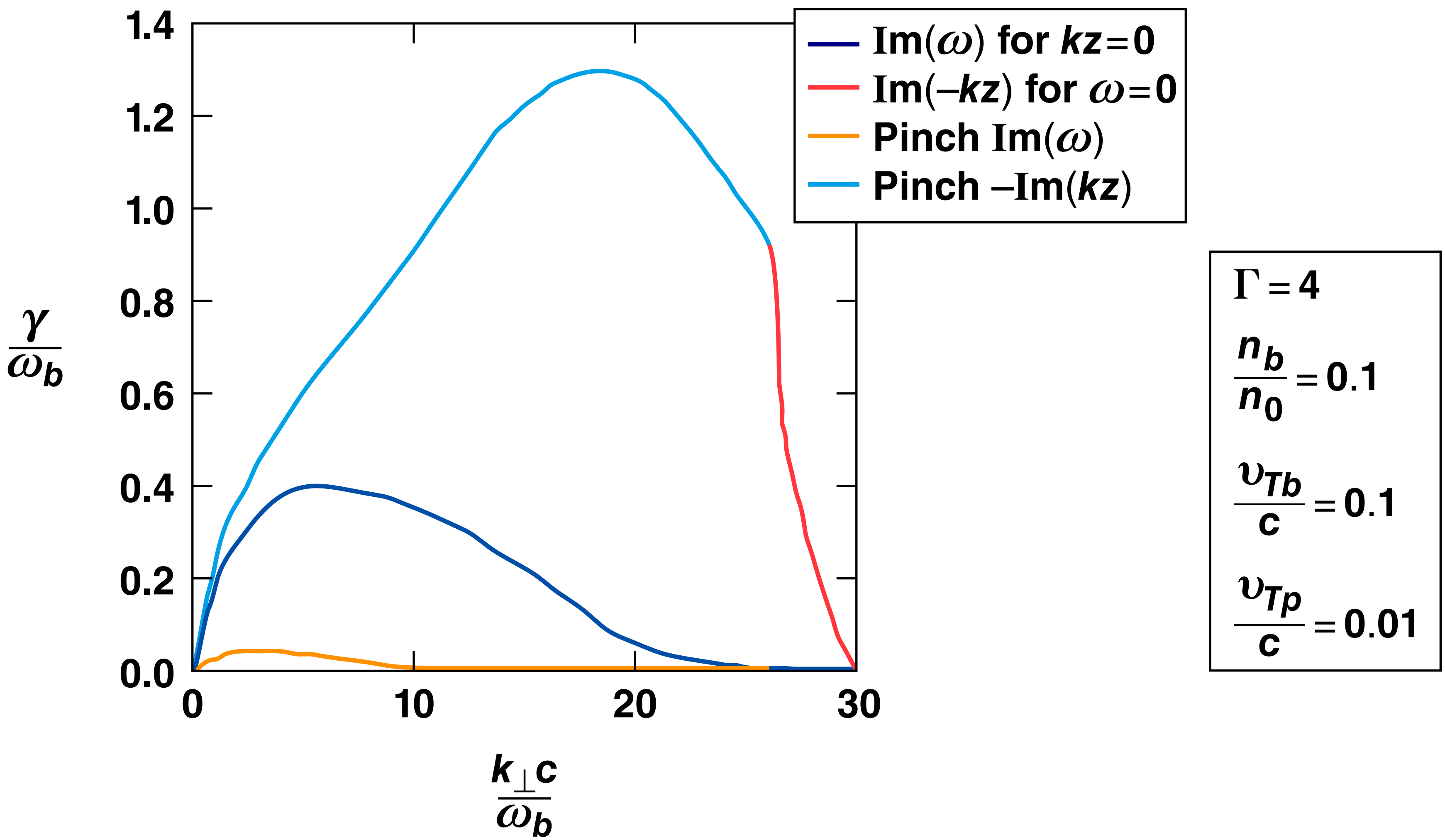
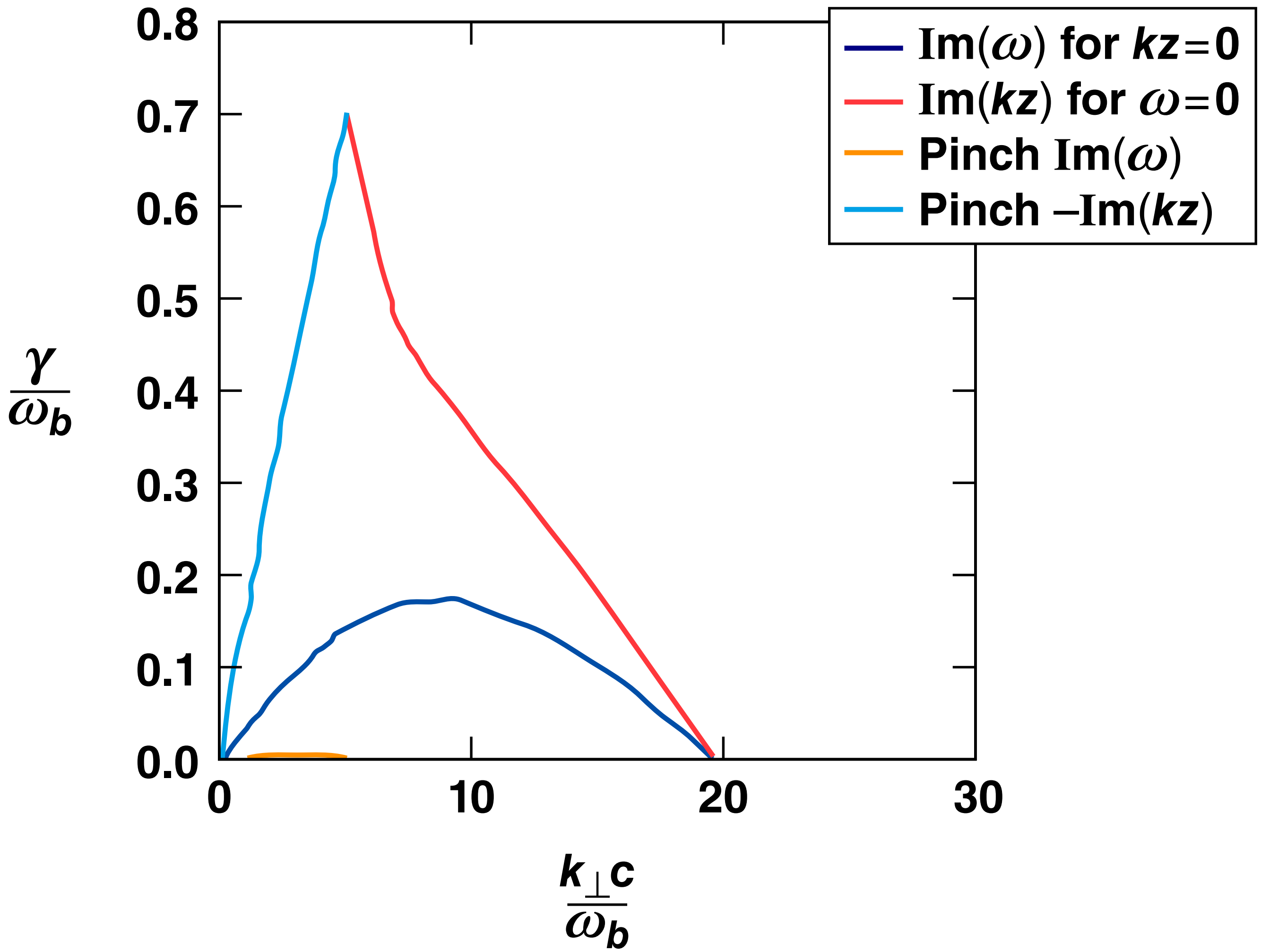
Mobile ions increase the range of instability



At smaller beam/plasma-density ratios only the kinetic model gives instability



$\Gamma = 4$

$\frac{n_b}{n_0} = 0.01$

$\frac{\upsilon_{Tb}}{c} = 0.1$

$\frac{\upsilon_{Tp}}{c} = 0.01$

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