Microinstabilities of Relativistic Electron Beams in Plasmas

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Summary

A general dispersion relation for relativistic electron-beam microinstabilities is useful in addressing several problems relevant to fast ignition

- Previous work on Weibel, two-stream, and related instabilities in relativistic electron beams has employed assumptions and approximations that limit applicability.

- A better determination of plasma and beam temperature effects is obtained using Maxwell–Boltzman–Jüttner distribution functions (or suitable approximations) rather than delta-function or waterbag distributions.

- Inclusions of off-diagonal elements in the dielectric tensor incorporates the electrostatic component of beam filamentation, an important diagnostic signature which can affect growth rates.

- The generalization to complex wave vectors in arbitrary directions allows calculations of spatial growth rates and investigations of absolute versus convective instability.

Most fast-ignition scenarios require propagation of a relativistic electron beam through a plasma

- Large-scale beam instabilities (kinking, pinching) develop slowly on the FI timescale.

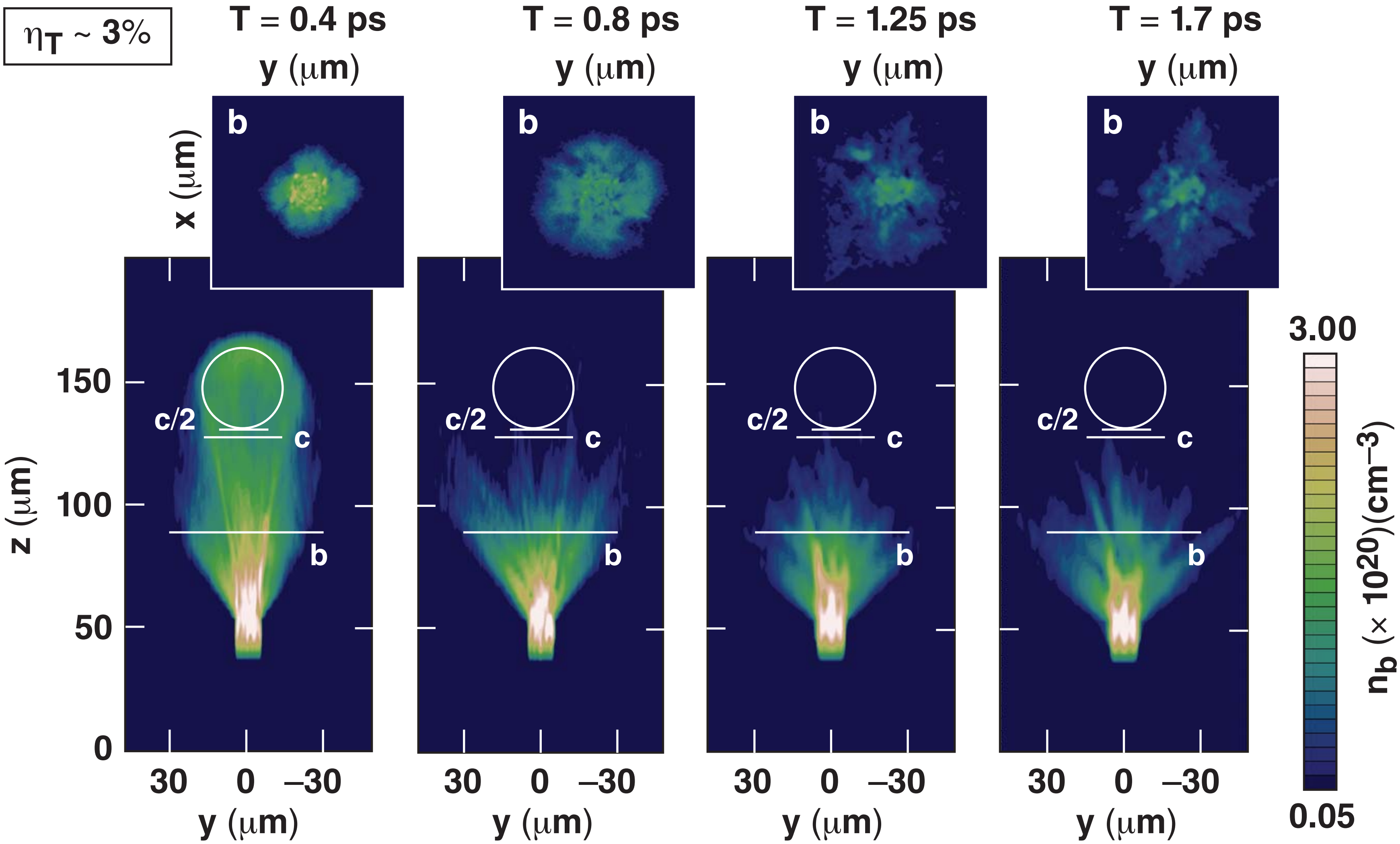
- Microinstabilities grow faster and include beam–plasma (electrostatic) and filamentation (electromagnetic or mixed) instabilities.

