Filamentation of Fast-Ignition Electron Transport in Plasmas: Spatial Growth and Absolute Modes

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Summary

Investigations of spatial growth and absolute forms of filamentation instabilities show significant differences from analyses based on temporal growth alone

- Previous work on Weibel-like filamentation instabilities of electron beams has been based on developing dispersion relations with real wave vectors and complex frequencies to get temporal growth rates.
- Generalizing the dispersion relation to complex wave vectors allows investigation of spatial growth and absolute instability.
- Spatial growth rates are found to peak at much larger transverse wavelengths.
- Absolute instability is also found in a region of larger transverse wavelengths; growth rates are significantly smaller than the corresponding purely temporal growth rates.

Most fast-ignition scenarios require propagation of a relativistic electron beam through a plasma

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- Large-scale beam instabilities (kinking, pinching) develop slowly on the FI timescale.
- Microinstabilities grow faster and include beam–plasma (electrostatic) and filamentation (electromagnetic or mixed) instabilities.
- These instabilities require impedance.
  - Reactive (electron inertia, Weibel and beam–plasma instability): dominant at low densities (few × critical).
  - Resisitive (collisional, resistive filamentation): dominant at high densities (compressed core).
  - A FI beam will transit both regions (reactive first).
- A fully relativistic treatment of the collisionless case has been carried out analytically; the collisional case is more difficult.

Instabilities can be treated as a perturbed equilibrium provided the growth times are shorter than the beam slowing time

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- Assume that, in equilibrium, the charge densities, currents, and fields vanish.
- All perturbed quantities have the space and time dependence $e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$
- Maxwell's equations relate the current to the perturbed electric field.

$$j = j_b + j_p = -\frac{ic^2}{4\pi\omega}\left(k^2\mathrm{I} \quad \underbrace{-\,\mathbf{kk}}_{\text{Longitudinal (electrostatic) term}} \quad \underbrace{-\,\frac{\omega^2}{c^2}\mathrm{I}}_{\text{From displacement current}}\right)\cdot E$$

- The rest of the problem consists of using the plasma properties to derive the perturbed current as a response to *E* (the dielectric tensor).

The simplest model treats the return current as purely resistive

- The beam current J_b is assumed collisionless and, in equilibrium, is balanced by a return current $J_p = -J_b$.
- In the resistive model (Gremillet *et al.*,*), the perturbed current is related to the field by $J_p = 1/\eta\, E$, where η is the resistivity.
- When the frequencies (real or growth rate) become comparable to η, inertial effects can be included using the result from a fluid treatment

$$\eta \rightarrow \frac{\omega}{\omega + k \cdot \upsilon_{0b}}\eta - \frac{4\pi i\omega}{\omega_p^2}$$
, where υ_{0b} is the equilibrium beam velocity.

- At low densities inertial effects dominate the perturbed return current, and a collisionless kinetic treatment is appropriate.

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*L. Gremillet, G. Bonnaud, and F. Amiranoff, Phys. Plasmas 9, 941 (2002).

The relativistic electron beam can be represented as a Maxwell–Boltzmann–Jüttner distribution

- The MBJ distribution is a relativistic generalization of the Maxwellian

  $f_0(w) = \dfrac{\xi n_0}{4\pi\Gamma K_2(\xi)} e^{-\xi\Gamma(\gamma - \beta \cdot w)}$, where $\xi \equiv \dfrac{mc^2}{k_B T} = \dfrac{c^2}{\upsilon_T^2}$, $w \equiv \dfrac{p}{mc} = \gamma \dfrac{v}{c}$, β

  is the average beam β, and $\Gamma \equiv \left(1 - \beta^2\right)^{-1/2} = \sqrt{1 + w^2}$.

- When the thermal spread is small compared to the beam velocity, this can be approximated as a drifting Maxwellian

$$f_0(p) = \frac{1}{(2\pi)^{3/2}} \frac{1}{(m\nu_T)^3 \Gamma} n_b e^{-\frac{\left(\frac{p_z}{\Gamma} - mc\beta\right)^2 + p_\perp^2}{2(m\upsilon_T)^2}}.$$

- These forms are also used to represent the return current in the collisionless case.

