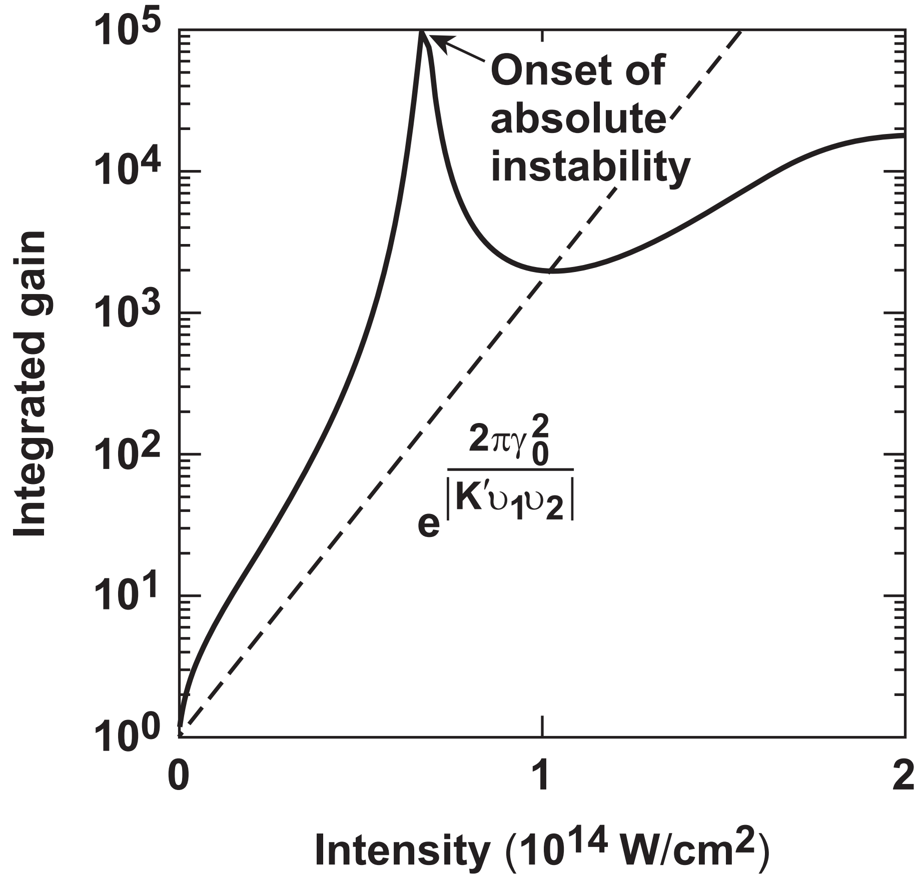
On the Convective Two-Plasmon-Decay Instability in Inhomogeneous Plasmas



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Summary

Accurate calculation of convective gain and absolute thresholds for TPD is greatly simplified in Fourier space

- **TPD is limited to a narrow range of densities, which are well approximated by a linear profile. For such profiles, the eighth-order differential equation describing TPD becomes second order in k-space.**

- **For small $k_{\perp}/k_0$, the spatial gain is roughly an order of magnitude larger than the Rosenbluth formula and the absolute instability threshold is low.**

- **For larger $k_{\perp}/k_0$, the Rosenbluth formula is a better fit to the gain and the absolute threshold is larger, but the instability may be "effectively absolute" at pump intensities well below this threshold.**

Outline

- Convective and absolute TPD in theory and experiment
- Advantages of the Fourier space approach
- Illustrative results
- Summary and conclusions

Both convective and absolute forms of the two-plasmon-decay (TPD) instability are expected to play a role in laser-fusion experiments

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- **Convective instability: Plasma waves arising from noise enter the interaction region, are amplified, and propagate out at an enhanced level. Spatial growth is essentially a steady-state process. Spatial growth $\rightarrow \infty$ represents the threshold of absolute instability.**

- **Absolute instability: Waves in the interaction region are amplified faster than they can propagate out; temporal growth continues until limited by nonlinear effects.**

- **Absolute instability predominates at small plasmon wave vectors; small group velocity, large phase velocity.**

- **Convective instability predominates at large wave vectors; large group velocity, smaller phase velocity (traps electrons more effectively).**