- These instabilities require impedance.
  - Reactive (electron inertia, Weibel and beam–plasma instability): dominant at low densities (few × critical).
  - Resisitive (collisional, resistive filamentation): dominant at high densities (compressed core).
  - A FI beam will transit both regions (reactive first.)

- A fully relativistic treatment of the collisionless case can be carried out analytically; the collisional case is more difficult.

An initially well-collimated beam has a poor transport efficiency due to the rapid onset of beam spraying

$\eta_T \sim 3\%$



Instabilities can be treated as a perturbed equilibrium provided the growth times are shorter than the beam slowing time

- **Assume that, in equilibrium, the charge densities, currents, and fields vanish and that all perturbed quantities have the space and time dependence $e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$**

- **Maxwell's equations relate the current to the perturbed electric field.**

$$\mathbf{j} = \mathbf{j}_b + \mathbf{j}_p = -\frac{ic^2}{4\pi\omega}\left(k^2\mathrm{I} \quad - \underset{\substack{\uparrow \\ \text{Longitudinal} \\ \text{(electrostatic)} \\ \text{term}}}{\mathbf{kk}} \quad - \underset{\substack{\uparrow \\ \text{From} \\ \text{displacement} \\ \text{current}}}{\frac{\omega^2}{c^2}}\mathrm{I}\right)\cdot\mathbf{E}$$

- **The rest of the problem consists of using the plasma properties to derive the perturbed current as a response to E (the dielectric tensor).**

The simplest model treats the return current as purely resistive

- The beam current $\mathbf{J_b}$ is assumed collisionless and, in equilibrium, is balanced by a return current $\mathbf{J_p} = -\mathbf{J_b}$.

- In the resistive model (Gremillet *et al.*, 2002), the perturbed current is related to the field by $\mathbf{E} = \eta \mathbf{J_p}$, where η is the resistivity.

- When the frequencies (real or growth rate) become comparable to η, inertial effects can be included using the result from a fluid treatment $\eta \rightarrow \frac{\omega}{\omega + \mathbf{k} \cdot \upsilon_{0b}} \eta - \frac{4\pi i \omega}{\omega_p^2}$, where υ_{0b} is the equilibrium beam velocity.

- At low densities, inertial effects dominate the perturbed return current, and a collisionless kinetic treatment is appropriate.

The relativistic electron beam can be represented as a Maxwell–Boltzmann–Jüttner distribution

- The MBJ distribution is a relativistic generalization of the Maxwellian

$$f_0\left(w\right) = \frac{\xi n_0}{4\pi\Gamma K_2\left(\xi\right)} e^{-\xi\Gamma(\gamma - \beta \cdot w)}, \text{ where } \xi \equiv \frac{mc^2}{k_B T_R} = \frac{c^2}{\upsilon_T^2},\ w \equiv \frac{p}{mc} = \gamma\frac{v}{c}, \beta$$

is the average β, and $\Gamma \equiv \left(1 - \beta^2\right)^{-1/2} = \sqrt{1 + w^2}$.

- When the thermal spread is small compared to the beam velocity, this can be approximated as a drifting Maxwellian

$$f_0\left(p\right) = \frac{1}{\left(2\pi\right)^{3/2}} \frac{1}{\left(mv_T\right)^3\Gamma} n_b e^{-\frac{\left(\frac{p_z}{\Gamma} - mc\beta\right)^2 + p_\perp^2}{2\left(m\upsilon_T\right)^2}}.$$

- These forms are also used to represent the return current in the collisionless case.