In the collisionless case, the perturbed currents are calculated from the relativistic Vlasov equation

- The linearized relativistic Vlasov equation can be written as

$$\frac{\partial f_{1\alpha}}{\partial t} + \frac{c\mathbf{w}}{\gamma} \cdot \frac{\partial f_{1\alpha}}{\partial \mathbf{x}} = -\frac{q_\alpha}{m_\alpha c}\left(\mathbf{E} + \frac{\mathbf{w}}{\gamma} \times \mathbf{B}\right) \cdot \frac{\partial f_{0\alpha}}{\partial \mathbf{w}}.$$

- Solving for the perturbed current gives

$$j_b = \frac{-ie^2}{m\omega}\left[\int \frac{\mathbf{p}}{\gamma}\frac{\partial f_0}{\partial \mathbf{p}} d^3p + \int \frac{\left(\mathbf{k} \cdot \frac{\partial f_0}{\partial \mathbf{p}}\right)\mathbf{pp}}{\gamma\left(\gamma m\omega - \mathbf{k}\cdot\mathbf{p}\right)} d^3p\right] \cdot \mathbf{E}.$$

- In the drifting Maxwellian approximation, the integrals can be expressed in terms of the usual plasma *Z*-function.

- The exact relativistic integrals can be expressed in terms of integrals of the form $\int_{-1}^{1} ds \frac{\gamma}{\omega - cks} \frac{K_{n/2}(z)}{z^{n/2}}$, where $\gamma = \left(1 - s^2\right)^{-1/2}$ and

$$z \equiv \xi\Gamma\sqrt{\left(1 - \beta_3 s\right)^2 \gamma^2 - \left(\beta_1^2 + \beta_2^2\right)}.$$

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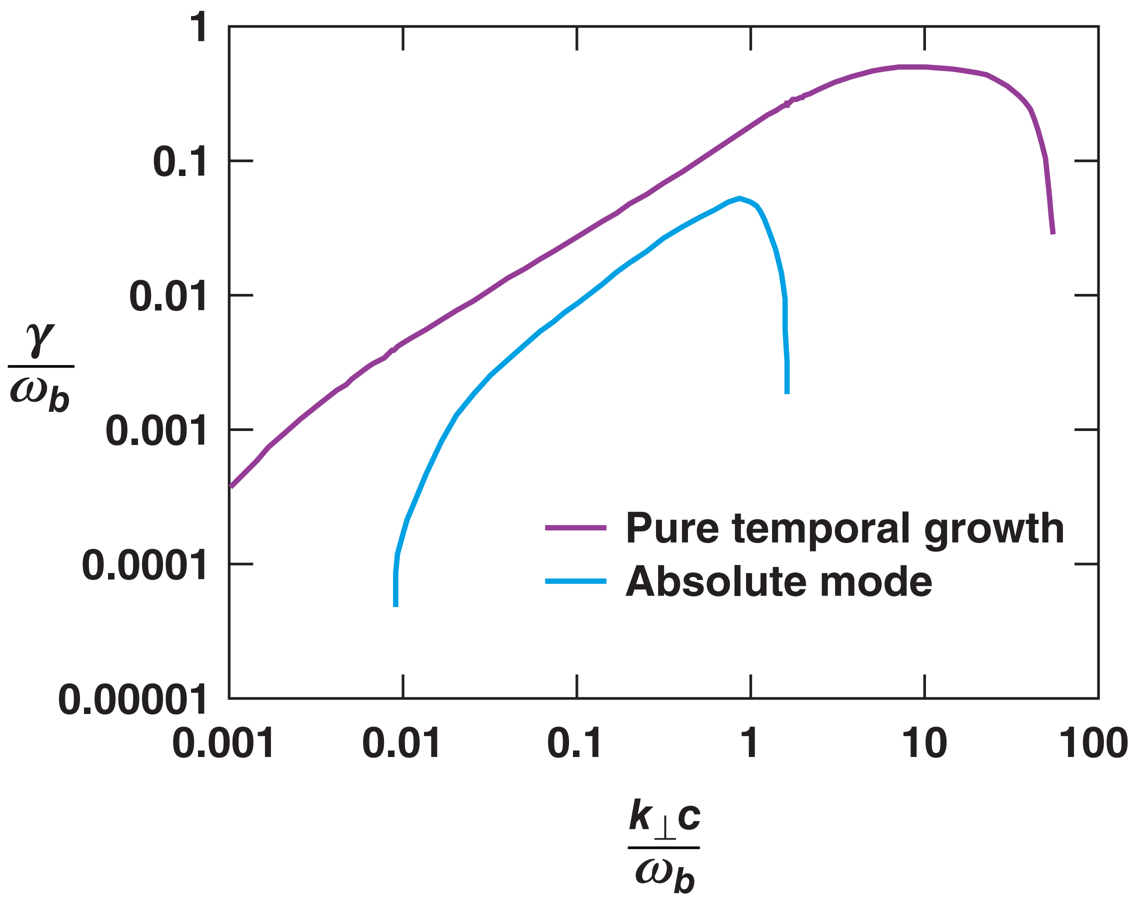
Generalizing the dispersion relation to complex *k* allows the study of spatial growth and absolute instability