The current TPD experiments allow for a rough estimate of the plasma-wave spectrum

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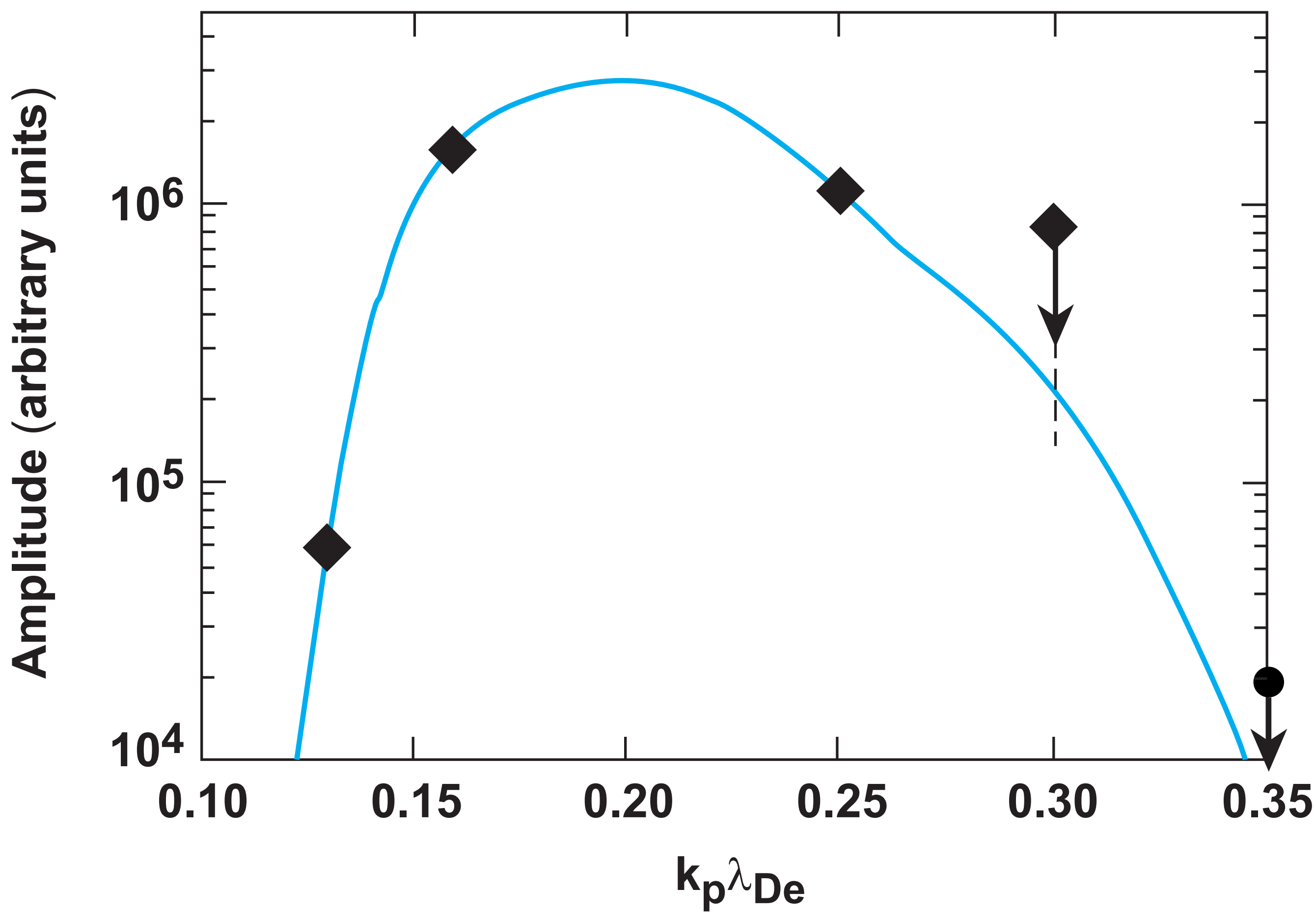

The absolute instability would be just above threshold for $k_p\lambda_{De} < 0.13$.