In the collisionless case, the perturbed currents are calculated from the relativistic Vlasov equation

- The linearized relativistic Vlasov equation can be written as
$$\frac{\partial f_{1\alpha}}{\partial t} + \frac{cw}{\gamma} \cdot \frac{\partial f_{1\alpha}}{\partial x} = -\frac{q_\alpha}{m_\alpha c}\left(E + \frac{w}{\gamma} \times B\right) \cdot \frac{\partial f_{0\alpha}}{\partial w}.$$

- Solving for the perturbed current gives
$$j_b = \frac{-ie^2}{m\omega}\left[\int \frac{p}{\gamma}\frac{\partial f_0}{\partial p} d^3 p + \int \frac{\left(k \cdot \frac{\partial f_0}{\partial p}\right) pp}{\gamma(\gamma m\omega - k \cdot p)} d^3 p\right] \cdot E.$$

- In the drifting Maxwellian approximation, the integrals can be expressed in terms of the usual plasma *Z*-function.

- The exact relativistic integrals can be expressed in terms of integrals of the form $\int_{-1}^{1} ds \frac{\gamma}{\omega - cks} \frac{K_{n/2}(z)}{z^{n/2}}$, where $\gamma = \left(1 - s^2\right)^{-1/2}$ and $z \equiv \xi \Gamma \sqrt{(1 - \beta_3 s)^2 \gamma^2 - \left(\beta_1^2 + \beta_2^2\right)}$.

The resulting dispersion relations are complicated algebraically but readily evaluated numerically

- **The dispersion relation is obtained from**

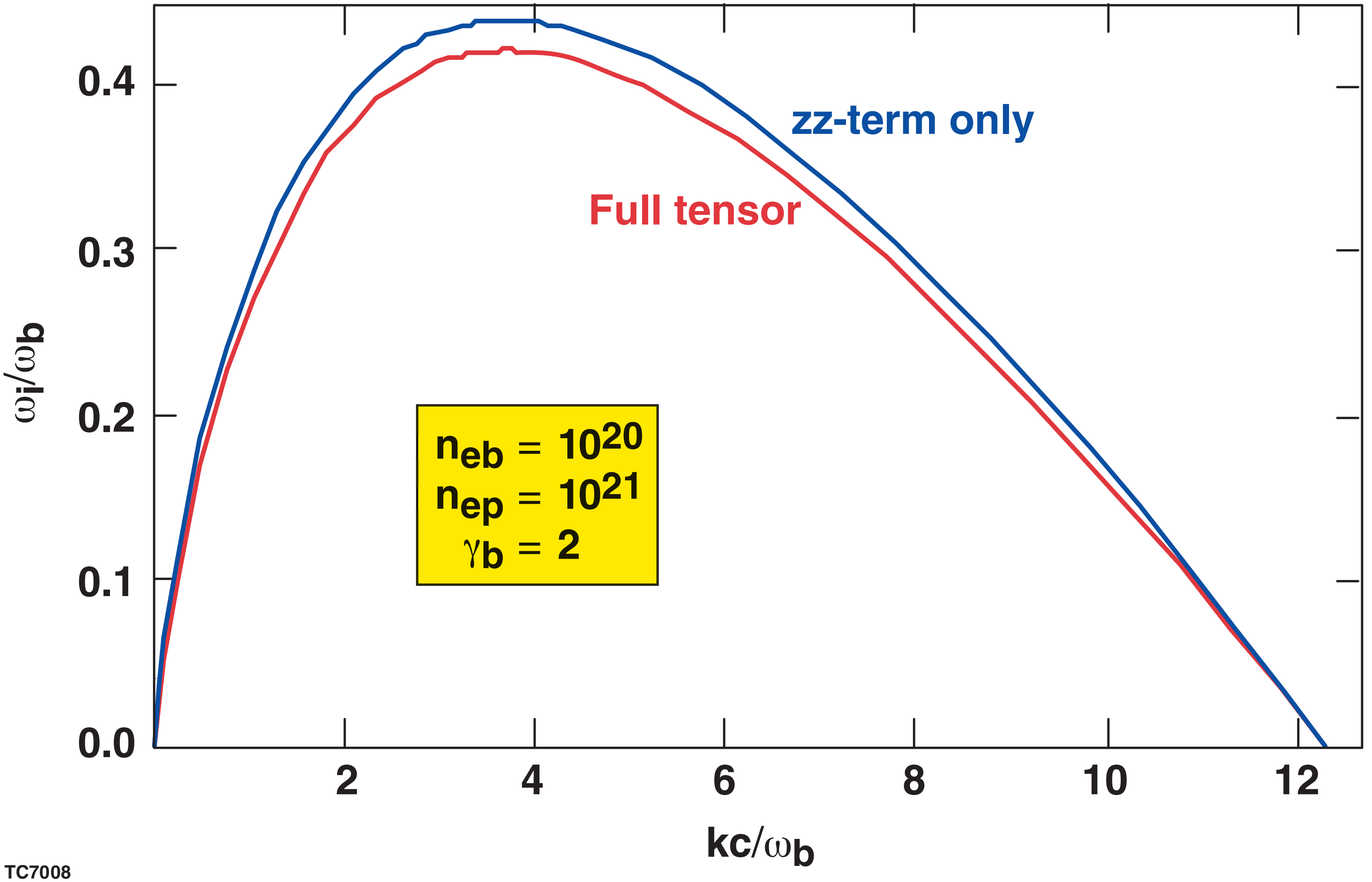
$$\left[\left(c^2 k^2 - \omega^2 + \frac{\omega_p^2}{\Gamma} - \frac{4\pi i \omega}{\eta}\right)\underset{\sim}{I} - c^2 kk - \frac{\omega_p^2}{\Gamma}\underset{\sim}{R}\right] \cdot E = 0.$$