- Once initiated, absolute modes grow in time indefinitely at any given point, until limited by nonlinear processes.
- Convective modes require a continuing source of perturbation; otherwise they eventually die away at any given point.
- The concepts and standard methods of analysis used to distinguish absolute and convective instability were originally developed for the problem of electron beams propagating in plasmas.*

*R. J. Briggs, *Electron Stream Interaction with Plasmas* (MIT Press, Cambridge, MA 1964).

The absolute mode peaks at much smaller perpendicular wave numbers



1-MeV beam

$n_b = 10^{20}\ \text{cm}^{-3}$

$\frac{n_b}{n_0} = 0.1$

$\frac{\upsilon_T}{c} = 0.03$

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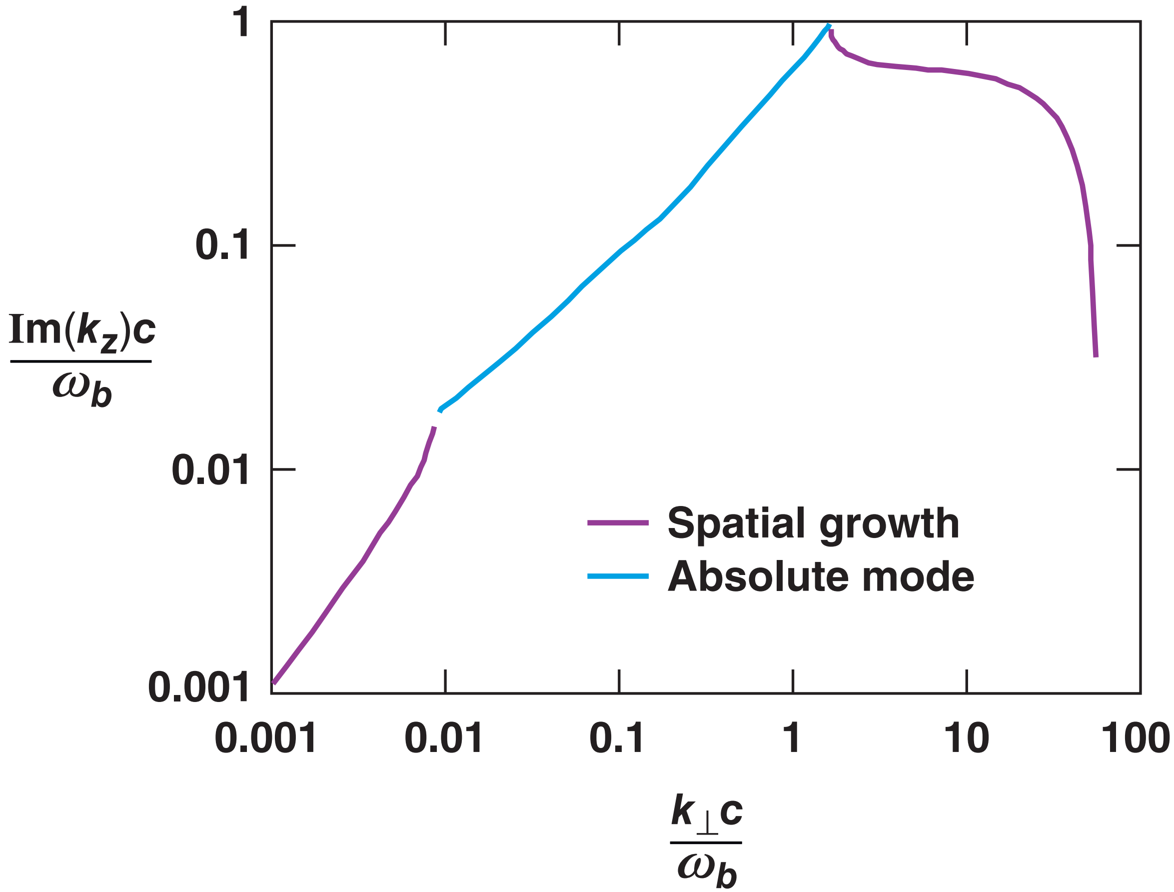
The spatial growth rates also peak at smaller wave numbers