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The equations describing TPD are difficult to treat in configuration space

- Using the velocity potential defined by $\mathbf{v} \equiv \nabla\psi$, the equations governing TPD can be written

$$\frac{\partial\psi}{\partial t} = \frac{e\phi}{m} - \frac{3\upsilon_e^2 n_1}{n_0} - \mathbf{v}_0 \cdot \nabla\psi \, ; \quad \frac{\partial n_1}{\partial t} + \nabla \cdot \left(n_0 \nabla\psi\right) + \mathbf{v}_0 \cdot \nabla n_1 = 0 ; \quad \nabla^2\phi = 4\pi e n_1 .$$

- These lead to an eighth-order ODE. Simplifications are of questionable validity near the plasma wave turning points.

- Simple generic three-wave convective instability theory gives the spatial gain formula $G = \exp\left(\frac{2\pi\gamma_0^2}{\left|K'\upsilon_1\upsilon_2\right|}\right)$. Constant parameters, and the exponential function of intensity must break down at the absolute threshold ($G \to \infty$ for finite intensity).

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For a linear density profile, a more sophisticated treatment is feasible using Fourier transforms

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- **TPD is confined to a narrow range of densities below quarter-critical, so a linear density profile should be a good approximation.**
- **For a linear density profile, Fourier transforming in space leads to two coupled first-order ODEs in k-space:**

$$\frac{dW_+}{d\kappa} = h(\kappa)W_-, \quad \frac{dW_-}{d\kappa} = -h^*(\kappa)W_+ \quad \text{for density profile} \quad \frac{n_1}{n_0} = 1 + \frac{x}{L};$$

coupling coefficient

$$h(\kappa) = \frac{\alpha\left(\frac{k_y}{k_0}\right)\kappa e^{i\alpha\sqrt{\beta}\kappa(\kappa-2\Omega)}}{\sqrt{\left[\kappa^2 + \frac{1}{4} + \left(\frac{k_y}{k_0}\right)^2\right]^2 - \kappa^2}}.$$

- **Previous studies have employed this k-space formulation to treat the absolute instability (Liu and Rosenbluth, 1976; Simon *et al*., 1983).**

Both absolute and convective forms of TPD can be studied using the k-space approach

- Absolute modes are found by searching for temporally growing modes localized in k-space. This involves complicated contour integrations in complex k-space for complex frequencies (Simon *et al*., 1983). It can be difficult to obtain accurate results near the threshold.

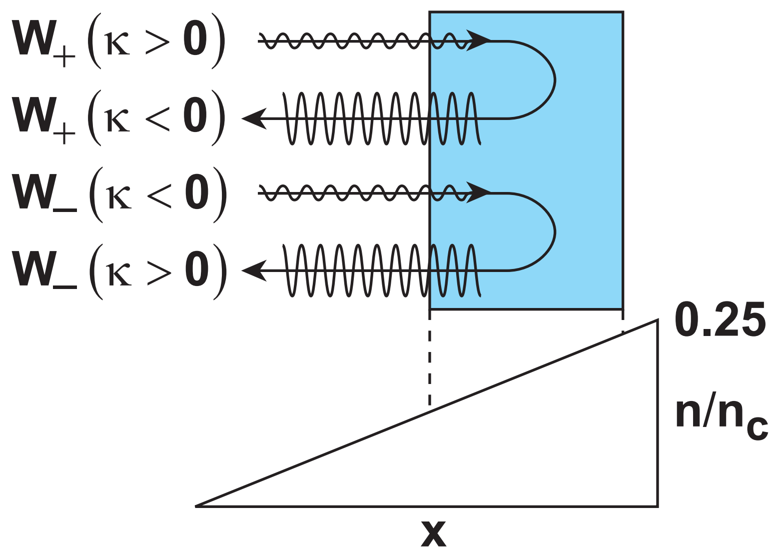
- The convective instability can be studied using real k and ω;
  the absolute threshold can be identified with divergent spatial gain.

- $\begin{pmatrix} W_+ \\ W_- \end{pmatrix}$ represents the plasma wave amplitudes at $\begin{pmatrix} k + k_0, \omega + \omega_0 \\ k - k_0, \omega - \omega_0 \end{pmatrix}$.

- Incoming waves at large negative x are represented by $W_\pm(\kappa \to \pm\infty)$ and outgoing waves by $W_\pm(\kappa \to \mp\infty)$.