- **A typical R-component in the drifting Maxwellian approximation with the beam propagating in the z-direction:** $R_{zz} \equiv \left(k_y^2 \upsilon_{T\perp}^2 + \Gamma^2 k_z^2 \upsilon_{Tz}^2\right)^{-2}$

$$\times \left[\begin{array}{l} [1 + \Omega Z(\Omega)]\left\{\begin{array}{l} \Gamma^2 \left(k_y^2 + k_z^2\right)\left[\left(\sqrt{2\Gamma k_z \upsilon_{Tz}^2 \Omega + \beta c \sqrt{k_y^2 \upsilon_{T\perp}^2 + \Gamma^2 k_z^2 \upsilon_{Tz}^2}}\right)^2 + k_y^2 \upsilon_{Tz}^2 \upsilon_{T\perp}^2\right] \\ + 2k_y^2 k_z^2 \upsilon_{Tz}^2 \left(\upsilon_{T\perp}^2 - \Gamma^2 \upsilon_{Tz}^2\right) \end{array}\right\} \\ + k_z \Gamma\left[\sqrt{2}\,\beta c k_y^2 \sqrt{k_y^2 \upsilon_{T\perp}^2 + \Gamma^2 k_z^2 \upsilon_{Tz}^2}\left(\upsilon_{T\perp}^2 - \Gamma^2 \upsilon_{Tz}^2\right) Z(\Omega) + \Gamma^3 \left(k_y^2 + k_z^2\right) k_z \upsilon_{Tz}^4\right] \end{array}\right]$$

where $\Omega \equiv \dfrac{\gamma\omega - \Gamma\beta c k_z}{\sqrt{2}\sqrt{\left(k_y \upsilon_{T\perp}\right)^2 + \left(\Gamma k_z \upsilon_{Tz}\right)^2}}$

The electrostatic component typically has little effect on filamentation growth rates, but is responsible for density perturbations seen in experiments