1-MeV beam

$n_b = 10^{20}\ \mathrm{cm}^{-3}$

$\frac{n_b}{n_0} = 0.1$

$\frac{\upsilon_T}{c} = 0.03$

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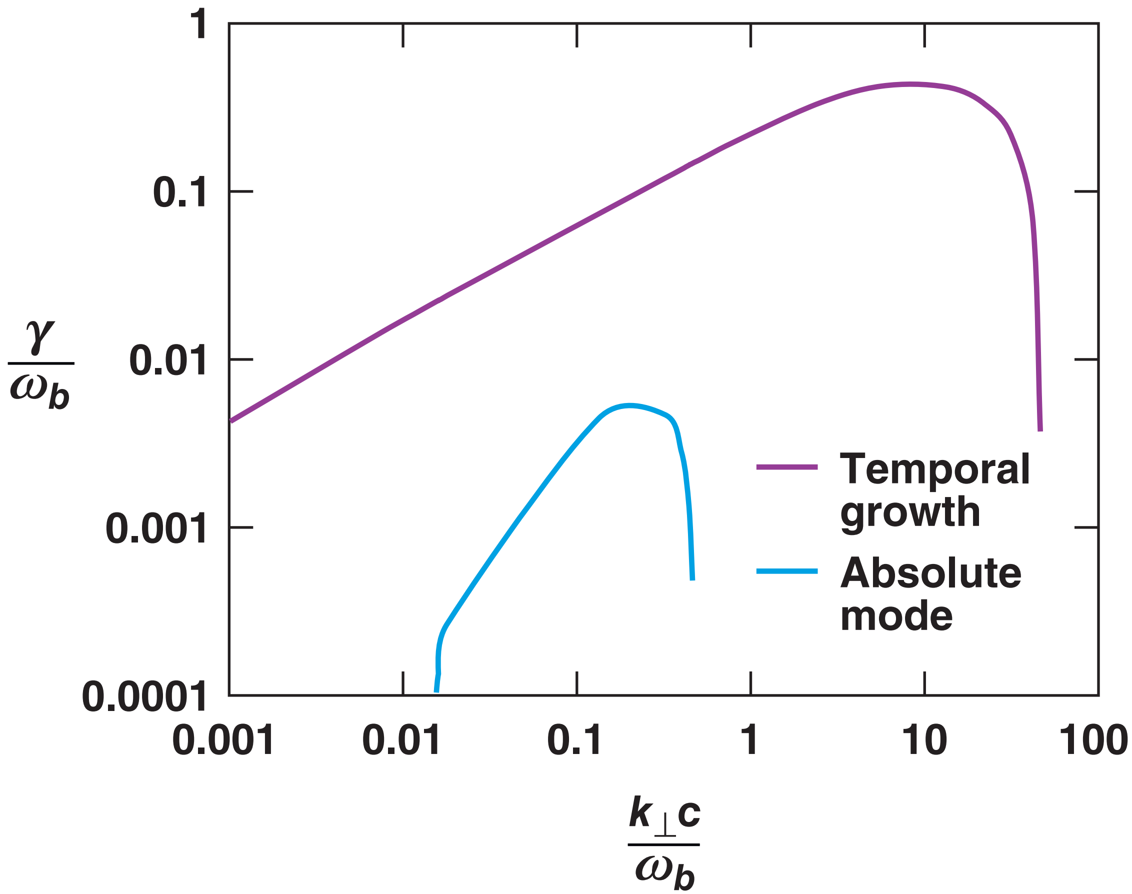
The absolute mode has relatively smaller growth rates at higher background densities



1-MeV beam

$n_b = 10^{20}\ \mathrm{cm}^{-3}$

$\frac{n_b}{n_0} = 0.01$

$\frac{\upsilon_T}{c} = 0.03$

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The general dispersion relation can be used to address several further problems of interest in FI research

- The growth rates and cutoffs can be used to benchmark simulations of beam propagation with codes such as *LSP*.
- These results can also be used to optimize such simulations—e.g., since spatial growth and absolute instability peak at smaller wave numbers, less resolution may be required to represent the most important modes in the FI problem.
- Arbitrary wave-vector directions allow comparison of two-stream and filamentation instabilities and identification of the most unstable mode, which may lie between these instabilities.*
- It should be possible to extend these results through WKB-type methods to include the effects of inhomogeneity.

*A. Bret and C. Deutsch, Phys. Plasmas 12, 082704 (2005).

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