TPD amplification factors can be obtained by numerical integration of the k-space equations

TPD resonant interaction region



- Define the growth factor $G = \dfrac{\text{out}}{\text{in}} = \dfrac{|W_+(\kappa \to -\infty)|^2 + |W_-(\kappa \to +\infty)|^2}{|W_+(\kappa \to +\infty)|^2 + |W_-(\kappa \to -\infty)|^2}$.

  If integration from $\begin{pmatrix} W_+(\kappa \to -\infty) \\ W_-(\kappa \to -\infty) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ gives $\begin{pmatrix} W_+(\kappa \to +\infty) \\ W_-(\kappa \to +\infty) \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$,

  then $G_{max} = \dfrac{1+|b|}{1-|b|}$.

Matching numerical integration to asymptotic expansions for large κ aids accurate growth calculation

- The solutions become highly oscillatory and thus difficult to integrate accurately at large κ.

- Asymptotic series for the solutions at large κ are

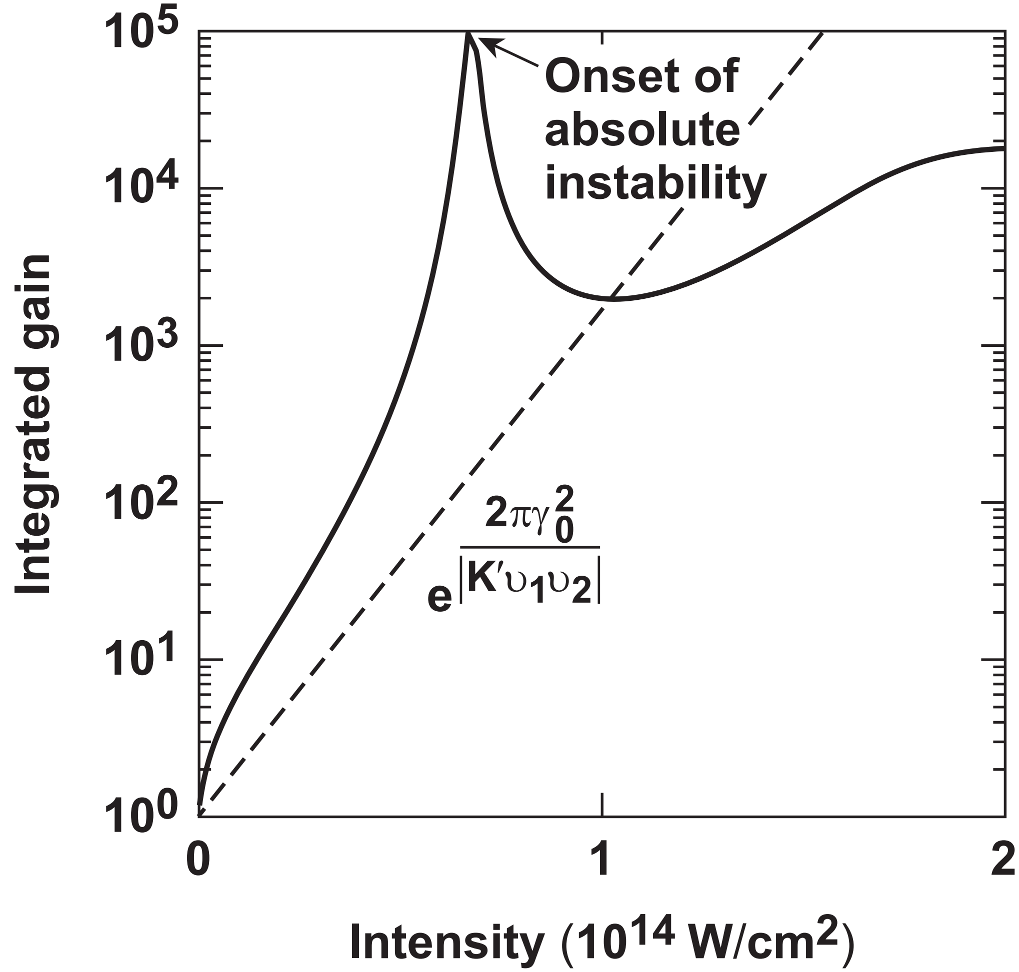
$$W_1(\kappa) \sim 1 + \frac{iq\alpha}{4\sqrt{\beta}\kappa^2} + \frac{iq\alpha\Omega}{6\sqrt{\beta}\kappa^3} + \ldots \qquad q \equiv \left(\frac{k_y}{k_0}\right)^2$$