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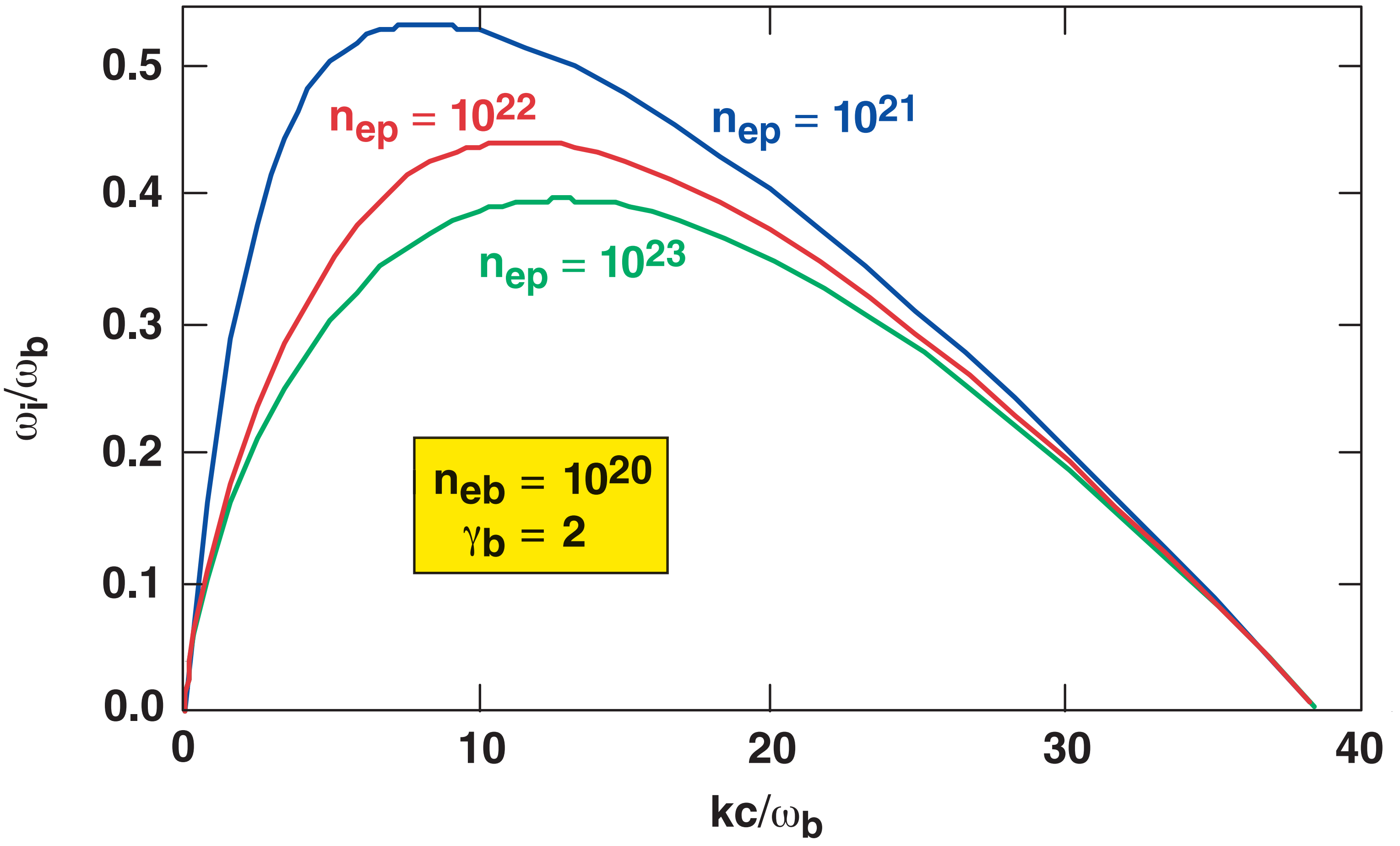
Inertial terms increase the growth rates at lower ratios of background to beam densities



The general dispersion relation can be used to address several further problems of interest in FI experiments

- Using real ω and complex k spatial growth rates can be calculated, which are of greater relevance to FI than the temporal growth rates.

- The transition from convective to absolute intabilities can be studied; this requires both ω and k to be complex.

- Arbitrary wave-vector directions allow the comparison of two-stream and filamentation instabilities and the identification of the most unstable mode, which may lie between these instabilities [A. Brett *et al.*, PRL 94 (2005).]

- The analytic theory can be used to benchmark simulation codes such as LSP and optimize simulation parameters.

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