and

$$W_2(\kappa) \sim \frac{e^{i\alpha\sqrt{\beta}\kappa(\kappa - 2\Omega)}\sqrt{\left(\kappa^2 + \frac{1}{4} + q\right)^2 - \kappa^2}}{\alpha\sqrt{q}\kappa} \left(\frac{iq\alpha}{2\sqrt{\beta}\kappa^3} + \frac{iq\alpha\Omega}{2\sqrt{\beta}\kappa^4} + \ldots\right).$$

- The numerical results can be matched to the asymptotic forms at moderate κ, allowing accurate calculation of the amplification.

At small values of k_y/k_0, spatial amplification is larger than predicted by the simple model and shows transition to absolute mode



$L_\mu = 400$

$T_{keV} = 2.0$

$k_y/k_0 = 0.1$

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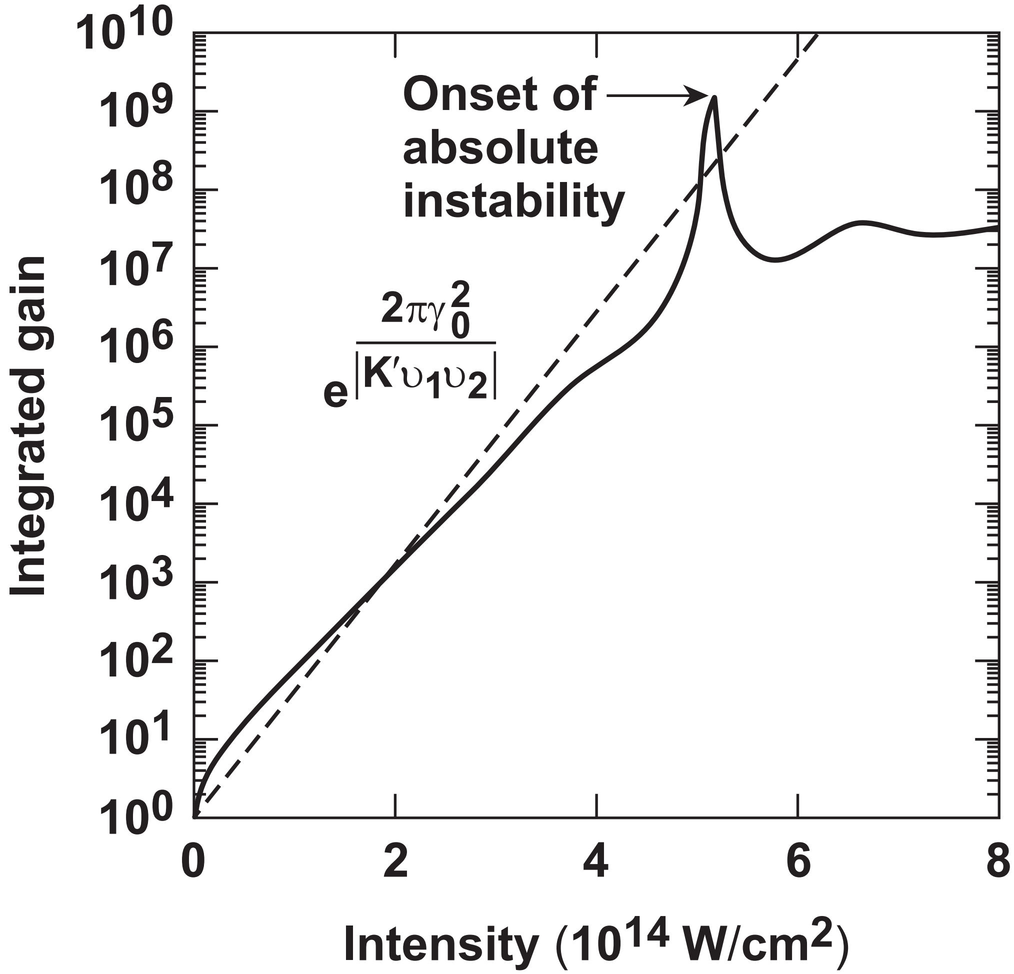
At larger values of k_y/k_0, spatial amplification is closer to the simple model; absolute threshold is higher



$L_\mu = 400$

$T_{keV} = 2.0$

$k_y/k_0 = 0.1$

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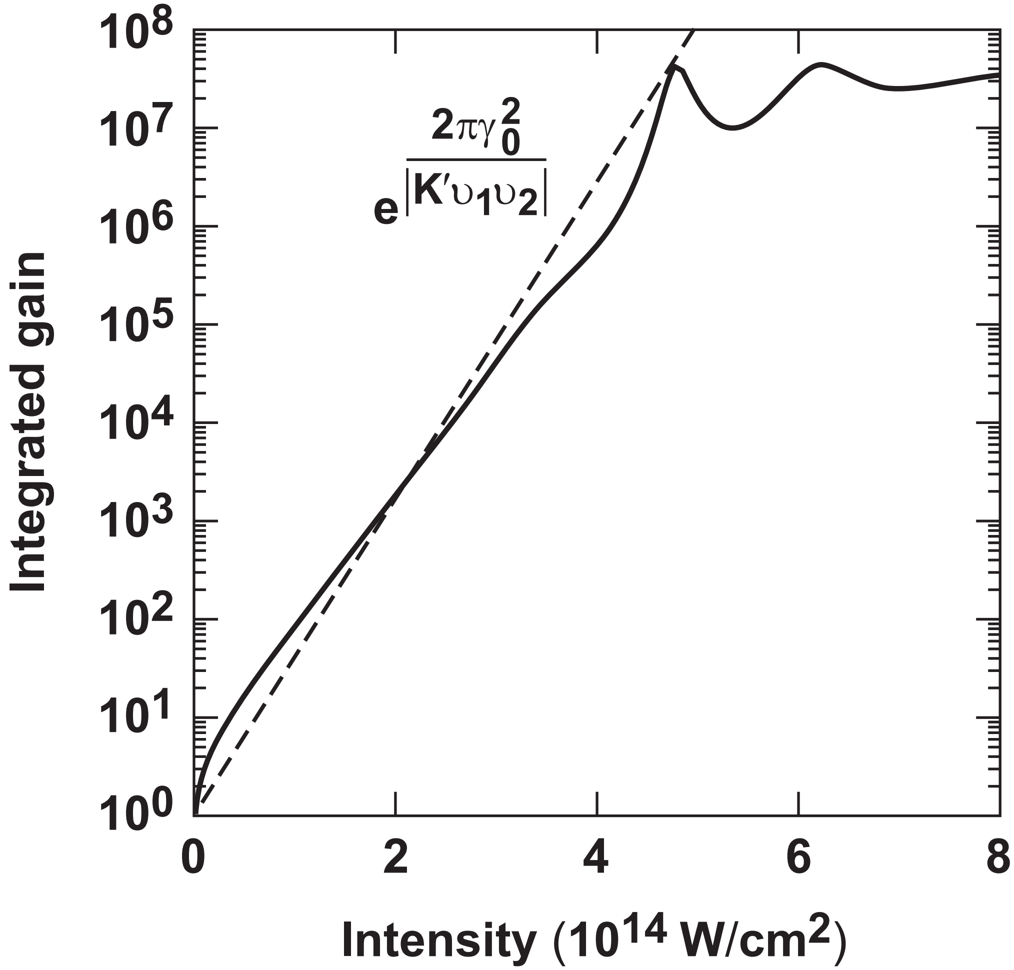
Off-resonance, instability remains convective, but gain may be large enough to make it effectively absolute



L_μ = 400

T_{keV} = 2.0

k_y/k_0 = 0.1

Off-resonant ω

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Future work

- Experiments at LLE indicate that TPD is often driven by multiple beams, so the model should be extended to overlapping beams at oblique incidence.
- Some mathematical questions remain: e.g., is there an absolute mode for arbitrary values of k_y/k_0?
- Benchmark simulation codes (A. V. Maximov, this session.)

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